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UAV TASK ASSIGNMENT WITH TIMING CONSTRAINTS

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14. ABSTRACT
This paper addresses the problem of task allocation for wide area search munitions. The munitions are required to search for, classify, attack, and verify the destruction of potential targets. We assume that target field information is communicated between all elements of the swarm. We generate a tour of optimal assignments for each vehicle using a Mixed Integer Linear Program, or MILP format. MILP can assign tasks that look infeasible, due to timing, by adding time to a UAV's path, and vehicle paths are then recalculated to match the required arrival times. The MILP formulation with variable arrival times provides an optimal solution to multiple-assignment problems for groups of UAVs with coupled tasks involving timing and task order constraints.

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UAV Task Assignment with Timing Constraints

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Abstract

This paper addresses the problem of task allocation for wide area search munitions. The munitions are required to search for, classify, attack, and verify the destruction of potential targets. We assume that target field information is communicated between all elements of the swarm. We generate a tour of optimal assignments for each vehicle using a Mixed Integer Linear Program, or "MILP" format. MILP can assign tasks that look infeasible, due to timing, by adding time to a UAV's path, and vehicle paths are then recalculated to match the required arrival times. The MILP formulation with variable arrival times provides an optimal solution to multiple-assignment problems for groups of UAV's with coupled tasks involving timing and task order constraints.

1 Introduction

Autonomous wide area search munitions (WASM) are small, powered air vehicles, each with a turbojet engine and sufficient fuel to fly for a short period of time. They are deployed in groups, or "swarms," from larger aircraft flying at higher altitudes. They are individually capable of searching for, recognizing, and attacking targets. Cooperation between munitions has the potential to greatly improve their effectiveness in many situations. The ability to communicate target information to one another will greatly improve the capability of future search munitions.

In [1], a time-phased network optimization model was used to perform task allocation for a group of powered munitions. The model is run simultaneously on all munitions at discrete points in time, and assigns each vehicle one or more tasks each time it is run. The model is solved each time new information is brought into the system, typically because a new target has been discovered or an already-known target's status has been changed. The network optimization model is run iteratively so that all of the known targets will be completely serviced by the resulting allocation. Classification, attack, and battle damage assessment tasks can all be assigned to different vehicles when a target is found, resulting in the target being more quickly serviced. A single vehicle can also be given multiple task assignments to be performed in succession, if that is more efficient than having multiple vehicles perform the tasks individually. In [2], variable path lengths are added to guarantee that feasible trajectories will be calculated for all tasks. This method is computationally efficient and can quickly assign all of the needed tasks to the available vehicles, however the iterative procedure is heuristic and does not guarantee that the solution is near optimal.

This paper proposes an optimal formulation for solving the coupled multiple-assignment problem. Formulating the problem in a Mixed Integer Linear Program, or MILP format, will allow the optimal solution to be found. MILP can assign tasks that look infeasible, due to timing, by adding time to a UAV's path. This allows all the tasks to be assigned giving an optimal solution. Solution times for non-trivial problems are much longer than for the heuristic iterative method used in [2], but the optimal solution can be used to evaluate the heuristically based solutions. This will help determine the value of solutions from other assignment procedures. The solution to this

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formulation will likely require too much computation and time for real-time use in a vehicle, except for extremely simple cases, but it can be used as a benchmarking and comparison tool to evaluate faster but non-optimal assignment methodologies. Details of the path-planning and path-lengthening algorithms used in this work can be found in [2,3].

2 MILP Model

The MILP model uses a discrete approximation of the real world based on nodes that represent discrete start and end positions for segments of a UAVs path. Nodes representing target positions range from 1…n and nodes for UAV positions range from 1+n…w+n. There is also an additional logical node for the sink n+w+1. The sink node is used when a UAV is not assigned to attack a target; it goes to the sink when it is done with all of its tasks. When a UAV enters the sink it is then used for searching the battlefield. The MILP model requires the costs or times for a UAV to fly from one node to another node. These flight times are constants represented by t_{vij}, the time it takes UAV v to fly from node i to node j. The flight times are positive real numbers, $t_{vij} \geq 0$.

In this MILP model the variable representations are as follows:

- n - the number of targets
- w - the number of UAVs
  - $w \geq n + 1$ - One or more UAVs than targets required in the model.
- v = 1…w - UAVs index
- k = 1,2,3 - Task index
  - 1 – Classify
    - Identify if the target is actually an eligible target.
  - 2 – Attack
    - Attempt to destroy the target. Attacks are terminal events.
  - 3 – Verification (Battle Damage Assessment)
    - Did we actually destroy the target?
- i = 1…n, n+v - Initial nodes for UAVs
  - A UAV can start at any target node, 1…n, and only its original start node n+v. It would not make sense for a UAV to start at any other UAVs start node because there are no tasks at start nodes.
- j = 1…n - Target nodes
- $k \neq 2 \Rightarrow i \neq j$ - If k does not equal an attack task node i cannot equal node j.
  - The nodes represent start and end positions for a segment of a UAVs path. If a UAV were attacking a target it just classified it would fly from a node to the same node to do the attack. This allows a UAV to do a classify/attack task at a single target. During a classify or VERIFICATION task the UAV would never fly from a node to the same node, hence the restriction on i and j.

- $x_{vkij} = \{0,1\}$ – Binary Decision Variables
  - If UAV v is assigned to do task k fling from node i to node j,
  - Then $x_{vkij} = 1$, otherwise it is zero
- $x_{vi,n+w+1} = \{0,1\}$ – Binary Sink Variables
  - If UAV v is assigned to go to the sink from node i
  - Then $x_{vi,n+w+1} = 1$, otherwise it is zero
- $t_{kj} = R^{+1}$ – Continuous Timing Variables
  - The time task k is performed on target j

3 MILP Formulation

The basic MILP formulation consists of three main parts: an optimization function, upper and lower bounds on all variables, and constraints using the variables. The variables are binary integer decision variables or continuous timing variables. The optimization function is the total time for all UAVs to do all tasks.

$$y = \sum_{v=1}^{w} \sum_{k=1}^{3} \sum_{\substack{i=1 \\ k \neq 2 \Rightarrow i \neq j}}^{1,\ldots,n,n+v} \sum_{j=1}^{n} \left(t_{vij} * x_{vkij} \right)$$

As described above t_{vij} is a constant describing the time for a UAV v to fly from node i to node j, this is multiplied by the binary decision variable x_{vkij}. Also from above x_{vkij} is either 0 or 1, if a UAV is flying from i to j and performing task k at j then the x_{vkij} will be 1 and the constant time multiplied by 1, will be added to the total time optimization function. If the UAV is not doing a task the x_{vkij} will be 0 and the constant t_{vij} will be multiplied by 0, which removes that time from the optimization function. There are other options for the optimization function such as maximizing the benefits, which would be similar to the benefit function used in [X].

The lower and upper bounds on variables describe the range of valid values for each variable. The ranges for the variables are described in Section 2. For example, the binary decision variables can take the values 0 or 1, and the continuous timing variables can take the values of the positive real numbers.

The formulation of the MILP is primarily based on the constraints. The constraints use the variables to make equations that describe the problem. For this research the constraints are as follows:

1. All tasks must be performed on all targets. This constraint is related to requiring, $w \geq n + 1$ or, that the number of UAVs is 1 more than the number of targets. If less then or equal number of UAVs to targets is used this constraint will fail and no solution will be found.

$$\sum_{v=1}^{w} \sum_{\substack{i=1 \\ k \neq 2 \Rightarrow i \neq j}}^{1,\ldots,n,n+v} x_{vkij} = 1 \quad \begin{matrix} k = 1,2,3 \\ j = 1,\ldots,n \end{matrix}$$

2. A UAV can only visit a target twice, to prevent looping. The UAV can do a classify task and then an attack task or a classify task and a verification task or a single one of these tasks at any target. But keeping in mind the attack task is a terminal event and that the attack has to occur before a verification can happen. A UAV does not have to visit a target any times so the times visited can be less then or equal to two or greater than or equal to zero.

$$\sum_{k=1}^{3} \sum_{\substack{i=1 \\ k \neq 2 \Rightarrow i \neq j}}^{1,\ldots,n,n+v} x_{vkij} \leq 2 \quad \begin{matrix} v = 1,\ldots,w \\ j = 1,\ldots,n \end{matrix}$$

3. A UAV can only enter the sink once. When a UAV is in the sink it is searching the battlefield for new targets, but it can't stop searching unless a reassignment occurs. This prevents inconsistencies with a UAV entering the sink and then immediately leaving the sink at a different target.

$$\sum_{i=1}^{1,\ldots,n,n+v} x_{vi,(n+w+1)} \leq 1 \quad v = 1,\ldots,w$$

4. If a UAV is doing a verification on a target it can't be assigned to also attack that target. The verification must occur after the attack at a target, and the attack is a terminal event. It is impossible for a UAV to attack a target and then to verification that target after it has attacked.

$$\sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{1,\ldots,n,n+v} x_{v2ij} \leq 1 - \sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{1,\ldots,n,n+v} x_{v3ij} \quad \begin{matrix} v=1,\ldots,w \\ j=1,\ldots,n \end{matrix}$$

5. If a UAV attacks a target it cannot attack any other target. These vehicles are munitions and are destroyed in an attack. The UAVs do not have to attack a target if it goes to the sink it is searching and can't leave the sink until a reassignment occurs. The UAV can do less then or equal to one attack.

$$\sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{1,\ldots,n,n+v} \sum_{j=1}^{n} x_{v2ij} \leq 1 \quad v=1,\ldots,w$$

6. If a UAV enters a target to perform task k = 1 or 3 it must also exit that target. Targets have flow in and out for keeping track of UAVs. If a UAV performs the classify or verification task at a target it must leave the target when it is done to do another task or go to the sink. If a UAV does an attack at a target it can't leave because an attack is a terminal event.

$$\sum_{k=1,3} \sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{1,\ldots,n,n+v} x_{vkij} = \sum_{k=1,3} \sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{n} x_{vkji} + \sum_{i=1}^{n} x_{v2ji} + x_{vj,(n+w+1)} \quad \begin{matrix} j=1,\ldots,n \\ k=1,2,3 \end{matrix}$$

7. If a UAV attacks a target it cannot be sent to any other targets to do any other tasks. This constraint is similar to constraint six. The attack is a terminal event and a UAV can't do anything after it attacks a target.

$$\sum_{k=1}^{3} \sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{n} x_{vkji} \leq 1 - \sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{1,\ldots,n,n+v} x_{v2ij} \quad \begin{matrix} v=1,\ldots,w \\ j=1,\ldots,n \end{matrix}$$

8. All UAVs must be assigned to leave the source node. A UAV has to do something either go to the sink immediately and search the battle field or do tasks that lead to an attack or go to the sink when the UAV is done with all of its non-attack tasks.

$$\sum_{k=1}^{3} \sum_{j=1}^{n} x_{vk(n+v)j} + x_{v(n+v)(n+w+1)} \quad v=1,\ldots,w$$

9. A UAV cannot do tasks if it goes to the sink from its start node. This helps prevent looping. A UAV could be assigned to go to the sink from its start node and to do a non-attack task at several targets.

$$\begin{matrix} x_{vkij} \leq 1 - x_{v(v+n)(n+w+1)} \\ v=1,\ldots,w \quad k=1,3 \quad i=1,\ldots,n \quad j=1,\ldots,n \\ i=1,\ldots,n \quad j=1,\ldots,n \quad i\neq j \end{matrix}$$

10. The segments of a UAVs path cannot be greater than the UAVs Endurance. Again the t_{vij} is multiplied by the binary decision variable x_{vkij} to determine if the constant value is added to the total UAV path.

$$T_v = \text{UAV Endurance}$$

$$\sum_{k=1}^{3} \sum_{\substack{i=1 \\ k\neq 2 \Rightarrow i\neq j}}^{1,\ldots,n,n+v} \sum_{j=1}^{n} \left(t_{vij} * x_{vkij}\right) \leq T_v \quad \begin{matrix} v=1,\ldots,w \\ T_v > 0 \end{matrix}$$

11. Timing for tasks performed after the UAV leaves the start node. These constraints only represent the first leg of a UAVs path after it leaves the start node.

$$\begin{aligned} t_{kj} &\le t_{1i} + t_{vij} + \left(1 - x_{vkij}\right) * T_v \\ t_{kj} &\le t_{3i} + t_{vij} + \left(1 - x_{vkij}\right) * T_v \\ t_{kj} &\ge t_{1i} + t_{vij} - \left(1 - x_{vkij}\right) * T_v \\ t_{kj} &\ge t_{3i} + t_{vij} - \left(1 - x_{vkij}\right) * T_v \end{aligned} \qquad \begin{aligned} v &= 1,\ldots,w \\ k &= 1,2,3 \\ i &= n+1,\ldots,n+w \\ j &= 1,\ldots,n \\ i &\ne j \end{aligned}$$

12. Timing for tasks performed on subsequent targets. These are the most complex attempt at coupling the paths of the UAVs from the first segment to the subsequent segments. These are soft constraints which have no effect, except when vehicle v is flying from node I to node j to perform task k – in that case the upper and lower bounds meet, enforcing an equality constraint on the timing for the task.

$$\begin{aligned} t_{kj} &\le t_{\{1,3\}i} + t_{vij} * \left(1 + \frac{1 - \sum_{\substack{l=1 \\ k \ne 2 \Rightarrow i \ne j}}^{1,\ldots,n,n+v} x_{v\{1,3\}li}}{\varepsilon} \right) + \left(1 - x_{vkij}\right) * T_v \\ t_{kj} &\ge t_{\{1,3\}i} + t_{vij} * \left(1 - \frac{1 - \sum_{\substack{l=1 \\ k \ne 2 \Rightarrow i \ne j}}^{1,\ldots,n,n+v} x_{v\{1,3\}li}}{\varepsilon} \right) - \left(1 - x_{vkij}\right) * T_v \end{aligned} \qquad \begin{aligned} v &= 1,\ldots,w \\ k &= 1,2,3 \\ i &= n+1,\ldots,n+w \\ j &= 1,\ldots,n \\ i &\ne j \end{aligned}$$

13. The time a classify task is performed on a target is ≤ the time an attack task is performed on a target which is ≤ the time the VERIFICATION is performed on a target. This constraint also prevents the VERIFICATION from happening before the classify and attack or the attack happening before the classify.

$$\begin{aligned} t_{1j} &\le t_{2j} \\ t_{2j} &\le t_{3j} \end{aligned} \quad j = 1,\ldots,n$$

Using these constraints the MILP solver can find an optimal assignment for the tasks that are to be done on a set of targets, with the timing variables also telling us if any of the pre-computed paths need to be lengthened.

4 Simulation Results

The iterative network flow task assignment methodology described above has been implemented in our multi-vehicle, multi-target coordinated-control simulation. The scenario has eight Wide Area Search Munitions performing a search for targets in a rectangular area. The WASM are using a simple "mowing the grass" search pattern. There are up to 5 different target

types possible in the simulation, including a "non-target" target type for objects that appear similar to targets but which may be distinguishable as non-targets by the ATR.

For the simulation results presented, eight vehicles are searching an area containing two targets. The targets have an orientation (facing) that has an impact on the ATR process and desired viewing angles, but this will not be discussed as it does not directly affect the task allocation. The search vehicles are initialized in a staggered row formation, with fifteen minutes of flight time remaining, out of a maximum thirty minutes. This assumes that the vehicles have been searching for fifteen minutes and then find a cluster of potential targets.

Figure 1 shows vehicle flight paths and target locations with variable-length paths. Targets are the grey objects numbered 1 and 2. UAV's are represented by small colored numbers and colored flight paths. The colored rectangles are vehicle sensor footprints. Whenever the minimum-length path does not satisfy the timing constraints, a new path that satisfies the constraints, and is near the minimum possible path length that satisfies the constraints, is calculated for each vehicle. All of the tasks assigned and completed successfully. There is, however, no guarantee that the method has produced a near-optimal result. The method does not give any quantification of how close to optimal the assignment might be.

5 Conclusions

We have presented a method for using a Mixed Integer Linear Program (MILP) formulation to find the optimal solution to a multipe-task assignment problem where the tasks are coupled by timing, precedence, and task order constraints. This formulation allows variation of vehicle flight paths to guarantee that timing constraints are satisfied, and directly incorporates the varying task completion times into the optimization. This is a promising formulation, which allows a true optimal solution for a very challenging problem. The MILP formulation results in a large optimization problem with many constraints that is not amenable to real-time solution, but it is usable for off-line assignment calculations, and for determining optimal solutions to evaluate the performance of more computationally tractable sub-optimal methods.

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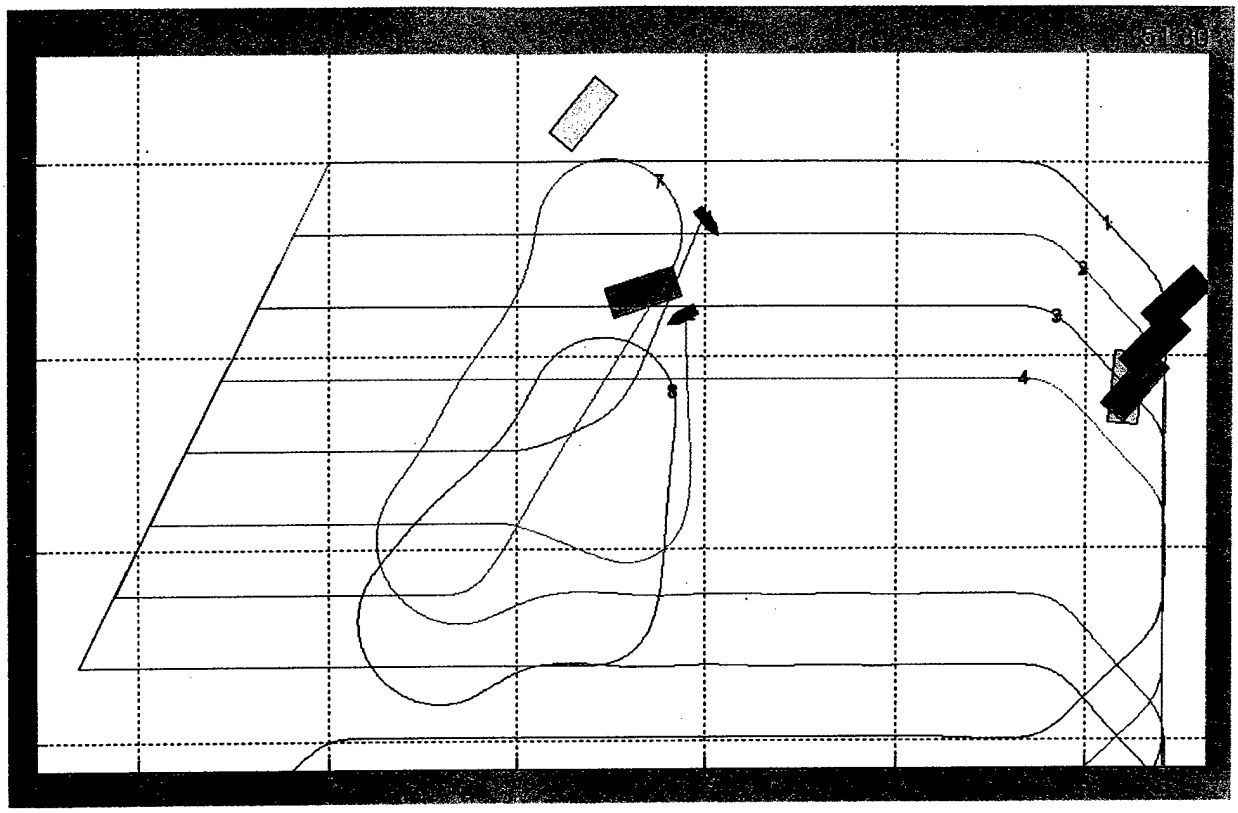

Figure 1 - Vehicle Paths and Target Locations with iterative network flow assignment