CHAPTER 8

RELIABILITY TEST PLANNING

INTRODUCTION

This chapter presents the techniques for determining the amount of test exposure required to satisfy previously established program reliability requirements. The reader will note that Chapter 7 addresses the topic of reliability data analysis. There, we assumed that the test data had already been gathered. We then used the available data to determine point estimates for reliability parameters and to stipulate the uncertainty associated with these estimates.

Chapter 8 presents techniques for designing test plans which can verify that previously specified reliability requirements have been achieved. We realize, of course, that the required test exposure and/or sample size may exceed the available resources. In such cases, alternative test plans, consistent with program constraints, must be developed. In this chapter, we also present methods which make it possible to clearly identify the inherent risks associated with a limited test program.

PRIMARY TEST DESIGN PARAMETERS

Upper and Lower Test Values

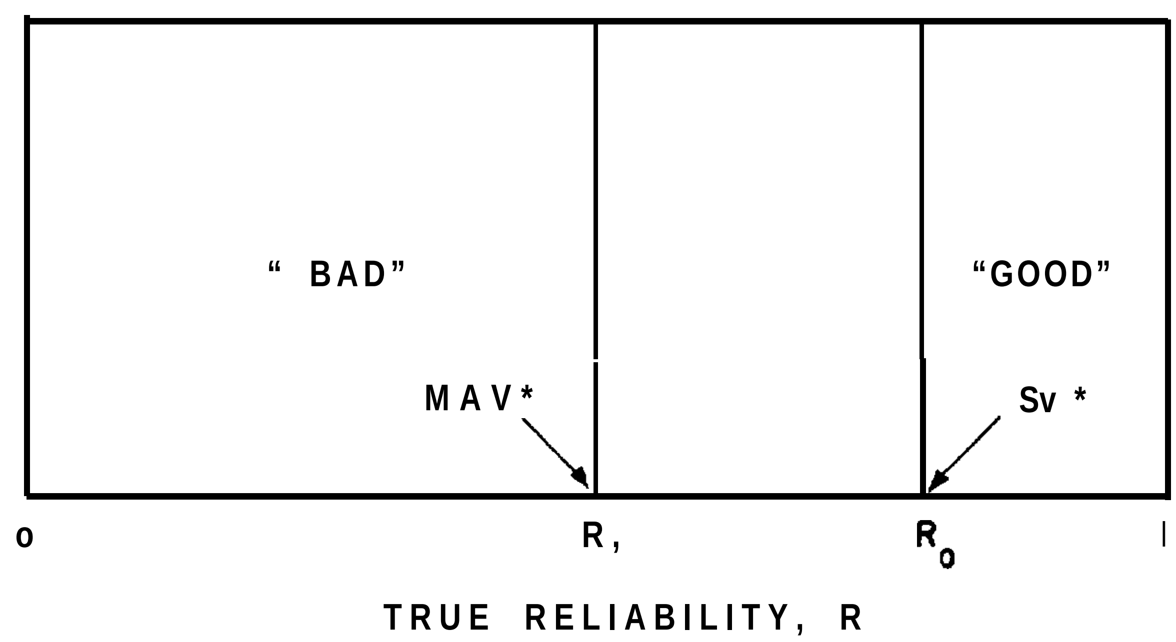
Two values of system reliability are of particular importance in the design of a reliability test plan. These are referred to as the upper test and lower test values. In some cases, only a single value is initially apparent, the second value being only implied. These two values and the risks associated with them determine the type and magnitude of testing required.

The upper test value is the hoped for value of the reliability measure. An upper test MTBF is symbolized as 6., and an upper test reliability is symbolized as R_o. A test plan is designed so that test systems whose true reliability parameters exceed θ_0 and R_o will, with high probability, perform during the test in such a way as to be "accepted."

The lower test value is commonly interpreted in two different ways that may initially appear contradictory. One interpretation is that this lower value of the reliability measure represents a rejection limit. The other interpretation is that this value is minimally acceptable. The apparent conflict is resolved by viewing the lower test value as the fine line between the best rejectable value and the worst acceptable value. A lower test MTBF is symbolized as θ_1, and a lower test reliability is symbolized as R_1. Systems whose true reliability parameters having values less than θ_1 and R_1 will, with high probability, perform in such a way as to be "rejected."

The upper and lower test values serve to divide the reliability, or MTBF, scale into three distinct regions as shown in Figure 8-1. Note that the region between R_1 and R_o is neither bad enough to demand rejection nor is it good enough to demand acceptance. This region is necessary since we will never precisely know the true reliability of the system.

FIGURE 8-1 REGIONS DEFINED BY R_o AND R_1



*MAY BE DIFFERENT DEPENDING ON ACTUAL MATURITY. SEE FOLLOWING PARAGRAPH.

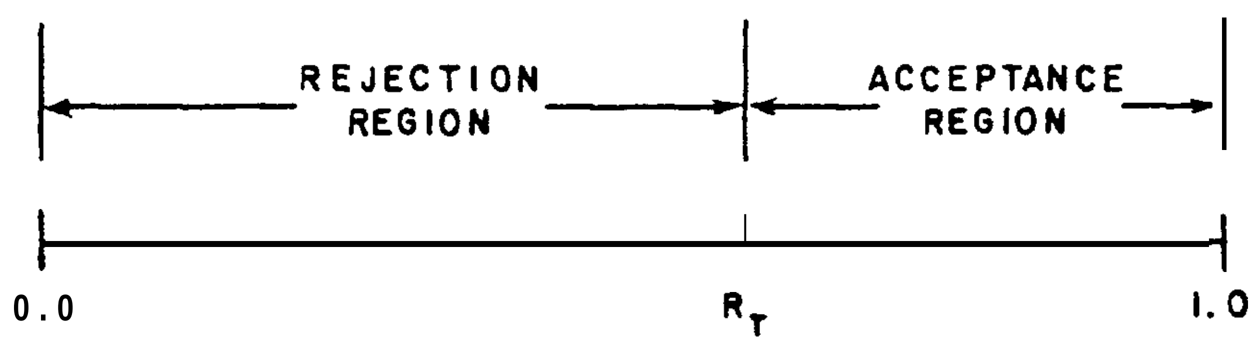
The user's reliability requirement should be stated as a minimum acceptable value (MAV); that is, the worst level of reliability that the user can tolerate and accept. The contractually specified value (SV) is a value somewhat higher than the MAV. For reliability qualification tests prior to production, the lower test value is the MAV, and the upper test value is the SV. Earlier in the development process, fixed configuration tests may be conducted to demonstrate the attainment of lower levels of reliability at specified milestones. In such cases, upper test and lower test values should be consistent with the stage of the development process.

In the above paragraphs, we have been discussing population parameter values only. These values are never known with absolute certainty, so we are forced to base an evaluation of system performance characteristics on sample data. Let us conclude this section with a discussion of sample reliability values and how we can interpret them to aid us in making our system reliability assessment.

One objective of this chapter is the determination of an accept/reject criterion for a test to be conducted. As an example, consider the value R_T in Figure 8-2 below. The term R_T is that value of the <u>sample</u> reliability which corresponds to the maximum allowable number of failures that can occur during testing and still result in acceptance of the system.

If we test our determined number of articles and find that R_{sample} is larger than R_T, then we accept the system because there is high probability that the

FIGURE 8-2 SAMPLE RELIABILITY RANGE



sample system(s) come from a population of systems whose true reliability R exceeds R_1, the minimum acceptable value (MAV) (see Figure 8-1) for this test. Note that when R_{sample} is larger than R_T, we have confidence that the true reliability exceeds the MAV. We should not interpret this result as an indication that the contractor has met the SV. Further, if R_{sample} is smaller than R_T, we will reject the system because there is high probability that the sample system(s) come from a population whose true reliability R is lower than R_0, the SV for this test. Note that when R_{sample} is smaller than R_T, we have confidence that the true reliability falls short of the SV. We should not interpret this result as an indication that the MAV has not been met, but rather that the MAV has not been demonstrated at a sufficiently high level of confidence.

Consumer's Risk (β) and Producer's Risk (α)

The consumer's risk (β) is the probability of accepting the system if the true value of the system reliability measure is less than the lower test value. It can be interpreted in the following ways:

1. β represents the maximum risk that the true value of the reliability measure is, in fact, less than the lower test value.

2. From an alternative viewpoint, if the acceptance criterion is met, there will be at least 100(1-β)% confidence that the true value of the reliability measure equals or exceeds the lower test value.

The producer's risk (α) is the probability of rejection if the true value of the reliability measure is greater than the upper test value. It can be interpreted in the following ways:

1. The probability of acceptance will be at least (1-α) if the upper test value is, in fact, met or exceeded.

2. From an alternative viewpoint, if there is a rejection decision, there will be at least 100(1-α)% confidence that the true value of the reliability measure is less than the upper test value.

Case study 8-1 illustrates the relationship between α and β.

Pre- and Post-Test Risk Considerations

Before proceeding on with the application of the consumer's and producer's risk concept, it is important to understand the contrast that exists between pre-and post-test risks.

The α and β risks represent uncertainties that exist in the test planning or pre-data environment discussed in this chapter. Once data has been gathered and we have accepted or rejected the system, we find that the risk environment is altered. For example, we take a sample and decide to accept the system. At this point the producer's risk is eliminated; the consumer's risk remains but is less than the maximum that would exist had the sample reliability, R_{sample}, been exactly equal to R_T.

If, on the other hand, R_{sample} is less than R_T, i.e., we reject the system, we find that the consumer's risk is eliminated since there is no risk of accepting a bad system. Likewise, the producer's risk is less than the maximum that would exist had the sample reliability R_{sample} been exactly equal to R_T.

In this chapter, we are concerned with pre-test risks. We determine the maximum α and β risks and then calculate the required test exposure and acceptable number of failures which will limit our risk to the maximum levels.

TEST DESIGN FOR DISCRETE TIME TESTING : BINOMIAL MODEL

Four values specify the plan for a binomial test. They are:

- the specified or desired proportion of failures (p_0),
- the maximum acceptable proportion of failures (p_1),
- the consumer's risk (β),
- the producer's risk (α).

The test plan itself consists of a sample size (n) and an acceptance criterion (c). The value c represents the maximum number of failures which still results in acceptance of the system. It is usually not possible to construct a plan which attains the exact values of α and β. There are however plans which attain risks which do not exceed α and β. We shall present methods for determining these types of plans, though in a real world situation, the user and producer may trade off some protection to achieve other goals.

The following paragraphs present exact and approximate procedures to be used in planning a Discrete Time-Binomial Model test program. The "exact procedure" presents the equations used to determine the two values required to specify a binomial test plan. These equations are presented here for the sake of completeness. The "approximate solution" procedure, which makes use of the binomial tables to simplify the procedure, is intended for use by our readers.

Exact Solution Procedure

The exact procedure for determining test plans for the four values listed above is to solve the following two inequalities simultaneously for c and n.

$$\sum_{k=0}^{c} \binom{n}{k} p_1^k (1-p_1)^{n-k} \leq \beta \tag{8.1}$$

$$\sum_{k=c+1}^{n} \binom{n}{k} p_0^k (1-p_0)^{n-k} \leq \alpha \tag{8.2}$$

There are an infinite number of solutions to this pair of inequalities. The plans of interest are, of course, those which minimize the sample size (n) required. Solving inequalities 8.1 and 8.2 directly is next to impossible without the aid of a computer. MIL-STD-105D contains numerous binomial test plans which may be used for reliability applications. We should point out that the user unfamiliar with this document will find it difficult to interpret, thus we present the following procedures.

Approximate Solution Procedures

The following so-called approximate procedures utilize the normal and Poisson distributions to obtain approximate solutions to equations 8.1 and 8.2 and thereby estimate values of the sample size (n) and the acceptance criterion (c). After approximate values for these parameters have been obtained, we may then use the values in conjunction with the binomial tables (Appendix B) and the previously selected and fixed values of α and β to "fine tune" the approximate values of n and c.

Test Planning Using Normal Approximation. The normal distribution provides good approximations for solving inequalities 8.1 and 8.2, especially for moderate values of p ($0.1 \leq p \leq 0.9$). Using this information, we obtain the approximate solutions for n and c as follows.

$$n = \frac{z_\alpha^2(p_0-p_0^2) + z_\beta^2(p_1-p_1^2) + 2z_\alpha z \sqrt{p_0p_1(1-p_0)(1-p_1)}}{(p_1-p_0)^2}, \tag{8.3}$$

$$c = z_\alpha \sqrt{np_0(1-p_0)} + np_0 - 0.5 . \tag{8.4}$$

Generally, the values computed using equations 8.3 and 8.4 are good approximations for the test plainer. When p_0 and p_1 are very small (less than 0.05), the procedure is not recommended. Fine-tuning of the test plan may still require solving the original inequalities or some bargaining with user and/or producer.

As an example, suppose that the minimum acceptable reliability of a system is 0.85 ($p_1 = 0.15$), while the contractually specified reliability is 0.95 (P. =

0.05). Consumer and producer risks of 0.11 are required, i.e., $\alpha = \beta = 0.11$. For $\alpha = 0.11$, $z_\alpha = 1.225$ and for $\beta = 0.11$, $z_\beta = 1.225$. (These values of z_α and z_β are obtained from Appendix B, Table 2.) Using the normal approximation, we have

$$n = \{(1.225)^2(0.05-0.0025) + (1.225)^2(0.15-0.0225)$$

$$+ 2(1.225)^2\sqrt{(0.05)(0.15)(0.95)(0.85)}\}/(0.15-0.05)^2$$

$$= 49.6$$

and

$$c = 1.225\sqrt{(49.6)(0.05)(0.95)} + (49.6)(0.05) - 0.5$$

$$= 3.9 .$$

The values of $n = 49.6$ and $c = 3.9$ are initial approximations. In order to fine tune the test plan, we round these values to $n = 50$ and $c = 4$ and use the binomial tables (Appendix B, Table 1). For an n of 50 and a c of 4, the probability of c or fewer failures when $p = p_1 = 0.15$ is 0.1121. In addition, the probability of c or fewer failures when $p = p_0 = 0.05$ is 0.8964. Thus, for the test using a sample size of 50 with a maximum acceptable number of failures of 4, the producer's risk $\alpha = 1 - 0.8964 = 0.1036$, and the consumer's risk $\beta = 0.1121$. Note that these values were obtained directly from Appendix B, Table 1. It would, however, have been difficult at best to decide where to begin looking in the binomial tables without having first used the normal approximation for guidance.

<u>Test Planning Using Poisson Approximation</u>. The Poisson distribution also provides reasonable approximations to inequalities 8.1 and 8.2. All this amounts to is substituting np for At or t/θ in the Poisson distribution equation. Consequently, approximate values for n and c are obtained by solving the following inequalities.

$$\sum_{k=0}^{c} \frac{(np_1)^k e^{-np_1}}{k!} \leq \beta. \tag{8.5}$$

$$\sum_{k=0}^{c} \frac{(np_0)^k e^{-np_0}}{k!} \geq 1 - \alpha. \tag{8.6}$$

Standard test plans and procedures for the Poisson (exponential) are readily available and may be used in lieu of solving inequalities 8.5 and 8.6. This subject is discussed in the "Sources of Exponential Test Plans" section of this chapter. To use these plans in this context, we let $\theta_0 = 1/p_0$, $\theta_1 = 1/p_1$, $n = T$, and use the acceptable number of failures as given.

As an example, suppose that the minimum acceptable reliability of a system is 0.9 (p_1 = 0.1) and the contractually specified reliability is 0.95 (p_0 = 0.05). Consumer and producer risks are to be 20%, i.e., $\alpha = \beta = 0.20$. To use the Poisson approximation, we define $\theta_0 = 1/p_0 = 1/0.05 = 20$ and $\theta_1 = 1/p_1 = 1/0.1 = 10$. The discrimination ratio, θ_0/θ_1, is 2. Note that test plan XIVC in Appendix B, Table 6, has α and β risks of 19.9% and 21.0%, respectively. This plan requires a test duration T, corresponding to n for this example, of $(7.8)(\theta_1)$ or 78, with five or fewer failures being acceptable.

The term "discrimination ratio" and the use of Appendix B, Table 6, test plans are discussed in detail in the following section.

Case Studies 8-1, 8-2, and 8-3 demonstrate the development of binomial test plans for a variety of α and β values.

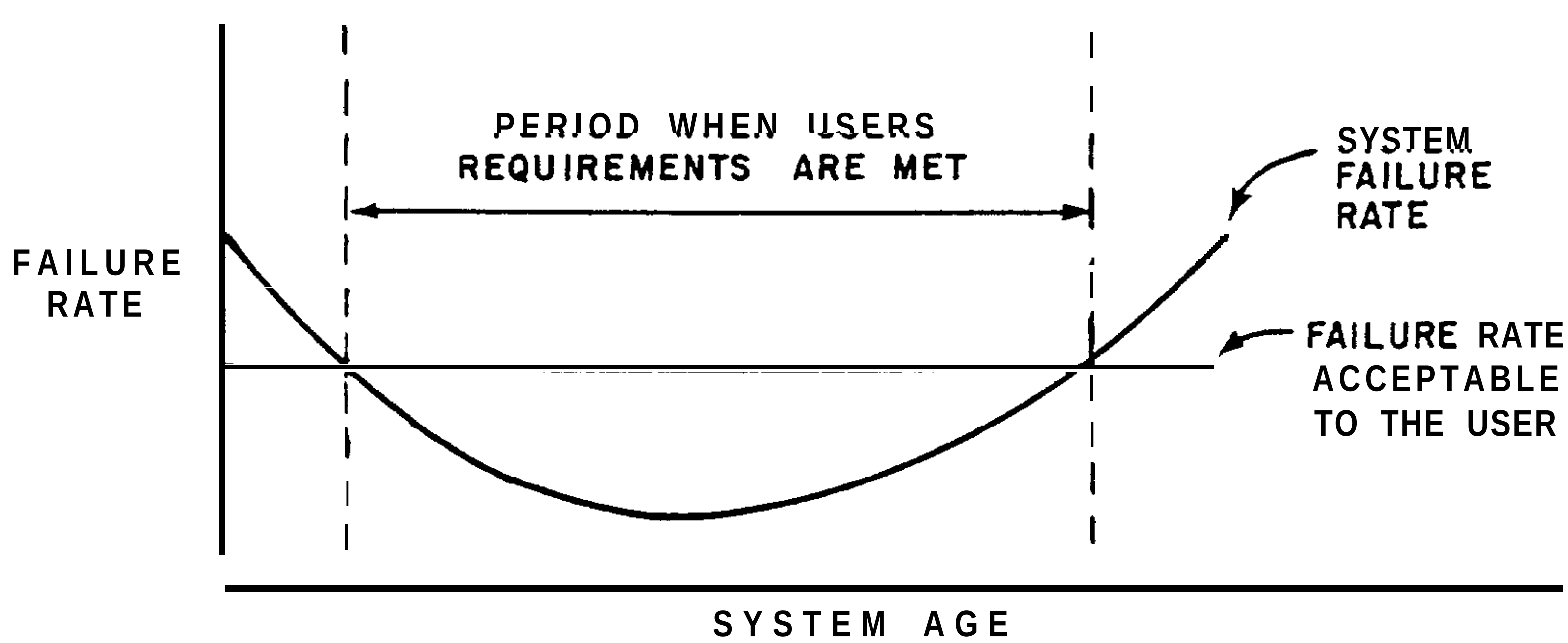
TEST DESIGN FOR CONTINUOUS TIME TESTING: EXPONENTIAL MODEL

The main feature of test planning for continuously operating systems based on the exponential distribution is the assumption that the systems have a constant failure rate.

Requirement Interpretation

When the user's requirement is stated in terms of an MTBF, there is an implication of a constant failure rate. This does not mean that the system must have a constant failure rate. It means, instead, that the need remains constant. Figure 8-3 illustrates that the user's needs may be met during only a portion of the time during the life of a system.

FIGURE 8-3 USER REQUIREMENTS vs SYSTEM PERFORMANCE



Constant System Failure Rate Assumption

The assumption that the system to be tested has a constant failure rate may not be a good one, but it is a practical necessity for determining the amount of testing required. In theory, with the constant failure rate assumption only the total test exposure is important. That is (in theory), one system could be tested for the required test exposure, or many systems could be tested for a short time.

In practice, a test should be planned with a moderate number of systems on test for a moderate period of time. This makes the test relatively insensitive to the constant failure rate assumption. For example, one organization recommends that at least three systems be tested for at least three times the MAV (each). These are constraints imbedded in the required total test exposure.

Discrimination Ratio

The discrimination ratio, $d = \theta_0/\theta_1$, is a parameter useful in test planning for the exponential model. (For the binomial model, it is necessary to consider the upper and lower test values, p_0 and p_1, explicitly along with the α and β risks.) An interesting feature of the exponential model is that only the ratio of the upper and lower test values, $d = \theta_0/\theta_1$, along with the α and β risks need to be considered. As a consequence, test plans for the exponential models address explicitly the discrimination ratio as a planning parameter.

Sources of Exponential Test Plans

There are numerous methods and references available for developing exponential test plans. Three such approaches are:

1. MIL-STD 105D and MIL-HBK 108.

2. MIL-STD 781C Test Plans.

3. Poisson Distribution Equations.

Reference to MIL-STD 105D and MIL-HBK 108 is included here solely for the sake of completeness. It is our intention that the reader become familiar with methods of exponential test planning using MIL-STD 781C and the Poisson distribution equations. These methods are described below. All the necessary excerps from MIL-STD 781C are provided in Appendix B, Tables 6 and 7.

MIL-STD 105D and MIL-HBK 108. MIL-STD 105D is a document devoted primarily to binomial and Poisson sampling plans, and as such, is mentioned in the previous section. The Poisson sampling plans may be used for continuous time reliability tests. MIL-HBK 108 is devoted to reliability testing based on the exponential distribution. However, it is limited in use for our purposes because it describes test plans for the situation when the test time per unit on test is preset and the number of units is determined. We iterate here that these documents are difficult to interpret, and as such, should only be used by a person familiar with their content.

MIL-STD 781C. The required excerpts from MIL-STD 781C are provided in Appendix B, Tables 6 and 7. Both tables provide information which enable the reader to design a test program which addresses established requirements. The following paragraphs detail the use of both tables.

Appendix B, Table 6: Exponential Test Plans for Standard Discrimination Ratios. This table presents information which supports the de-velopment of a test plan based on discrimination ratios of 1.5, 2.0, and 3.0. For each of these discrimination ratios, four test plans are provided which attain approximate α and β risks of 10% and 10%, 10% and 20%, 20% and 20%, and 30% and 30%. Figure 8-4, in conjunction with the following example problem, illustrates the use of Appendix B, Table 6.

FIGURE 8-4. HOW TO USE APPENDIX B, TABLE 6

1. Identify rows corresponding to the specified "d". (If not in Appendix B, Table 6, use Table 7 plans.)
2. Identify desired α and β risks.
3. Identify test plan number (for reference).
4. Identify test duration multiplier.
5. Determine total test time as θ_1 times multiplier.
6. Identify accept/reject criteria.

Test Plan	True Decision Risks α	True Decision Risks β	Discrimination Ratio θ_0/θ_1	Test Duration Multiplier (M) $T = M\theta_1$	Accept-Reject Failures: Reject (Equal or More)	Accept-Reject Failures: Accept (Equal or Less)
IXC*	12.0%	9.9%	1.5	45.0	37	36
XC	10.9%	21.4%	1.5	29.9	26	25
XIC	17.8%	22.4%	1.5	21.1	18	17
XIIC	9.6%	10.6%	2.0	18.8	14	13
XIVC	19.9%	21.0%	2.0	7.8	6	5
XVC	9.4%	9.9%	3.0	9.3	6	5
XVIC	10.9%	21.3%	3.0	5.4	4	3
XVIIC	17.5%	19.7%	3.0	4.3	3	2

*NOTE : C refers to Revision C of MIL-STD-781.

How To Use Appendix B, Table 6. As an example, suppose that the upper test MTBF is 900 hours and the lower test MTBF is 300, so that the discrimination ratio (d) is 900/300 = 3. Consumer and producer risks of approximately 10% for each are required. Now, as shown in Figure 8-4, test plan XVC has α and β risks of 9.4% and 9.9%, respectively, and the discrimination ratio is 3. Test plan XVC requires a test length (T) of 9.3 times the lower MTBF of 300, so T = (9.3)(300) = 2790 hours. The acceptance criterion is to accept with 5 or fewer failures and reject with 6 or more failures. Note that if the upper test MTBF had been 90 and the lower test MTBF had been 30, the same test plan is appropriate. However, in this situation the test duration (T) is (9.3)(30) or 279 hours, whereas the accept/reject criterion remains the same.

Case Study 8-4 is another example illustrating the use of this table.

Appendix B, Table 7: Supplemental Exponential Test Plans. This table presents information which supports the development of a test plan based on combinations of α and β risks of 10%, 20%, and 30%. Figure 8-5, in conjunction with the following example problem, illustrates the use of Appendix B, Table 7.

How to Use Appendix B, Table 7. Concerning Figure 8-5 and Appendix B, Table 7, note the following:

If the discrimination ratio, d, is not given exactly in the tables, going to the next lower value will give a conservative (i.e., longer) test time requirement.

Consider once again, the example where θ_0 = 900, θ_1 = 300, and the desired α and β risks are 10% each. Recall that the discrimination ratio was 3. To select a test plan from Figure 8-5, we search the column labeled as a 10% producer's risk to find the number closest to 3. In this case, test plan 10-6 has a discrimination ratio of 2.94. The test duration is (9.27)(300) or 2781 hours with 5 being the maximum acceptable number of failures. Note how this plan compares with test plan XVC which has the same acceptance criterion, requires 2790 hours of test time, and has a discrimination ratio of 3. Case Study 8-5 further illustrates the use of this table.

Graphical Representation of Test Planning Parameters. Figure 8-6 graphically illustrates the interaction between α and β risks and test length for three-commonly used discrimination ratios. The graphs do not provide complete test planning information since no acceptance criterion is specified. These curves are useful tools for conducting tradeoff analyses between risk levels and test length. Note that some of the specific test plans presented in Appendix B, Tables 6 and 7 are displayed on the curves, i.e., 30-7, 10-19, 20-7, etc.

To illustrate how the graphs may be used, consider that θ_1 = 100 hours, θ_0 = 150 hours, and a test duration of 2500 hours is affordable. To enter the

FIGURE 8-5. HOW TO USE APPENDIX B, TABLE 7

1. Identify desired β risk (10% β risk table shown below).
2. Identify column for desired α risk.
3. Identify row for d.
4. Identify test plan number (for reference).
5. Identify total test time multiplier.
6. Determine total test time as θ_1 times multiplier.
7. Identify accept/reject criteria.

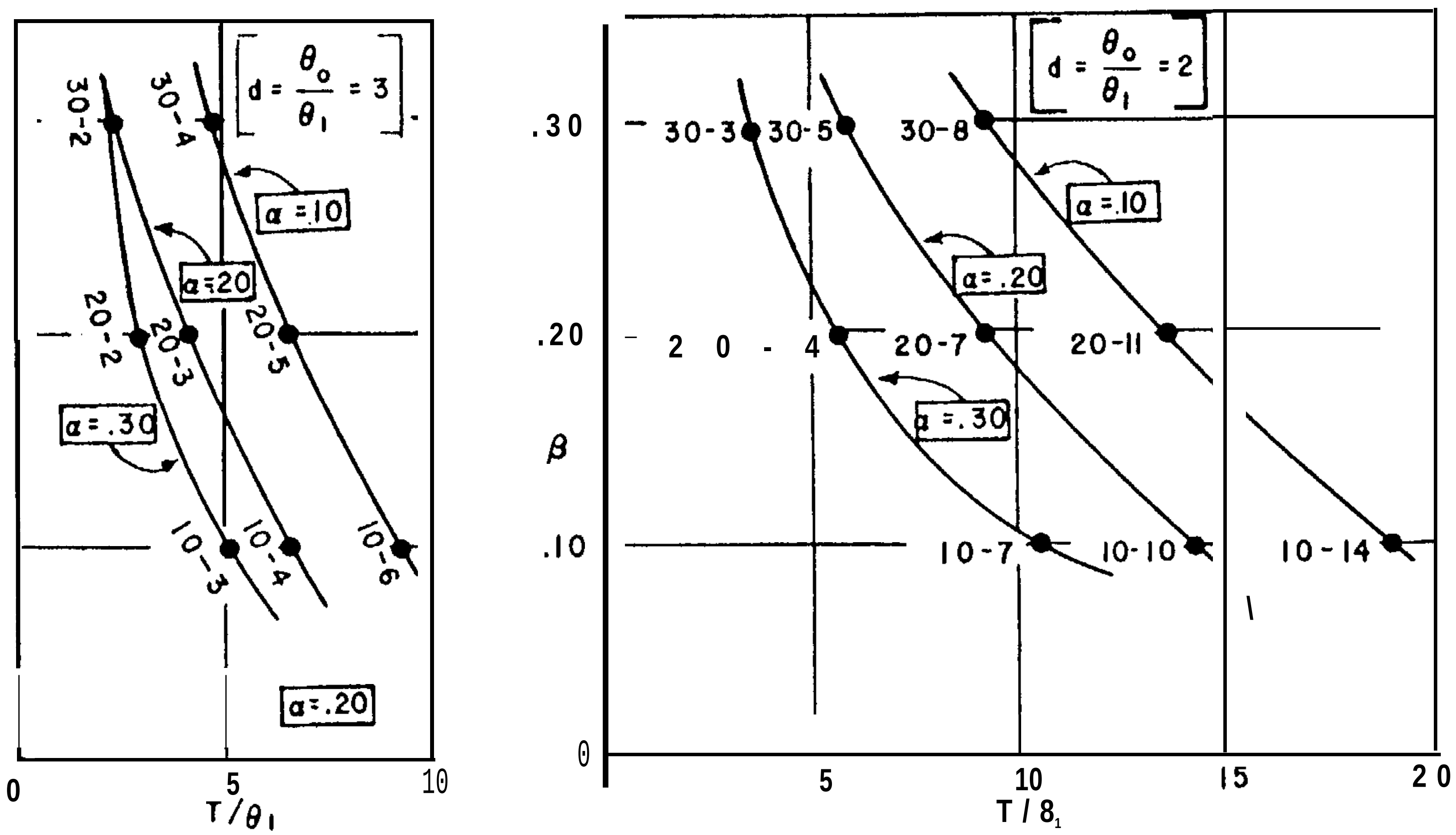
Test Plan No's	No. Failures Acc.	Rej.	Test Duration Multiplier (M) $T = M\theta_1$	Discrimination Ratio θ_0/θ_1 For Producer's Risk 30%	20%	10%
10-1	0	1	2.30	6.46	10.32	21.85
10-2	1	2	3.89	3.54	4.72	7.32
10-3	2	3	5.32	2.78	3.47	4.83
10-4	3	4	6.68	2.42	2.91	3.83
10-5	4	5	7.99	2.20	2.59	3.29
10-6	5	6	9.27	2.05	2.38	2.94
10-7	6	7	10.53	1.95	2.22	2.70
10-8	7	8	11.77	1.86	2.11	2.53
10-9	8	9	12.99	1.80	2.02	2.39
10-10	9	10	14.21	1.75	1.95	2.28
10-11	10	11	15.41	1.70	1.89	2.19
10-12	11	12	16.60	1.66	1.84	2.12
10-13	12	13	17.78	1.63	1.79	2.06
10-14	13	14	18.96	1.60	1.75	2.00
10-15	14	15	20.13	1.58	1.72	1.95
10-16	15	16	21.29	1.56	1.69	1.91
10-17	16	17	22.45	1.54	1.67	1.87
10-18	17	18	23.61	1.52	1.62	1.84
10-19	18	19	24.75	1.50	1.62	1.81
10-20	19	20	25.90	1.48	1.60	1.78

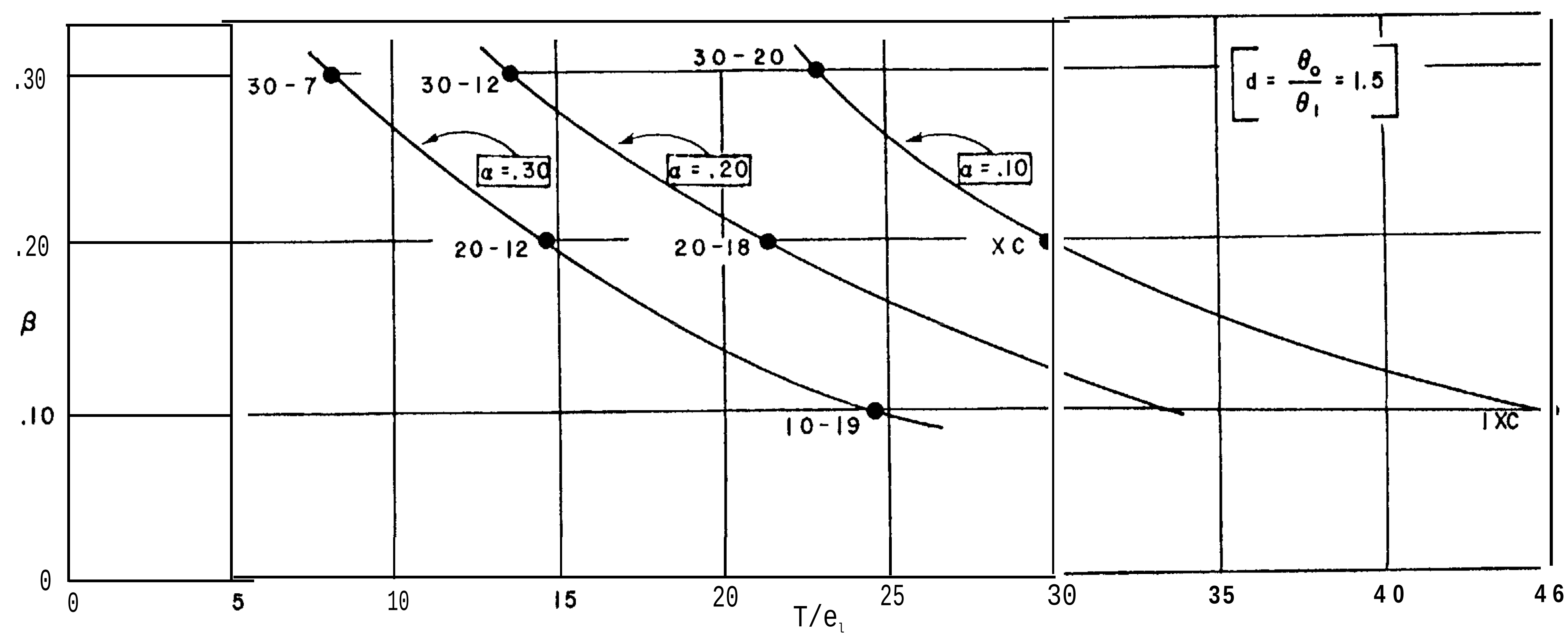
FIGURE 8-6

GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS

T = TOTAL TEST EXPOSURE

NOTE :
PLOTTING POINTS ARE CODED TO INDICATE THE MIL-STD 781C TEST PLAN
THESE PLANS ARE CONTAINED IN APPENDIX B, TABLES 6 & 7.





graphs, use $d = \theta_0/\theta_1 = 150/100 = 1.5$ and $T/\theta_1 = 2500/100 = 25$. Reading up through the three curves , we find the following risk combinations:

$\alpha = 0.30 \qquad \beta = 0.10$

$\alpha = 0.20 \qquad \beta = 0.16$

$\alpha = 0.10 \qquad \beta = 0.26$

If one of these combinations is tolerable, the test length is adequate. To reduce one or both risks, the test duration must be increased. Tolerating greater risks permits reduction of the test duration. Case Study 8-6 further illustrates the use of these graphs.

Figure 8-7 is a graphical portrayal of the interaction between test length and risk when α and β risks are equal. Curves for each of the three values (1.5, 2.0, 3.0) of the discrimination ratio appear on the same graph. Case Study 8-6 illustrates the use of Figure 8-7.

Poisson Distribution Equations. When a standard test plan for a specific combination of $\theta_0, \theta_1, \alpha$, and β is not available, the test designer may use the Poisson equations to develop a test plan.

The following notation is used in the discussion of the Poisson equation technique.

T = Total test exposure

θ = True MTBF

c = Maximum acceptable number of failures

θ_0 = Upper test MTBF

$'_1$ = Lower test MTBF

α = Producer's risk

β = Consumer's risk

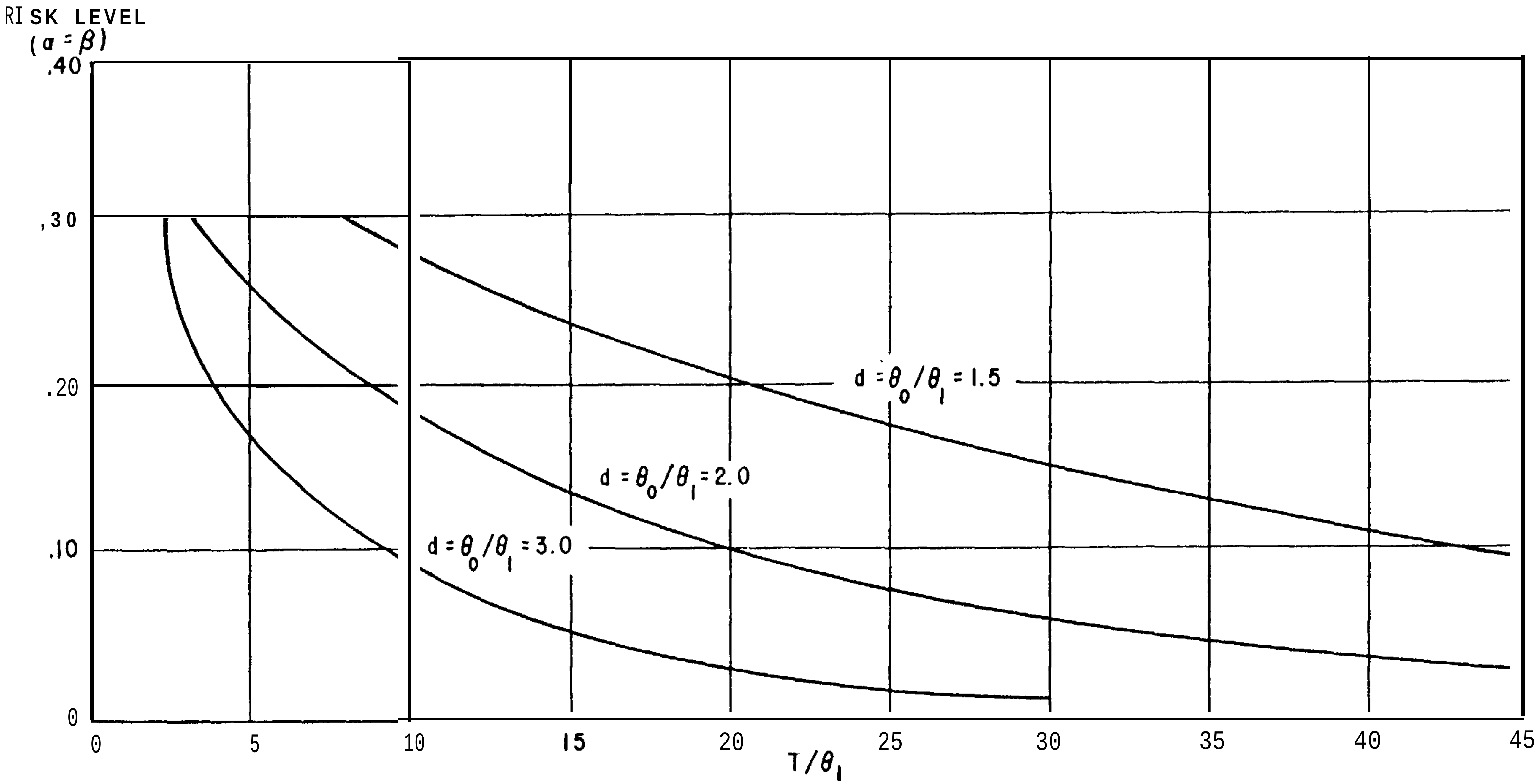
$P(ac|\theta)$ = Probability of accepting the system assuming the true MTBF is θ.

$P(rej\ \theta)$ = Probability of rejecting the system assuming the true MTBF is e.

The probability of acceptance is the probability that no more than a certain (acceptable) number of failures will occur. This probability can be computed using the equation:

$$P(ac|\theta) = \sum_{k=0}^{c} \frac{(T/\theta)^k e^{-(T/\theta)}}{k!} \tag{8.7}$$

FIGURE 8-7 GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS FOR $\alpha = \beta$



This is the Poisson Distribution Equation. This distribution and assumptions regarding its applications are discussed in Chapter 5.

The consumer's risk (β) is the probability that during the test no more than the acceptable number of failures will occur when the true MTBF is θ_1. Consequently,

$$\beta = P(ac\ \theta=\theta_1)$$

$$= P(c \text{ or fewer failures } \theta=\theta_1),$$

where c is the maximum acceptable number of failures. Thus,

$$\beta = \sum_{k=0}^{c} \frac{(T/\theta_1)^k e^{-(T/\theta_1)}}{k!} \quad (8.8)$$

The producer's risk (α) is the probability that during the test more than the acceptable number of failures will occur when the true MTBF is θ_0. Consequently,

$$\alpha = P(rej\ \theta=\theta_0)$$

$$= P(c + 1 \text{ or more failures } \theta=\theta_0).$$

Since

$$P(c + 1 \text{ or more failures } \theta=\theta_0) = I - P(c \text{ or fewer failures } \theta=\theta_0),$$

we have that

$$\alpha = 1 - \sum_{k=0}^{c} \frac{(T/\theta_0)^k e^{-(T/\theta_0)}}{k!},$$

or equivalently,

$$1 - \alpha = \sum_{k-0}^{c} \frac{(T/\theta_0)^k e^{-(T/\theta_0)}}{k!} \quad (8.9)$$

In order to determine the complete test plan, we must solve equations 8.8 and 8.9 simultaneously for T and c.

Solving these equations directly without the aid of a computer is too tedious and time consuming to be considered practical. We therefore present the following graphical solution procedure which utilizes the Poisson Chart, Chart No. 1 in Appendix B.

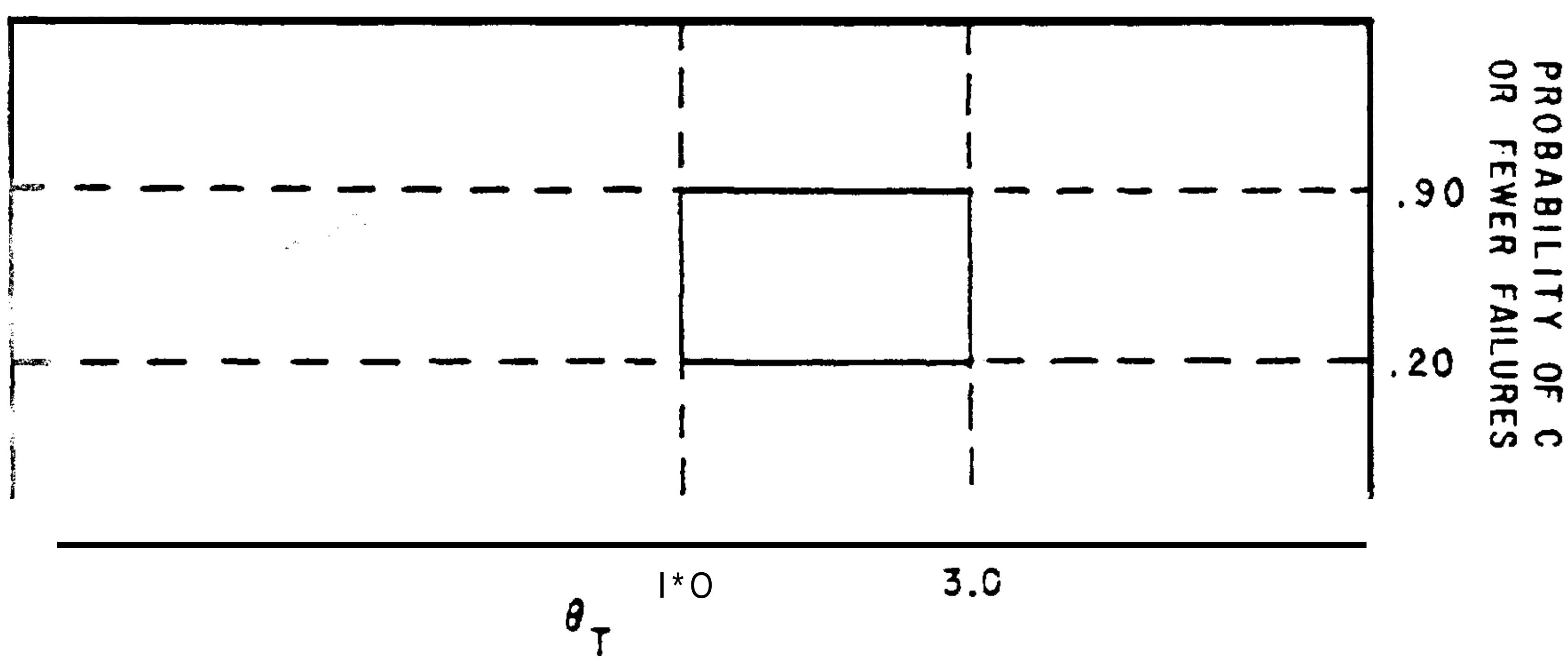
Graphical Poisson Solution Procedure. We wish to find a test exposure, T, and an acceptable number of failures, c, such that the probability of acceptance is β when $\theta = \theta_1$ and $1 - \alpha$ when $\theta = \theta_0$. This is done graphically with the use of a transparent overlay.

On an overlay sheet, draw vertical lines at $\theta/T = 1$ and $\theta/T = \theta_0/\theta_1$. Draw horizontal lines at probabilities β and $1 - \alpha$, forming a rectangle. Slide the overlay rectangle horizontally until a curve for a single value of c passes through the lower left and upper right corners. (It may not be possible to hit the corners exactly. Conservative values of c will have curves that pass through the horizontal lines of the rectangle.) This value of c is the acceptable number of failures. Read the value of θ/T corresponding to the left side of the rectangle. Divide θ_1 by this value to find T, the required test exposure. The following numerical example illustrates the use of the graphical Poisson Solution Procedure.

We wish to find the required test exposure, T, and acceptable number of failures c; such that when the MTBF, $\theta = \theta_1 = 100$ hours, the probability of acceptance, β, will be 0.20 and then $\theta = \theta_0 = 300$ hours the probability of acceptance, $1 - \alpha$, will be 0.90.

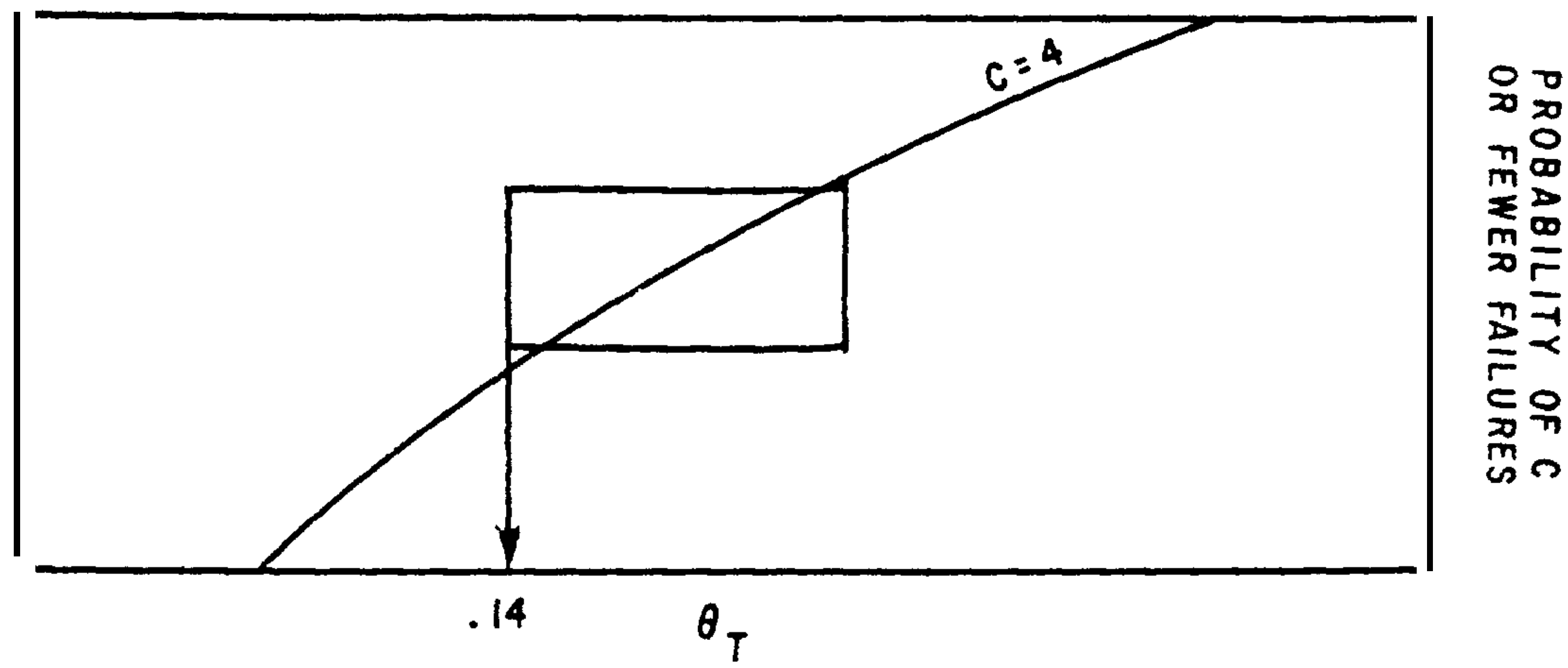
An overlay rectangle is constructed as shown.

FIGURE 8-3 OVERLAY CONSTRUCTION TECHNIQUE



Sliding the rectangle to the left, we find that when c = 3 the fit is close, but slightly higher risks must be tolerated. Going to c = 4, the curve passes through the horizontal lines of the rectangle. At the left of the rectangle, $\theta/T = 0.14$, so the required test exposure is approximately 100/0. 14 = 714 hours and the acceptance criterion is 4 or fewer failures.

FIGURE 8-9 OVERLAY CURVE MATCHING PROCEDURE



OPERATING CHARACTERISTIC (OC) CURVES

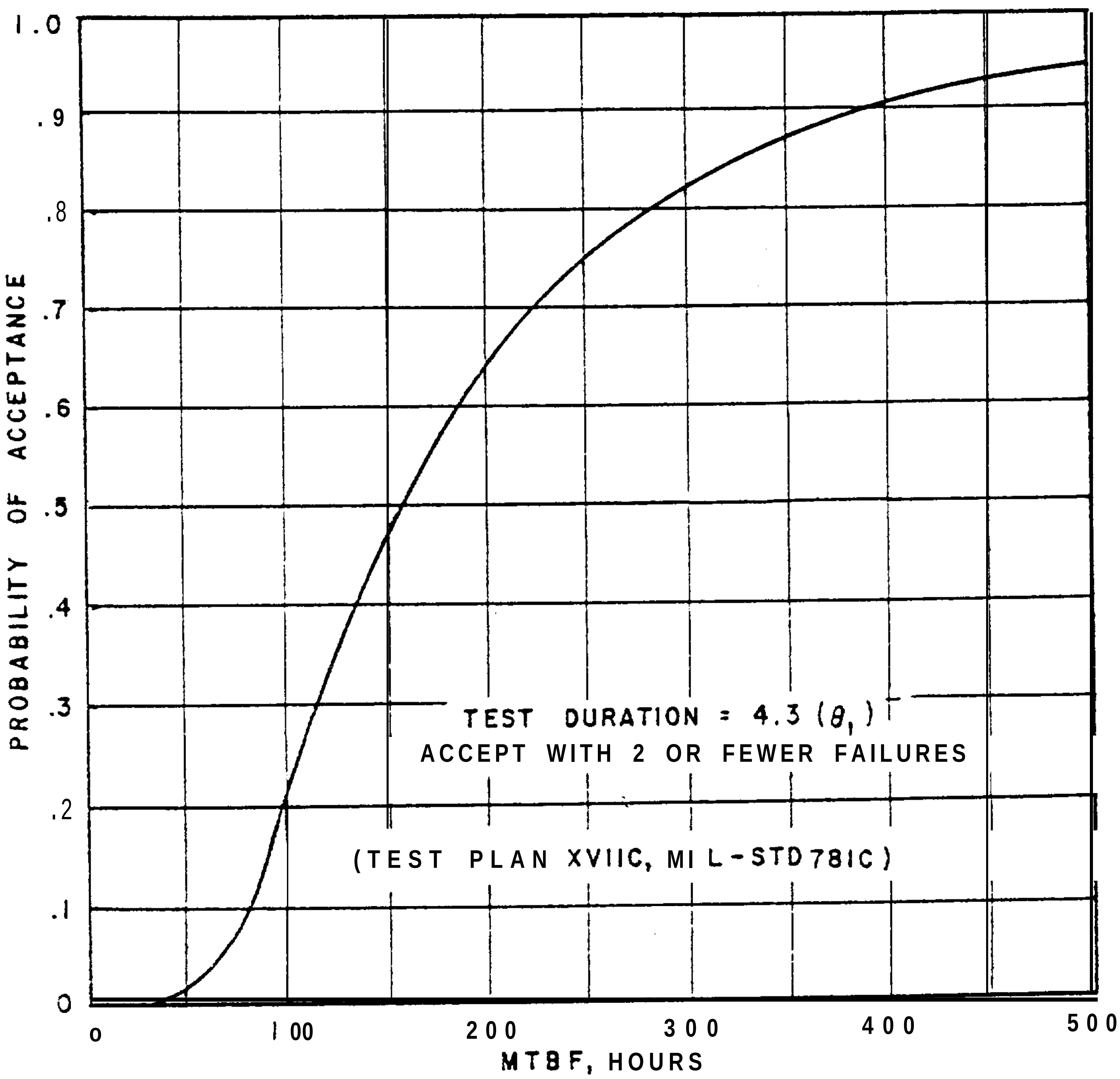
Introduction

In the previous sections of this chapter, we have discussed methods for developing test plans which achieve required α and β risks. The test plan itself is specified by the test exposure and the maximum acceptable number of failures. For a test plan developed using the methods in this chapter, we know that the producer's risk (the probability of rejecting a good system) for the specified value (SV) is α and the consumer's risk (the probability of acceptance) for the minimum acceptable value (MAV) is β. In addition, to assess a test plan proposed by another party, we have shown methods for computing the producer's risk and the consumer's risk for the SV and MAV, respectively. A graphical tool which provides more complete information about a specific test plan is the operating characteristic (OC) curve. The OC curve displays both acceptance and rejection risks associated with all possible values of the reliability parameter and not merely the SV and MAV. By definition, an OC curve is a plot of the probability of acceptance (the ordinate) versus the reliability parameter value (the abscissa) .

Figure 8-10 contains the operating OC curve for test plan XVICC from MIL-STD 781C, with θ_1, the lower test MTBF, assumed to be 100 hours.

Consider a single point on the curve, say an MTBF of 200 hours and a probability of acceptance of 0.63. This means that for test plan XVIIC (test duration of 430 hours, accept with 2 or fewer failures) , a system which has a true MTBF of 200 hours has a 63% chance of passing this test, i.e. , being accepted. A system requires an MTBF of around 400 hours in order for the producer to be at least 90% confident that the system will be accepted. A system whose true MTBF is about 80 hours has only a 10% chance of being accepted.

FIGURE 8-10 OPERATING CHARACTERISTIC (OC) CURVE



FOR THIS EXAMPLE θ_1 IS ASSUMED TO EQUAL 100 HOURS

Operating characteristic curves for all the test plans in Appendix B, Table 6 of this text can be found in Appendix C of MIL-STD 781C. However, OC curves for the test plans in Appendix B, Table 7, of this text are not available in MIL-STD 781C.

OC Curve Construction

The OC curve shown in Figure 8-10 is a representation of the mathematical model used to compute the reliability for a system. We have discussed two basic models in previous sections. The Poisson/exponential model is used for systems undergoing continuous time testing and the binomial model is used for discrete time tests.

The OC curve specifically displays the relationship between the probability of acceptance and MTBF.

For the Poisson/exponential model, we indicated in equation 8.7 that

$$P(ac|\theta) = \sum_{k=0}^{c} \frac{(T/\theta)^k e^{-(T/\theta)}}{k!} \tag{8.10}$$

where

k = Number of failures

T = Total test time

θ = Values of MTBF

c = Maximum acceptable number of failures

Referring to Figure 8-10, let us assume that θ_1 = 100 hours and that we have selected test plan XVIIC from Appendix B, Table 6 which permits a maximum of two failures. Using θ_1 = 100 hours, which corresponds to a test duration of 430 hours for test plan XVIIC (T = 4.301), and c = 2, we can determine points on the curve by calculating $P(ac\ \theta)$ for different values of θ. As an example, for θ= 215 hours

$$P(ac|\theta=215) = \frac{\left(\frac{430}{215}\right)^0 e^{-\left(\frac{430}{215}\right)}}{0!} + \frac{\left(\frac{430}{215}\right)^1 e^{-\left(\frac{430}{215}\right)}}{1!} + \frac{\left(\frac{430}{215}\right)^2 e^{-\left(\frac{430}{215}\right)}}{2!}$$

$$= \frac{2^0 e^{-2}}{0} + \frac{2^1 e^{-2}}{1} + \frac{2^2 e^{-2}}{2}$$

$$= 0.135 + 2(0.135) + 2(0.135)$$

$$= 0.676\ .$$

By choosing a sufficient number of values for θ between 0 and 500 and computing the probability of acceptance for each, we can construct a smooth curve.

For the binomial model, the probability of acceptance is expressed by the equation

$$P(ac|p) = \sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=0}^{c} \frac{n!}{[k!\ (n-k)!]} p^k (1-p)^{n-k}$$

where

n = Number of trials

c = Maximum acceptable number of failures

p = Probability of failure on any trial.

By inserting specific values for n, k and by varying the probability of failure on any trial, p, we can compute values of the probability of acceptance which permit us to construct an OC curve.

For example, by letting $n = 5$ and $c = 2$, calculate the probability of acceptance for $p = 0.1$.

$$P(ac \mid p=0.1) = \frac{5!}{0!(5-0)!}(0.1)^0(1-0.1)^{5-0} + \frac{5!}{1!(5-1)!}(0.1)^1(1-0.1)^{5-1}$$

$$+ \frac{5!}{2!(5-2)!}(0.1)^2(1-0.1)^{5-2}$$

$$= \frac{(120)(1)(0.9)^5}{120} + \frac{(120)(0.1)(0.9)^4}{24} + \frac{(120)(0.1)^2(0.9)^3}{(2)(6)}$$

$$= (0.9)^5 + (0.5)(0.9)^4 + (0.1)(0.9)^3$$

$$= 0.59 + 0.33 + 0.07 = 0.99.$$

Thus ,

$$P(ac \mid p=0.1) = 0.99 .$$

As expected, the probability of acceptance is very high since we have designed a relative easy test to pass.

CASE STUDY NO. 8-1

Background

A mechanical system which controls the aiming point of a large-caliber gun is under development. The specified and minimum acceptable values for the probability of aiming correctly are 0.85 and 0.70 respectively. Testing requires that expensive projectiles be fired for each trial, and only 20 rounds are allotted for testing.

Determine

1. Propose a test plan which equalizes the consumer's and producer's risks. What are the risks?

2. The user can tolerate a risk of no worse than 5%. What test plan gives the best (smallest) producer's risk?

Solutions

1. As mentioned in Chapter 6, "Statistical Concepts", consumer' s risk increases when producer's risk decreases, and vice versa, when the sample size is fixed. Theoretically, there is a point where they are equal or almost equal.

It is also important to understand that the analytical interpretation of producer' s and consumer' s risk when determining the solution to question no. 1. The producer' s risk is the probability of rejecting a system which meets the SV of 0.15 proportion of failures (reliability of 0.85). For a given accept/reject criterion (determined by a value c which represents the maximum number of failures which results in acceptance of the system) , the producer' s risk, α, is the probability that c + 1 or more failures occur. The consumer's riks is the probability of accepting a system which exceeds the MAV of 0.30 proportion of failures (reliability of 0.70) . For the same accept/reject criterion, the consumer' s risk, β, is the probability that c or fewer failures occur. Below is a section of binomial tables for n = 20, extracted from Appendix B, Table 1.

c	β = P(c or fewer failures) p_1 = 0.30	$1-\alpha$ = P(c or fewer failures) p_0 = 0.15	α = P(c+1 or more failures) p. = 0.15
0	0.00	0.04	0.96
1	0.00	0.18	0.82
2	0.03	0.40	0.60
3	0.11	0.65	0.35
4	0.24	0.83	0.17
5	0.42	0.93	0.07
6	0.61	0.98	0.02
7	0.77	0.99	0.01

The proposed test plan is to accept with 4 or fewer failures and reject with 5 or more failures. The consumer's and producer's risks are 0.24 and 0.17, respectively.

2. From the above table, we see that test plans which have the maximum acceptable number of failures (c) of O, 1, and 2, and satisfy the consumer's risk of no more than 5%. The best (smallest) producer' s risk occurs when c = 2, the risk being 0.60.

CASE STUDY NO. 8-2

Background

A new, highly reliable missile system is under development. The specified reliability (SV) is 0.98, and the minimum acceptable reliability (MAV) is 0.85.

Determine

1. Design test plans for producer's risks of 5%, 10%, and 20%, with a consumer's risk of 5%.

2. Design test plans for a producer's risk of 10% and for a consumer's risk of 10%.

3. Redo number 2 if the MAV is 0.92 instead of 0.85.

Solutions

Note that a reliability of 0.98 corresponds to a proportion of failures, or unreliability, of 0.02, and a reliability of 0.85 corresponds to a proportion of failures, or unreliability, of 0.15. Thus, we list our test planning parameters p_0 and p_1 as 0.02 and 0.15, respectively.

1a. $p_0 = 0.02$ $p_1 = 0.15$ $\alpha = 0.05$ $\beta = 0.05$

i. Normal Approximation. In order to determine a starting point for our analysis, we calculate approximate values of n and c using equations 8.3 and 8.4. For values of z_α and z_β, use Appendix B, Table 2.

$$n = \{(1.645)^2(0.02-0.0004) + (1.645)^2(0.15-0.0225)$$

$$+ 2(1.645)^2 \sqrt{(0.02)(0.15)(0.98)(0.85)}\}/(0.15-0.02)^2$$

$$= 39.6$$

$$c = (1.645)\sqrt{(39.6)(0.02)(0.98)} + (39.6)(0.02) - 0.5$$

$$= 1.7$$

ii. Poisson Approximation (Appendix B, Chart 1, with np = T/El, the reciprocal of θ/T)

c	np_1	n	np_0	β	1-α	α
o	3.0	20.0	0.40	0.05	0.69	0.32
1	4.7	34.3	0.63	0.06	0.88	0.12
2	6.3	42.0	0.84	0.05	0.94	0.06
3	7.8	52.0	1.04	0.05	0.97	0.03

NOTE : α = P(c + 1 or more failures)
β = P(c or fewer failures)
1-α = P(c or fewer failures)

iii. Proposed Test Plans. It appears from i and ii above that a good starting point for fine tuning is an n of 40 and c of 2. Using Appendix B, Table 1 to fine tune, we propose the following test plans.

n	c	α	β
40	2	0.04	0.05
39	2	0.04	0.05
38	2	0.04	0.06
37	2	0.04	0.07
* 36	2	0.03	0.08

*The protection afforded by this plan seems to be adequate though the consumer's risk is 8% (slightly above the required 5%).

lb. $p_0 = 0.02$ $p_1 = 0.15$ $\alpha = 0.10$ $\beta = 0.05$

i. Normal Approximation

$$n = \{(1.28)^2(0.02-0.0004) + (1.645)^2(0.15-0,0225)$$

$$+ 2(1.28)(1.645)\sqrt{(0.02)(0.15)(0.98)(085)}\ ]/(0.15-0.02)^2$$

$$= 34.8$$

$$c = (1.28)\sqrt{(34.8)(0.02)(0.98)} + (34.8)(0.02) - 0.5$$

$$= 1.3$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	1-α	α
0	3.0	20.0	0.40	0.05	0.69	0.31
1	4+7	34.3	0.63	0.05	0.88	0.12
2	6.3	42.0	0.84	0.05	0.94	0.06
3	7.8	52.0	1.04	0.05	0.97	0.03

iii. It appears that a good starting point for fine tuning is an n of 35 and c of 1. The following test plans are proposed.

	n	c	α	β
	35	1	0.15	0.03
	34	1	0.15	0.03
	33	1	0.14	0.03
	32	1	0.13	0.04
	31	1	0.13	0.04
	30	1	0.12	0.05
	29	1	0.11	0.05
*	28	1	0.11	0.06

'The actual risks exceed the required risks of 10% and 5% but not to any significant extent.

l.c. $p_0 = 0.02 \quad p_1 = 0.15 \quad \alpha = 0.20 \quad \beta = 0.05$

i. Normal Approximation

$$n = \{(0.84)^2(0.02-0.0004) + (1.645)^2(0.15-0.0225)$$

$$+ 2(0.84)(1.645)\sqrt{(0.02)(0.15)(0.98)(0.85)}\,)/(0.15-0.02)^2$$

$$= 29.4$$

$$c = (0.84)\sqrt{(29.4)(0.02)(0.98)} + (29.4)(0.02) - 0.5$$

$$= 0.72$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	1-α	α
0	3.0	2.0	0.40	0.05	0.69	0.31
1	4.7	34.3	0.63	0.05	0.88	0.12

iii. It appears that a good starting point for fine tuning is an n of 25 and c of 1. The following programs are proposed.

	n	c	α	β
*	25	1	0.09	0.09
	25	0	0.40	0.02

$'Generally, there is no reasonable test plan for the input values given. A very large sample size is required to achieve an α of 0.20. (For n = 40, α = 0.19, and β = 0.01, with a c of O.) This sample size seems unwarranted. Our recommendation is to use the n of 25 and the c of 1.

2. $p_0 = 0.02$ $p_1 = 0.15$ $\alpha = 0.10$ $\beta = 0.10$

i. Normal Approximation

$$n = \{(1.28)^2(0.02-0.0004) + (1.28)^2(0.15-0.0225)$$

$$+ 2(1.28)^2 \sqrt{(0.02)(0.15)(0.98)(0.85)}\}/(0.15-0.02)^2$$

$$= 24.2$$

$$c = (\ .28) \sqrt{(24.2)(0.02)(0.98)} + (24.2)(0.02) - 0.5$$

$$= 0\ 86$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_o	β	$1-\alpha$	α
0	2.3	15.3	0.31	0.10	0.74	0.26
1	3.9	26.0	0.52	0.10	0.90"	0.10
2	5.2	35.0	0.70	0.10	0.96	0.04

iii. It appears that a good starting point for fine tuning is an n of 25 and c of 1. The following programs are proposed.

	n	c	α	β
*	25	1	0.09	0.09
	24	1	0.08	0.11

*The test plans with a sample size of 25 fits well. The sample size can be reduced by 1 to 24 if the consumer allows his risk to be 11%.

3. $p_0 = 0.02$ $p_1 = 0.08$ $\alpha = 0.10$ $\beta = 0.10$

i. Normal Approximation

$$n = \{(1.28)^2(0.02-0.0004) + (1.28)^2(0.08-0.0064)$$

$$+ 2(1.28)^2 \sqrt{(0.02)(0.08)(0.98)(0.92)}\}/(0.08-0.02)^2$$

$$= 77.0$$

$$c = (1.28)\sqrt{(77.0)(0.02)(0.98)} + (77.0)(0.02) - 0.5$$

$$= 2.6$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
1	3.9	48.8	0.87	0.10	0.75	0.25
2	5.3	66.3	1.32	0.10	0.88	0.12
3	6.7	83.8	1.67	0.10	0.91	0.09
4	8.0	100.0	2.00	0.10	0.94	0.06

iii. It appears that a good starting point for fine tuning is an n of 75 and c of 3. The following programs are proposed.

n	c	α	β
75	3	0.06	0.14
76	3	0.07	0.13
77	3	0.07	0.13
78	3	0.07	0.13
79	3	0.07.	0.11
80	3	0.08	0.10

CASE STUDY NO. 8-3

Background

An operational test is being considered for a disposable survival ratio which must work for at least three hours. The ratio has a specified mission reliability of 0.85 and a minimum acceptable mission reliability of 0.7. A number of radios will be put on test for-three hours each and the-number of failures recorded.

Determine

1. Propose some test plans for a producer’s risk of about 10% and consumer’s risks of about 10%, 20%, and 30%.

2. Propose a test plan for a minimum acceptable reliability of 0.5 with a user risk of 2% and a producer risk of 20%.

Solutions

Note that a reliability of 0.85 corresponds to a proportion of failures or “unreliability” p. of 0.15.

la. $p_0 = 0.15 \quad p_1 = 0.3 \quad \alpha = 0.10 \quad \beta = 0.10$

i. Normal Approximation

$$n = \{(1.28)^2(0.15-0.0225) + (1.28)^2(0.3-0.09)$$

$$+ 2(1.28)^2 \sqrt{(0.15)(0.3)(0.85)(0.7)}\}/(0.3-0.15)^2$$

$$= 48.4$$

$$c = 1.28\sqrt{(48.4)(0.15)(0.85)} + (48.4)(0.15) - 0.5$$

$$= 9.9$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
7	12.0	40.0	6.0	0.10	0.75	0.25
8	13.0	43.3	6.5	0.10	0.80	0.20
9	14.0	46.7	7.0	0.10	0.83	0.17
10	**15.5**	**51.7**	**7.7**	**0.10**	**0.85**	**0.15**
11	16.5	55.0	8.2	0.10	0.88	0.12
12	18.0	60.0	9.0	0.10	0.89	0.11
13	19.0	63.0	9.5	0.10	0.90	0.10

iii. It appears that a good starting point for fine tuning is an n of 50 and c of 10. The following programs are proposed.

n	c	α	β
50	10	0.12	0.08
49	10	0.11	0.09
48	10	0.10	0.10

1b. $p_0 = 0.15$ $p_1 = 0.3$ $\alpha = 0.10$ $\beta = 0.20$

i. Normal Approximation

$$n = \{(1.28)^2(0.15-0.0225) + (0.84)^2(0.3-0.09) + 2(0.84)(1.28)\sqrt{(0.15)(0.3)(0.85)(0.7)}\}/(0.3-0.15)^2$$

$$= 31.5$$

$$c = (1.28)\sqrt{(31.5)(0.15)(0.85)} + (31.3)(0.15) - 0.5$$

$$= 6.78$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
6	9.1	30.7	4.6	0.20	0.82	0.18
7	10.3	34.3	5.1	0.20	0.85	0.15
8	11.5	38.3	5.7	0.20	0.88	0.12
9	12.5	41.7	6.3	0.20	0.90	0.10
10	13.8	46.0	6.9	0.95	0.20	0.05

iii. It appears that a good starting point for fine tuning is an n of 35 and c of 7. The following programs are proposed.

n	c	α	β
35	7	0.14	0.13
34	7	0.12	0.16
33	7	0.11	0.19
32	7	0.10	0.21

1.c. $p_0 = 0.15$ $p_1 = 0.30$ $\alpha = 0.10$ $\beta = 0.30$

i. Normal Approximation

$$n = \{(1.28)^2(0.15-0.0225) + (0.526)^2(0.3-0.09)$$

$$+ 2(0.526)(1.28)\sqrt{(0.15)(0.3)(0.85)(0.7)}\,]/(0.3-0.15)^2$$

$$= 21.7$$

$$C = (1.28)\sqrt{(21.7)(0.15)(0.85)} + (21.7)(0.15) - 0.5$$

$$= 4.9$$

ii. Poisson Approximation (Appendix B, Chart 1)

c	np_1	n	np_0	β	$1-\alpha$	α
4	5.9	19.7	2.9	0.30	0.82	0.18
5	7.0	23.3	3.5	0.30	0.87	0.13
6	8.4	28.0	4.2	0.30	0.89	0.11

iii. It appears that a good starting point for fine tuning is an n of 22 and c of 5. The following programs are proposed.

n	c	α	β
22	5	0.10	0.31
23	5	0.12	0.27

2. $p_0 = 0.15$ $p_1 = 0.5$ $\alpha = 0.20$ $\beta = 0.02$

i. Normal Approximation

$$n = \{(0.84)^2(0.15-0.0225) + (2.06)^2(0.5-0-25)$$

$$+ 2(0.84)(2.06)\sqrt{(0.15)(0.5)(0.85)(0.5)}\}/(0.5-0.15)^2$$

$$= 14.4$$

$$C = (0.84)\sqrt{(14.4)(0.15)(0.85)} + (14.4)(0.15) - 0.5$$

$$= 2.8$$

ii. Poisson Approximation (Appendix B, Chart 6)

c	np_1	n	np_0	β	1-α	α
2	7.5	15.0	2.25	0.02	0.63	0.37
3	9.1	18.2	2.70	0.02	0.73	0.27
4	10.6	21.2	3.20	0.02	0.80	0.20

iii. It appears that a good starting point for fine tuning is an n of 15 and c of 3. The following programs are proposed.

n	c	α	β
15	3	0.18	0.02
14	3	0.16	0.03

CASE STUDY NO. 8-4

Background

A communication system has minimum acceptable value (MAV) of 100 hrs MTBF, and a specified value (SV) of 150 hrs MTBF.

Determine

How many hours of test are required for design qualification prior to a production decision if we desire α and β risks of 10% each?

Solution

$$\theta_0 = 150, \; \theta_1 = 100, \; d = \frac{150}{100} = 1.5, \; \alpha \cong \beta \cong 0.10$$

In this case, a "standard" test plan may be selected from Appendix B, Table 6. Test plan IXC satisfies these inputs. The required test duration is

$$T = (45.0)(\theta_1) = (45.0)(100) = 4500 \text{ hrs.}$$

The accept/reject criterion for this test plan is to accept if we encounter 36 or fewer failures. Bear in mind though, that the acceptance criteria may require violation, depending on the nature of the failures and the verification of corrective action.

Commentary

1. In order to make the test less sensitive to the constant failure rate assumption, it would be desirable to have at least 3 systems tested for at least 3(100) = 300 hours each. The remainder of the 4500 hours may be satisfied with these or other systems. See section entitled "Constant Failure Rate Assumption" for a discussion of this topic.

2. The test duration of 4500 hours is very long! (The equivalent of 187.5 24-hour days). Putting more systems on test will reduce the calendar time requirement, but 4500 hours of test exposure are still required. The required test exposure is high because of the low discrimination ratio, d, and the relatively low α and β risks. Plans with higher risks may be worth consideration to see how much the amount of testing may be reduced.

CASE STUDY NO. 8-5

Background

An air defense system has a minimum acceptable value (MAV) of 80 hours MTBF and a specified value (SV) of 220.

Determine

How many hours of testing are required to give the user 80% assurance that his needs have been met? The producer requires 90% assurance that his product will be accepted if it meets the SV.

Solution

The 80% user assurance is equivalent to a consumer's risk of $\beta = 0.20$, and the 90% contractor assurance is equivalent to a producer's risk of $\alpha = 0.10$.

$$\theta_0 = 220, \ \theta_1 = 80, \ d = \frac{220}{80} = 2.75.$$

Because the discrimination ratio is 2.75, the "standard" test plans from Appendix B, Table 6, cannot be used. Appendix B, Table 7, will be considered.

For the 20% β risk, and entering the 10% α risk column, we find a discrimination ratio of 2.76 available, which is very close to 2.75. This is test plan number 20-5. The required test duration is

$$T = (6.72)(\theta_1) = (6.72)(80) = 537.6 \text{ hrs.}$$

The accept/reject criterion is to accept with 4 or fewer failures.

CASE STUDY NO. 8-6

Background

A radar system is under development. A test is to be run which will be essentially a fixed configuration test. At this stage of development, an MTBF of 200 hours is planned, but assurance is desired that the MTBF is not lower than 100 hours. For cost and schedule reasons, a test exposure of 300 hours has been proposed.

Determine

Is this amount of test exposure adequate? If not, what is an adequate amount of testing?

Solution

The upper test value, θ_0, is 200, and the lower test value, θ_1, is 100. Test exposure, T, is 300 hrs.

For a quick and simple look at the adequacy of the proposed test, we will use Figure 8-7. (For the convenience of the reader, the graphs in Figures 8-6 and 8-7 have been reproduced and annotated below.) Entering Figure 8.7 with T/θ_1 = 300/100 = 3 and d = θ_0/θ_1 = 200/100 = 2, we find that the proposed test exposure results in risks slightly above 30%. The amount of testing proposed is minimally adequate for this stage in the program.

We may use Figure 8-7 to determine an adequate test duration. At about 580 hours, $\alpha = \beta = 0.25$. At about 900 hours, $\alpha = \beta = 0.20$. At about 2000 hours, $\alpha = \beta = 0.10$. At 580 hours, the risks are fairly high. A β risk of 25% is perhaps tolerable, but an α risk of 25% means a 25% chance of an erroneous "back to the drawing board" decision.

A test duration of about 900 hours looks reasonable, particularly if we reduce α by letting β increase. To investigate this possibility, we may use Figure 8-6. From the graph for d = 2, we find that test plan 30-8 with $\alpha = 0.10$, $\beta = 0.30$ and T = 981 looks reasonable. (Test plan 30-5 with $\alpha = 0.20$, $\beta = 0.30$ and T = 589 is another attractive possibility) . A test duration of about 900 hours is recommended.

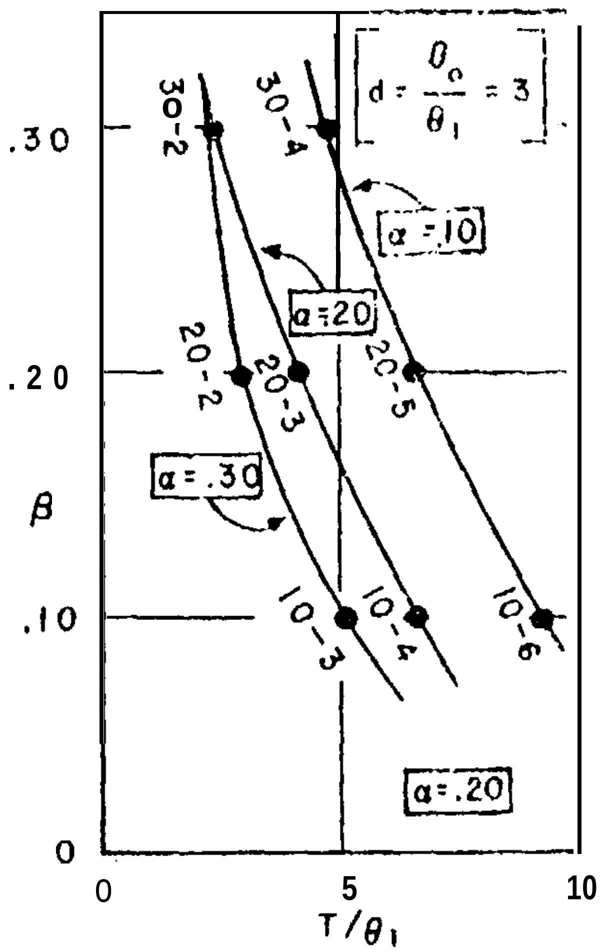
Commentary

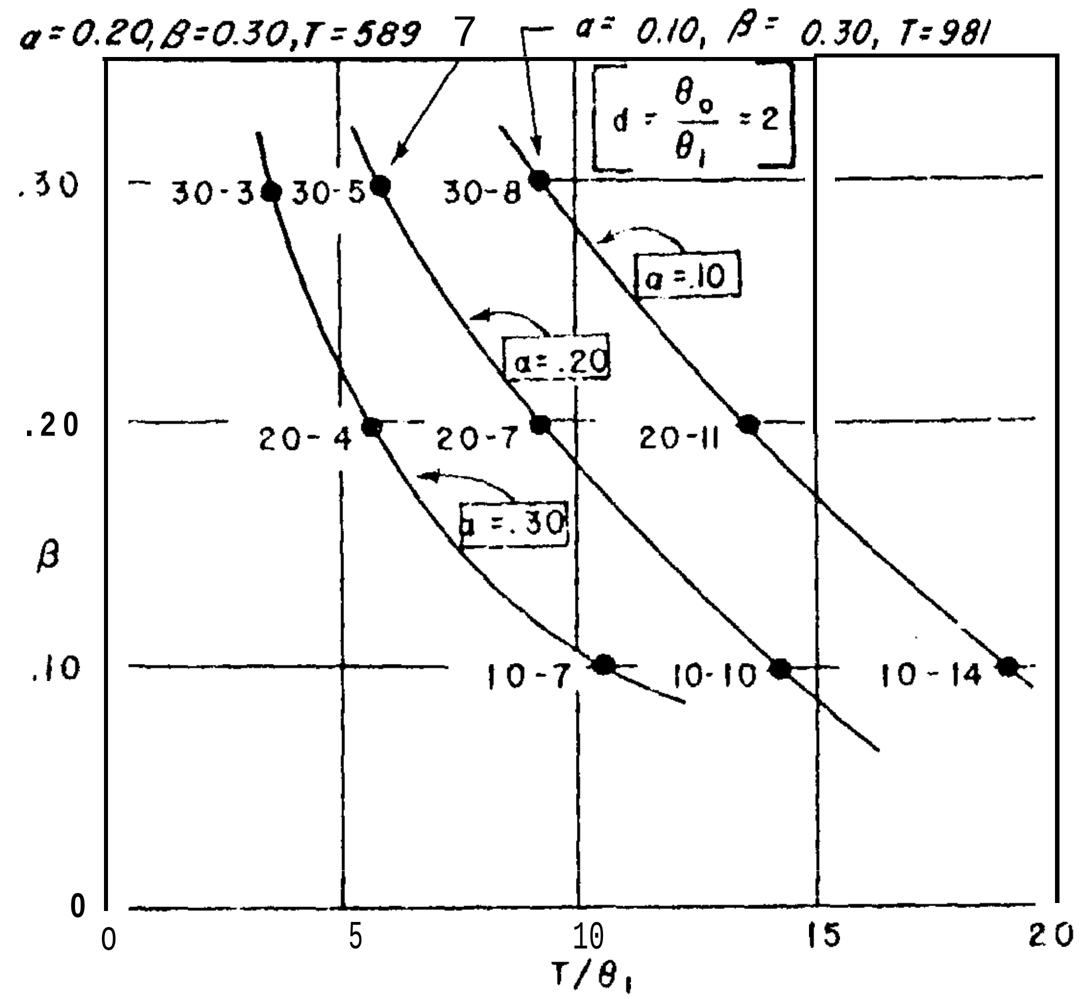
The process of trading test length for testing risks is inherently somewhat subjective. Actual problems of the type illustrated in this case should, of course, explicity address time, cost, and other penalties associated with the test.

FIGURE 8-6

GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS

T = TOTAL TEST EXPOSURE





NOTE.
PLOTTING POINTS ARE CODED TO INDICATE THE MIL-STD 781C TEST PLAN THESE PLANS ARE CONTAINED IN APPENDIX B, TABLE. S 6L7.

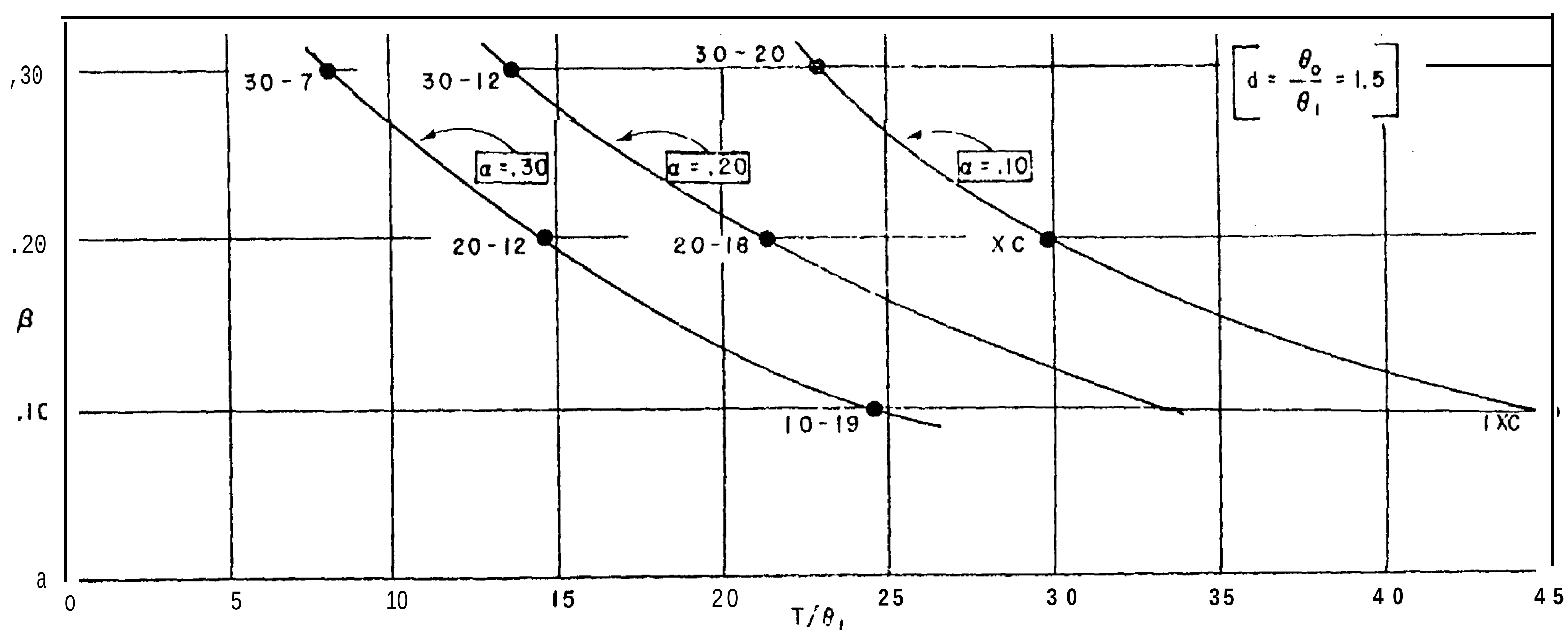
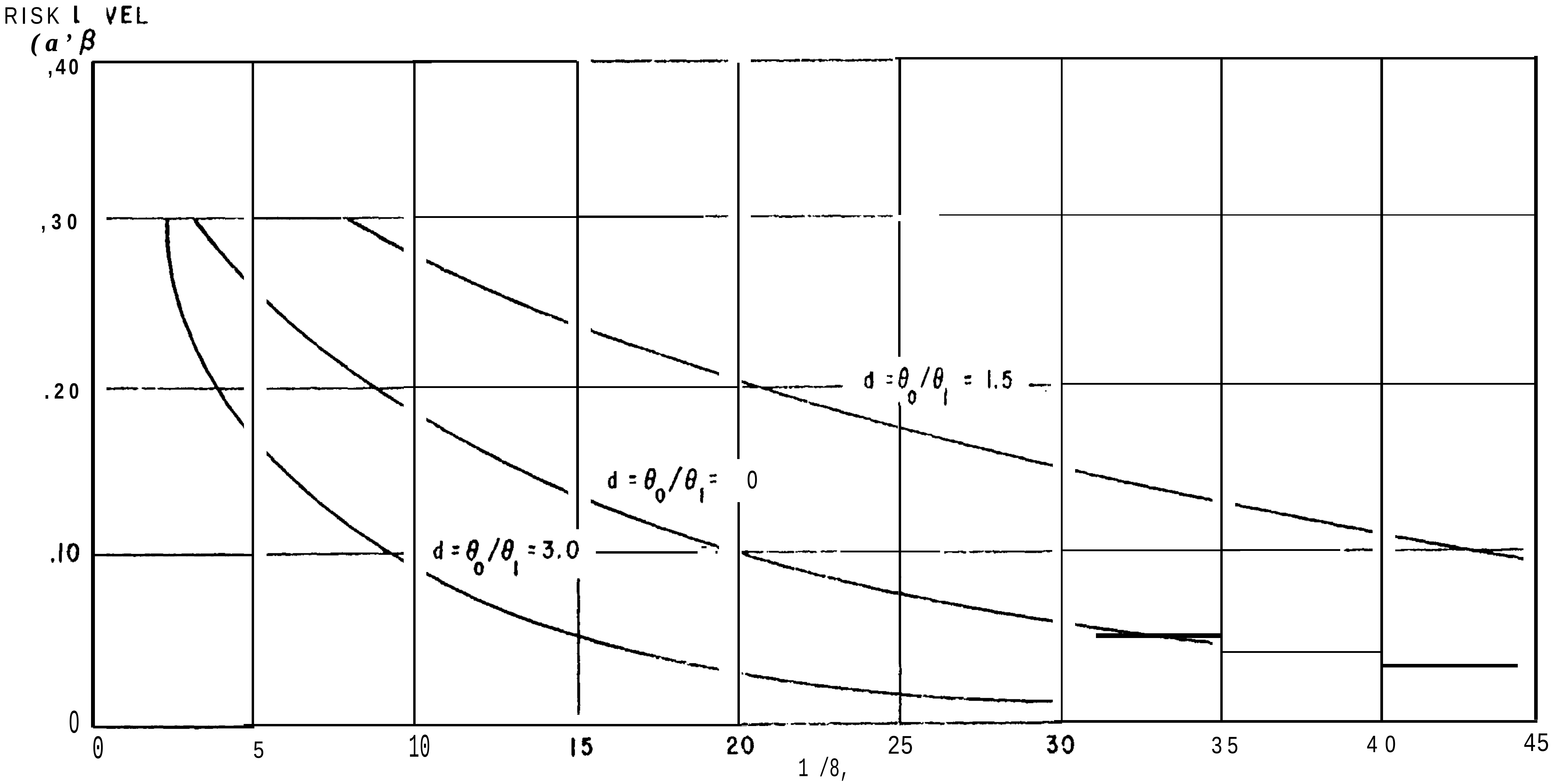


FIGURE 8-7 GRAPHICAL REPRESENTATION OF TEST PLANNING PARAMETERS FOR $\alpha = \beta$



CASE STUDY NO. 8-7

Background

A program manager desires to demonstrate an MTBF of at least 200 at a 90% confidence level.

Determine

The minimum test length required, and an evaluation of the proposed test plan.

Commentary

The correct interpretation of the background statement is that the MAV is 200 hours and the consumer's risk is 10%.

Solution

The absolute minimum test length is that which permits making the desired confidence statement with zero failures. Applying inequality 7. 10a we have:

$$\theta \geq \frac{2T}{\chi^2_{\alpha,2r+2}}$$

$$200 = \frac{2T}{\chi^2_{0.10,2}}$$

$$T = \frac{200(4.60)}{2} = 460.0 \text{ hours}$$

of test exposure.

NOTE : Values of $\chi^2_{\alpha, 2r+2}$ are found in Appendix B, Table 5.

An MTBF of 200 can be demonstrated at a 90% confidence level by completing 46o.o hours of test exposure with zero failures.

To evaluate this proposed test plan, we will use an OC curve. To construct the OC curve, we use equation 8.10.

$P(ac|\theta)$ = Probability of acceptance for a given value of θ

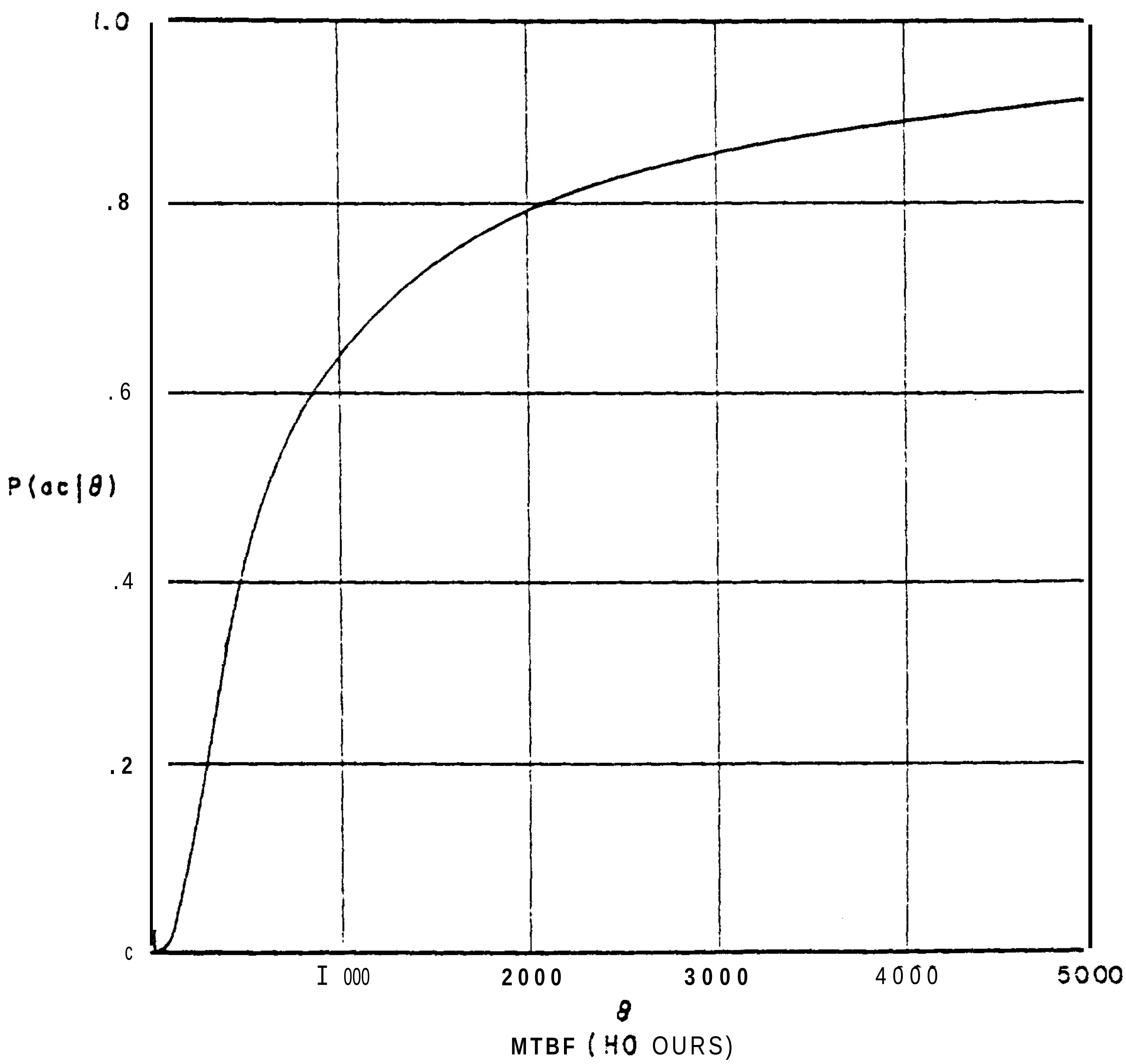
$$P(ac|\theta) = \sum_{k=0}^{c} \frac{(T/\theta)^k e^{-(T/\theta)}}{k!} .$$

For T = 460.0 and C = 0

$$P(ac\ \theta) = e^{-(460.0/\theta)}$$

A few of the points for plotting the OC curve are tabulated below.

θ	W
100	0.010
200	0.100
500	0.398
1000	0.631
2000	0.794
3000	0.858
4000	0.891



Commentary

The curve shows that the proposed test plan does , in fact, achieve a consumer's risk of 10% for the MAV of 200 hours. Let us now examine the OC curve for this test plan through the eyes of the contractor.

The curve shows that a system whose true MTBF is 650 hours has only a 50% chance of passing the test, i.e., being accepted. In addition, for a system to have a 90% chance of being accepted, it must have a true MTBF of 4,400 hours. In other words, for the contractor to obtain a producer' s risk of 10%, he needs to manufacture a system whose true MTBF is 4,400 hours. To have a 50/50 chance of passing the test, he needs to manufacture a system whose true MTBF is 650 hours. The lesson to be learned here is that consideration must be given to both the upper test value (SV) , the lower test value (MAV) , and the risks associated with them in designing or evaluating a test plan- The test planner or evaluator should be concerned with obtaining the minimal test exposure plan which protects both the consumer and producer. To ignore either aspect can be a dangerous policy.