Why Impacted Yarns Break at Lower Speed Than Classical Theory Predicts

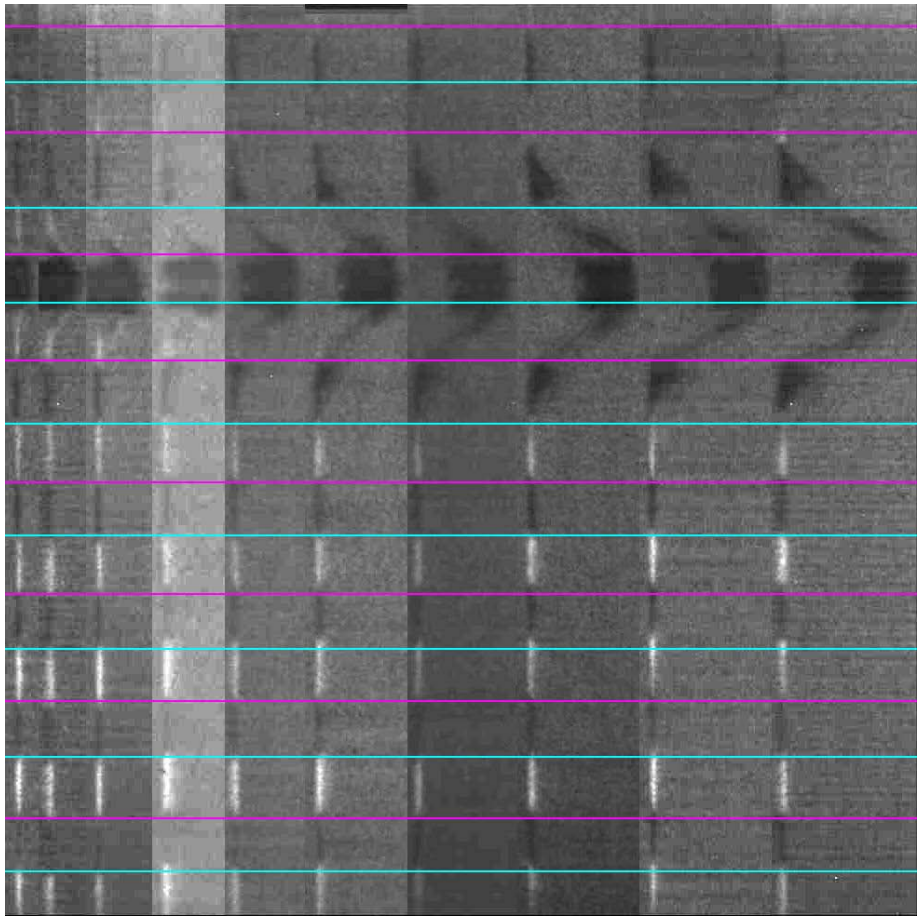
James D. Walker and Sidney Chocron

Southwest Research Institute

San Antonio, Texas 78228

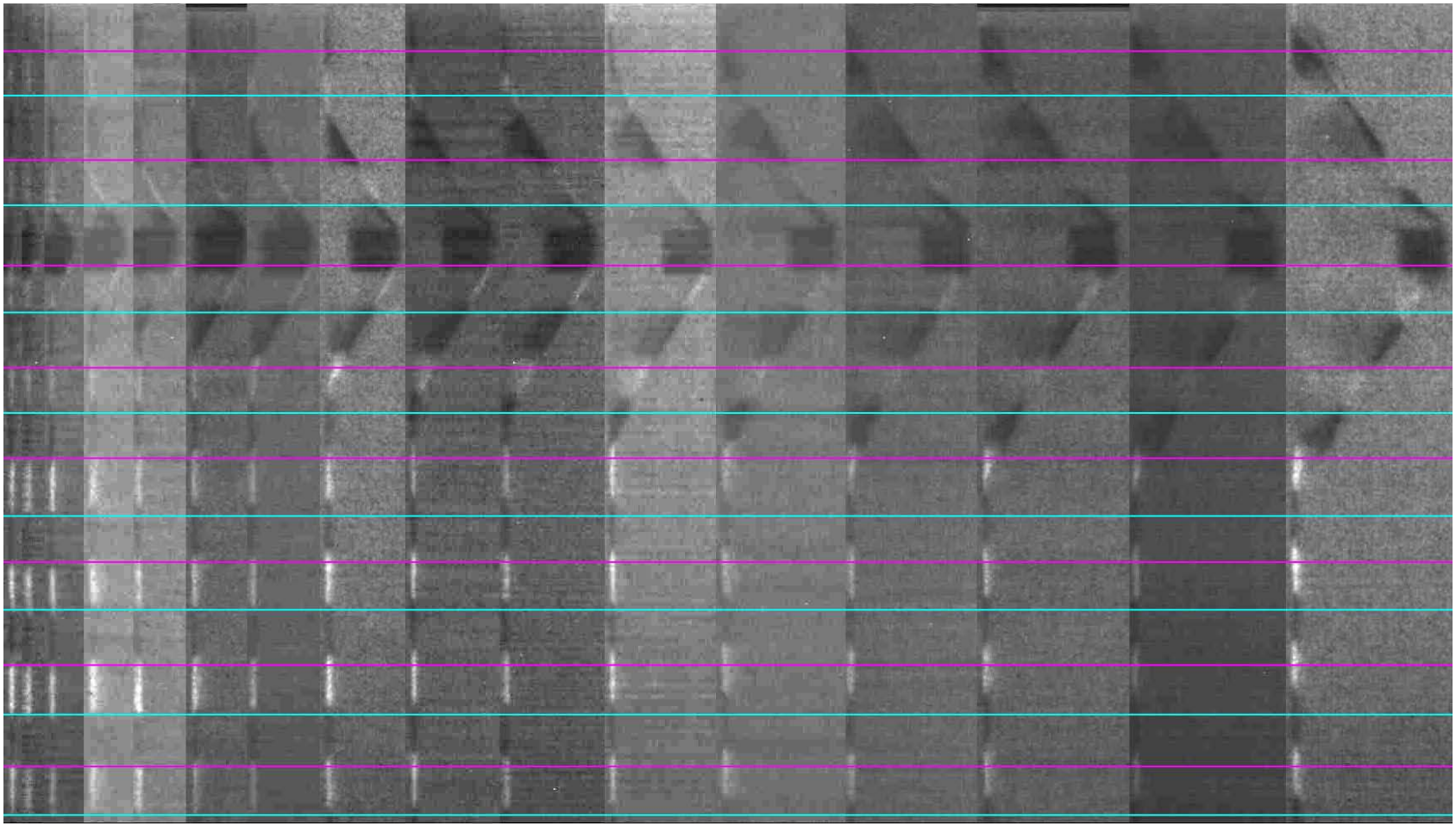
j:\engdyn\jdw\aps2001i.ppt

Genesis of the Talk



- In September 2010 in Quebec City at PASS, Harm van der Werff of DSM pointed out to Sidney Chocron the fact in the title.
- Sidney returned to Southwest Research Institute and confirmed with already existing SwRI data that it was indeed the case.
- Sidney told James about it – James felt we had the analytic tools to understand the effect.
- In particular, the authors had included the dynamic deformation of yarns in their Advanced Solid Mechanics course they were teaching at the University of Texas at San Antonio that semester.

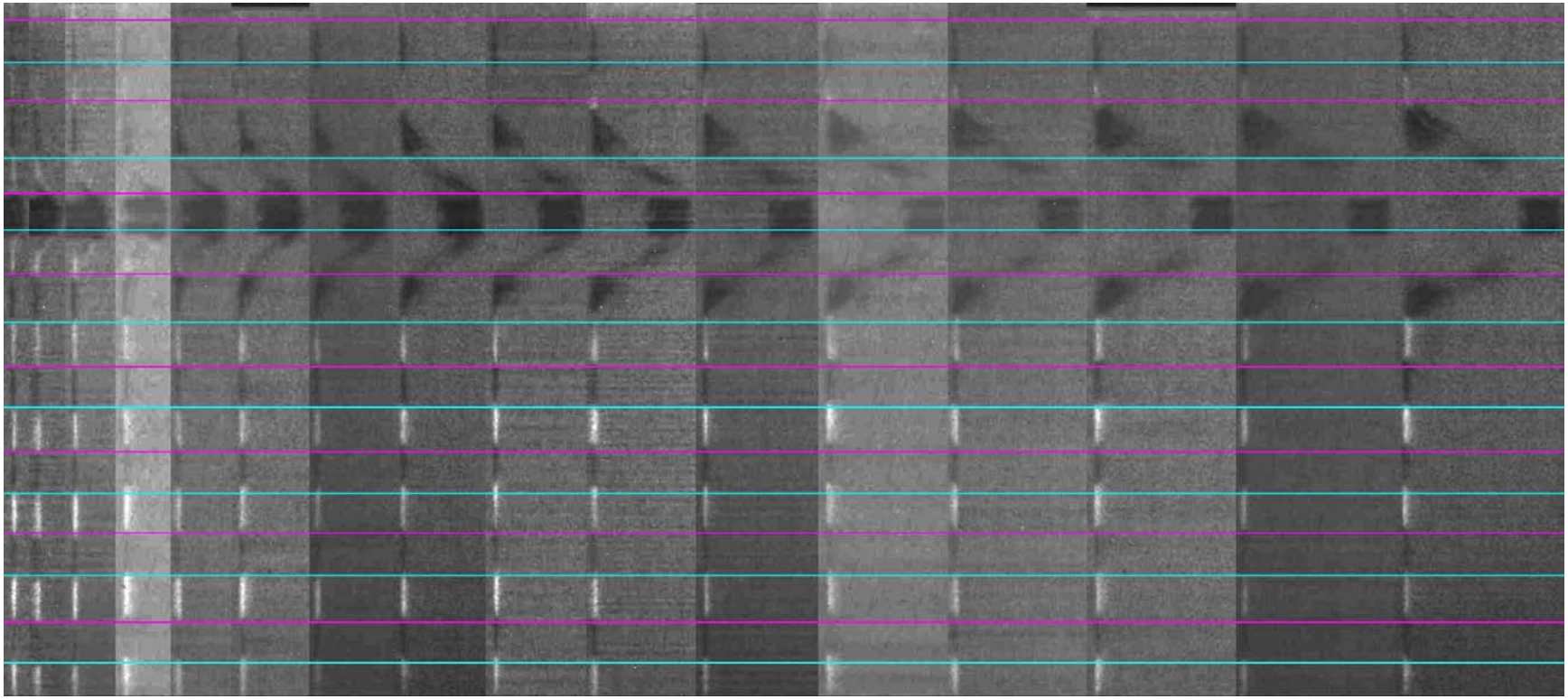
Yarn 12 – Dyneema – 517m/s
4 μs per frame



No failure

3

Yarn 11 – Dyneema – 583m/s
4 μs per frame



Immediate failure

4

Fact Observed by Other Researchers

1970: **Wilde, Ricca, Cole and Rogers**. Dynamic response of a constrained fibrous system subjected to transverse impact Part 1 – Transient responses and breaking energies of nylon yarns. Technical Report, Army Materials and Mechanics Research Center, Watertown, Massachusetts, 02172. Report no. AMMRC TR 70-32.

1982: **Carr**. Failure mechanisms of yarns subjected to ballistic impact. *Journal of Material Science Letters*, 18(7):585-588.

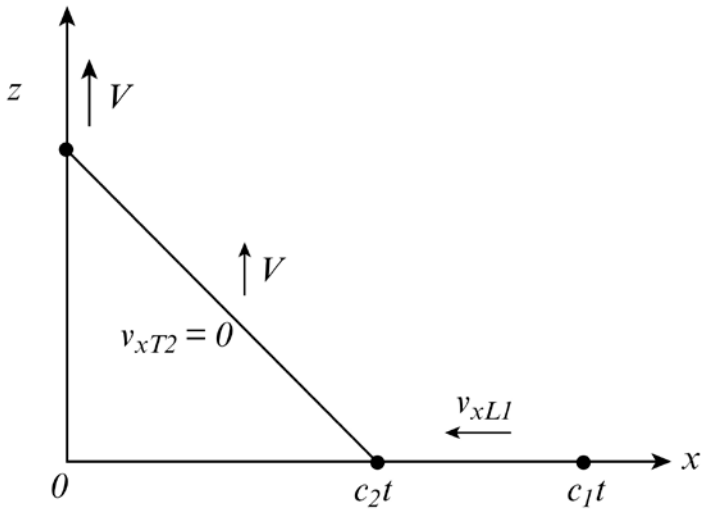
1990: **Field and Sun**. A high speed photographic study of impact on fibres and woven fabrics. In SPIE Vol. 1358, 19th International Congress on High-Speed Photography and Photonics.

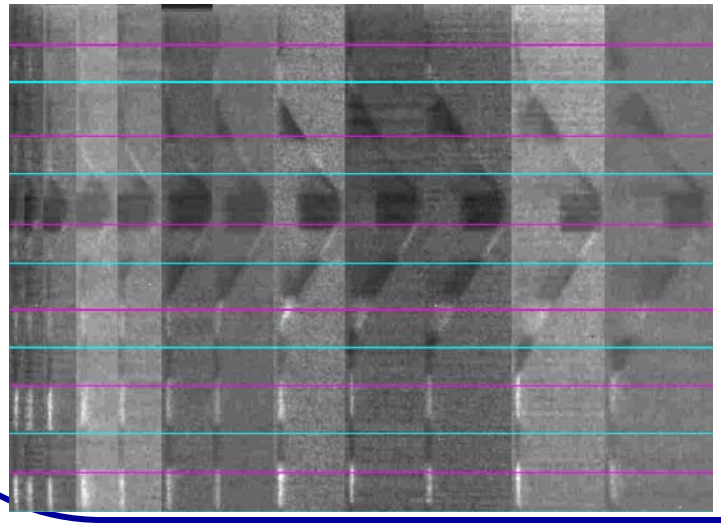
1992: **Wang, Field and Sun**. Dynamic behaviour of pre-stressed high strength polymeric yarns transversely impacted by a blade. In Proceedings of the Int. Symp on Intense Dynamic Loading and Its Effects, pages 354-359. Chinese Society of Theoretical and Applied Mechanics, Sichuan Univ. Press, Chengdu, China.

2001: **Bazhenov, Dukhovskii, Kovalev and Rozhkov**. The fracture of SVM aramide fibers upon a high-velocity transverse impact. *Polymer Science*, Ser. A, 1:61-71. [Had an explanation, which did not seem to work.]

2010: **van der Werff and Heisserer**. Personal communication.

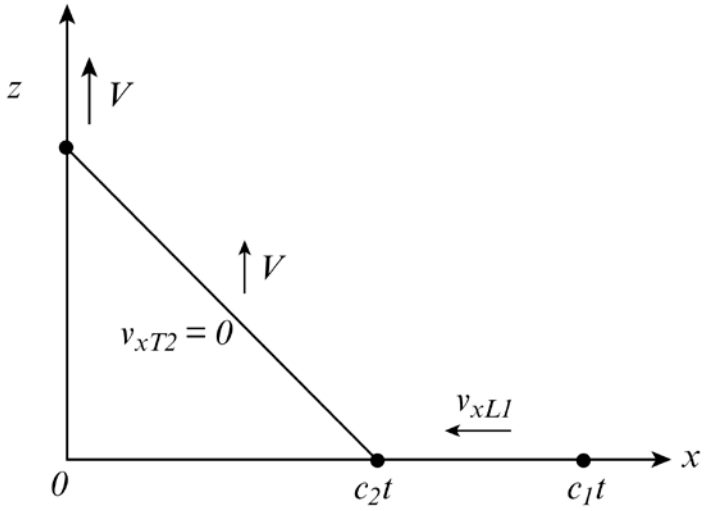
Classical Theory of Smith





- The classical theory of yarn deformation developed by Smith and published in 1958 dealt with a yarn that was deformed by a perpendicular motion at one end.
- Two waves develop:
  - A tensile longitudinal wave in the yarn, and
  - A transverse wave in the yarn.
- It is possible to explicitly solve for these waves.

New Approach to Classical Theory



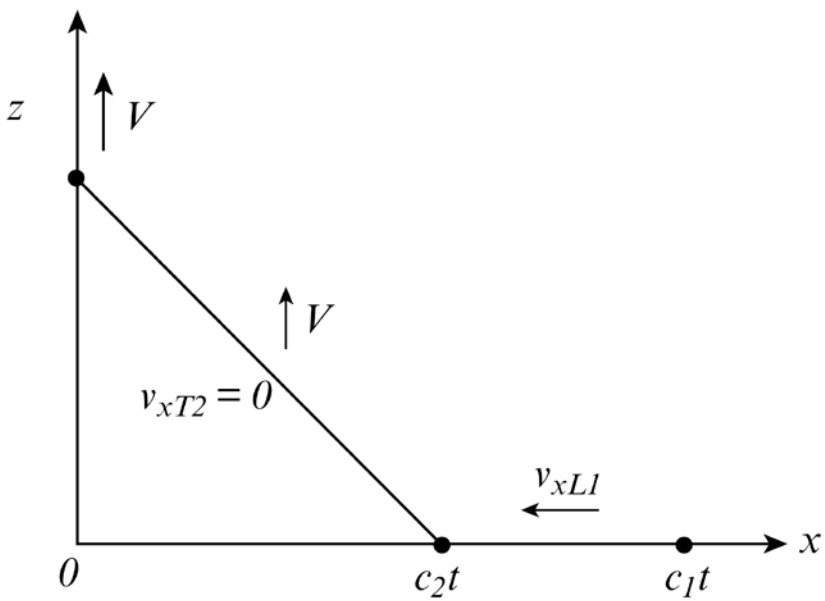
$$\sum_{j=1}^{3} \frac{\partial S_{ji}}{\partial x_j} + \rho_0 b_i = \rho_0 \frac{\partial v_i}{\partial t} = \frac{\partial}{\partial t}(\rho_0 v_i)$$

$$[-S_{ji}] = c[\rho_0 v_i]$$

$$S = \frac{\rho_0}{\rho} F^{-1}\sigma \quad \text{where} \quad F = \delta_{ij} + \frac{\partial u_i}{\partial x_j}$$

- A new approach to solving the equations for the deformation of the yarn is to do it in the original, undeformed frame of reference of the yarn.
  - Yields an exact solution, even for large strains.
  - Framework for solving the harder problem.
- The stress when written in this reference frame is referred to as the Piola-Kirchhoff stress.
- In particular, we use what is referred to as the first Piola-Kirchhoff stress tensor, which is not symmetric.
- The stress is the force with respect to the undeformed area.
- The wave motion can be solved by the Hugoniot jump conditions at the two wave fronts (the longitudinal and the transverse).

The Yarn is Assumed Linear Elastic



- The yarn deformation is assumed to be linear elastic, based on energy, with a constant modulus E and a strain to failure ε_f. The conjugate variable to the first Piola-Kirchhoff stress is the transpose of the deformation gradient, F^T.

$$e = \frac{1}{2} E (E_{11})^2$$

$$(E_{ij}) = \frac{1}{2}(F^T F - I)$$

$$S_{ij} = \frac{\partial e}{\partial F_{ij}^T} = \frac{\partial e}{\partial F_{ji}}$$

$$S_{11} = \frac{\partial e}{\partial F_{11}} = E E_{11} F_{11}, \quad S_{13} = \frac{\partial e}{\partial F_{31}} = E E_{11} F_{31}$$

- In these expressions, the first Piola-Kirchhoff stress has been written in terms of the deformation gradient F and the Lagrangian (Green) strain E_{ij}.

The Exact Solution

- Solving with the linear stress-strain relation, assuming a boundary condition of an applied tangential velocity at one end of the yarn, yields an exact solution (this applies even to large strain), most easily expressed in terms of strain in the deformed section of yarn E_{11}:

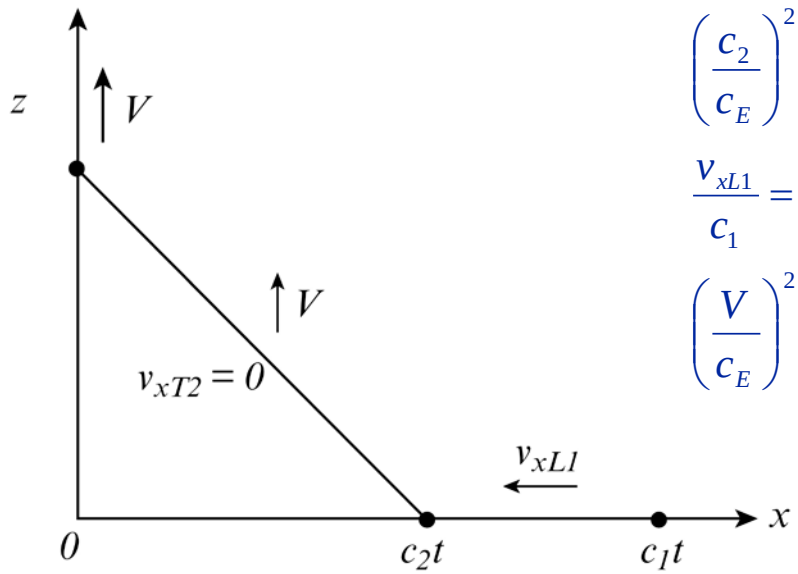
$$\left(\frac{c_1}{c_E}\right)^2 = \frac{1}{2} + \frac{1}{2}\sqrt{1+2E_{11}} + E_{11},$$

$$\left(\frac{c_2}{c_E}\right)^2 = E_{11},$$

$$\frac{v_{xL1}}{c_1} = 1 - \sqrt{1+2E_{11}},$$

$$\left(\frac{V}{c_E}\right)^2 = \sqrt{E_{11}}\left(\sqrt{1+2E_{11}} - 1\right)\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1+2E_{11}} + E_{11}}$$

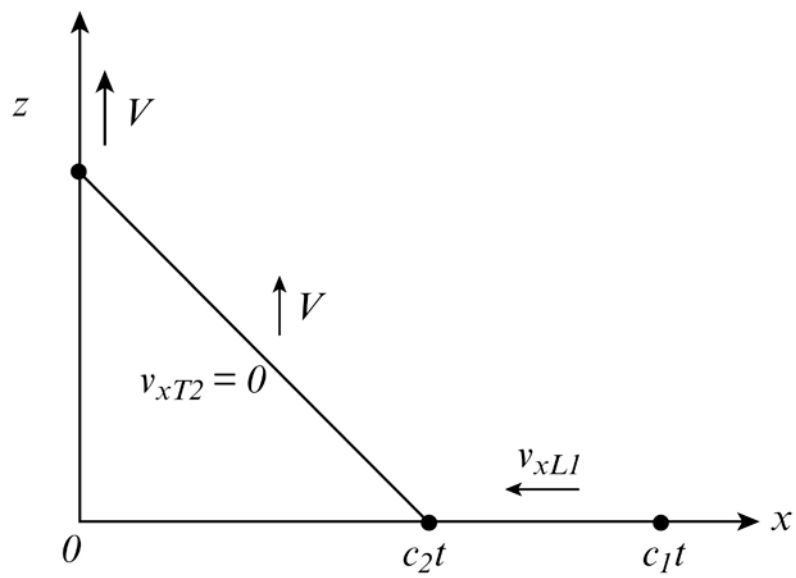
$$\times\left\{2\sqrt{1+2E_{11}} + \left(1-\sqrt{1+2E_{11}}\right)\sqrt{\frac{1}{2E_{11}} + \frac{\sqrt{1+2E_{11}}}{2E_{11}} + 1}\right\}.$$



For Small Strain, the Exact Solution Reduces to Smith's Solution

$$\frac{c_1}{c_E} = 1, \quad \frac{c_2}{c_E} = \sqrt{E_{11}}, \quad \frac{v_{xL1}}{c_1} = -E_{11}, \quad \frac{V}{c_E} = (E_{11})^{3/4}\sqrt{2-\sqrt{E_{11}}}.$$

$$V / c_E = \sqrt{\varepsilon(2\sqrt{\varepsilon(1+\varepsilon)} - \varepsilon)}$$



- The experimental transverse wave speed, which is a function of impact velocity and the longitudinal wave speed in the yarn, has been shown to agree with this result for various yarns.
- The natural idea is to assume that during impact the projectile speed that will break a yarn is given by setting $E_{11} = \varepsilon_f$.
- However, that doesn't agree with experiment.

Experimental Impact Speeds that Break Yarn do not Agree with Classical Theory

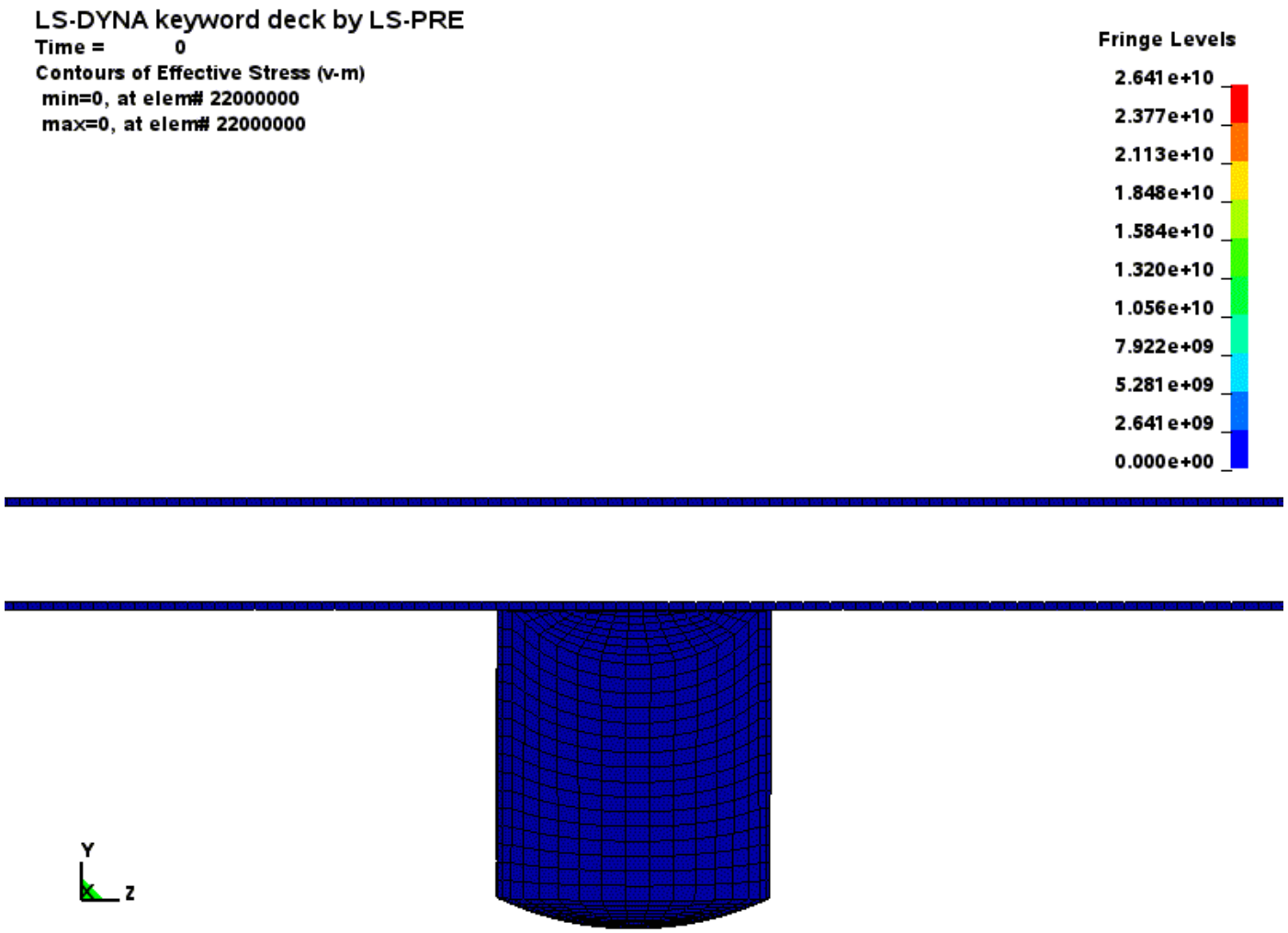
- These are experiments performed at SwRI by Chocron, et al. (reported Tuesday in the first general session).
- Clearly there is a large discrepancy.

Yarn	Longitudinal sound speed in the yarn	Yarn breaking strain ε_f	Breaking speed from experiment [Chocron, et al.]	Breaking speed from classical theory
	(km/s)	(%)	(m/s)	(m/s)
KM2 S5705	7.45	4.25	Between 621 and 634	934
Dyneema SK-65	9.89	3.60	Between 517 and 583	1100
PBO	10.7	3.25	Between 523 and 610	1105

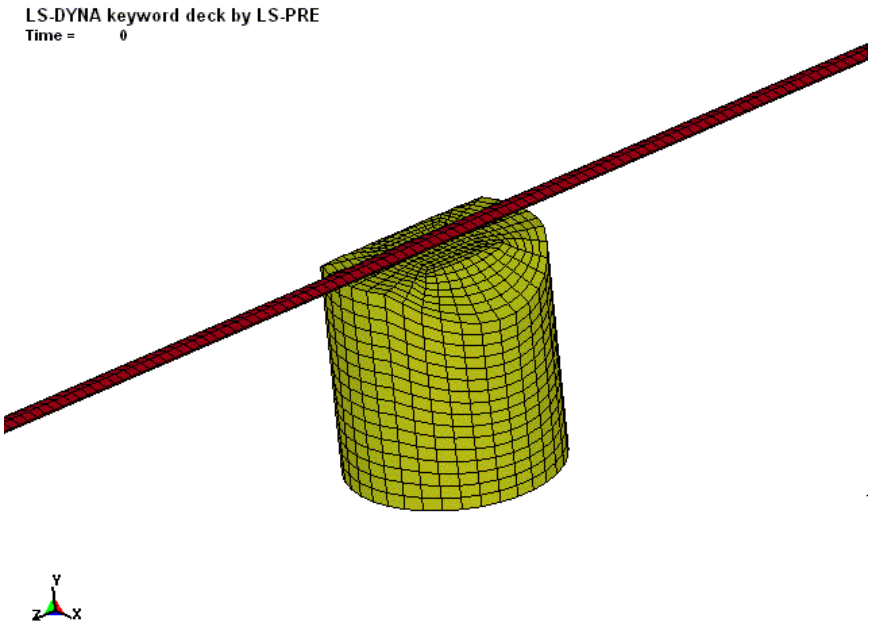
Why is there a Discrepancy?

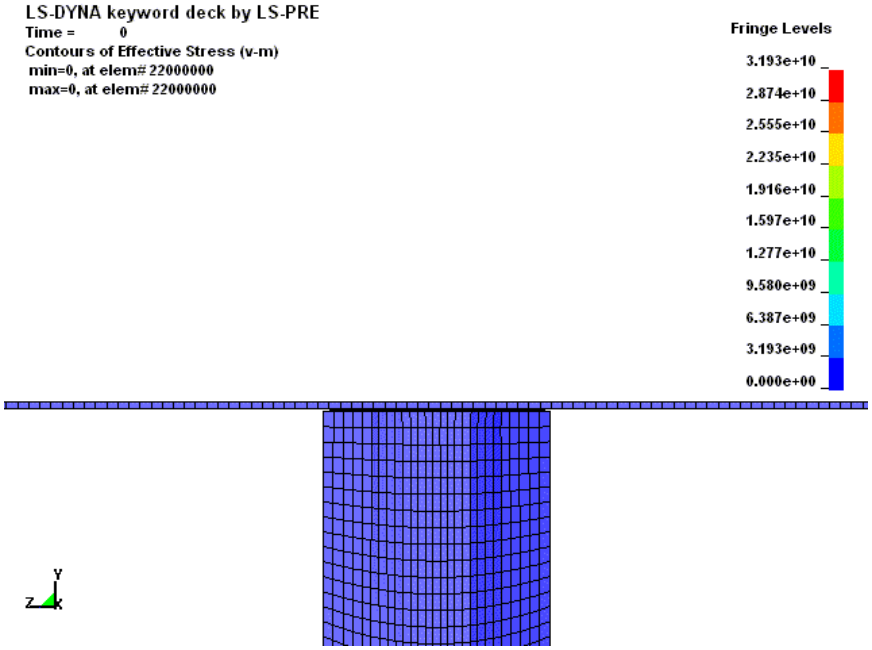
- If the classical theory agrees for transverse wave speed, why does it not agree for the impact speed that breaks the yarn?
- The reason is that for a the boundary condition that occurs during impact of a flat-faced projectile is not the same as the applied boundary condition from the classical (Smith) theory.
- However, for a small projectile and when the yarn does not break, a few microseconds after impact the boundary condition approaches the Smith boundary condition.

The Initial Boundary Condition is Not the Same (impact speed 555 m/s)

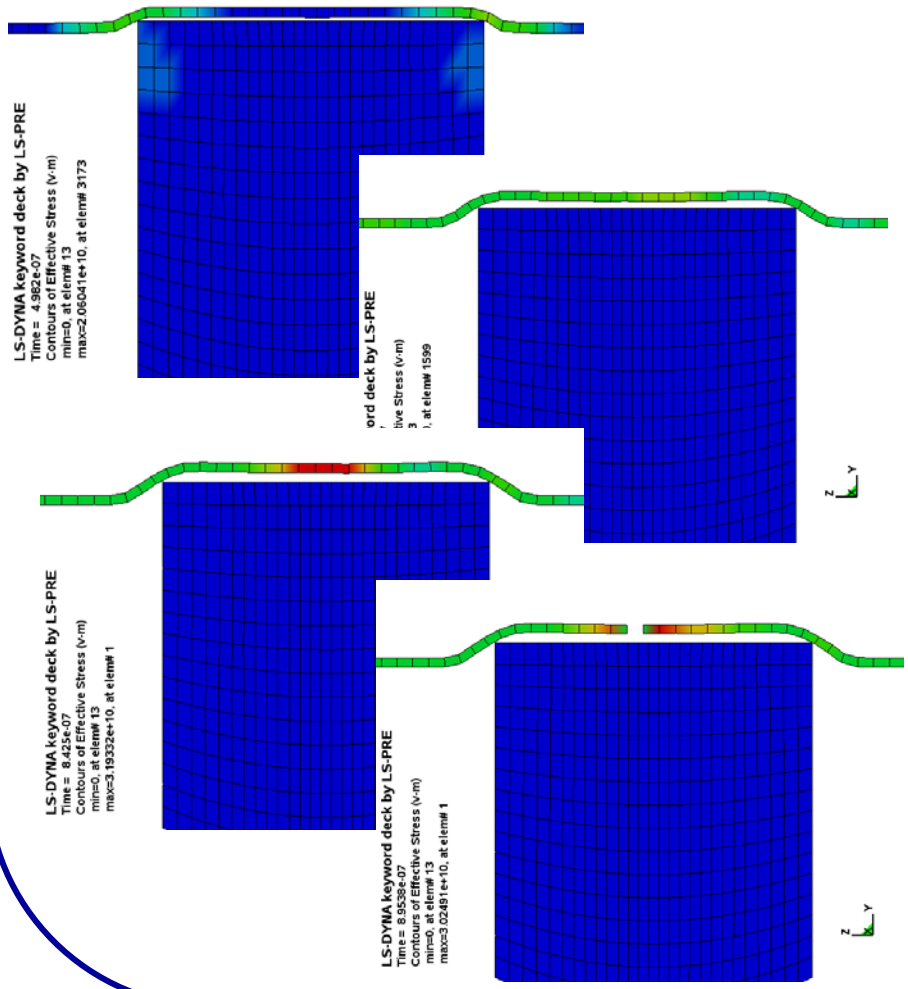


For A Higher Impact Speed, the Yarn Breaks ALMOST Immediately



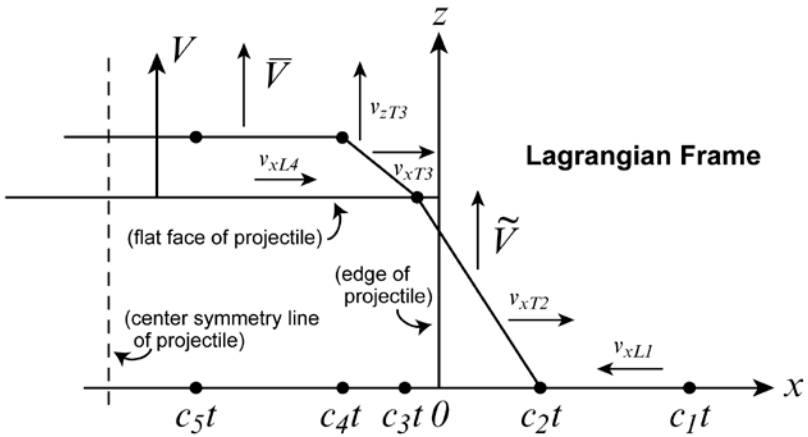


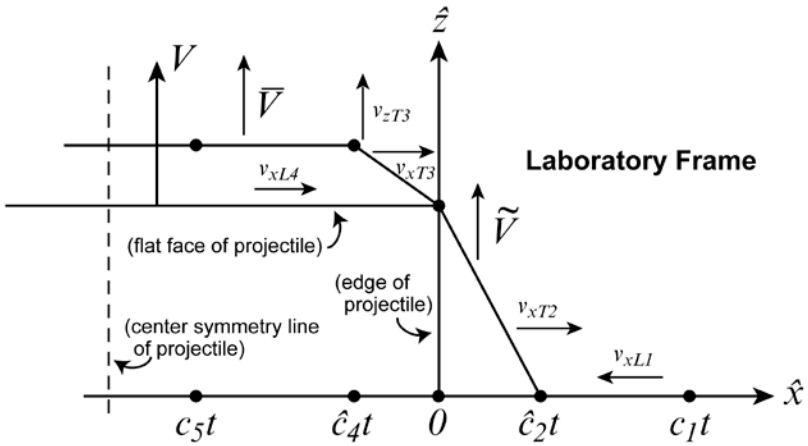
Flat-face Projectile Impact on Yarn



- The flat projectile face strikes the yarn.
- The yarn bounces off the surface in the direction of impact, at twice the impact speed for an elastic impact.
- From both sides where the yarn touches the projectile, longitudinal and transverse waves emanate along the yarn in both directions.
- Two longitudinal waves meet above the center of the projectile, doubling the stress and strain.
- If the stress and strain are high enough, the yarn breaks.

Solving for the More Complicated Condition





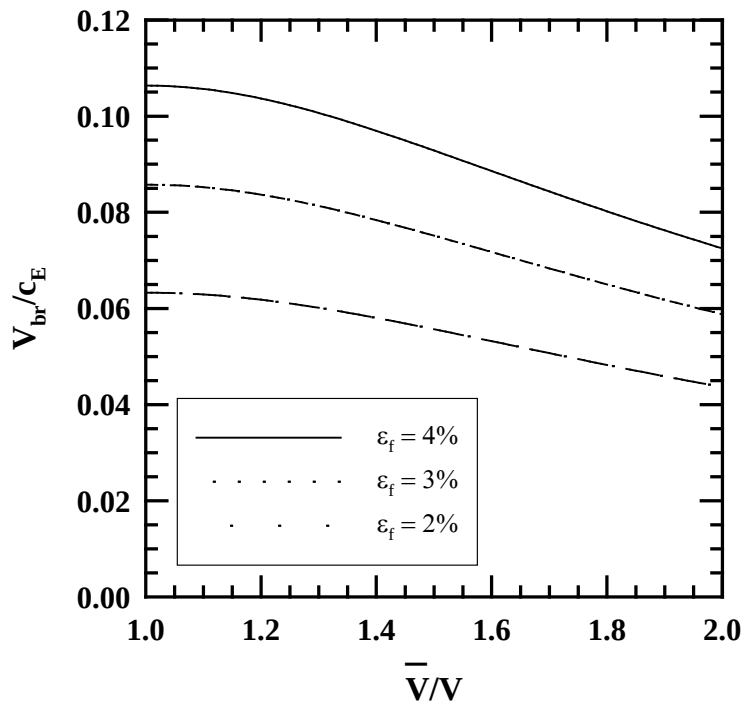
- We focus on just one side of the projectile.
- Four waves emanate along the yarn from that point, two longitudinal waves and two transverse waves.
- The particle velocity and wave speed of these fronts can be explicitly determined by solving the Hugoniot jump condition at each wave front.

The Assumptions

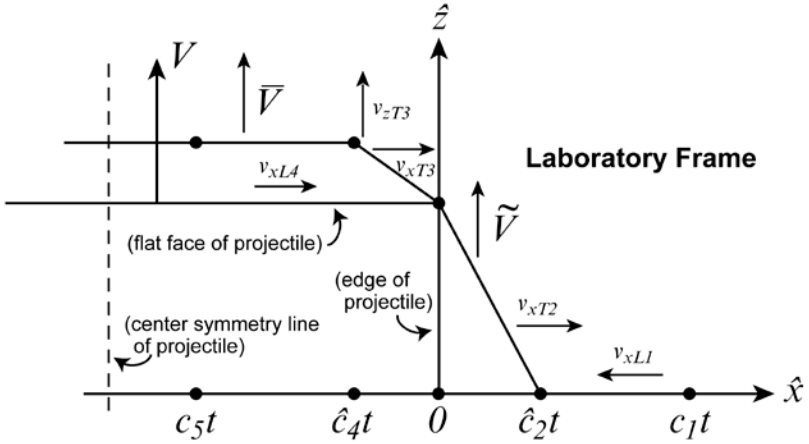
- We include the fact that the yarn “bounces” off the face of the projectile in varying amounts, from no bounce (stays on surface) to elastic bounce (twice the impact speed). Yarn bounce plays a significant role in the yarn breaking speed.
- We assume that the strain in the yarn does not change where it touches the edges of the projectile, thus yielding the result that strain is uniform in the deformed region of yarn.
- The previous assumption allows us to use a geometric approximation to determine the strain in the yarn.
- The strain in the yarn due to the initial impact doubles when the waves in the yarn meet from the opposite edges of the projectile.

The Solution

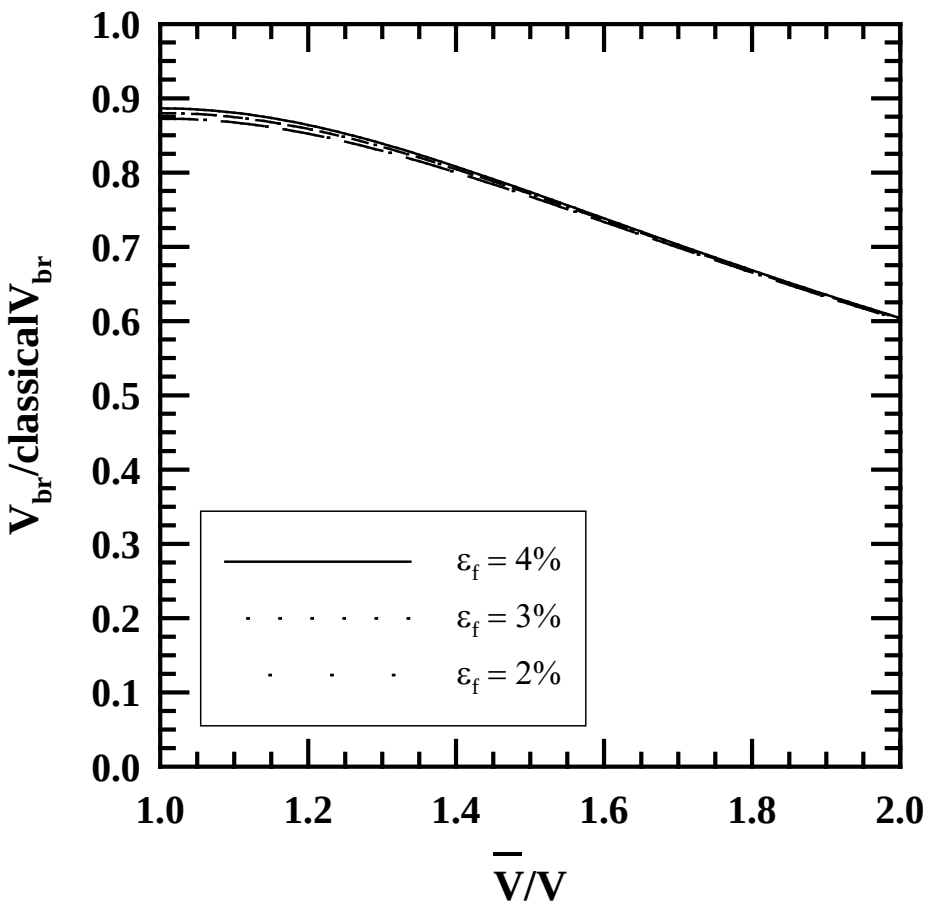
$$E_{11}^{5/2}(2-\sqrt{E_{11}}) = \frac{E_{11}}{2}\left\{\left(\frac{V}{c_E}\right)^2 + \left(\frac{\overline{V}-V}{c_E}\right)^2\right\} - \frac{1}{16}\left\{\left(\frac{V}{c_E}\right)^2 - \left(\frac{\overline{V}-V}{c_E}\right)^2\right\}^2$$



- This equation relates the yarn bounce, the impact speed, and the strain in the yarn.
- If we set $E_{11} = \varepsilon_f$ in this equation, then the resulting $V = V_{br}$ is the impact speed that breaks the yarn.



The Solution



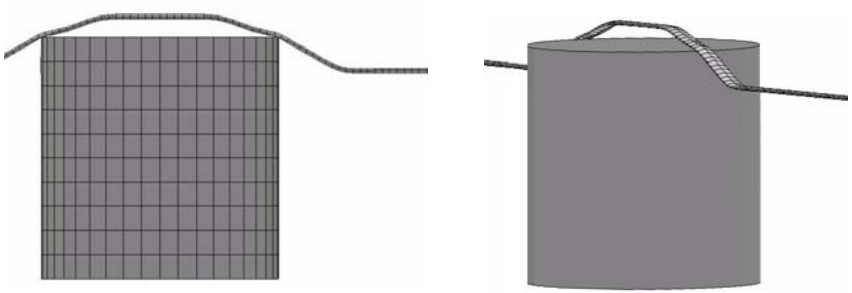
- This plot shows that the decrease in velocity for no bounce is 11% - thus, this is the largest possible breaking speed due to impact with a flat-faced projectile.
- If the bounce is at twice the impact velocity (an elastic bounce) then the decrease in impact speed that breaks the yarn is 40%.

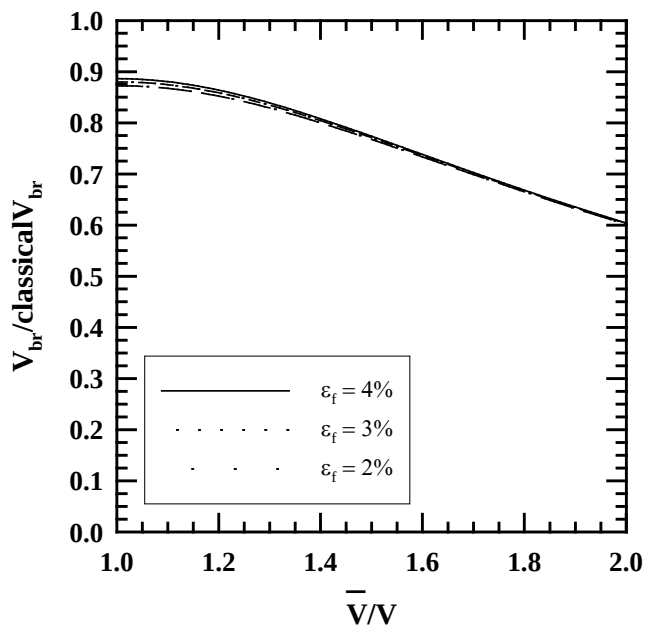
Comparison with Data

Yarn	Breaking speed from experiment [Chocron, et al.]	Breaking speed from classical theory	Breaking speed with no bounce	Breaking speed with elastic bounce
	(m/s)	(m/s)	(m/s)	(m/s)
KM2 S5705	Between 621 and 634	934	829	565
Dyneema SK-65	Between 517 and 583	1100	972	664
PBO	Between 523 and 610	1105	973	666

- Thus, we are have reasonable agreement for an elastic bounce.
- As an aside, computations with LS-DYNA give the yarn breaking speeds as 557±3 m/s, 672±3 m/s, and 692±3 m/s, showing that the analytical model and DYNA agree.

Conclusions





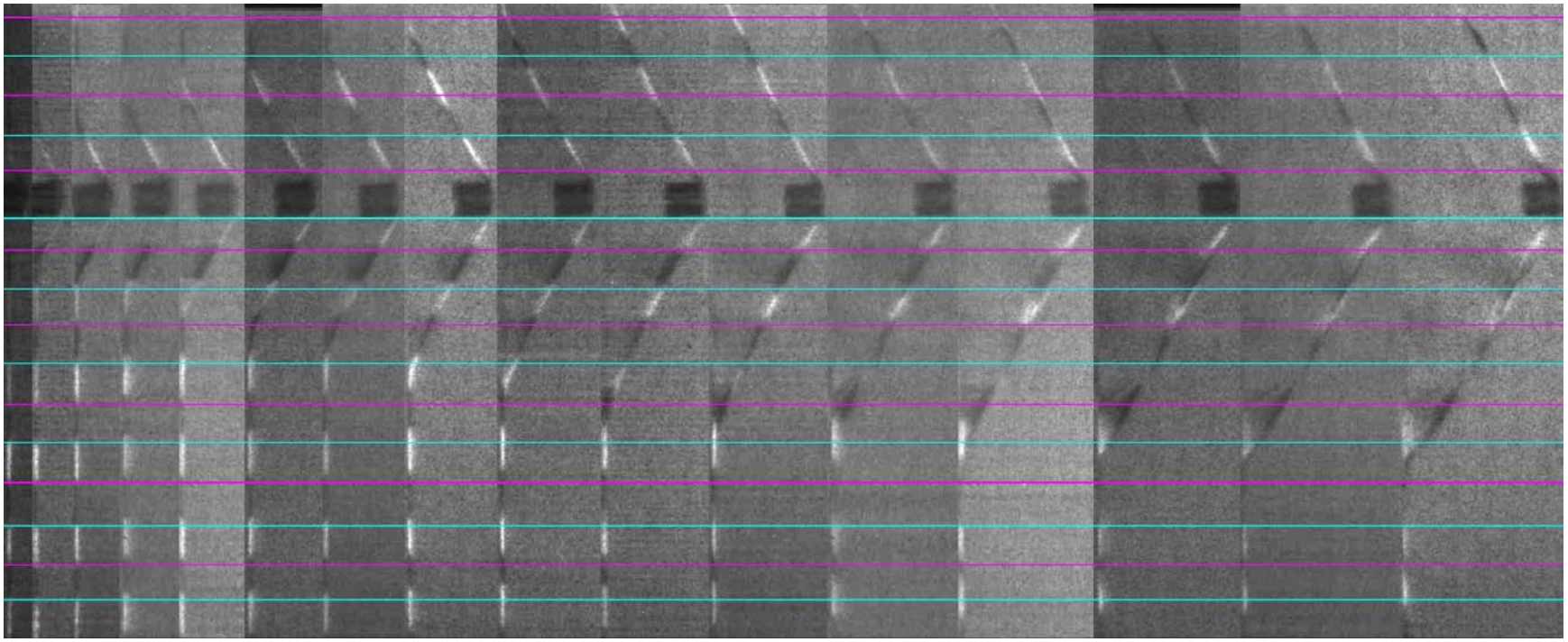
- The boundary condition when a flat-faced projectile strikes a yarn is not the same as applying a velocity to the end of the yarn.
- Working through the details yields a reduction in yarn breaking impact speed of 11% to 40%.
- These new predicted yarn breaking speeds values have reasonable agreement with experiment.

Acknowledgements

- The discrepancy in yarn breaking impact speeds between classical theory and experiment was originally pointed out to the authors at the PASS conference by
  - Harm van der Werff of DSM Dynemma (Geleen, The Netherlands) and
  - C. Leigh Phoenix of Cornell University (Ithaca, New York, USA).
- There was some discussion of this effect with van der Werff, Phoenix, and Uli Heisserer, also of DSM Dyneema.
- The LS-DYNA computations were performed by Trenton T. Kirchdoerfer (SwRI).

Extra Slides

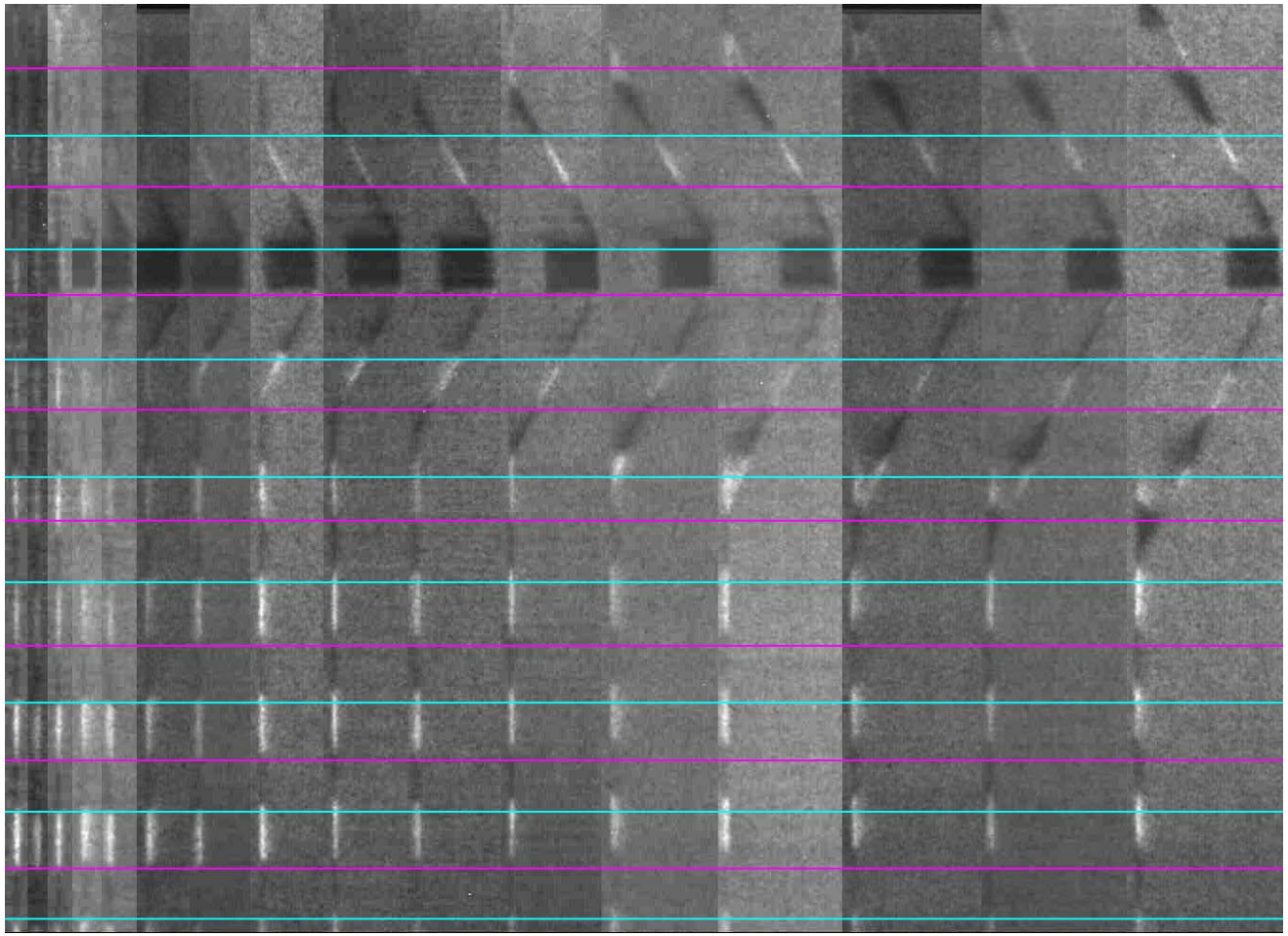
Yarn 03 – Dyneema – 477m/s
5 us per frame



No failure

24

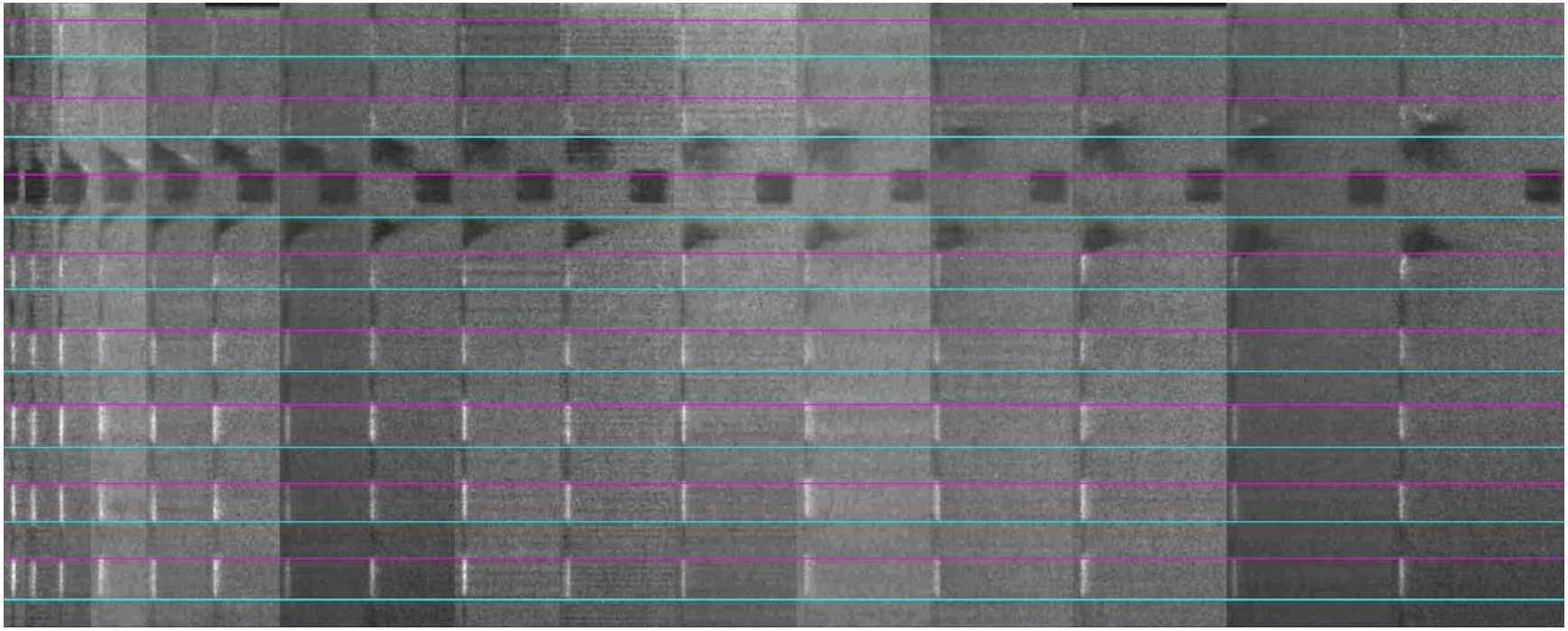
Yarn 06 – Dyneema – 474m/s
4 us per frame



No failure

25

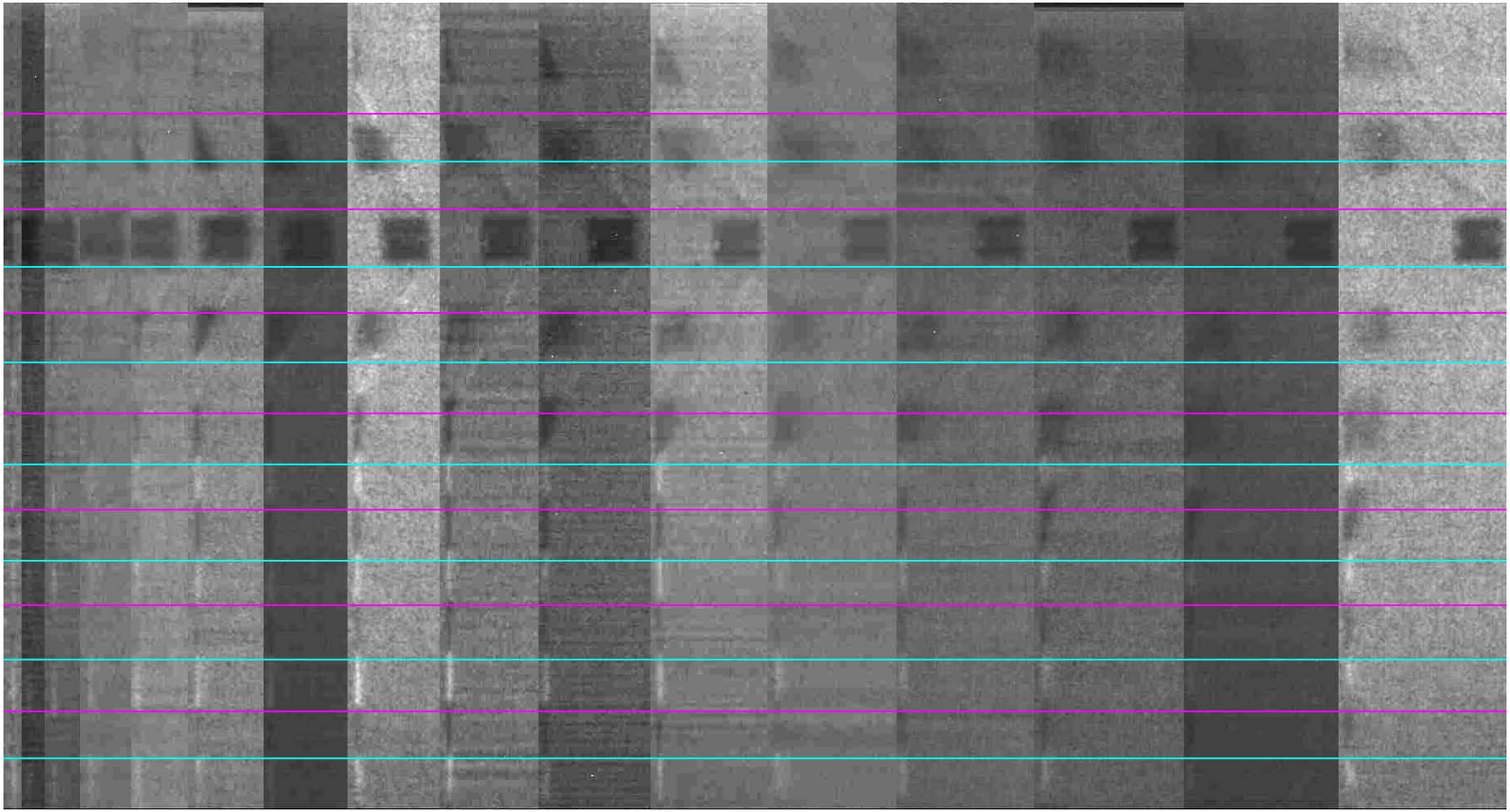
Yarn 09 – Dyneema – 672m/s
4 us per frame



Immediate failure

26

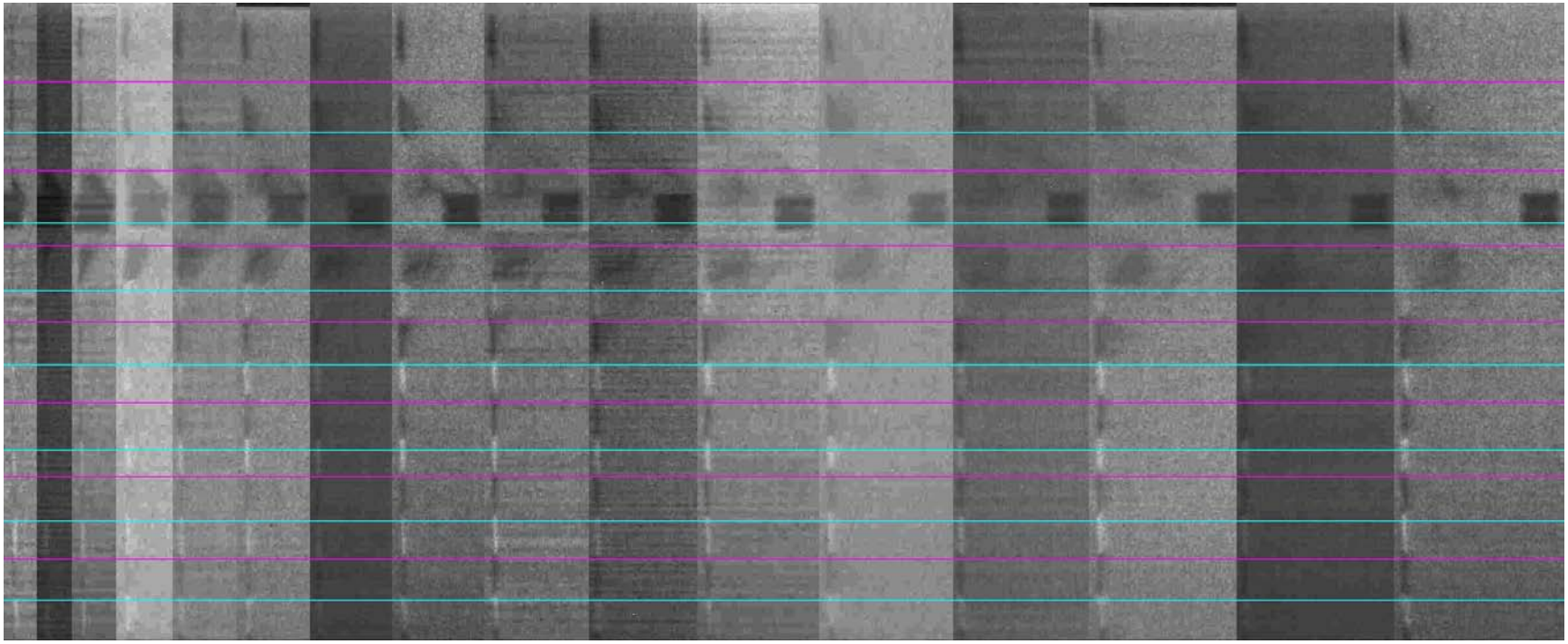
Yarn 13 – PBO – 523m/s
4 us per frame



No failure

27

Yarn 18 – PBO – 610m/s
4 us per frame



Immediate failure

28

End of Slides