Coordinated Resource Allocation Among Multiple Agents with Application to Autonomous Refueling and Servicing of Satellite Constellations

Final Report
AFOSR Grant Number FA9550-04-1-0135

by

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The objective of this work was to develop new methods for high-level decentralized control of multiple space agents (i.e., satellites and spacecraft) with the objective of coordinated action and decision making. The blanket underlying assumption in this work was the sharing of a common resource (information, consumables, fuel, etc) so that all agents satisfy their own needs in a time-critical, cost-effective, optimal fashion. As a specific example of interest to the US Air Force we have addressed the problem of coordinated refueling between several satellites in a constellation. Satellite refueling has the potential to revolutionize future spacecraft operations. Apart from eliminating the need to replace (otherwise perfectly operating) satellites due to depletion of onboard fuel, a satellite constellation with refueling capabilities could easily change orbital planes or even have satellites move in *non-Keplerian orbits*. As a matter of fact, true formation "flying" (as opposed to orbiting) of spacecraft requires continuous thruster firing and the subsequent depletion of onboard fuel. Having the capability to continuously change the orbit of the satellites in a completely unpredictable manner will give unprecedented advantages to the US intelligence community.

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COORDINATED RESOURCE ALLOCATION AMONG MULTIPLE AGENTS WITH APPLICATION TO AUTONOMOUS REFUELING AND SERVICING OF SATELLITE CONSTELLATIONS

Final Report

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Executive Summary

Therein we summarize the research results developed under AFOSR Grant number FA9550-04-1-0135. The period of performance for this research award was from January 2004–December 2007. The objective of this work was to develop new methods for high-level decentralized control of multiple space agents (i.e., satellites and spacecraft) with the objective of coordinated action and decision making. The blanket underlying assumption in this work was the sharing of a common resource (information, consumables, fuel, etc) so that all agents satisfy their own needs in a time-critical, cost-effective, optimal fashion. As a specific example of interest to the US Air Force we have addressed the problem of coordinated refueling between several satellites in a constellation. Satellite refueling has the potential to revolutionize future spacecraft operations. Apart from eliminating the need to replace (otherwise perfectly operating) satellites due to depletion of onboard fuel, a satellite constellation with refueling capabilities could easily change orbital planes or even have satellites move in *non-Keplerian orbits*. As a matter of fact, true formation "flying" (as opposed to orbiting) of spacecraft requires continuous thruster firing and the subsequent depletion of onboard fuel. Having the capability to continuously change the orbit of the satellites in a completely unpredictable manner will give unprecedented advantages to the US intelligence community.

As part of this work the novel paradigm of peer-to-peer (P2P) refueling and/or servicing between the satellites in a satellite constellation was introduced, demonstrating the efficacy of the proposed methods. It is shown that P2P refueling strategies can be naturally incorporated as part of a mixed refueling strategy, which often outperforms single-vehicle refueling scenarios. Distributed methods for deciding the optimal pairings in a P2P strategy using auction algorithms were developed and the optimal scheduling of P2P maneuvers in order to minimize the constellation down-time was investigated. Several extensions of the baseline P2P refueling strategy were proposed (asynchronous P2P, coasting allocation, egalitarian P2P and cooperative P2P), all of which lead to further reductions in the overall fuel consumption during the ensuing orbital transfers.

As a result of the support received from this award, one student received his MS degree and another student will receive his Ph.D. degree. Eleven papers, in addition to a PhD dissertation and an MS thesis, document in great detail the results of this work.

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1 Introduction

1.1 Research Objectives and Summary of Accomplishments

It has long been recognized that servicing and refueling spacecraft in orbit has the potential to revolutionize spacecraft operations by extending the useful lifetime of the spacecraft, by reducing launching and insurance cost, and by increasing operational flexibility and robustness. Fuel is also a concentrated source of energy that can be used – in addition to satellite station-keeping – for powering "power-hungry" systems in space.

The main objective of this work has been the development of *efficient, distributed* algorithms for the replenishment of a large number of space assets with consumables (i.e., propellant). We have used methodologies from the operations research literature to solve this large-scale optimization problem. This work is one of the first to apply operational-theoretic ideas and graph-search methods in the area of astrodynamics.

Most of the previous studies in the literature that have dealt with the problem of refueling have limited the discussion to the specific mechanisms of exchanging fuel in a zero-gravity environment, and consider the simple case of a single satellite. Even when refueling of multiple satellites is discussed, it has always been assumed that a single spacecraft alone undertakes the task of refueling the whole satellite constellation. That is, a single service spacecraft plays the role of the sole supplier of fuel. One of the innovations of this work is the introduction of an alternative scenario for distributing fuel amongst a large number of satellites. In this scenario, no single spacecraft is in charge of the complete refueling process. Instead, all satellites share the responsibility of refueling each other on an equal footing. We call this the peer-to-peer (P2P) refueling strategy.

A P2P refueling strategy is, by definition, a *distributed* method for redistributing fuel/propellant within a constellation of spacecraft. Consequently, it offers a great degree of robustness and protection against failures. For instance, with a P2P strategy a failure of a single spacecraft will have almost no impact on the refueling of the rest of the constellation. On the contrary, a failure of the service vehicle in a single-spacecraft scenario will result in the failure of the whole mission. Several extensions of the baseline P2P refueling strategy were developed, all of which improved on the original strategy in terms of fuel savings.

1.2 Goals of this Report

The goal of this report is to summarize the results obtained under this research program. Since most of the technical results have appeared or will soon appear in 11 archival journal and conference publications, below we only provide a brief summary of these results, and remark on their significance and their interrelationship.

2 Description of Work Accomplished

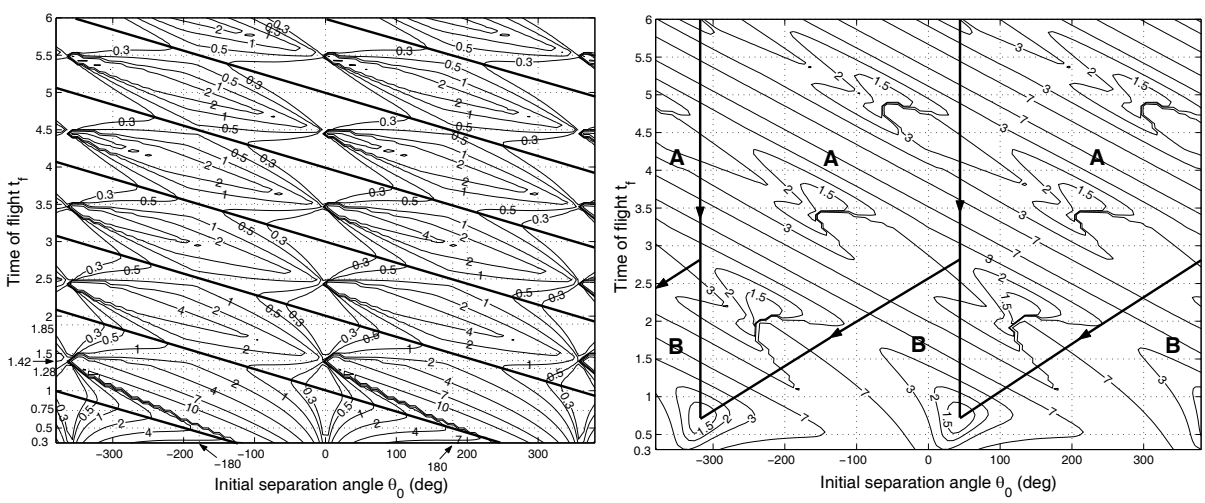
The following research accomplishments were achieved over the duration of this project (January 2004–December 2007).

2.1 P2P and Mixed Refueling Strategies

Pure P2P refueling for circular spacecraft constellations was originally proposed in Ref. [P1] as a means to equalize fuel. The P2P refueling problem can alternatively be formulated by imposing a minimum fuel requirement on each satellite in order to remain operational [P6,P7]. In this context, the satellites having at least the required amount of fuel are called *fuel-sufficient*, while those which do not have the required amount of fuel are called *fuel-deficient*. We seek to determine the optimal assignment of satellites so that all satellites end up being fuel-sufficient after the refueling process is over. The objective is to achieve fuel-sufficiency for all satellites by expending as little fuel as possible during the ensuing orbital transfers. A P2P refueling strategy seeks to match fuel-deficient with fuel sufficient satellites, while minimizing the orbital transfer cost.

A necessary ingredient for solving the fuel-optimal, time-constrained, multi-rendezvous satellite problem is the requirement of having an efficient method to calculate the fuel-optimal, time-constrained, single-rendezvous problem between two satellites in the same or different orbits. The problem of finding the transfer orbit between to points in space within a specified transfer time is known as the Lambert's problem.

As a first step towards a multi-satellite scheduling problem we have developed an efficient algorithm to solve the multi-revolution Lambert problem that quickly and efficiently identifies the optimal (minimum-ΔV) solution without the need to calculate all $2N_{\max} + 1$ transfer orbits, as is the current practice. The result of this investigation is a family of isocost contours (parameterized by the separation angle and total time), shown in Fig. 1.



(a) Contour of ΔV when $r_1 = r_2$

(b) Contour of ΔV when $r_1 = 1, r_2 = 1.5$

Figure 1: Non-dimensionalized, isocost optimal transfers between two circular orbits. The key parameters is the initial separation angle and allowed maneuver time.

These contour plots, along with a standard sliding rule, facilitate the task of finding the optimal initial and terminal coasting periods, and hence obtaining the globally optimal solution for the moving-target rendezvous problem.

The P2P refueling problem is formulated as an assignment problem on the so-called constellation graph

having as nodes the satellites in the constellation. The edges in the constellation graph indicate a feasible P2P refueling rendezvous between the corresponding satellite pair. The results of Fig. 1 also show that the rendezvous cost between two satellites in the same circular orbit decreases monotonically as the total time to conduct the rendezvous increases. As a result, the number of edges in the constellation graph involved in the optimal matching changes, depending on the refueling period. Figure 2 shows how the solution to the P2P refueling problem evolves as a function of the total refueling period.

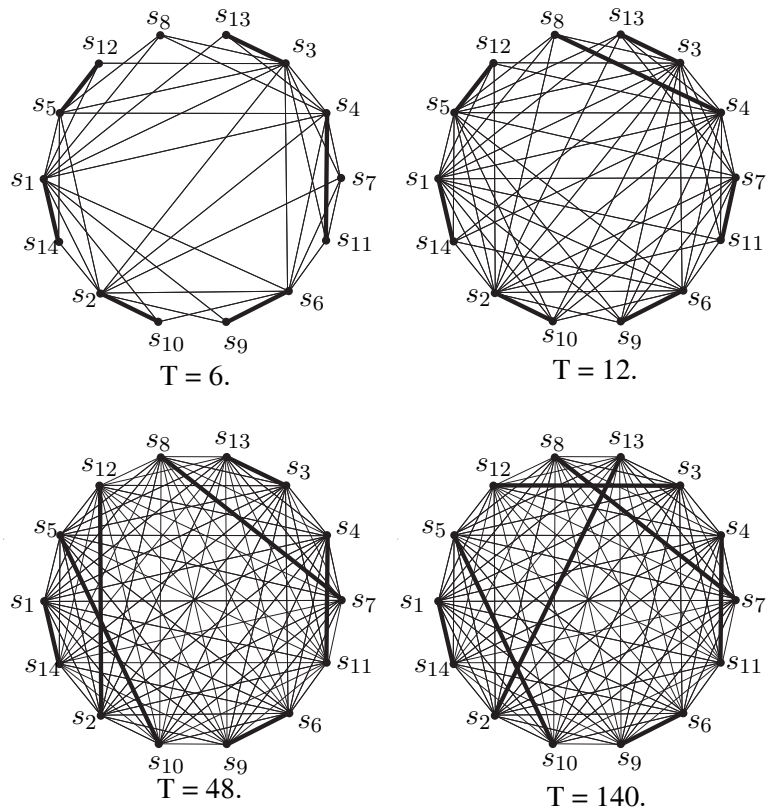
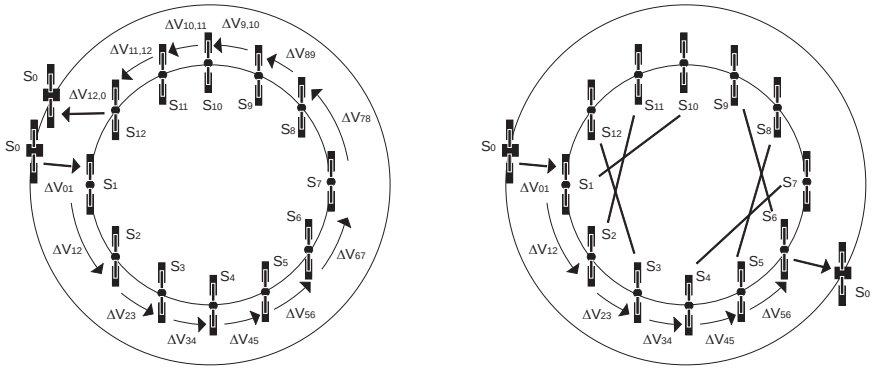


Figure 2: The evolution of solution with respect to the total refueling time.

Although a stand-alone P2P scenario may seem unconventional at first glance, it arises naturally as an essential component of a *mixed refueling strategy*. By mixed refueling strategy we mean a strategy which involves at least two stages. During the first stage a single spacecraft refuels only a certain fraction (perhaps half) of the satellites. During the second stage the satellites that received fuel during the first stage act as go-betweens, and distribute the fuel to the rest of the constellation in a P2P manner. That is, a P2P refueling strategy can be implemented as the final distribution phase of a single-vehicle refueling strategy. A pictorial comparison between the single-vehicle and mixed refueling strategies is depicted in Fig. 3.

In Refs. [P5,P1] it has been shown that a mixed refueling strategy may be more fuel-efficient than a single-spacecraft strategy, especially for a large number of satellites, and for short refueling periods. As a matter of fact, it is not difficult to come up with cases for which the single-spacecraft scenario is infeasible (due to the time constraint), while a mixed refueling strategy is still possible. Figure 4 depicts the results from the comparison between these two refueling strategies as the number of satellites in the constellation



(a) Single-spacecraft scenario.

(b) Mixed refueling scenario.

Figure 3: Single-vehicle and mixed refueling strategies.

varies, while keeping the total refueling time constant.

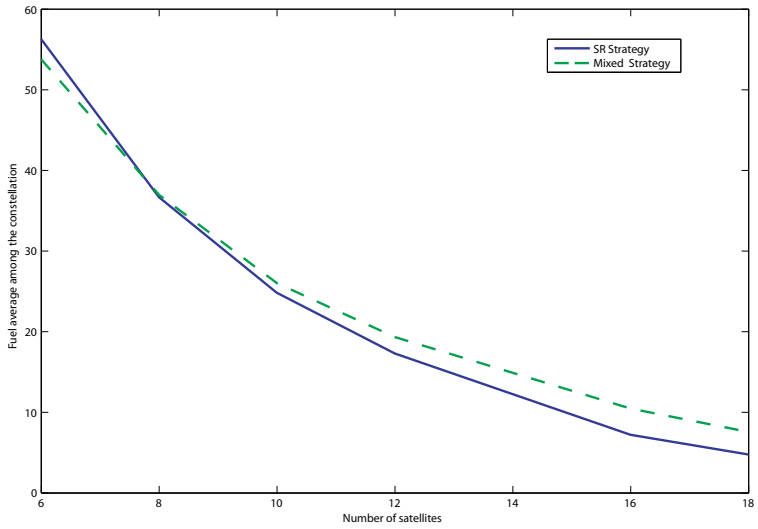


Figure 4: Comparison between the single-refueler and mixed refueling strategies for various numbers of satellites in the constellation.

2.2 Auction Algorithms for P2P Assignments

The baseline P2P problem is an asymmetric assignment problem. Of the many existing methods for solving assignment problems, the auction algorithm naturally fits the P2P refueling problem because of its inherent distributed nature. A decentralized approach that uses auctions in order to determine the optimal assignments has been proposed in Refs. [P6,P7]. The method is also immune to time delays and asynchronous or bad communication links between the satellites that may result in out-of-date bids. Moreover, auction algorithms tend to be far superior than other methods when the underlying graph structure is sparse, as is typically the case with satellite constellations.

The methodology consists of assigning to each fuel sufficient/fuel deficient satellite pair (i, j) a benefit a_{ij} for matching the two. Each fuel sufficient satellite has a *price* π_j. Fuel deficient satellites bid on fuel sufficient satellites so that they maximize their *profit* $a_{ij} - \pi_j$, that is,

$$a_{ij_i} - \pi_{j_i} = \max_{j \in \mathcal{N}(i)} \{a_{ij} - \pi_j\}, \tag{1}$$

where $\mathcal{N}(i)$ consists of all fuel sufficient satellites that fuel deficient satellite i can be bid on. Successive bids are calculated so that at each iteration the so called *complementary slackness* condition is satisfied. At the end, we obtain an assignment of the fuel deficient satellites to the fuel sufficient ones, with minimal exchange of information amongst the satellites.

The auction algorithm is highly parallelizable, and is immune to communication delays. This nice property of the auction algorithm is confirmed in Figure 5 where the update probability of each bid by the fuel deficient satellites is shown as a function of the required number of iterations for convergence. It is clear that the algorithm converges even for very small update rates.

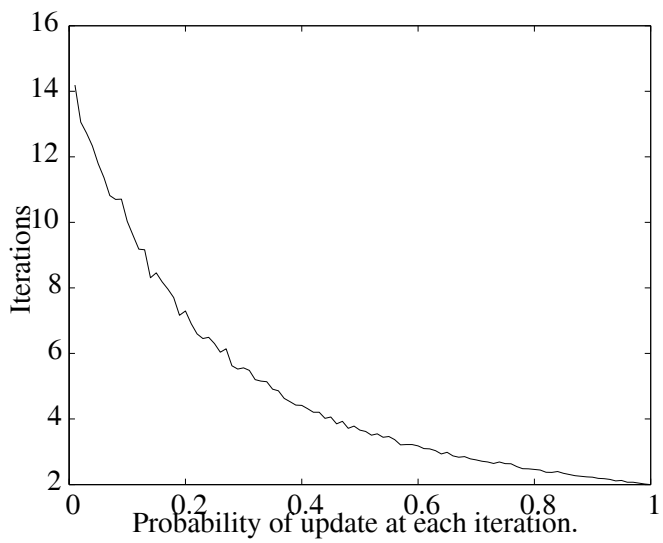


Figure 5: The number of iterations compared with the probability of each fuel deficient satellite having updated price information after each iteration during the auction algorithm.

2.3 Optimal Scheduling of P2P Rendezvous

Most often than not, simultaneous maneuvering of more than one satellite in a constellation will lead to constellation downtime. It is therefore of interest to develop optimal scheduling of a sequence of P2P maneuvers in order to minimize the constellation downtime. To this end, we have introduced the following three operability constraints for a constellation: (i) outside world connectivity (OWC) constraints, which model communications between the constellation and the outside world, (ii) skeleton crew requirement (SCR) constraints, which model the requirement that a certain number of satellites from a given subset be operational at any given time, and (iii) inter-constellation connectivity (ICC) constraints, which model the requirement that certain subsets of satellites maintain communication connectivity.

Given these sets of constraints, we have developed explicit formulas to calculate the violation of the OWC, SCR and ICC constraints, hence the constellation downtime. We have proposed a new heuristic that schedules a set of maneuvers over a given interval of time $[t_0, t_f]$ so that the overlap of maneuvers that are incompatible with each other is minimized. The heuristic is based on the observation that the only points of interest are the initial and final times of each maneuver, hence the optimal schedule must consist of maneuver sequences that are "anchored" at the initial time t_0 and/or the final time t_f. A sequence of k-maneuvers $r_1, r_2, \ldots, r_k$ with respective initialization and termination times t_i^I and t_i^F $(i = 1, \ldots, k)$ is said to be anchored to time τ if $t_1^I = \tau$ or $t_1^F = \tau$ and for each $j = 2, \ldots, k-1$ one of the following is true: either $t_i^F = t_{i+1}^F$ or $t_i^I = t_{i+1}^I$ or $t_i^F = t_{i+1}^I$ or $t_i^I = t_{i+1}^F$; see Fig. 6 for an example for a sequence of four maneuvers anchored to t_0.

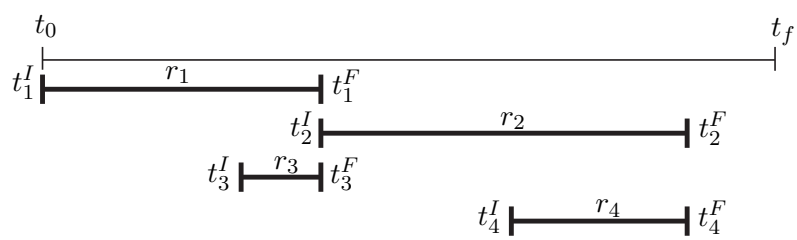


Figure 6: An example of a sequence of four maneuvers anchored at t_0.

2.4 Coasting Time Allocation Strategy

It is well known that coasting (period of no thrust) can significantly reduce the fuel expenditure during an orbital rendezvous between two satellites. Therefore, during each transfer, initial or final coasting intervals play an important role in the overall optimal rendezvous cost. Figure 7 shows a typical variation of the rendezvous cost (non-dimensionalized ΔV) with respect to the transfer time. The dashed line is the cost when no coasting is used and the solid line is the cost when initial or final coasting are used. Clearly, the use of coasting reduces the overall fuel consumption, as it is not necessary for the satellite to enter into a higher transfer orbit in order to meet the terminal time constraint. In [P2,P4] we have used this observation to reduce the fuel for each P2P rendezvous by adding suitable initial and final coasting intervals, either for the forward, or return trips.

The main idea behind the formulation of a fuel-reducing strategy using coasting is to allow for unequal time distribution between the forward and the return legs for each fuel transaction. To this end, we consider the following three cases: Case-I: $t_{ij}^f = t_{ij}^r = t_{ij}/2$; Case-II: $t_{ij}^f = t_{ij}/2 - t'_{ij}$ and $t_{ij}^r = t_{ij}/2 + t'_{ij}$; Case-III: $t_{ij}^f = t_{ij}/2 + t''_{ij}$ and $t_{ij}^r = t_{ij}/2 - t''_{ij}$, where t_{ij}^f and t'_{ij} denote the total time and the coasting time for the forward journey, respectively, and t_{ij}^r and t''_{ij} denote the total time and the coasting time for the return journey; clearly, $t_{ij} = t_{ij}^f + t_{ij}^r$. In case of an equal partition of the total time between the forward and return transfers, we have $t_{ij}^f = t_{ij}^r = t_{ij}/2$. Let also $p_i^{j\mathrm{I}}$, $p_i^{j\mathrm{II}}$ and $p_i^{j\mathrm{III}}$ denote the fuel spent for satellite s_i to rendezvous with s_j and return back to its original position, for each of the previous three cases, respectively. We then choose the optimal time partition for the forward and return transfers, the one that satisfies

$$p_i^{j*} = \min\{p_i^{j\mathrm{I}}, p_i^{j\mathrm{II}}, p_i^{j\mathrm{III}}\}. \tag{2}$$

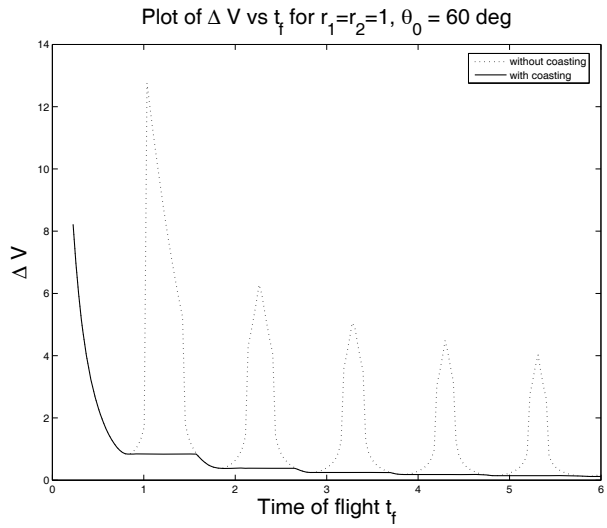


Figure 7: Variation of rendezvous cost with transfer time. Transfer from and to a circular orbit with an initial separation angle of 60 deg.

The corresponding time allocation is then given by

$$(t_{ij}^f, t_{ij}^r) = \begin{cases} (t_{ij}/2, t_{ij}/2), & \text{if } p_i^{j*} = p_i^{j\mathrm{I}}, \\ (t_{ij}/2 - t'_{ij}, t_{ij}/2 + t'_{ij}), & \text{if } p_i^{j*} = p_i^{j\mathrm{II}}, \\ (t_{ij}/2 + t''_{ij}, t_{ij}/2 - t''_{ij}), & \text{if } p_i^{j*} = p_i^{j\mathrm{III}}. \end{cases}$$

We can similarly compute the cost of a single fuel transaction for the case s_i is the passive satellite in the rendezvous and s_j is the active satellite. Finally, the optimum fuel consumption between any two satellites s_i, s_j in the constellation is given by

$$p_{ij}^* = \begin{cases} p_i^{j*}, & \text{if } s_i \text{ can be active, but } s_j \text{ cannot be active}, \\ p_j^{i*}, & \text{if } s_j \text{ can be active, but } s_i \text{ cannot be active}, \\ \min\{p_i^{j*}, p_j^{i*}\}, & \text{if either } s_i \text{ or } s_j \text{ can be active}, \\ \infty, & \text{if neither } s_i \text{ nor } s_j \text{ can be active}. \end{cases}$$

Figure 8 shows a comparison between the three cases as a function of the separation angle between the two satellites. For all separation angles, an equal time allocation for the forward and return legs of a fuel transaction (Case I) always results in more or equal fuel expenditure than an unequal time allocation (Cases II or III).

2.5 Asynchronous P2P Refueling

In the original mixed-P2P strategy we assumed only synchronous implementation for the P2P second stage, that is, all P2P maneuvers during the second stage of the mixed refueling scenario, occur simultaneously and

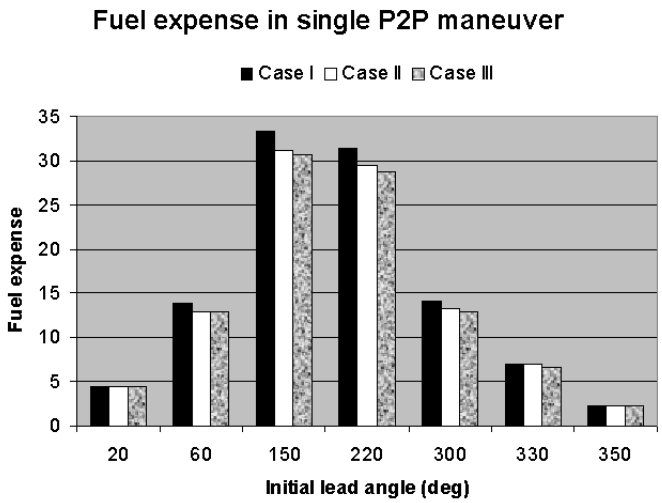


Figure 8: Effect of CTA algorithm to a single P2P maneuver.

they all take the same time to be completed. However, we can further improve on the fuel savings incurred during the second stage by allowing asynchronous P2P (A-P2P) maneuvers, as described next [P2,P4]. To demonstrate the A-P2P strategy, assume that the total time allowed for refueling of the whole constellation be T (given) and let $\mathcal{I}_1$ denote the index set of the satellites refueled during the first stage by the service vehicle s_0 in a mixed strategy, and let $\mathcal{I}_2 = \mathcal{I} \backslash \mathcal{I}_1$ denote the remaining satellites which are to be refueled during the second stage, where $\mathcal{I} = \{1, 2, \ldots, 2n\}$ denotes the index set of all satellites in the constellation. Without loss of generality, we may assume that $\mathcal{I}_1 = \{1, 2, \ldots, n\}$ and $\mathcal{I}_2 = \{n+1, n+2, \ldots, 2n\}$. Let also $T^{(1)}$ denote the time allotted for the first stage and $T^{(2)} = T - T^{(1)}$ the time allotted for the second (P2P) stage in a mixed strategy.

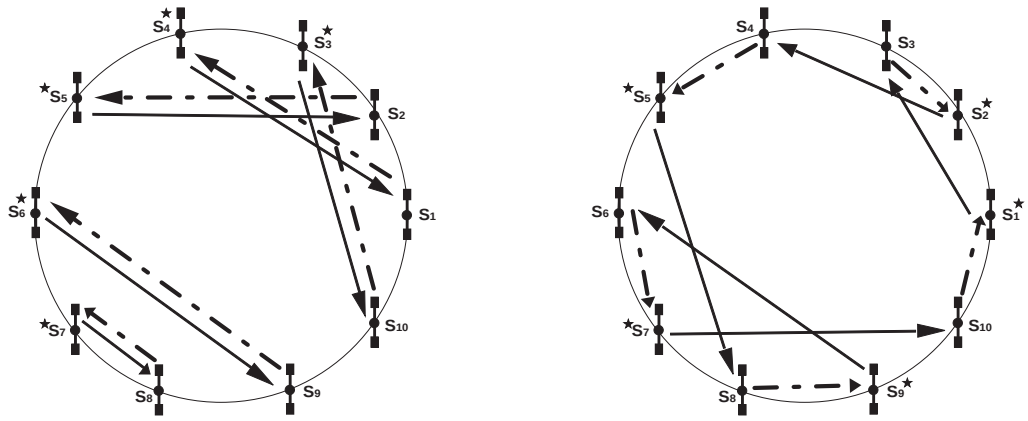
During now $T^{(1)}$ the service vehicle s_0 delivers fuel (perhaps sequentially) to the n satellites s_i $(i \in \mathcal{I}_1)$ in an optimal fashion. The optimal time distribution for these transfers, denoted by $t^{(1)}_{i,i+1}$ $(i = 1, \ldots, n-1)$ then satisfies

$$T^{(1)} = \sum_{i=1}^{n-1} t^{(1)}_{i,i+1}, \tag{3}$$

where the optimal values $t^{(1)}_{i,i+1}$ are calculated by solving a binary integer programming problem [10].

In a synchronous mixed-P2P scenario all the satellite rendezvous take place simultaneously. In a mixed refueling strategy, this implies that all fuel deficient satellites (at the end of the first stage) are refueled within the time $T^{(2)}$. Note, however that the time $T^{(2)}$ is binding only for satellite s_n (the last satellite to be visited by s_0 during the first stage of a mixed strategy). All other satellites s_i $(i = 1, \ldots, n-1)$ have available $T^{(2)} + \sum_{k=i}^{n-1} t^{(1)}_{k,k+1}$ time units to perform their fuel transactions. Thus, the time available for s_i to complete the P2P maneuver with its matching satellite s_j is given by

$$t^{(2)}_{ij} = \begin{cases} T^{(2)} + \sum_{k=i}^{n-1} t^{(1)}_{k,k+1}, & \text{if } i \in \mathcal{I}_1 \backslash \{n\}, \\ T^{(2)}, & \text{if } i = n. \end{cases} \tag{4}$$



(a) Baseline P2P strategy

(b) E-P2P strategy

Figure 9: Optimal assignments for a sample constellation.

Since $t_{ij}^{(2)} \geq T^{(2)}$ for all satellite pairs, and due to the monotonic dependence of transfer time with fuel expenditure (when coasting is allowed) (see Fig. 7), it is clear that each rendezvous between two satellites will require less fuel than a synchronous implementation. Consequently, the overall fuel consumption for the whole constellation will also be reduced by using an asynchronous P2P implementation.

2.6 Egalitarian P2P Refueling

In all of the above-mentioned studies [P1,P2,P4,P5,P6,P7] it has been assumed that all active satellites return to their original orbital slots after the refueling process is over. References [P3,P8,P9] aimed at relaxing this constraint by allowing the active satellites to return to any of the available orbital slots left vacant by other (active) satellites. The underlying assumption behind such a consideration is that all satellites are similar and perform the same functions, so that any satellite can be used in lieu of any other satellite in the constellation. We call this the egalitarian P2P (E-P2P) refueling strategy. Figure 9 compares a P2P with an E-P2P solution for the same constellation.

The E-P2P refueling problem can be formulated as a three-index assignment problem. A class of algorithms that has been developed for solving the three-index assignment problem includes the Greedy Random Adaptive Search Procedure (GRASP), which generates good quality solutions for the three-index assignment problem by constructing low cost feasible solutions. These feasible solutions can then be improved by performing a local search about these solutions. In Ref. [P8] we have used the GRASP method to determine the optimal assignments for E-P2P refueling. Alternatively, the E-P2P refueling problem can be modeled as a minimum cost flow problem in an appropriately constructed constellation network [P2,P9].

The network flow formulation for the solution of the E-P2P refueling problem is particulary appealing, since it gives a good quality solution much faster than the GRASP algorithm. We have formulated the E-P2P refueling problem as a minimum cost flow problem in an appropriately constructed network. An optimal flow in this network yields a set of E-P2P maneuvers that has the minimum ΔV cost. Recognizing that the real objective is to minimize the total fuel expenditure (as opposed to minimizing total ΔV), we have

propose a local search method to improve the obtained solution. Local search methods have been found to be very useful for improving the cost of solutions for the three-index assignment problem. We have also derived upper and lower bounds on the fuel expenditure corresponding to the optimal assignments for E-P2P refueling. The lower bound can be calculated by solving a separate two-index assignment problem, and is useful for providing a measure of suboptimality of the E-P2P solution. A P2P solution, in which the active satellites are constrained to return to their original orbital slots, provides an upper bound for the E-P2P solution.

The network flow formulation minimizes the total ΔV rather than the total fuel expenditure, and hence the results obtained using this method are suboptimal when compared to those obtained by GRASP. However, from a computational point of view, the network flow formulation generates the optimal E-P2P assignments several times faster than the (non-parallelized) GRASP method. Figure 10 compares the results from the P2P and E-P2P strategies in terms of fuel consumption, including the calculated lower optimality bound (the baseline P2P always provides an upper bound).

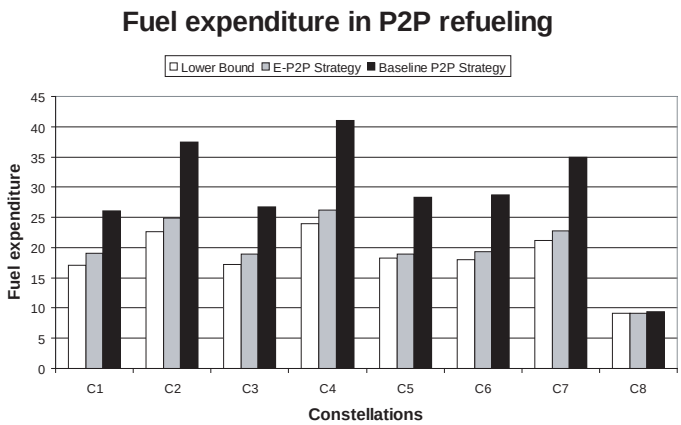
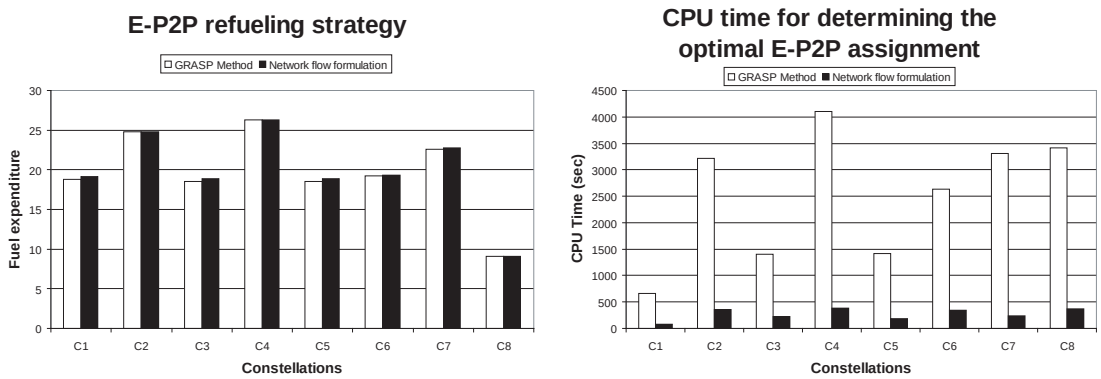


Figure 10: Comparison of E-P2P and baseline P2P refueling strategies.

Figure 11 shows the optimality of the network flow formulation when compared with the GRASP method. As seen from the left plot of Fig. 11, the network flow formulation is a slightly suboptimal when compared to the GRASP, however the CPU time taken to compute the solution is orders of magnitude smaller than the time taken using GRASP (see the right plot of Fig. 11).

2.7 Cooperative P2P Refueling

For the case of fixed-time impulsive maneuvers, cooperative rendezvous may be advantageous when the time allotted for the maneuver is relatively short. Examples show that a non-cooperative solution becomes cheaper once the time allotted for the rendezvous is large enough for Hohmann transfers to be feasible. Unlike the non-cooperative case, the amount of fuel exchanged between the satellites in the cooperative case affects the return trips of both the active satellites. Hence, a natural question that arises is how to obtain the amount of fuel that must be shared between the two satellites. In this context, we have shown that if two satellites engaging in a cooperative P2P maneuver have engines with the same specific thrust, the optimal



(a) Optimality Comparison

(b) CPU Time Comparison

Figure 11: Comparison of network flow formulation with GRASP method.

fuel exchange takes place when the satellite making the costlier ΔV transfer returns with just enough fuel to be fuel-sufficient. This observation allows us to determine the amount of fuel exchange that leads to minimum fuel expenditure during the maneuver.

The C-P2P problem formulation involves the introduction of an undirected bi-partite graph, with the first partition being the set of edges between fuel-sufficient and fuel-deficient satellites, and the second partition being the allowable orbital slots where these two satellites can meet (see Fig. 12).

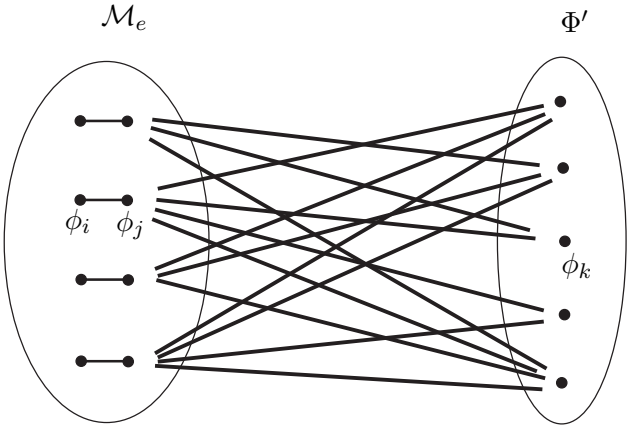


Figure 12: Bipartite graph for determining the C-P2P solution given the fuel transactions $\mathcal{M}_e$.

We also need to impose additional constraints that will eliminate cases such as

i) Cooperative rendezvous corresponding to the two edges occur at the same orbital slot, or

ii) Cooperative rendezvous corresponding to one edge occurs at the slot of the passive satellite corresponding to another edge.

Figure 13 compares the optimal P2P and C-P2P fuel expenditure for sample constellations. For the constellations C_1 and C_4, the optimal non-cooperative P2P solution is the cheapest way to redistribute fuel

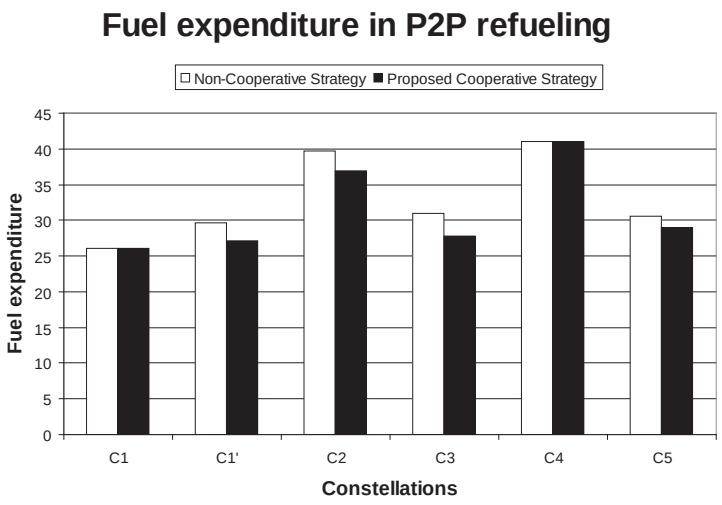


Figure 13: Comparison of P2P and C-P2P refueling strategies.

in the constellation. For these, the fuel-deficient satellites have enough fuel to complete a non-cooperative rendezvous. Whenever this is not possible, as in case of the remaining constellations, cooperative maneuvers turn out to be beneficial.

3 Research Personnel Supported

Faculty

Prof. Panagiotis Tsiotras, Principal Investigator

Graduate Students

Atri Dutta, Ph.D., August 2008 (expected)
Alexandros Salazar-Cardozo, M.S., December 2006

In addition to the previous personnel, the following graduate students were involved in several aspects of this research project, although their financial support did not come directly from this AFOSR award.

Mark Hunkele, M.S., May 2005
Arnaud de Nailly, M.S., December 2004

4 Interactions and Transitions

4.1 Participation and Presentations

The following conferences and workshops were attended:

- Malcolm D. Shuster Astronautics Symposium, Buffalo, NY, June 13–15, 2005 (invited).
- Infotech at Aerospace Conference, Crystal City, DC, September 26–29, 2005
- Georgia Tech Space Systems Engineering Conference, Atlanta, GA, November 8–10, 2005, Paper No. GT-SSEC.F.5 (invited).
- American Control Conference, Minneapolis, MN, June 14–16, 2006 (invited).
- AAS Spaceflight Mechanics Meeting, Sedona, AZ, Jan. 28-Feb. 1, 2007.
- F. Landis Markley Astronautics Symposium, Chesapeake Bay, June 28–July 2, 2008 (invited).
- AIAA Space Conference and Exhibit, San Diego, Sept. 9-11, 2008.

Furthermore, the conference articles [P4-P11] were (will be) presented in these conferences.

The following invited presentations used results obtained from AFOSR-sponsored research, and were given during the period of this award.

- "Some New Advances in Controlling Space Systems," Space Vehicles Directorate, Air Force Research Laboratory, Kirtland AFB, Albuquerque, NM, February 16, 2006.
- "Nonlinear Control in Aerospace - IPACS Design: Theory and Experiments," Department of Mechanical Engineering, The University of New Mexico, *Mechanical Engineering Excellence Seminar Speaker* series, February 17, 2006.
- "Putting the Dynamics Back in Control: The IPACS Case," Department of Mechanical and Aerospace Engineering, UCLA, June 9, 2006.
- "Coordinated Multi-rendezvous for Satellite Constellations and Related Problems," Guidance and Control Branch, Jet Propulsion Laboratory, Pasadena, CA, May 9, 2007.
- "Putting the Dynamics Back in Control: The IPACS Design Case," Dept. of Electrical, Computer and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY, October 22, 2007.

4.2 Transitions

Parts of this work have been used by the group of Dr. Alan Lovell of AFRL/VSES (Tel: (505) 853-4132, Email: thomas.lovell@kirtland.af.mil) to support work on coordinated spacecraft control. The following AFRL personnel served as points of contact with PI.

Dr. Erwin Scott, Space Vehicles Directorate, AFRL/VSSV, 3550 Aberdeen Ave SE, Kirtland AFB, New Mexico 87117-5776, Tel: (505) 846-9816, Fax: (505) 846-7877, Email: richard.erwin@kirtland.af.mil. Met with the PI during his visit at AFRL on February 16, 2006.

Dr. Alan Lovell, Space Vehicles Directorate, AFRL/VSES, 3550 Aberdeen Ave SE, Kirtland AFB, New Mexico 87117-5776, Tel: (505) 853-4132, Fax: (505) 846-6053, Email: thomas.lovell@kirtland.af.mil. Met with the PI during his visit at AFRL on February 16, 2006.

Several emails and telephone exchanges were also undertaken with the above AFRL personnel during the period of performance of this award.

5 Honors and Awards

- Promoted to the rank of Full Professor, School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia (2005)
- Reappointed Associate Editor (third 3yr-term) for the *AIAA Journal of Guidance, Control, and Dynamics* (2005).
- Invited Speaker, Mechanical Engineering Excellence Series, Department of Mechanical Engineering, The University of New Mexico, February 17, 2006.
- Reappointed Associate Editor for the *IEEE Control Systems Magazine* (2005, 2007).
- Luther Long Award in Engineering Mechanics, for best doctoral thesis at Georgia Tech (advisor of E. Velenis) (2006).
- Best session paper award, *Multi-Vehicle Control I* session, American Control Conference, Minneapolis, MN (2006).
- Best session paper award, *Linear Matrix Inequalities* session, 13th IEEE Mediterranean Conference on Control and Automation, Limassol, Cyprus (2005).
- Elected member of the AIAA Navigation and Control Technical Committee (2007).

6 Acknowledgment/Disclaimer

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7 Research Publications and Presentations under this AFOSR Award

7.1 Doctoral Dissertations and Master's Theses

The following dissertations and theses directly benefited from the support of this AFOSR award.

1. "Optimal Cooperative And Non-Cooperative P2P Maneuvers For Refueling Satellites In Circular Constellations," Ph.D. dissertation, (Atri Dutta), School of Aerospace Engineering, Georgia Institute of Technology, August 2008, (expected).
2. "A High-Level Framework for the Autonomous Refueling of Satellite Constellations," M.S. thesis, (Alexandros Salazar-Kardozo), School of Aerospace Engineering, Georgia Institute of Technology, December 2006.

7.2 Journal and Conference Publications

P1. Shen H. and Tsiotras, P., "Peer-to-Peer Refueling for Circular Satellite Constellations," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 6, pp. 1220–1230, 2005.

P2. Dutta, A., and Tsiotras, P., "Asynchronous Optimal Mixed P2P Satellite Refueling Strategies," *Journal of the Astronautical Sciences*, Vol. 54, No. 3-4, 2006.

P3. Dutta, A., and Tsiotras, P., "An Equalitarian Peer-to-Peer Satellite Refueling Strategy," *Journal of Spacecraft and Rockets*, (accepted December 2007).

P4. Dutta, A. and Tsiotras, P., "Asynchronous Optimal Mixed P2P Satellite Refueling Strategies," *Malcolm D. Shuster Astronautics Symposium*, Buffalo, NY, June 13–15, 2005, AAS Paper 2005-474.

P5. Tsiotras, P. and de Nailly, A., "Comparison Between Peer-to-Peer and Single-Spacecraft Refueling Strategies for Spacecraft in Circular Orbits" *Infotech at Aerospace Conference*, (Crystal City, DC), September 26–29, 2005, AIAA Paper 2005-7115.

P6. Salazar A. and Tsiotras, P., "An Auction Algorithm for Optimal Satellite Refueling," *Georgia Tech Space Systems Engineering Conference*, Atlanta, GA, November 8–10, 2005, Paper No. GT-SSEC.F.5.

P7. Salazar A. and Tsiotras, P., "An Auction Algorithm for Allocating Fuel in a Satellite Constellation Using Peer-to-Peer Refueling," *American Control Conference*, Minneapolis, MN, June 14–16, 2006, pp. 4214–4219.

P8. Dutta, A., and Tsiotras, P., "A Greedy Random Adaptive Search Procedure for Optimal Scheduling of P2P Satellite Refueling," *AAS Spaceflight Mechanics Meeting*, Sedona, AZ, Jan. 28-Feb. 1, 2007, AAS Paper 07-150.

P9. Dutta, A., and Tsiotras, P., "A Network Flow Formulation for an Egalitarian P2P Satellite Refueling Strategy, " *AAS Spaceflight Mechanics Meeting*, Sedona, AZ, Jan. 28-Feb. 1, 2007, AAS Paper 07-151.

P10. Dutta, A., and Tsiotras, P., "A Cooperative P2P Refueling Strategy for Circular Satellite Constellations," *AIAA Space Conference and Exhibit*, San Diego, Sept. 9-11, 2008.

P11. Dutta, A., and Tsiotras. P., "Hohmann-Phasing Cooperative Rendezvous Maneuvers," *F. Landis Markley Astronautics Symposium*, Chesapeake Bay, June 28–July 2, 2008.