Global Variance Reduction in Monte Carlo Simulation of Systems Engineering Reliability Analysis

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Introduction

- System Reliability
  - Use of Monte Carlo Simulation
  - Variance Reduction Methods
  - Maintaining a “Fair Game”
- Particle Physics Simulation
  - Background and Use of Monte Carlo Simulation
  - Standard Variance Reduction Methods
  - Global Variance Reduction Methods
- System Reliability in 3D
  - Description and Use
  - Comparison to the Analog Standard System Reliability Method
- Global Variance Reduction for System Reliability
  - Implementation Process
  - Advantages Over Standard System Reliability Method

Research Goals

- This research seeks to demonstrate a new method of converting system engineering reliability simulations to particle physics simulations, that yields results equivalent to those from the standard technique
- It will show that results obtained using global variance reduction are not statistically different from results obtained using an analog process
- It will also demonstrate that the new global variance reduction method can obtain these results in significantly less computational time than the analog method

System Reliability

- System Reliability is the probability that a system will not fail before a given time *t*
- Figure 1 depicts a simple system with three components, that fails only if C and one or both of the other two components fail
- Calculating the system reliability given the individual component probability of failure is trivial initially, but once the ability to repair components is included, the solution becomes significantly more difficult

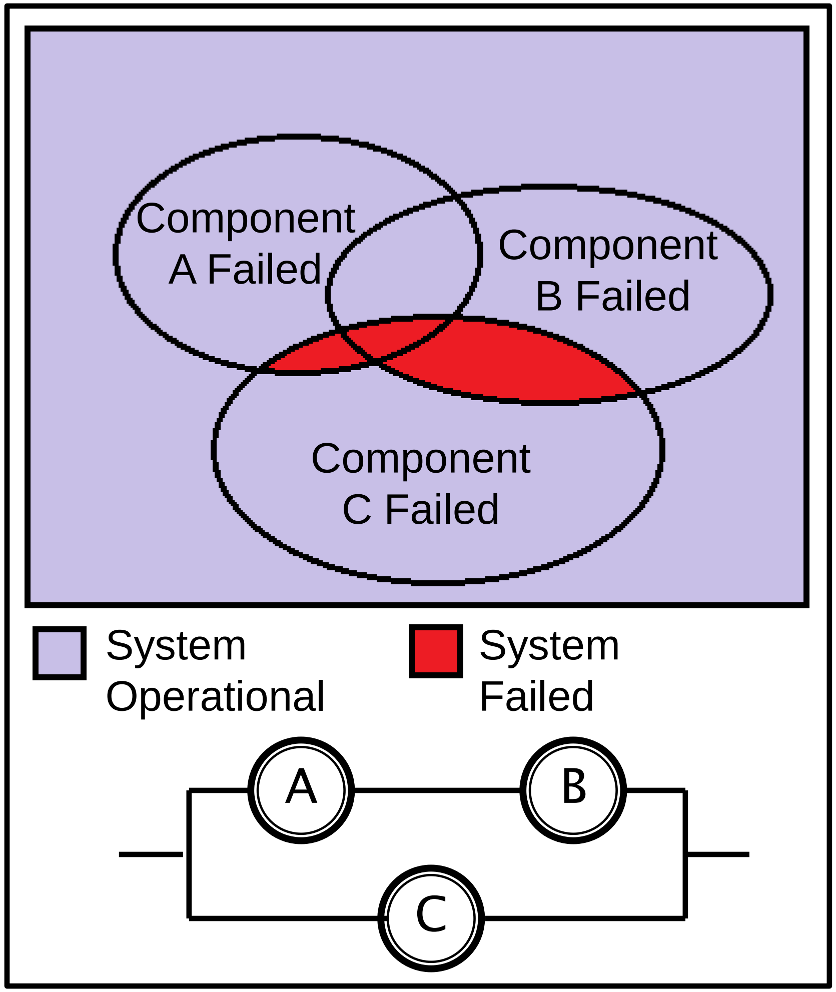

Figure 1: System Reliability

Monte Carlo Simulation

- For more complex systems or systems with component repair, Monte Carlo simulation using the component failure and repair probabilities can be used to determine system reliability
- A large number of individual simulations or *trials* are performed using a random number generator and the Cumulative Distribution Function (CDF) for the system, to determine the time of the next failure/repair until the system fails, or time *t* is reached
- As individual components fail or repair, they change *state*, which refers to the status of the component
  - 1 for operable, and 0 for failed
- The status of the entire system can be simplified to a single state vector, composed of 1's and 0's
  - [1 1 1] would represent all three components operating properly
  - [1 0 1] represents A and C operating while B has failed

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Monte Carlo Simulation (cont.)

- When a system reaches a state that causes failure, also known as a *cut set*, the individual trial is usually ended
  - For the system shown in figure 1, the cut sets are [1 0 0], [0 1 0], and [0 0 0]
- The system reliability is then obtained simply by dividing the number of trials that reached time *t* without system failure, by the total number of trials performed

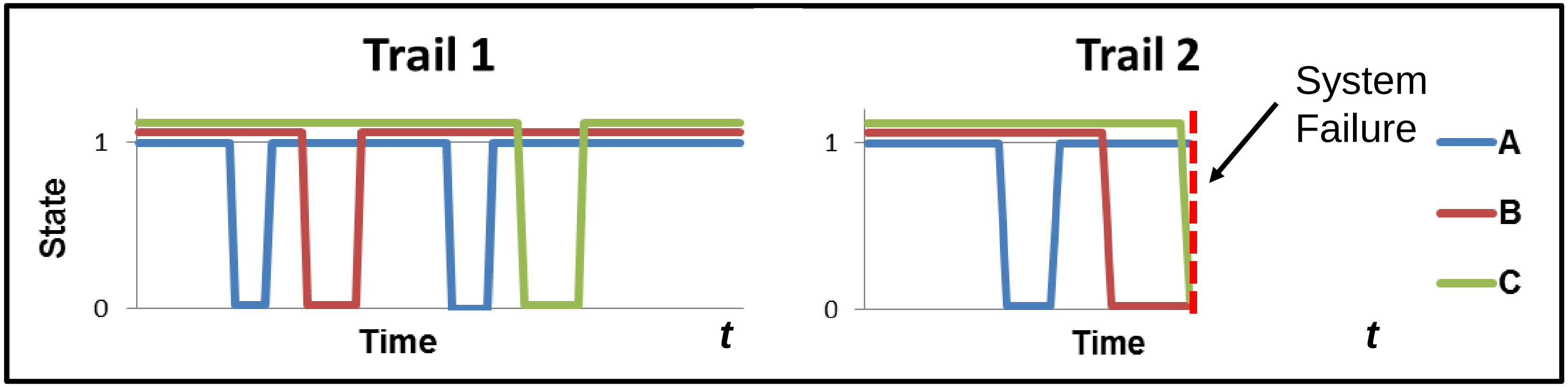

Figure 2: Monte Carlo Simulation

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Analog Standard Monte Carlo Limitations

- This simple implementation of Monte Carlo simulation, which I'll refer to as the *Analog Standard Method,* provides a high level of accuracy in a short time for most systems
- When individual system state probabilities are also needed, or for systems with a high level of reliability, the computational requirements increase substantially
- The accuracy of the simulation can be determined using the relative standard deviation from Equation 1, where:
  - P is the probability of the state
  - N is the total number of trials performed

$$\sigma_r = 100 * \sqrt{\frac{1-P}{P * N}}$$

Equation 1: Relative Standard Deviation

Analog Standard Monte Carlo Limitations (cont.)

- As the number of components *n* grows, the time required to perform each trial increases due to the increased probability of state changes
- With each additional component, the number of possible system states double, diluting the number of trials that reach each individual state
- For systems with low failure probabilities on the order of 10^{-6}, standard Monte Carlo simulations running at 10^4 attempts per second require approximately 278 hours of processing to reach a 1% relative standard deviation
- In order to reduce the relative standard deviation by half, the total number of trials performed must be increased by a factor of 4
- These large computing requirements can be overcome either by brute force with parallel computing, or through methods that increase the number of trials that reach rare system states

Increasing Accuracy Through Variance Reduction

- The accuracy of a Monte Carlo simulation is tied to the variance of the results
- In order to decrease the error without increasing the computational time, the variance of the results can be reduced by increasing the number of trials that reach rare states, or by reducing the average time per trial
- **Forced Events Biasing** – The probability of a failure is artificially increased to produce a corresponding rise in the probability of reaching rare states
- **Transition Biasing**– The probability of failure in the next event is increased, while the probability of a component repair is decreased, to more frequently reach states with multiple failures
- **Roulette** – Trials determined to have a low probability of reaching rare states are ended early and a new trial is initiated

Maintaining a Fair Game

- A *fair game*, where final probability values are not influence, must be maintained by each variance reduction method, and is achieved by weighting trials whose probabilities are modified
- When the probability of an event is increased and occurs, the weight of the trial is reduced by the reciprocal of the change in probability
- When the event does not occur, the weight is increased by the reciprocal of the change in probability that the event would not occur
- When a trail is rouletted, a random selection is used to determine if the trial is ended
- If the roulette attempt is successful, then a new trial begins, but if the roulette attempt fails, the trial weight is increased
- Regardless of the technique used, the expected value for each trial must remain 1

Particle Physics Simulation

- Particle physics simulations of neutron and gamma radiation also utilize Monte Carlo simulation to determine radiation dose rates
- Each trial begins with a particle created by a radioactive source, and is randomly assigned a direction and an energy, based on the source's properties
- The particle is then tracked along its path, using material and particle property information to determine when it interacts with the material it is traveling through
- In the event of an interaction, the particle is then either absorbed by the material, ending the trial, or assigned a new direction and energy based on the interaction

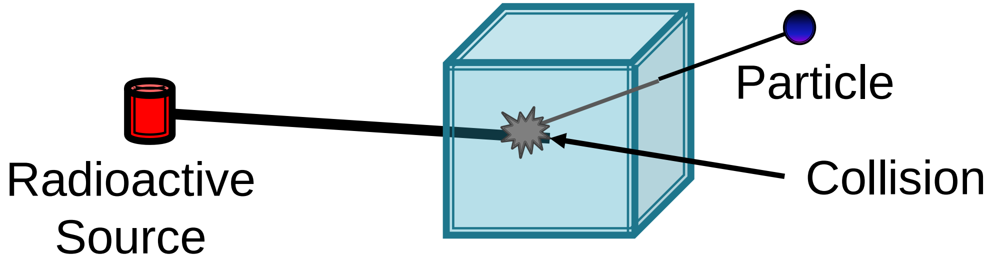

Figure 3: Particle Interaction

Variance Reduction in Particle Physics

- Virtually all radiation sources that warrant particle physics simulations are enclosed within shielding, which causes the majority of trials to be absorbed before exiting the shielding and reaching an area of interest
- This significantly increases the computing time required to perform radiation simulations, and led to the development of variance reduction techniques similar to those performed in system reliability simulations
- The major difference between the methods is that particle physics variance reduction can be governed by simple geometric values such as distance and direction

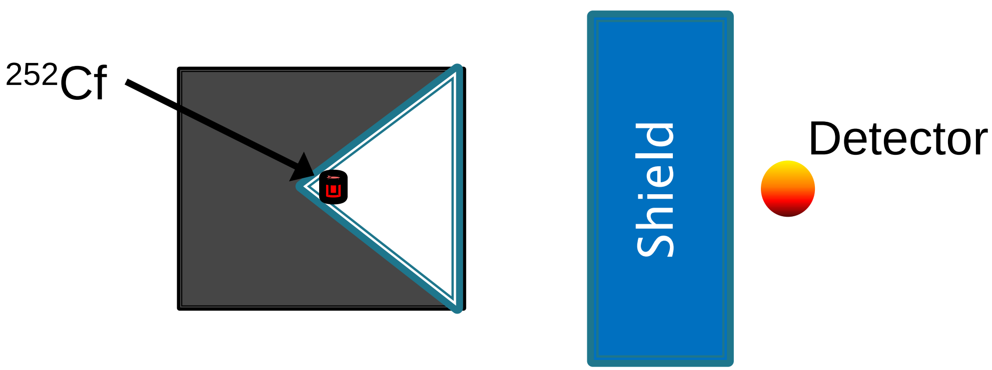

Figure 4: Particle Simulation

Variance Reduction in Particle Physics (cont.)

- **Collision Biasing** – Similar to Forced Events Biasing in that the probability of interaction with the material the particle is traveling though is adjusted up or down based on the need
- **Sample Biasing**– Similar to Transition Biasing in that the probability of sampling a particle direction or energy upon collision is modified
- **Source Biasing** – Equivalent to Sample Biasing, in that energy and direction of particles generated at the beginning of each trail are modified
- **Roulette** – Identical to the implementation in system reliability, cutting trials short based on low likelihoods of reaching rare states
- **Splitting** – As a particle enters an area near a rare location, it is split equally into smaller particles to increase the probability that one or more will reach it

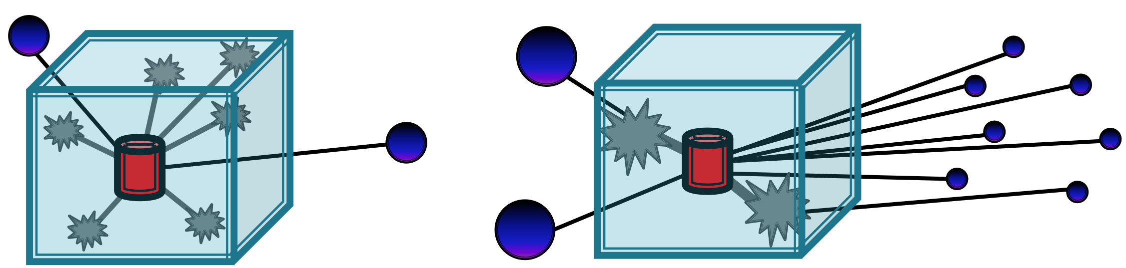

Figure 5: Particle Variance Reduction (before and after)

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Area Particle Physics Simulations

- For radiation simulations where specific locations of interest are unknown or so numerous that they fill the space being simulated, a mesh methodology is implemented that determines the average dose within small subdivisions (or elements) of the entire area of interest
- The distance traveled by a particle within each element of the mesh is recorded and used to determine the average dose rate within the volume, as shown in Figure 6
- Since all locations are now potentially important, traditional variance reduction techniques are difficult to apply

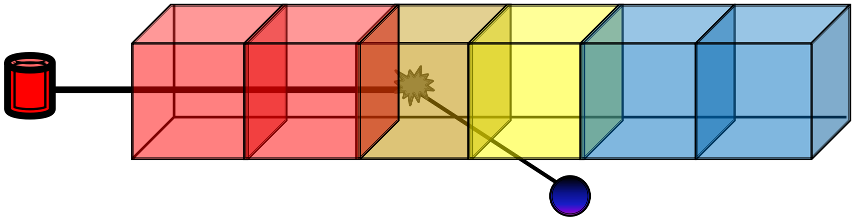

Figure 6: Mesh Tracking

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Global Variance Reduction in Particle Physics

- In order to reduce processing time when performing mesh simulations, global variance reduction techniques are required to:
  - Determine which elements are rare
  - Identify which trials have the best probability of reaching rare elements
  - Reduce computational time requirements
- First, a deterministic routine is implemented that approximates the probability of particles reaching each element, identifying which elements are rare
- This is used to develop an importance map that estimates the probability of particles, within each element, reaching rare elements
- During the standard simulation, the importance map is then used to determine which variance reduction, if any, to use as the trial enters each element

System Reliability in 3D Space

- In order to implement particle physics style global variance reduction methods on system reliability simulations, system reliability equivalents for location and direction must be developed
- By arranging the system reliability by time (broken into a total number of time steps *S*), by number of failed components, and individual system state, a 3D representation for the simulation is created
- Each system reliability trial can now be represented as a particle moving through different materials, with a collision probability equivalent to the combined failure/repair rate for the element

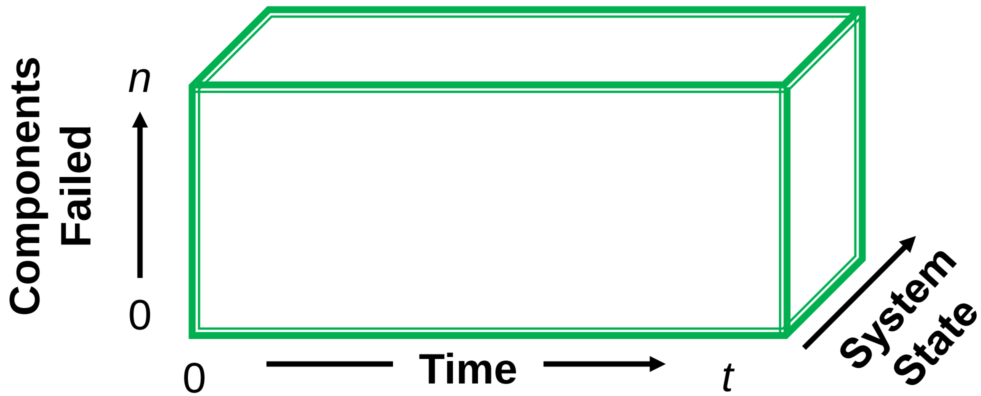

Figure 7: System Reliability in 3D

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System Reliability in 3D Space (cont.)

- Using this methodology, the example system simulation from Figure 1 becomes the following

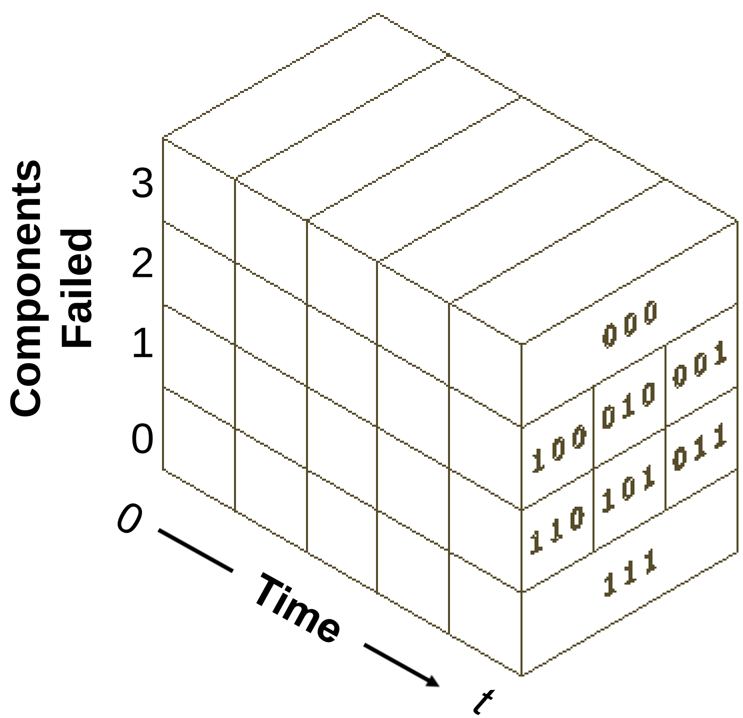

Figure 8: Example in 3D

Comparison of 3D to Standard

- Information can be stored on individual simulation elements to provide the user with greater insight into the system over time, including:
  - Probability of entering an element
  - Average time spent in an element
  - Frequency of entering an element
- The simulation can be performed without knowing which states cause a system failure or a precise value for the final time *t*, allowing it to be performed earlier in the system design phase
- System maintenance requirements and dependability can also be determined without the need for additional simulations
- Due to the additional information being saved during the simulation, the 3D method requires additional computational time however

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System Reliability in 3D: Example

- Simple visuals that can easily be created to better understand the system from a reliability standpoint:

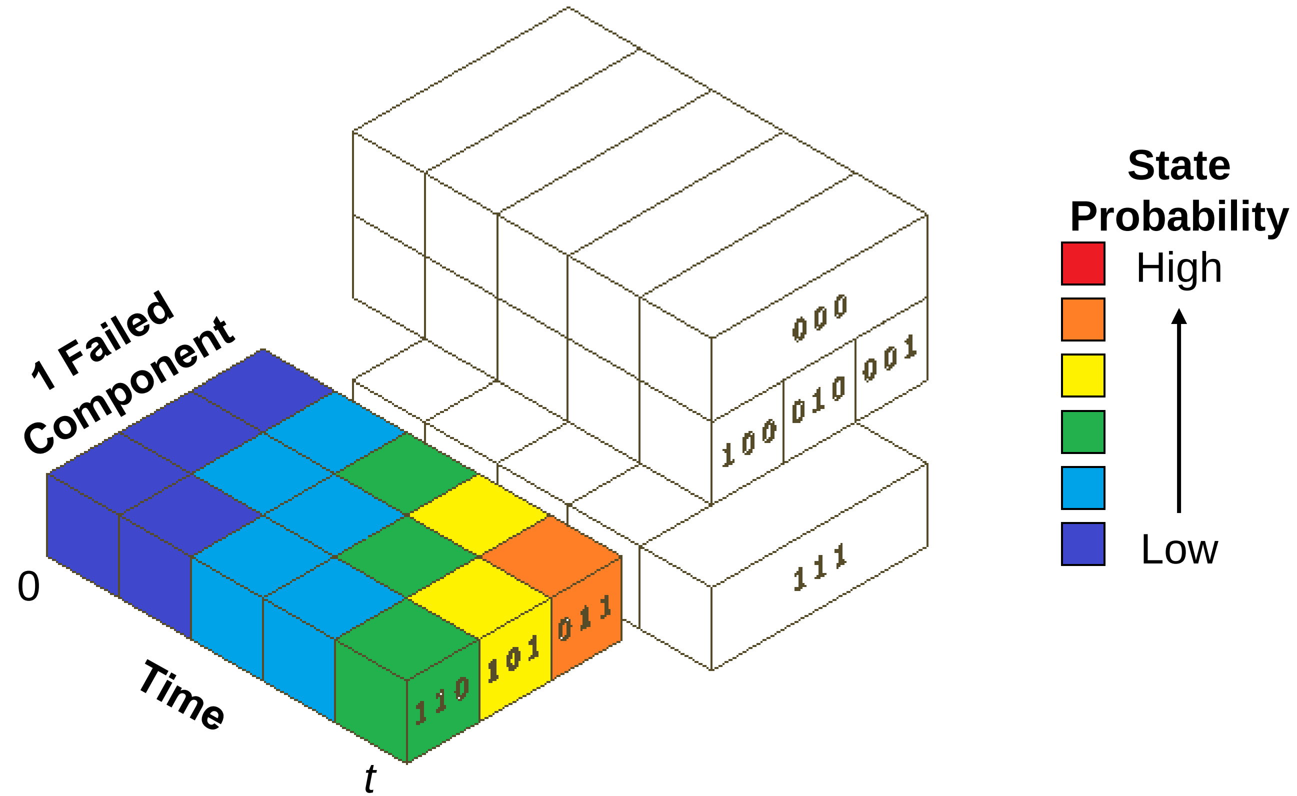

Figure 9: Single Component Rates

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System Reliability in 3D Example (cont.)

- Expected system states within sub-time intervals can be obtained

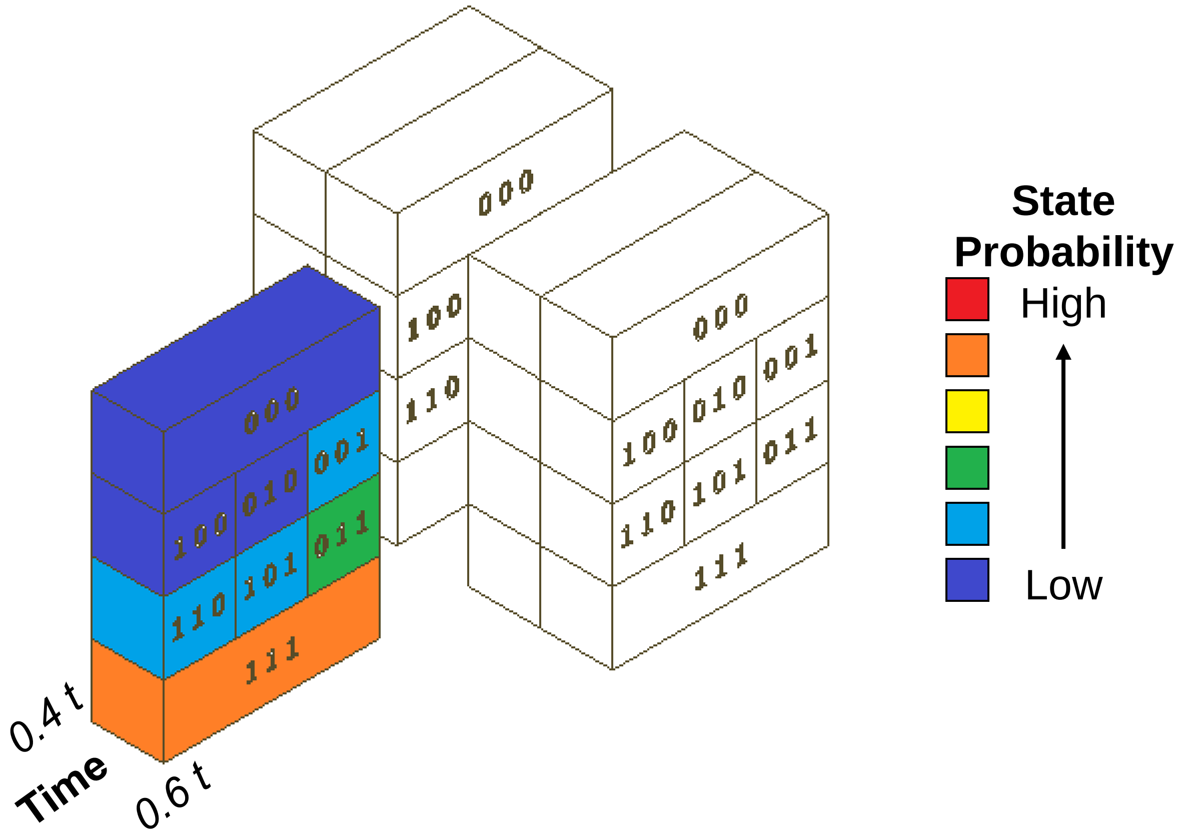

Figure 10: System State 0.4 to 0.6 *t*

Global Variance Reduction for System Reliability Simulation

- Now that the reliability simulation can be expressed in terms of a particle traveling through material, a variation of the particle physics method for global variance reduction can be applied to decrease the computational time required
- By applying global variance reduction techniques, the user ensures that system states or elements are not overlooked, and no initial knowledge of failure states or rare states is required
- The overall steps of implementing the method remain equivalent to the particle physics procedure, but the process of implementing these steps changes greatly due to the differences between the two simulations
- Since all trials begin in the same location with the same "direction," variance reduction techniques may be applied prior to the initial probability estimate

Initial Variance Reduction

- First, the probability of interaction within each element is determined using known failure and repair rates associated with each element
- For elements with no failed components, Forced Events Biasing is applied to modify the probability of interaction to the value found using Equation 2, where *i* is the position of the element as counted from time = 0
  - This ensures that each element without failed events is "seeded" with an equal number of interactions, helping to get trials to reach all elements
- Weighting factors to maintain a fair game are also calculated for each of these elements

$$Modified\ Probability = \frac{1}{S + 2 - i}$$

Equation 2: Modified Probability of Elements Without Failed Components

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Estimation of Element Trial Probabilities

- Next, an estimation of the probability of trials reaching each element is made using a deterministic algorithm
- The percentage of trials that interact with the first element [1 1 1] in the time line are split equally among the elements directly above, using Transition Biasing, while the portion that do not interact “time out” and are applied to the next element in the state
- Weighting factors are calculated for each element, to account for the Transition Biasing
- This process continues for each of the elements, moving up level by level, until the top state of all failures [0 0 0] is reached
- The process then begins again with the next element in [1 1 1], and continues until an estimate is made for all elements

Importance Map Creation

- The estimated trial probabilities found are then used to generate an importance map to determine when additional variance reduction methods are needed
- Starting with the last element in the fully failed state [0 0 0] and working down, each element is checked to determine if it is predicted to be rare or semi-rare, based on the user inputs
- If a rare state or semi-rare state is identified, its neighbors below, that feed into it, are tagged with either a 2 for important, or 1 for semi-important, depending on whether the element is rare or semi-rare
- This is repeated from top to bottom, working backward along the time line, until all elements are checked, and all neighbors to rare and semi-rare states are tagged

Simulation and Applying Additional Variance Reduction Techniques

- The simulation is now performed using additional variance reduction techniques applied based on the importance assigned to the elements
- As each trial enters an element, the importance map is checked to determine the new importance level and whether or not it is changing
- If the importance levels are zero the simulation continues as normal
- If the new importance level is 1, Collision Biasing is applied to increase the probability of a collision, based on a user-supplied value
  - If a collision occurs, the weight is decreased accordingly
  - If the trail times out into the next element, the weight is increased accordingly
- If the new importance level is 2, Collision Biasing is applied along with Splitting, separating the trial's current weight equally into a user-defined number of sub-trials
- If the importance level changes from 1 or 2 down to 0, a Roulette is performed to attempt to end the trial early

Current Status of Research

- A program to benchmark final results has been completed that utilizes the analog standard Monte Carlo process to determine system state probabilities and system reliability
- Trials using the 3D method of System Reliability simulation have begun, and are demonstrating results equivalent to those obtained using the analog standard Monte Carlo method
- The program that applies global variance reduction to the 3D System Reliability simulation is currently under development and will be used to:
  - Demonstrate the robustness of the methodology
  - Verify the accuracy of the results
  - Determine the reduction in calculation time

Questions?

Thank you for attending

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