MIXED-INTEGER CONIC LINEAR PROGRAMMING - CHALLENGES AND PERSPECTIVES

TAMAS TERLAKY

LEHIGH UNIVERSITY

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Final Report

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AIR FORCE RESEARCH LABORATORY
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AIR FORCE MATERIEL COMMAND
## Mixed-Integer Conic Linear Programming-Challenges And Perspectives

**Fundamental Disjunctive Conic Cut (DCC) methodology for Mixed-Integer Conic Linear Optimization (MISOCO) was developed.** To describe the convex hull of the intersection of a convex set $E$ and a linear disjunction is at the fundamental problem, and that served as the core of solution techniques for Mixed Integer Conic Optimization (MICO). It was proved that if there exists a cone $K$ that has the same intersection with the boundary of the disjunction as the convex set $E$, then the convex hull is the intersection of $E$ with $K$. While uniqueness of a DCC is proved for general MICO, the existence of such a cone is difficult to prove for the general case. Thorough analysis of a parametric family of quadrics all to prove the existence and uniqueness of a second order cone, when $E$ is the intersection of an affine space and a second order cone. An efficiently computable method for finding that cone, which provided novel and powerful DCCs for MISOCO that may be used in branch-and-cut algorithms when solving MISOCO problems. All special and degenerate cases are carefully analyzed and easy to compute criteria are developed to compute the DCC for all cases. Limited, but thorough computational experiments gave strong indication of the power of the DCCs.

### Mixed integer conic optimization; Conic optimization; Discrete optimization; Disjunction; Disjunctive conic cut.
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PI: Tamás Terlaky, Lehigh University, Bethlehem, PA

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Summary:

Fundamental Disjunctive Conic Cut (DCC) methodology for Mixed-Integer Conic Linear Optimization (MICO) was developed. To describe the convex hull of the intersection of a convex set $E$ and a linear disjunction is the fundamental problem, and that served as the core of solution techniques for MICO. It was proved that if there exists a cone $K$ that has the same intersection with the boundary of the disjunction as the convex set $E$, then the convex hull of the disjunction is the intersection of $E$ with $K$. While uniqueness of a DCC is proved for general MICO, the existence of such a cone is difficult to prove for the general case. Thorough analysis of a parametric family of quadrics allows to prove the existence and uniqueness of a second order cone, when $E$ is the intersection of an affine space and a second order cone. An efficiently computable method was developed for finding that cone, which provided novel and powerful DCCs for Mixed Integer Second Order Cone Optimization (MISOCO), which can be used in branch-and-cut algorithms when solving MISOCO problems. All special and degenerate cases are carefully analyzed and easy to compute criteria are developed to compute a DCC for all cases. Limited, but rigorous computational experiments gave strong indication of the power of the DCCs.

Research Results:

In this project the foundations of developing a comprehensive computational theory for efficient solution of MICO and MISOCO problems were established. A comprehensive family of efficiently computable Disjunctive Conic Cuts (DCCs) and Disjunctive Cylindrical Cuts (DCyC) for MISOCO problems. The fundamental idea is that instead of adding linear constraints as customary in Mixed Integer Linear Optimization (MILO), we add Second Order Cones (SOCs) for tightening the relaxed MISOCO problem obtained when one relaxes the integrality constraints.

The first step was to investigate properties and algebraic descriptions of parametric families of quadrics which have fixed intersections with two given hyperplanes. Both the case when the two hyperplanes are parallel and the case when they are nonparallel were studied under...
the assumption that the feasible set of the SOCO problem is bounded. Later the theory was extended to the case of unbounded intersections as well. Under mild assumptions, we proved that the family of quadrics with the desired properties can be described with only one parameter. Further, we demonstrated that the quadrics are evolving as the parameter changes, and we gave efficiently computable procedures to compute the parameter value which provides the unique second order cone, or cylinder that contains the convex hull of the disjunction.

Then, using the above outlined theory of quadrics, under realistic assumptions, we developed novel, valid disjunctive conic and cylindrical cuts for MISOCO. We showed that under mild conditions one can find a unique second order conic cut $K$ which gives the convex hall of the disjunctive sets. Additionally, we presented a procedure for finding $K$. By some simple examples we have demonstrated that our DCC is different from Atamturk and Narayan’s ”nonlinear conic mixed-integer rounding inequality”. This comparison demonstrates that our DCC is new, and frequently stronger than the Nonlinear conic mixed-integer rounding inequality.

This way we have made the first complete development of an efficiently computable family of DCC’s and DCyC’s for MISOCO. The fundamental theory is summarized in paper [1]. The existence of the disjunctive cut for general convex cones, and the efficiently computable construction for MISOCO is presented in paper [2]. All special and degenerate cases are carefully analyzed and easy to compute criteria are developed to compute DCCs and DCyCs for all cases. The complete description of the novel cuts is presented in the technical report [3].

We have made significant progress in building a test-set library for MISOCO problems. The test set library includes randomly generated MISOCO problems, facility location problems from available literature, robust versions of standard MILO test problems, and Round-lot Portfolio Optimization Problems. The novel DCCs for MISOCO may be used in branch-and-cut algorithms when solving MISOCO problems. The experimental software CICLO was developed to perform limited, but rigorous computational experiments. The CICLO solver utilizes continuous SOCO solvers, MOSEK, CPLES or SeDuMi, builds on the open source CHIPS (available in COIN-OR) branch-and-cut software, and implements the addition of a limited number of DCCs and DCyC’s within the branch-and-cut framework. The results of the experiments gave strong indication of the power of the DCCs. The problem sets and the computational experiments are described in the working papers [4-5]. All the results are summarized in Julio C. Góez’s Ph.D. thesis.

Participants:
Tamás Terlaky (PI, Lehigh), Julio Goez (Ph.D. Student, Lehigh), Yu Fu (Ph.D. Student, Lehigh), Sertalp Cay (Ph.D. Student, Lehigh), Imre Pólik (SAS), Ted Ralphs (Lehigh), and Pietro Belotti (Clemson)

Sincerely,

Prof. Tamás Terlaky, Department Chair
George N. and Soteria Kledaras ’87 Endowed Chair Professor
Publications

Ph.D. Thesis

Papers:


Posters:


Technical Reports, Working Papers:


Software:

1. **CICLO**: Integer conic linear optimization package. Authors: J.C. Góez, T.K. Ralphs, Y. Fu, and T. Terlaky

Presentations:


11. *Cone Linear Optimization (CLO): From LO, SOCO and SDO towards mixed integer CLO*. Department of Mechanical and Industrial Engineering, University of Toronto, ON, Canada, October 28, 2011. Presented by T. Terlaky.


