Optimizing Guidance Strategy for Gun Launched Underactuated Weapons

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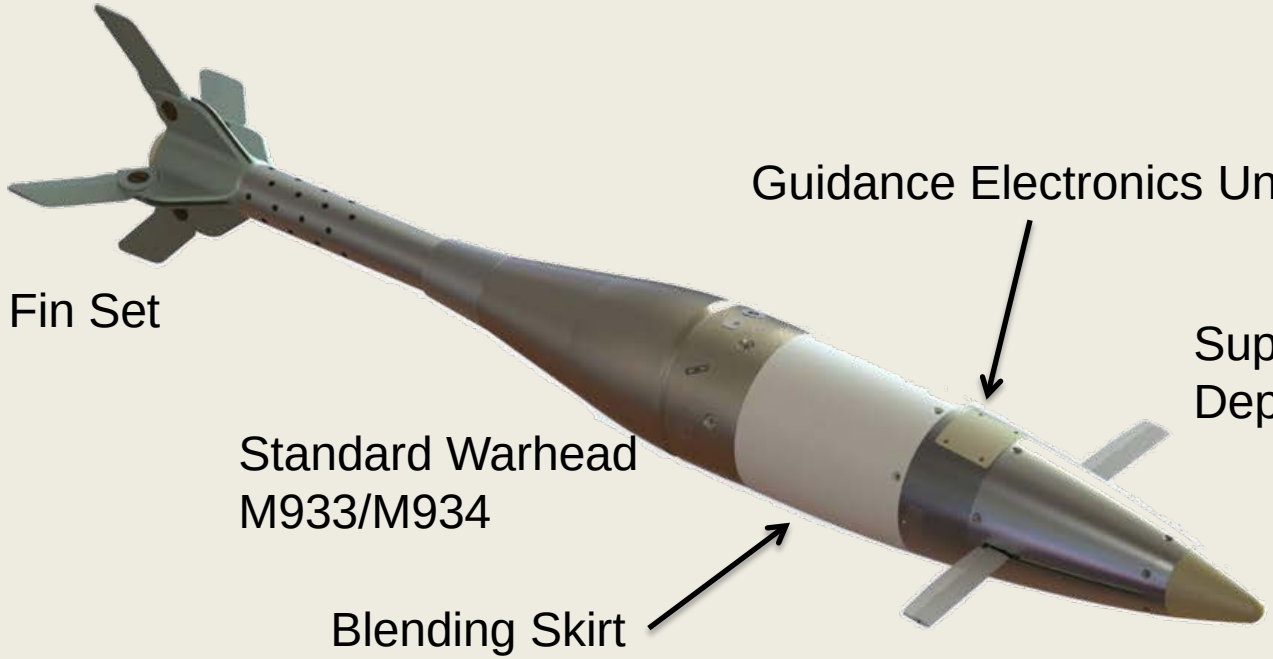
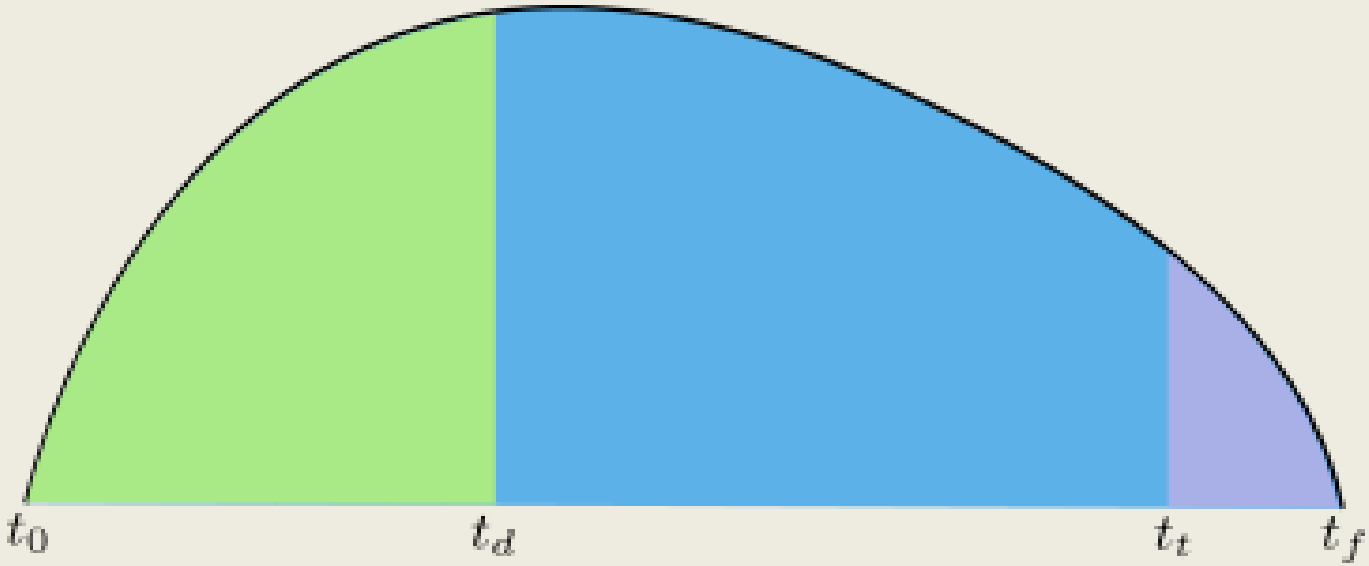
System Overview



<u>Guidance Electronics Unit</u>

- Flight Processor
- ARL Inertial Navigation Unit
- GPS Receiver
- Power Electronics

Problem Statement



Ballistic

- Internal System Checks
- Calibrations

Mid Course Guidance

- Deploy Canards
- Extend Range
- Seeker Basket Engagement

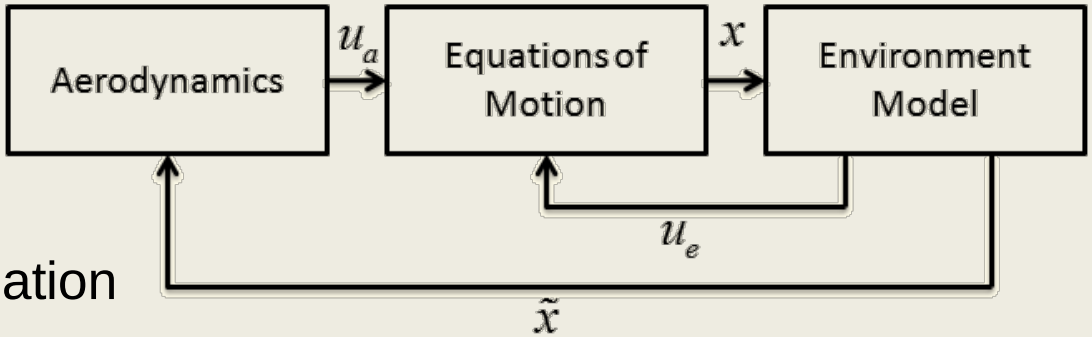
Terminal Guidance

- Minimize CEP
- Maximize Angle of Fall

Aerodynamics Model

Aerodynamics:

- CFD
- Wind Tunnel
- Free Flight Characterization



6DOF State Vector: $x = \begin{bmatrix} x_g & v_b & \omega_{gb} & \mathbf{q} \end{bmatrix}'$

Equations of Motion:

$$\dot{v}_b = \frac{f_b}{m} - \omega_{gb} \times v_b$$

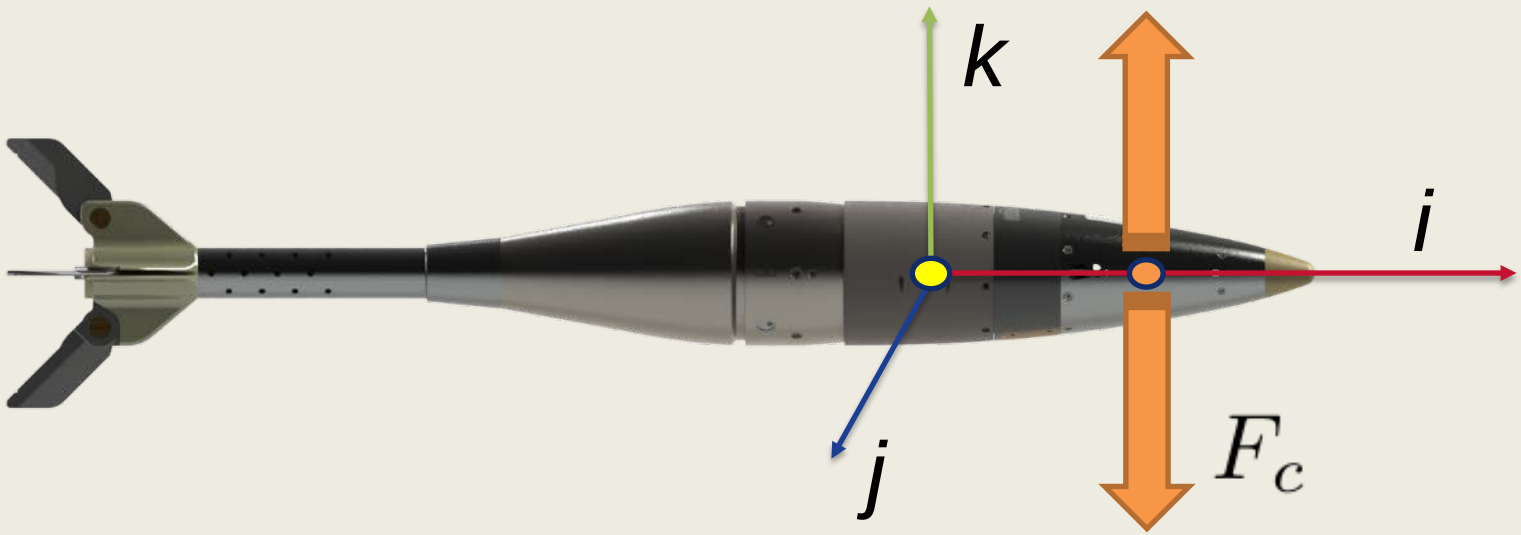
$$\dot{\omega}_b = I^{-1}\left(M_b - \omega \times I\omega_b\right)$$

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q} \otimes \omega_{\mathbf{gb}}$$

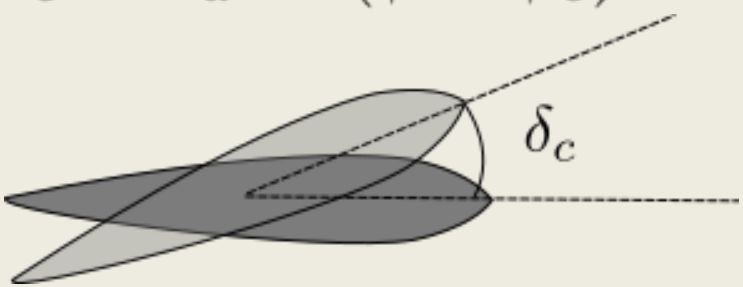
Environmental Model:

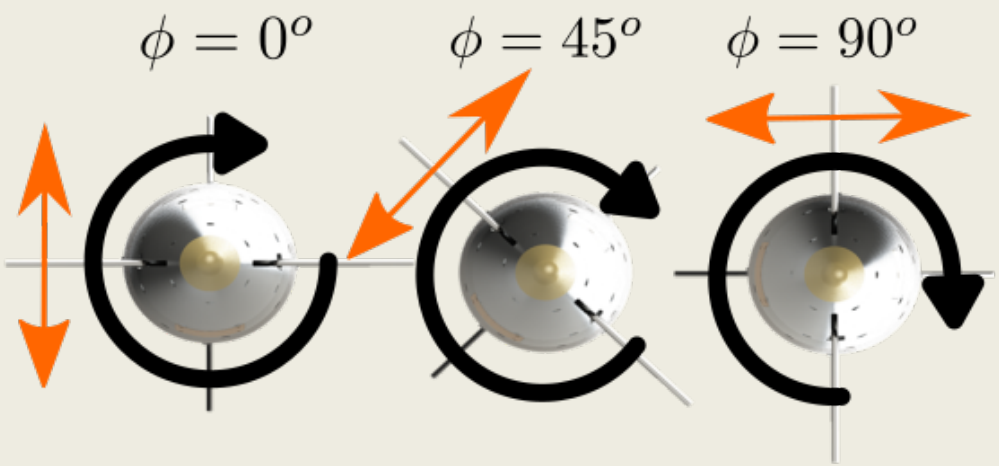
- Standard Lapse Rate Model
- Constant Wind Models

Control Strategy



$$\delta_c = \delta_a sin(\phi + \phi_c)$$





Guidance System

Guidance Strategies

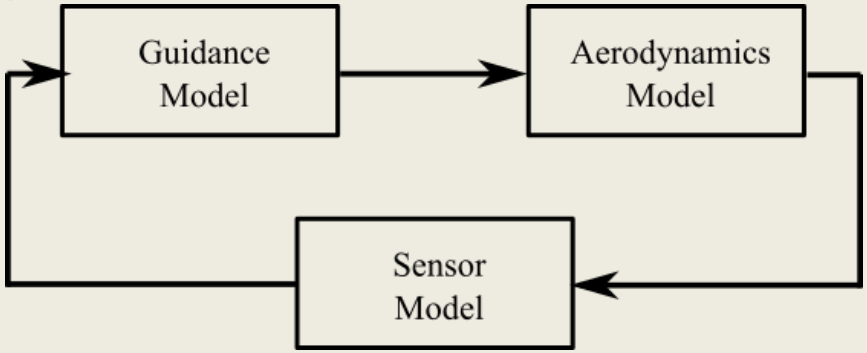
- Proportional Navigation (PN)
- Impact Point Prediction (IPP)
- Generalized Vector Explicit Guidance (GENEX)

Canard Displacement from Acceleration Command

$$\delta_c = f_g(x, a_c)$$

Generalized System Model:

$$\dot{x} = f(x, u)$$



Sensor Model:

Assumes Truth State Feedback

- Position
- Velocity
- Roll Angle + Roll Rate

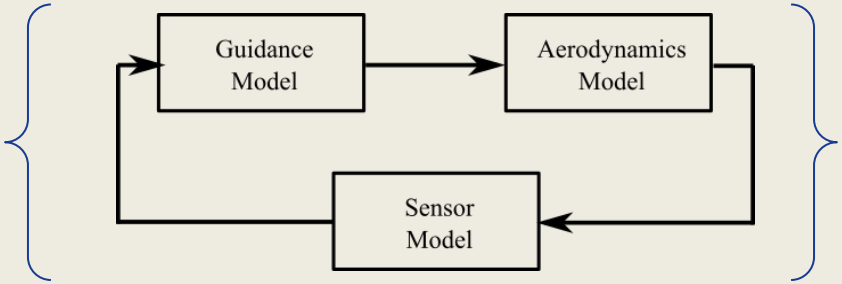
Optimization Formulation

Objective: $$\min_{p} J(x,p) \quad s.t. \quad \underline{p} <= p <= \bar{p}$$

Cost Function: $$J(x,p) = (x_f - x_t)^T Q (x_f - x_t)$$

Objective States: $$x_t = \{x_{g_t}, \theta_t\}$$

System: $$f_g(x,p,t)$$



Navigation Functions / Parameters

Common Parameter Optimization

$$\{\theta_0, t_d\}$$

Proportional Navigation

$$f_P = K_p \frac{2}{t_{go}^2}[r_p + v_p t_{go}]$$

Generalized Vector Explicit Guidance

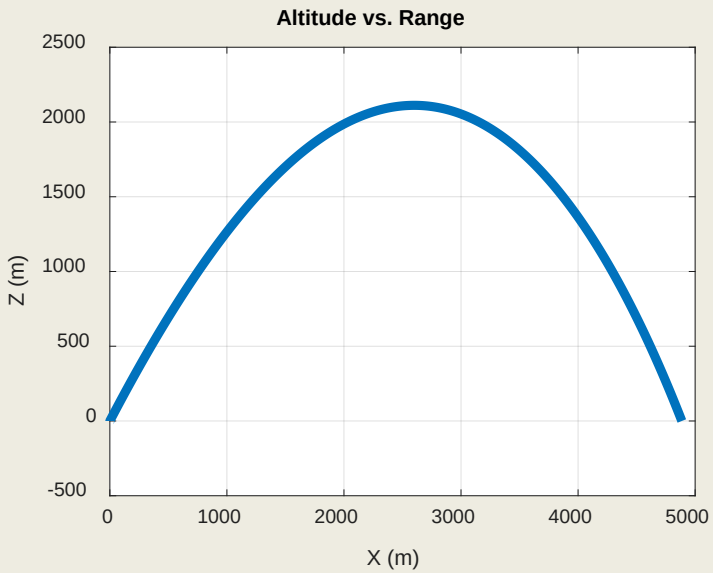
$$f_G = \frac{\|v_p\|^2}{\|r_p\|}(K_p(r - v(v \cdot r)) + K_v(v_f - v(v_f \cdot v)))$$

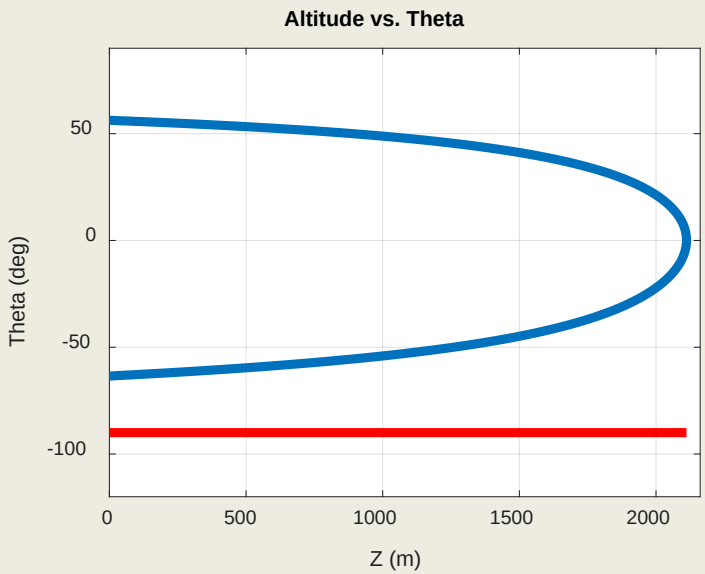
Impact Point Predictor

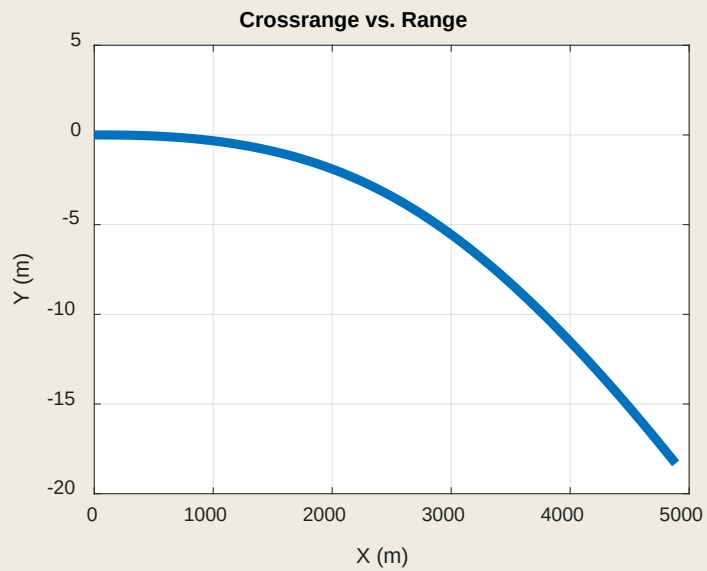
$$f_I = \begin{cases} \frac{K_{pg} r_{p2} + K_{dg} v_{p2}}{-r_{p3}} & x_{i_1} <= r_{thresh} \\ K_{pt} e_\lambda \dot{\phi}^2 & t > t_g \end{cases}$$

Optimization Setup

Variable	Initial Condition	Units
x_0	[0, 0 ,0]	m
v_{b0}	[275,0,0]	$\frac{m}{s}$
ω_{b0}	[0 0 0]	$\frac{rad}{s}$
θ_0	56.2500	deg
azimuth	0	deg
x_t	[5000 300 0]	m







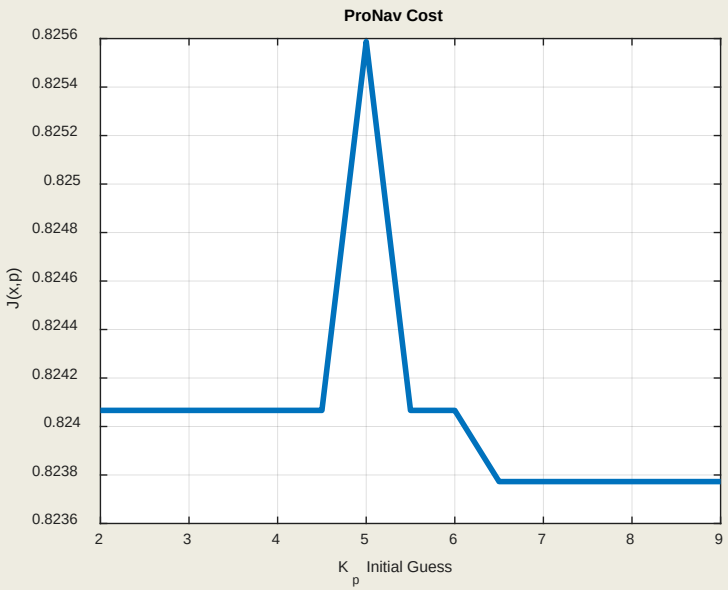
PN Results

Objective:
Minimize Cost Function with Parameters

Parameter	Lower Bound	Initial Value	Upper Bound
K_p	2	3	10
θ_0	0	65	67.50
t_d	7	15	20

Optimization Solution:

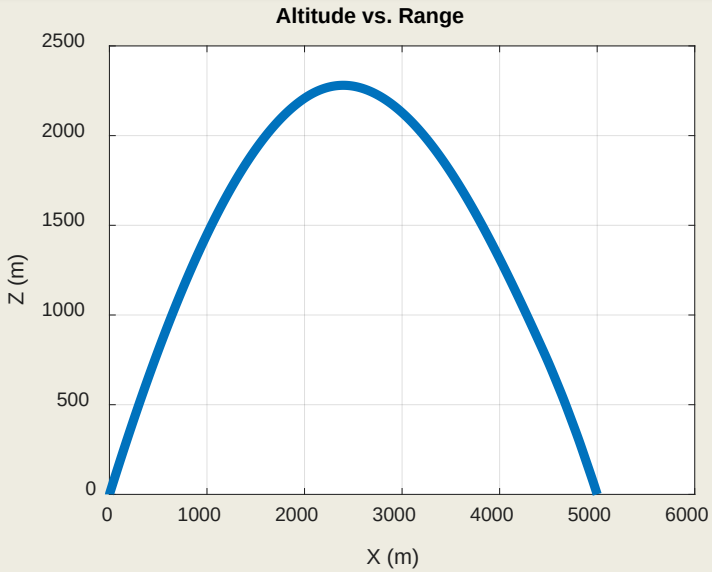
Parameter	Solved Value
K_p	3
t_d	16.7149
θ_0	60.1082

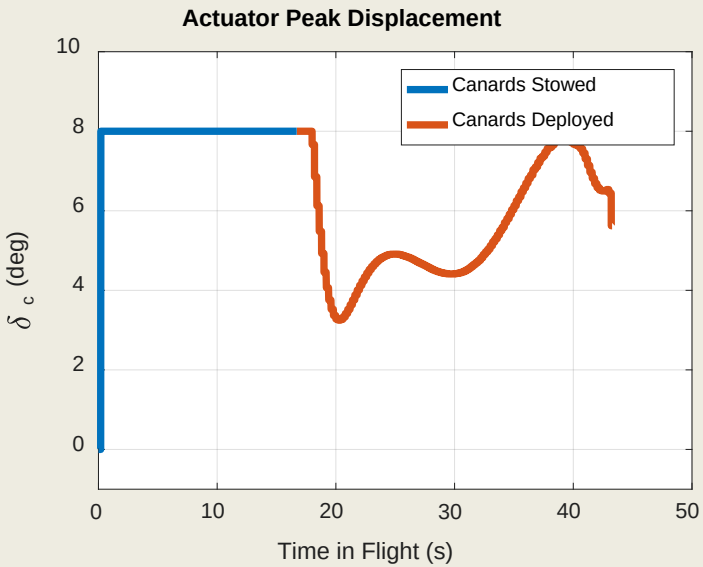


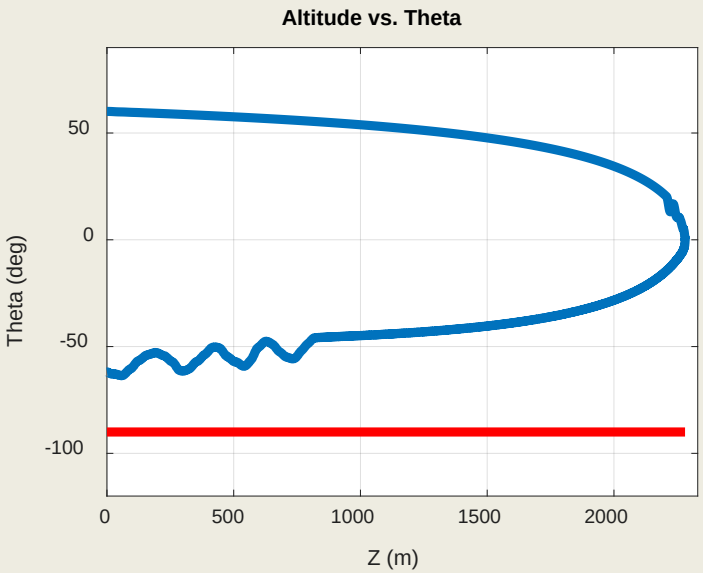
ProNav Trajectory

ProNav

- Engages Target, Error [-0.03, .58, 0] m
- Actuation System not Saturated
- Biggest Drivers
  - QE
  - Muzzle Velocity







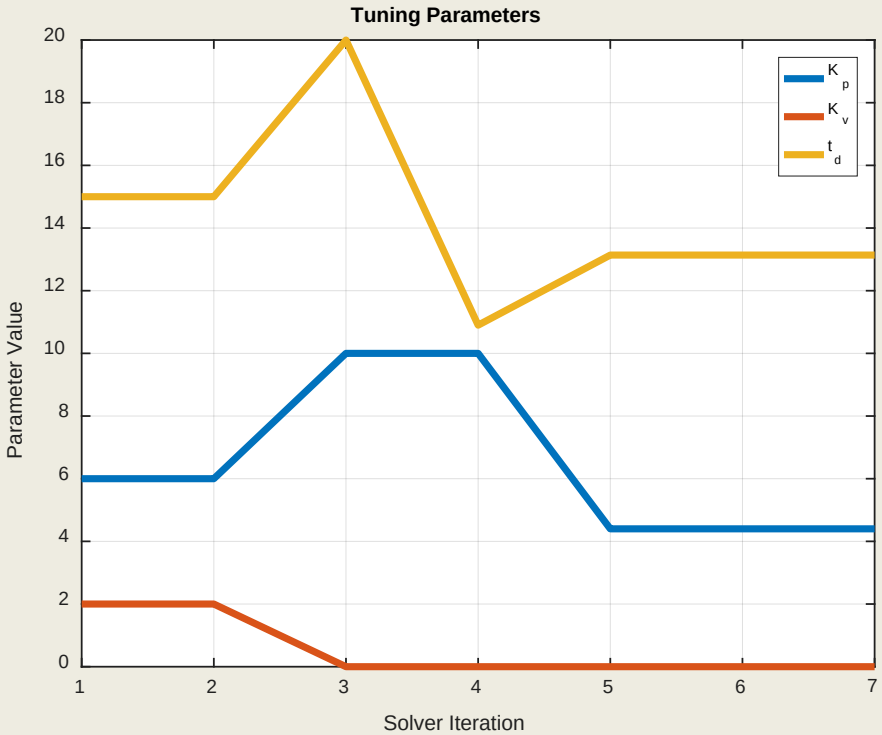
GENEX Results

Objective:
Minimize Cost Function with Parameters

Parameter	Lower Bound	Initial Value	Upper Bound
K_p	2	6	10
K_v	0	2	5
θ_0	0	65	67.5
t_d	7	15	20

Optimization Solution:

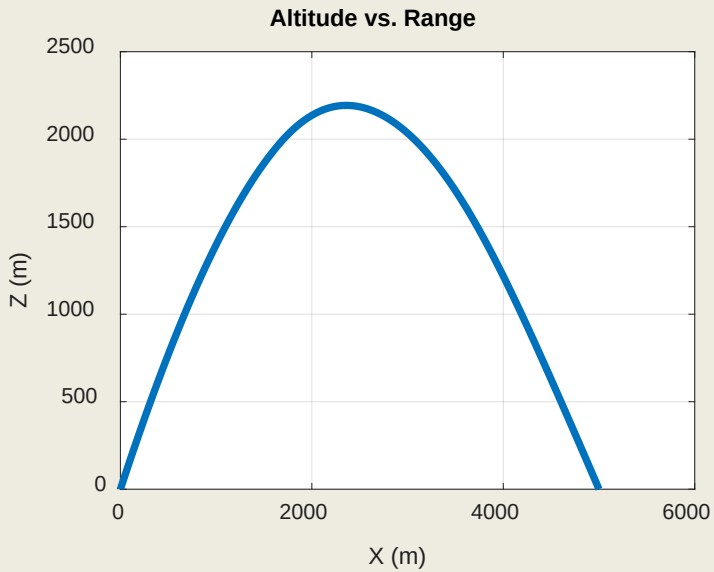
Parameter	Solved Value
K_p	4.4
K_v	.005
t_d	13.1375
θ_0	59.0625

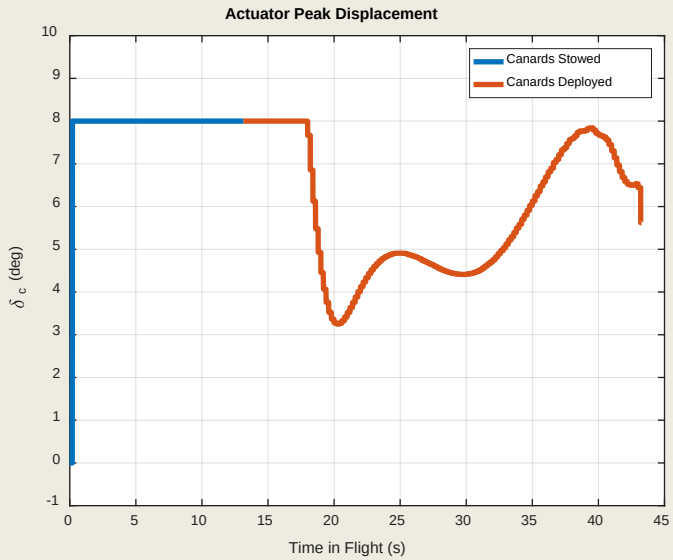


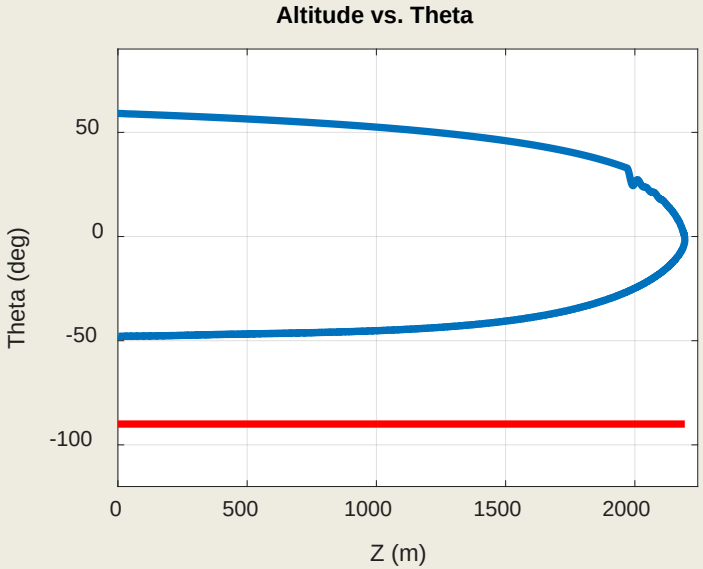
GENEX Trajectory

GENEX

- Engages Target, Error [-0.013, 0.13, 0] m
- Actuation System not Saturated
- Kv Term Little Impact on Performance







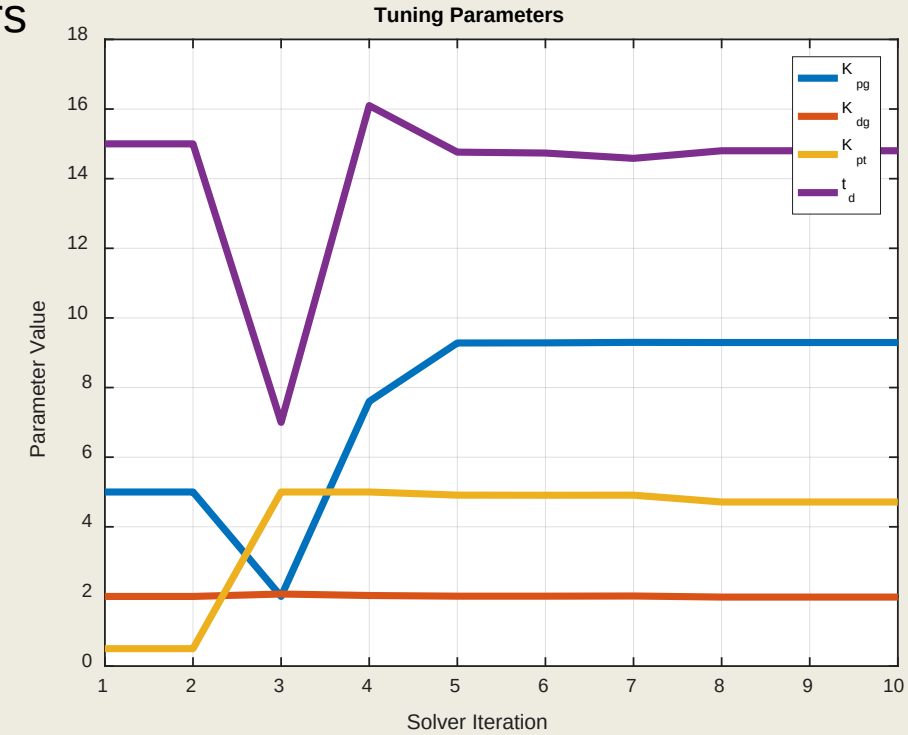
IPP Results

Objective:
Minimize Cost Function with Parameters

Parameter	Lower Bound	Initial Value	Upper Bound
K_{pg}	2	3	10
K_{dg}	0	1	5
K_{pt}	.05	.5	5
θ_0	0	65	67.5
t_d	7	15	20

Optimization Solution:

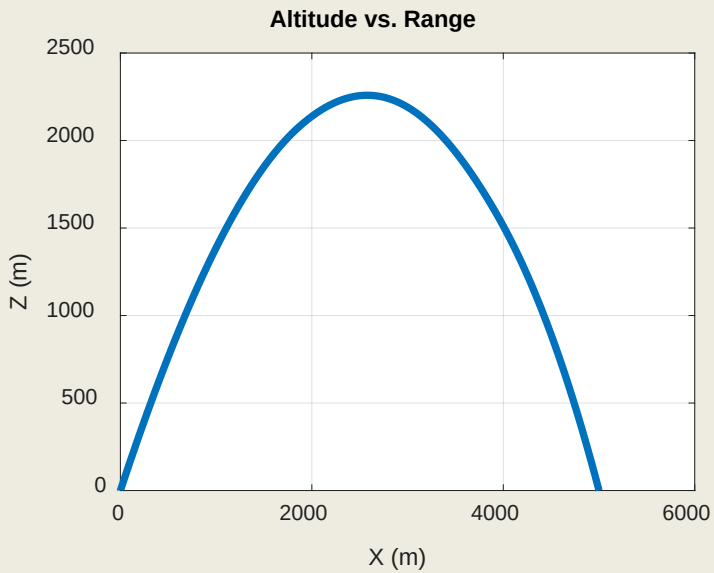
Parameter	Solved Value
K_{pg}	9.29344
K_{dg}	1.981
K_{pt}	4.713
t_d	14.802
θ_0	58.8724

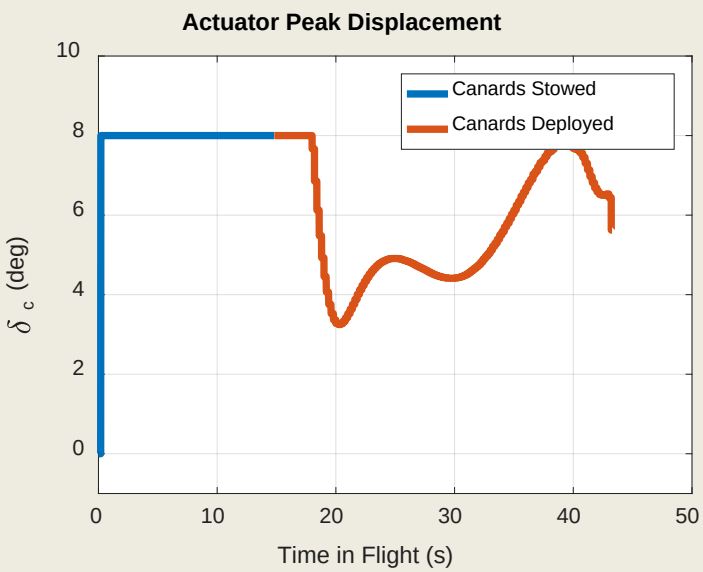


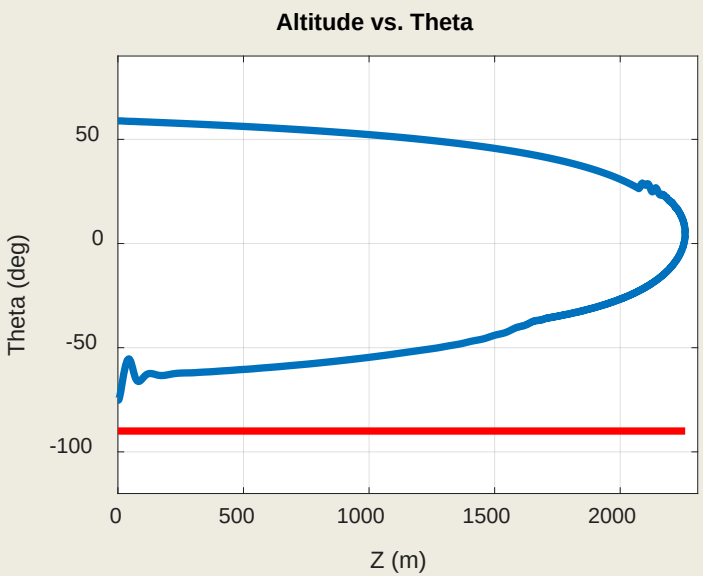
IPP Trajectory

GENEX

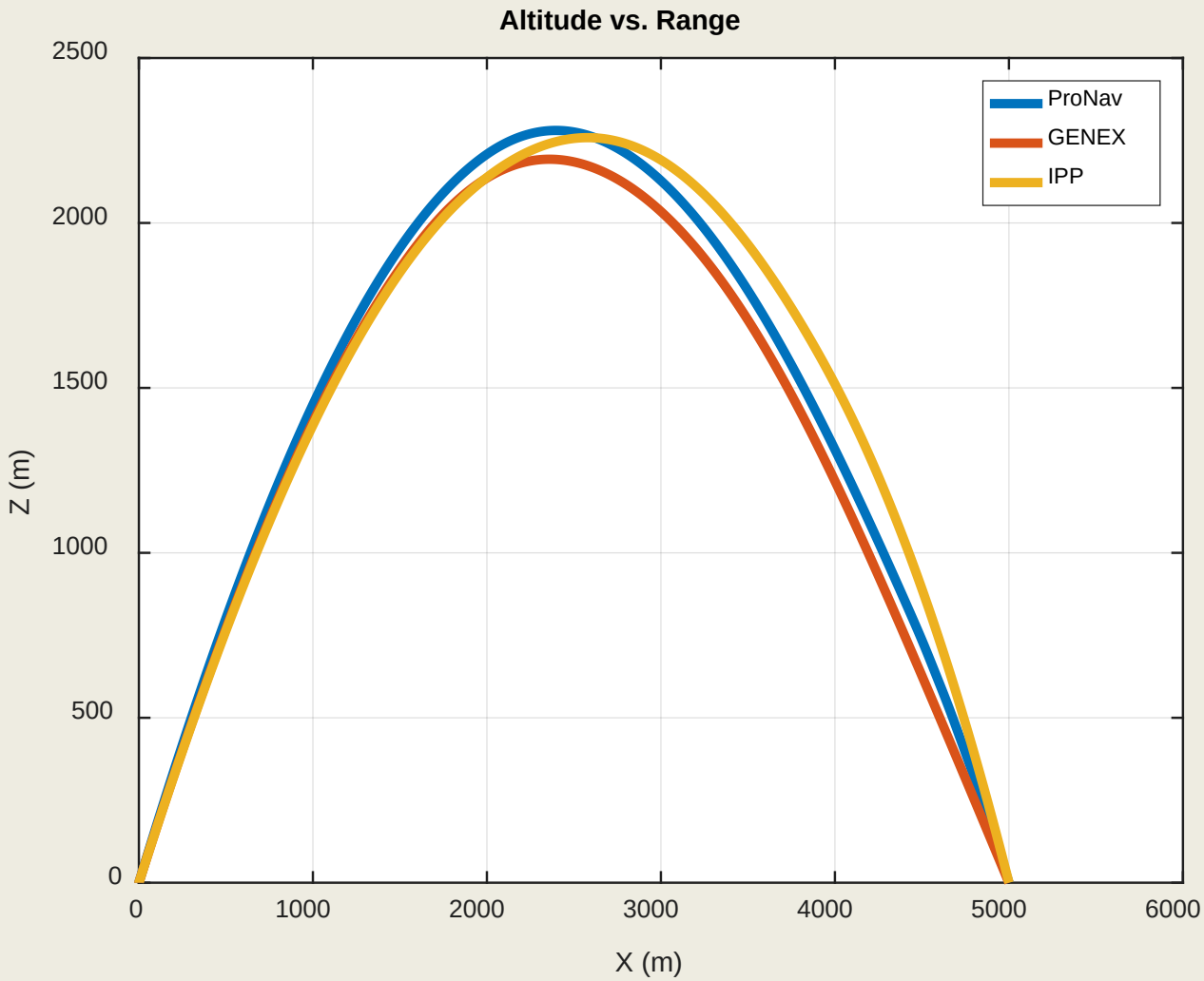
- Engages Target, Error [-0.22 , 0.01, 0] m
- Actuation System not Saturated
- Least Impactful Term







Trajectory Summary



Monte Carlo Analysis

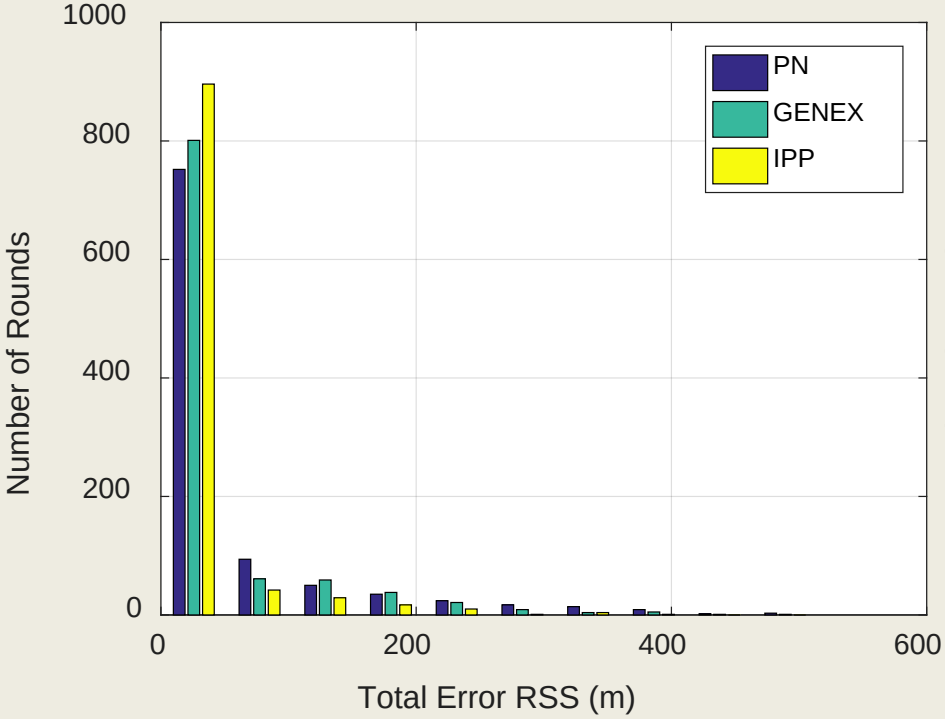
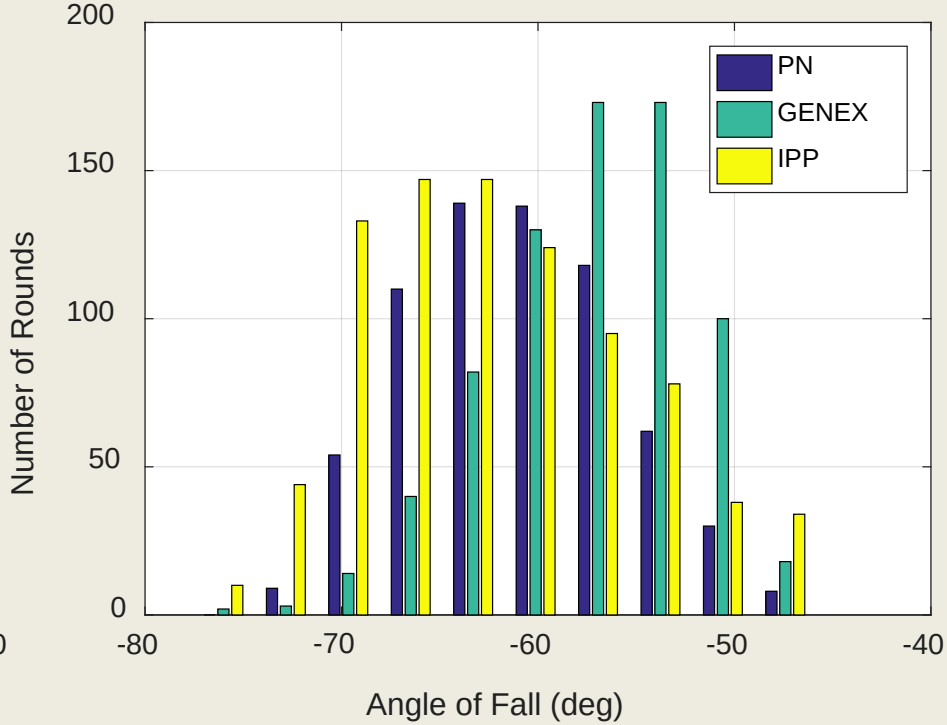
1000 Runs Each Configuration using Optimized Parameters

- PN
- GENEX
- IPP

Initial Conditions / Environmental Variations

Description	Parameter	Mean Value	Standard Deviation	Units
Muzzle Velocity	m_v	275	5	$\frac{m}{s}$
Gun QE	θ_0	θ_o	1	deg
Ground Temperature	T_0	100	5	^oF
Wind Amplitude	w_a	0	7	$\frac{m}{s}$
Wind Direction	w_d	0	π	rad

Objective

- Target near effective range of maneuver for this system
- Determine effectiveness of each guidance strategy

Monte Carlo Results