a. REPORT
Decentralized Estimation and Vision-based Guidance of Fast Autonomous Systems with Guaranteed Performance in Uncertain Environments

b. ABSTRACT
We propose development of distributed estimation and control algorithms for design of reliable guidance, navigation and control systems for autonomous vehicles with the use of sensor networks that can enable more precise navigation laws, lower cost and increased reliability. The technical focus is the integration of estimation and control algorithms for multiple vehicles that would lead to guaranteed performance bounds in uncertain, and possibly occluded, environments in the presence of communication losses and network failures.

c. THIS PAGE

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrived as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

15. SUBJECT TERMS
Uncertain system, Multivehicle autonomous systems, Sensor networks, Decentralized estimation, Adaptive navigation and guidance, Guaranteed performance

16. SECURITY CLASSIFICATION OF:

17. LIMITATION OF ABSTRACT

18. NUMBER OF PAGES

19a. NAME OF RESPONSIBLE PERSON
Naira Hovakimyan

19b. TELEPHONE NUMBER
217-244-1672
Report Title
Decentralized Estimation and Vision-based Guidance of Fast Autonomous Systems with Guaranteed Performance in Uncertain Environments

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We propose development of distributed estimation and control algorithms for design of reliable guidance, navigation and control systems for autonomous vehicles with the use of sensor networks that can enable more precise navigation laws, lower cost and increased reliability. The technical focus is the integration of estimation and control algorithms for multiple vehicles that would lead to guaranteed performance bounds in uncertain, and possibly occluded, environments in the presence of communication losses and network failures.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received Paper


TOTAL: 5

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)
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**TOTAL:** 11

**Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):**

(d) Manuscripts
This paper addresses the problem of steering a fleet of UAVs along desired 3-D paths, while meeting stringent spatial and temporal constraints. A representative example is the challenging mission scenario where the UAVs are tasked to cooperatively execute collision-free maneuvers and arrive at their final destinations at the same time, or at different times, so as to meet a desired inter-vehicle schedule. In the proposed framework, the UAVs are assigned nominal spatial paths and speed profiles along those, and then the vehicles are requested to execute cooperative path following, rather than “open-loop” trajectory-tracking maneuvers. This strategy yields robust behavior against external disturbances by allowing the UAVs to negotiate their speeds along the paths in response to information, exchanged over the dynamic inter-vehicle communications network. The proposed approach addresses explicitly the situation where each vehicle transmits, coordination-relevant information to only a subset of the other vehicles, as determined by the communications topology. Furthermore, the paper considers the case where the graph that captures the underlying communications topology is disconnected during some interval, of time or even fails to be connected at all times. Conditions are given under which the complete time-critical cooperative path-following closed-loop system is stable and yields convergence of a conveniently defined cooperation error to a neighborhood of the origin. Flight test results of a coordinated road search mission demonstrate the efficacy of the, multi-UAV cooperative control framework developed in the paper. Time-Critical Cooperative Path Following of Multiple UAVs over Time-Varying Networks, Journal of Guidance, Control, and Dynamics (09 2011)
Awards

Naira Hovakimyan was selected to receive the 2011 American Institute of Aeronautics and Astronautics (AIAA) Mechanics and Control of Flight Award

Enric Xargay received Roger A. Strehlow Memorial Award by the Aerospace Engineering Department of UIUC in May 2011.

Dr. Hovakimyan gave a keynote at ICNPA in Vienna and was honored with a technical achievement award for her contributions to ICNPAA on July 27, 2012. http://naira.mechse.illinois.edu/2012/07/27/icnpaa/

Prof. Hovakimyan has been recognized as University Scholar in UIUC on September 28, 2011. The program recognizes excellence while helping to identify and retain the university’s most talented teachers, scholars and researchers. http://naira.mechse.illinois.edu/2011/09/28/prof-hovakimyan-named-university-scholar/

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Graduate Students

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### Student Metrics
This section only applies to graduating undergraduates supported by this agreement in this reporting period

- The number of undergraduates funded by this agreement who graduated during this period: 0.00
- The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields: 0.00
- Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00
- Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

### Names of Personnel receiving masters degrees

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### Sub Contractors (DD882)

### Inventions (DD882)
Scientific Progress

Decentralized estimation and control algorithms for design of reliable guidance, navigation and control systems for autonomous vehicles with the use of sensor networks that can enable more precise navigation laws, lower cost or smaller future systems, and increased reliability, are the objectives of this research. The technical focus is the integration of estimation and control algorithms for multiple vehicles that would lead to guaranteed performance bounds in uncertain, and possibly occluded, environments in the presence of communication losses and network failures.

During the contract period we explored design of distributed cooperative control laws for multi-agent systems that take the uncertainties in the agent dynamics into consideration.

1. We proposed a cascaded control structure for multi-agent coordination in the presence of uncertain agent dynamics and disturbances, which resolved the coupling between the communication topology and the system dynamics. We implemented and tested the proposed algorithms on the UIUC multi-robot testbed.

2. We extended the cascaded control structure for state-dependent (relative-position-induced) network topology to enable a decoupled design for distributed control with event-triggered sampling. The decoupled architecture helps to leverage tools from network theory and robust control to enable the operation of the large-scale system with guaranteed performance bounds and robustness margins.

3. We developed a PI consensus algorithm for a multi-agent system with disturbances in each agent's dynamics operating over time-varying communication topologies and with quantized feedback.

4. We designed time-coordinated 3D path-following algorithms for multiple quadrotors and flight-tested the algorithms at the Naval Postgraduate School.

5. L1 adaptive controller is the core technology that enables distributed control design for multi-agent systems with uncertainties and guaranteed performance. We have made several modifications to the standard L1 adaptive controller algorithm to relax the CPU requirements, making it more suitable for systems with limited computation and sensing capabilities.

6. Throughout our research, we have been considering coordination under a very realistic communication setup, i.e., we have assumed distance-based network or network with an integral type of connectivity (which is a much weaker assumption than the point-wise connectivity), and shown the robustness of our coordination algorithms. We start to consider stochastic network which is in nature closer to the physical communication link used in practice.
Robust Architectures for Multi-Agent Systems

Army Research Office Final Report

Naira Hovakimyan, Dusan Stipanovic

Coordinated Science Laboratory

University of Illinois at Urbana-Champaign
**Outline**

- Brief Literature Overview
- Decoupled Design for Distributed Control of Uncertain Networked Control Systems
- Cooperative Missions of UAVs
Objectives of Distributed Multi-Agent Systems

**Distributed Stabilization**

- Weak physical coupling
- Decentralized control — Siljak 1991
- Strong physical coupling
- Distributed control — Bamieh 2002 TAC, D’Andrea 2003 TAC, Dunbar 2007 TAC, Rice 2009 TAC

**Cooperative Control**

- Consensus — Olfati-Saber 2005CDC, Ren 2007CS
- Flocking — Olfati-Saber 2006TAC, Tanner 2003CDC, Blondel 2005CDC
- Formation — Tabuada 2001ACC, Tanner 2002IFAC, Egerstedt 2001TRA
- Maximal coverage — Bullo 2005CC, Cortes 2004TRA, Martinez 2007CSM
- Optimization — Rabbat 2004 IPSN, Palomar 2007TAC, Lee 2004CL

- **Perfect Communication**
- **Perfect Model**
  - No Physical Uncertainty
Flocking:
- Increase the chance of detecting enemies or targets

Self-assembled network (Olfati-Saber 2006)
- No assumptions on network topology (but needs a common objective for the group), more realistic for low cost agents with limited communication range
- Ideal double integrator dynamics
- Uncertain dynamics (Li ACC 2011)

Nonholonomic agents (more realistic and challenging)
- Tanner 2004: inter-agent velocity alignment, assumptions on network topology
Main Contributions

- Multiple nonholonomic agents
  - Uncertain dynamics and disturbances

- Common goal: target tracking

- Self-assembled flocking and collision avoidance

- Cascaded control structure
  - Coordination and uncertainties decoupled and addressed independently
  - Guaranteed performance bounds
Problem Formulation

- **N** Mobile (heterogeneous) agents

\[
\begin{align*}
\dot{q}^i(t) &= \begin{bmatrix} \dot{x}^i(t) \\ \dot{y}^i(t) \end{bmatrix} = v^i(t) \begin{bmatrix} \cos(\theta^i(t)) \\ \sin(\theta^i(t)) \end{bmatrix} \\
\dot{\theta}^i(t) &= \omega^i(t) \\
v^i(s) &= G_n^i(s)(u_n^i(s) + z_n^i(s)) \\
\omega^i(s) &= G_\omega^i(s)(u_\omega^i(s) + z_\omega^i(s))
\end{align*}
\]

- **Target:** constant velocity, \(q_t, \theta_t, v_t\) are known to agents

- **Communication:** inter-agent communication range \(r\)

- **Objective:** track the target forming a flock and avoid collision
Preliminaries from graph theory

- General definition

- Graph induced by agents' positions
Cascaded Control Structure :: Overview

“Simulator” (coordination) + Local Tracking Law (uncertainties)

- Simulator: virtual ideal kinematic model
  - Running in each agent’s computer
  - Driven by a cooperative guidance law that achieves flocking and collision avoidance
  - Each agent exchanges information of its simulator with neighbors; this information is uncertainty-free, and is used in the guidance law
  - Position and velocity of the simulator serve as reference inputs for the real agent to track

- Local tracking controller: track the simulator
  - Outer-loop: guidance law for the kinematic model
  - Inner-loop: adaptive controller for uncertain dynamics
Cascaded Control Structure :: Overview

- A demo of the simulator-follower structure
Cascaded Control Structure :: Overview

Exchange (uncertainty-free) simulator states

Controller
Simulator: Flocking Algorithm for Ideal Kinematic Model

- Ideal kinematic model

\[
\begin{align*}
\dot{q}_s^i &= \begin{bmatrix} \dot{x}_s^i \\ \dot{y}_s^i \end{bmatrix} = v_s^i \begin{bmatrix} \cos \theta_s^i \\ \sin \theta_s^i \end{bmatrix} \\
\dot{\theta}_s^i &= \omega_s^i \\
x_s^i(0) &= x^i(0), \ y_s^i(0) = y^i(0), \ \theta_s^i(0) = \theta^i(0)
\end{align*}
\]

- Flocking algorithm

\[
\begin{align*}
v_s^i &= v_t - k_v \nabla_{q_s^i} V^i(q_s) \cdot \begin{bmatrix} \cos \theta_s^i \\ \sin \theta_s^i \end{bmatrix}^T \\
\omega_s^i &= -k_\omega (\theta_s^i - \theta_d^i) \\
\theta_d^i &= \text{angle}(-\nabla_{q_s^i} V^i(q_s) + \vec{v}_t)
\end{align*}
\]

Smooth transition from aggregation to heading alignment.

- Collective potential function

\[
V^i(q_s) = \sum_{j \in N^r_{i}(q)} V_a(||q_s^i - q_j^i||) + V_l(||q_s^i - q_t||)
\]
Potential Functions

- **Inter-agents**

  \[ V_a(z) = \begin{cases} 
  \infty \\ 
  \log \frac{z - r_c}{r_d - r_c} + \frac{r_d - r_c}{z - r_c} - 1 \\
  h + h \sin \frac{r - (r_c + r_d)/2}{r_d - r_c} \pi \\
  h 
  \end{cases} \]

- **Target-agent**

  \[ V_t(z) = \begin{cases} 
  \infty \\ 
  \log \frac{z - R_c}{R_d - R_c} + \frac{R_d - R_c}{z - R_c} - 1 \\
  -(z - R_d) + \\
  (z - R_d + 1) \log(z - R_d + 1) 
  \end{cases} \]
Track the Simulator: Outer-loop

- **Objective**: the real agent tracks the simulator, i.e., $|p_e| \to 0$

- **Tracking error**

  $p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_s - x \\ y_s - y \\ \theta_s - \theta \end{bmatrix}, \quad \det(T) \neq 0.$

  $\dot{p}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \omega y_e - v + v_s \cos \theta_e \\ -\omega x_e + v_s \sin \theta_e \\ \omega_s - \omega \end{bmatrix}$

- **Velocity commands** *(Kanayama et al., 1991)*

  $u_c = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} k_x x_e + v_s \cos \theta_e \\ \omega_s + k_y y_e + k_{\theta} \sin \theta_e \end{bmatrix}$
Track the Simulator: Outer-loop

- For **perfect velocity tracking**: \( \nu = \nu_c, \omega = \omega_c \)

\[
\dot{p}_e = f(p_e, u_s) \triangleq \begin{bmatrix}
(\omega_s + k_y y_e + k_\theta \sin \theta_e)y_e - k_x x_e \\
-(\omega_s + k_y y_e + k_\theta \sin \theta_e)x_e + v_s \sin \theta_e \\
-k_y y_e - k_\theta \sin \theta_e
\end{bmatrix}
\]

- \( v_s, \omega_s \) constant: asymptotic stability

- **Key result**: **imperfect velocity tracking**:

  - If \(|\nu - \nu_c| < \gamma_\nu, |\omega - \omega_c| < \gamma_\omega\), then \(|p_e| < \gamma_e\), where
Objective: \( v(s) \approx M_v(s)v_c \), \( \omega(s) \approx M_\omega(s)\omega_c(s) \)

\[
v^i(s) = G^i_v(s)(u^i_v(s) + z^i_v(s))
\]

\[
\omega^i(s) = G^i_\omega(s)(u^i_\omega(s) + z^i_\omega(s))
\]

Solution: L1 Adaptive Controller

Main features of L1 Adaptive Controller

- Guaranteed transient response for system’s both input and output:
Track the Simulator : Inner-loop

- **L1 adaptive controller for \( v \), \( M_v(s) = \frac{m_v}{s + m_v} \)
  - **Output predictor:** \( \dot{\hat{v}}(t) = -m_v \hat{v}(t) + m_v(u_v(t) + \hat{\sigma}_v(t)) \)
  - **Adaptive law:** \( \dot{\hat{\sigma}}_v(t) = \Gamma_v \text{Proj}(\hat{\sigma}_v(t), -\hat{\tilde{v}}(t)) \)
  - **Control law:** \( u_v(s) = C_v(s)(v_c(s) - \hat{\sigma}_v(s)) \)

- **L1 stability condition:**
  \[
  H_v(s) \triangleq \frac{G_v(s)M_v(s)}{C_v(s)G_v(s)+(1-C_v(s))M_v(s)} \quad \text{stable}
  \]
  \[
  \|H_v(s)(1-C_v(s))\|_{L_1 L_v} < 1
  \]

- **Key result:** If \( v_c \) is bounded and \( \dot{v}_c \leq \gamma \dot{v}_c \), then
  \[
  \|v - v_c\|_{L_\infty} \leq \frac{\gamma_v^{\text{ref}}}{\sqrt{\Gamma_v}} + \gamma_v^{\text{des}} + \frac{1}{m_v} \gamma \dot{v}_c
  \]
|v − v_c|, |ω − ω_c| small ⇒ |p_e| bounded ∀t

L1 adaptive controller: |v − v_c|_{L\infty}, |ω − ω_c|_{L\infty} small

Agent can track the simulator with transient performance guarantees!
Coupling between dynamics and topology

- Coupling exists because:
  - Communication is induced by motion, no artificial assumptions
  - Input to the simulator is based on $G^r(q)$, not $G^r(q_s)$

- Without transient guarantees, it is possible that simulators “move” to the same position

- With transient guarantees, we can select

\[(i, j) \in \mathcal{E}^r(q_s) \Rightarrow (i, j) \in \mathcal{E}^r(q)\]

- The guaranteed transient behavior of the controller is the key to resolve the coupling issue.

Real agents can’t talk, simulators “collide”, and generate infinite $v_s, o_s$

Real agents are always close to simulator, and well behaving
Simulation: 3 agents, without L1 adaptive controller

- Algorithm designed for ideal dynamics

Response for ideal dynamics

\[ G_v^i(s) = G_\omega^i(s) = \frac{1}{s}, z_v^i = z_\omega^i = 0 \]

Response for slow dynamics

\[ G_v^i(s) = G_\omega^i(s) = \frac{0.2}{s + 0.2}, z_v^i = z_\omega^i = 0 \]

Almost collide
Simulation: 3 agents, with L1 adaptive controller

- Trajectories
Simulation: 3 agents, with L1 adaptive controller

- Velocity tracking

\[ v^l, v_c^l \]

\[ \omega^l, \omega_c^l \]

\[ \theta^l \]
Simulation: 3 agents, with L1 adaptive controller

- Animation
Simulation: 20 agents, with L1 adaptive controller

- Animation
Experimental results

- Ground robots
Experimental results

- Experimental configuration

Camera 1 → Image

Fusion Center

Wireless Network

Information of all robots

Agent 1 → Image

Camera 4

Agent N

Target
Experimental results

- A test run with 2 followers
Coordinated Road Search and Target Tracking – The Concept

2DoF P/T gimbal with a **video camera** enables vision based UAV guidance and **target tracking**.

Single DOF gimbal with **high resolution camera** delivers satellite quality imagery well suited for **target ID**.

Thermal seeking **soaring gliders** is used as a **Flying Antenna** to extend communication range.
Coordination in “Road Search” mode (CPF) is represented by the results of stitching 4 consecutive high resolution frames taken from the UAV above:
- UAVs are looking at the same virtual target on the road
- Successful coordination results in “encapsulating” second UAV in each frame

Flight imagery of 4 consecutive frames with coordinating UAV below
Flight Test: Coordinated Road Capture and Search

**Coordinated Road Capture**
- two UAVs generate real-time capture paths
- coordination is used to robustly achieve simultaneous arrival

**Coordinated Road Search**
- two UAVs follow the road search paths
- coordination is used to guarantee nonzero intersection of the FOVs between two cameras
If coordination is successful then both cameras look at the same place

The video at the top is from the UAV with a standard 2DOF gimbaled camera

The video at the bottom was generated by the high rez camera stabilized by a 1DOF gimbal (this was accounted for in the path generation)
Once the target is designated by the user

- two UAVs switch from road search to coordinated target tracking mode

- the pre-assigned phase separation is π/2, maintained by coordination
Conclusions for Coordinated Road Search

- Coordinated Control of Multiple UAVs: Theoretical Framework / Practice

![Diagram of coordinated control system]

- Path Generation
- Path following
- Coordination
- Onboard A/P + UAV (Inner loop)
- Network

- Desired path
- Coordination variables
- Velocity command
- Pitch rate
- Yaw rate commands
Real-Time Issues

Limited Communication Resource

- Transmission delays, discrete data transmission, packet loss, quantization

Sampled-Data Systems (Single Plant)

- Periodically sampled system --- Chen 1995
- Event-triggered control --- Årzén 1999IFAC, Tabuada 2007TAC
  - Saves Communication Resource, but requires hardware detector
  - Predict the next execution time based on the past information. No need for hardware!

Real-Time Communication in Multi-Agent Systems

- Physically Decoupled
  - Consensus, Dimarogonas 2009CDC, 2011IFAC, Yu2011CDC
  - Maximum Coverage, Nowzari, 2011Automatica 2012ACC
- Physically Coupled
  - Stabilization, Wang 2009TAC 2011TAC
  - Network Utility Maximization, Wan 2009IPSN
  - Perfect Model
    - No Physical Uncertainty
Robust Control and Communication Co-Design

- Input-to-state stability for robustness
  - Periodic Transmission — Nesic 2004TAC
  - Event-triggering broadcast — Wang 2008ACC, Mazo 2009CDC
  - ISS only provides an upper bound on the state trajectory, without quantifying the *transient performance*.

- L1-adaptation-based event-triggering for performance
  - State Feedback, linear interconnection, matched uncertainty, fixed communication model — (presented in AFOSR in 2010)

**Drawback:** Event-triggering was defined to transmit the **real system outputs** over the network, often measured by **noisy sensors**.

**Desired Solution:**

1. **Network must be used for communication of information**, **NOT affected** by system uncertainties or noise.

2. **The design of control and communication can be decoupled to the maximum extent so that the existing techniques can be easily leveraged.**
Problem Formulation

**Ideal Model:**

\[
\begin{align*}
\dot{x}_i^{id} &= A_i^m x_i^{id} + B_i u_i^{id} + f_i(t, y_i^{id}, y_{-i}^{id}) \\
y_i^{id} &= C_i x_i^{id}, \quad x_i^{id}(0) = x_i^{0}
\end{align*}
\]

**Ideal Controller:**

\[
\begin{align*}
\dot{z}_i^{id} &= g_i(t, z_i^{id}, y_i^{id}, y_{-i}^{id}) \\
u_i^{id} &= h_i(t, z_i^{id}, y_i^{id}, y_{-i}^{id})
\end{align*}
\]

- Subsystems are physically coupled
  - Unmatched uncertainties in the system dynamics and interconnections
  - Uncertainties are Locally Lipschitz
  - Output feedback
  - Nonlinear ideal interconnection
- Limited Communication
  - Discrete data transmission
  - Delay and packet loss may exist

**Real Subsystem:**

\[
\begin{align*}
\dot{x}_i &= A_i x_i + B_i u_i + \Delta_i(t, y) \\
y_i &= C_i x_i, \quad x_i(0) = x_i^{0}, \quad \text{Input is computed based on discrete data over networks}
\end{align*}
\]

Uncertainties
To what level can we decouple the design of control and communication?

Solution:

Reference Model

\[ \dot{x}_i^s = A_i^m x_i^s + B_i u_i^s + f_i(t, y_i^s, \hat{y}_i^s) \]
\[ y_i^s = C_i x_i^s \]
\[ x_i^s(0) = x_i^0 \]
\[ \dot{z}_i^s = g_i(t, z_i^s, y_i^s, \hat{y}_i^s) \]
\[ u_i^s = h_i(t, z_i^s, y_i^s, \hat{y}_i^s) \]

- Doesn’t contain physical uncertainties.
- The information is “clean”.

Network Data

local data based on the data from the reference model.
Control Design

- Regard the real subsystem as a single system, with the signals from its reference model as the reference signals;
- Robust adaptive control ensures the closeness between signals inside the real dynamics and the reference model inside each individual agent.
Communication Design

- Regard the real subsystem as a single system, with the signals from its reference model as the reference signals;
- Robust adaptive control ensures the closeness between signals inside the real dynamics and the reference model inside each individual agent.

- No physical uncertainties are involved in the communication model design;
- If the signals over the network are bounded, the closeness between the real dynamics and the reference model will **NOT** be affected by
  - The type of communication models
  - The network effects, such as delays, packet loss, etc.
Design Procedure

Given a multi-agent system
1. Model the system dynamics and obtain the ideal model
2. Design the ideal control algorithm based on the ideal model
3. Design the communication model based on the ideal model so that
   - The resulting closed-loop system (reference model) fulfills the same objective that the ideal model does
   - The input and the output of the resulting reference system are bounded.
4. With the bounds obtained in Step 3, design the distributed local robust adaptive controller

The only coupling between the control and the communication design is that the bounds on the input and the output of the reference model will be used in selecting the parameters of the control generator.
The Difference between Reference and Ideal Models

The reference model:

\[
\begin{align*}
\dot{x}_i^s &= A_i^m x_i^s + B_i u_i^s + f_i (t, y_i^s, \hat{y}_i^-) \\
y_i^s &= C_i x_i^s \\
\hat{z}_i^s &= g_i (t, z_i^s, y_i^s, \hat{y}_i^-) \\
u_i^s &= h_i (t, z_i^s, y_i^s, \hat{y}_i^-).
\end{align*}
\]

The combined closed-loop reference model:

\[
\dot{w}^s = \Psi (t, w^s(t), \hat{y}^s(t))
\]

The closed-loop ideal model:

\[
\dot{w}^{id} = \Psi (t, w^{id}(t), y^{id}(t))
\]

The closed-loop reference model can be written as:

\[
\dot{w}^s = \underbrace{\Psi (t, w^s(t), y^s(t))}_{\text{Ideal Model}} + \underbrace{\Psi (t, w^s(t), \hat{y}^s(t)) - \Psi (t, w^s(t), y^s(t))}_{\text{Perturbation}}
\]

The key challenge in the communication model design is to ensure that the difference between \( \hat{y}^s \) and \( y^s \) is not TOO large.
Assume that

- \( \Psi(t, w^s, y^s) \) and its first partial derivatives are continuous, bounded, and Lipschitz in \( w^s \), uniformly in \( t \), for all \( t \geq 0 \) and \( w^{id} \) in a compact set;

- \( \Psi(t, w^s, \hat{y}^s) \) is piecewise continuous in \( t \), locally Lipschitz in \( y^s, \hat{y}^s \);

- there exist continuous, positive definite class \( \mathcal{K} \) functions \( \alpha_1, \alpha_2, \alpha_3 \) and a continuously differentiable function \( V \) such that

  \[
  \alpha_1(\|w^{id}\|) \leq V(t, w^{id}) \leq \alpha_2(\|w^{id}\|)
  \]

  \[
  \frac{\partial V}{\partial t} + \frac{\partial V}{\partial w^{id}} \Psi(t, w^{id}) \leq -\alpha_3(\|w^{id}\|);
  \]

- \( w^{id} = 0 \) is an exponentially stable equilibrium point of the ideal model.

Then there exist \( \beta, \eta \geq 0 \), such that if

\[
\sum_{i \in \mathcal{N}} \|y^s_i - \hat{y}^s_i\|_{\ell_\infty} \leq \eta,
\]

we have

\[
\|w^{id} - w^s\|_{\ell_\infty} \leq \beta \eta.
\]
Enforcing Communication Constraint

Assume that

**Example (to enforce the communication constraint):**

**Event Triggered Data Transmission:**

Transmit when the local measurement error $\|y_i^s(t) - \hat{y}_i^s(t)\|$ exceeds a pre-specified positive constant.

- The sampled output of the reference model is transmitted. The real output is not involved.

$w^{id} = 0$ is an exponentially stable equilibrium point of the ideal model.

Then there exist $\beta, \eta \geq 0$, such that if

$$\sum_{i \in \mathcal{N}} \|y_i^s - \hat{y}_i^s\|_{L_\infty} \leq \eta,$$

we have

$$\|w^{id} - w^s\|_{L_\infty} \leq \beta \eta.$$
This $L_1$ adaptive control architecture uses data from the reference model that carries the information from the network.
Main Results

Assume that the input and the output of the $i$th reference model are bounded by $\rho_i^{u_i}$ and $\rho_i^{y_i}$, respectively. If $\exists \rho_y$ such that

$$
\rho_y > \phi_i(\|x_i^0\|) + \psi_i(\rho_i^{u_i}) + \gamma_i\left(\max_{i \in N_i} \rho_i^{y_i}\right)
+ \|(I - F_i(s)) \hat{H}_i(s)\|_{L_1} \left(a_i \rho_y + b_i\right) + \frac{1}{\alpha_i(\sigma_{\min}(A_i^m + \Gamma_i C_i))}
$$

for any $i$, then $\|y(t)\|$ and $\|u(t)\|$ are uniformly bounded and

$$
\|y - y^s\|_{L_\infty} \leq \rho_{\Delta_i} \left(\|(I - F_i(s)) \hat{H}_i(s)\|_{L_1} + \frac{1}{\beta(\sigma_{\min}(A_i^m + \Gamma_i C_i))}\right),
$$

where $\phi_i$, $\psi_i$, $\gamma_i$, $\alpha_i$, $\beta$ are class $K_\infty$ functions and $\sigma_{\min}(\cdot)$ denotes the minimal singular value of a matrix.

- We can always find $\rho_y$ such that the stability condition holds.
- The performance bound can be rendered arbitrarily small by increasing the bandwidth of the low-pass filter.
- Large bandwidth leads to small stability margin, which suggests a tradeoff between performance and robustness.
Real-Time $L_1$ Adaptive Control Generator

When implementing the controller in digital processors, we seek real-time $L_1$ adaptive controller and consider the problem of computational resource management.

Assume that the $L_1$ stability condition holds. For each agent, if the time intervals between two consecutive receptions of the data from the network are lower bounded by a positive constant, then there exists a continuous function $\alpha_i$ satisfying $\alpha_i(0, 0, 0, 0, 0) = 0$ and a positive constant $\gamma_i$ such that if

$$\alpha_i \left( \begin{array}{c} \text{Comp. Period; Sensing Period; Actuation Period; } \\
\text{Sensing Delay; Actuation Delay} \end{array} \right) < \gamma_i$$

Sensing Period > Sensing Delay
Actuation Period > Actuation Delay

hold, then $\|y_i - y_i^s\|_{L_\infty} < \gamma_i$.

- The performance is subject to the hardware limitations.
New Results on Cooperative Missions

- **Time-critical** applications for multiple vehicles with spatial constraints:
  - Sequential auto-landing (UAVs)
  - Coordinated reconnaissance missions
  - Simultaneous arrival at multiple locations

Coordinate on the **arrival** of a leader (or group of leaders) subject to collision-avoidance, communications, and spatial **constraints**
Overall Conceptual Architecture: Decoupling Space and Time

Xargay, Dobrokhodov, Kaminer, Pascoal, Hovakimyan, & Cao 2012
Consensus problem: reach an agreement on some distributed variables of interest (coordination states) as well as their rates.

\[
\begin{align*}
x_i(t) - x_j(t) & \xrightarrow{t \to \infty} 0, \quad \forall i,j \in \mathcal{I}_n \\
\dot{x}_i(t) & \xrightarrow{t \to \infty} \rho, \quad \forall i \in \mathcal{I}_n
\end{align*}
\]

Reference rate

Synchronize in both ‘position’ and ‘speed’

New results:

• New coordination states that accommodate time-varying desired speed profiles;

• Performance guarantees in the presence of temporary link losses and disconnected graphs:
  - Lower bound on the (exponential) convergence rate...
  - ... as a function of the QoS of the network and the number of leaders;

• In the presence of quantization, existence of equilibrium points corresponding to undesirable steady solutions with zero ‘speed’:
  - Derivation of a bound on the quantizer step size to ensure that these equilibria do not exist;

• Performance improvement under low connectivity through ‘onboard estimators’;

• Performance improvement via emergent leaders.
Key Ideas & Previous Work

**PI protocol:**
- Critical to coordinate ‘speed’;
- Allows agents to learn a reference rate command and reject constant disturbances.
  - Kaminer, Yakimenko, Pascoal, & Ghabcheloo 2006
  - Carli, Chiuso, Schenato, & Zampieri 2008
  - Bai, Arcak, & Wen 2008 (generalized PI protocol)

  Fixed connected graphs;
  No guaranteed convergence rate

**Virtual leaders:**
- Allows to track a constant reference rate command with zero steady-state error;
- Drawback: adding agents reduces the connectivity level.
  - Shi, Wang, & Chu 2006
  - Ren & Beard 2007
  - Su, Wang, & Lin 2009

  The ‘virtual leader’ is implemented as an isolated node providing a reference trajectory
  ‘Extra agent’ imposes a reference rate and its dynamics are affected by other agents

**Quantized consensus:**
- Quantization can create equilibrium points corresponding to undesirable steady solutions with zero ‘speed’.
  - Kashyap, Basar, & Srikant 2007
  - Censi & Murray 2009
  - Nedic, Olshevsky, Ozdaglar, & Tsitsiklis 2009
  - Ceragioli, De Persis, & Frasca 2011

  In the ‘conventional’ agreement problem, quantization leads to ‘practical consensus’

**Guaranteed rate of convergence:**
- Critical to ensure successful execution of the mission.
  - Olfati Saber & Murray 2003
  - Kashyap, Basar, & Srikant 2007
  - Nedic, Olshevsky, Ozdaglar, & Tsitsiklis 2009

Convergence rates derived only for ‘conventional’ consensus problems
Coordination States :: Time-Varying Speed Profiles

- For **constant desired speed profiles**:
  - Normalized curvilinear abscissas:
    \[ \ell_i'(t) \equiv \frac{\ell_i(t)}{\ell_{fi}} \]

- For **time-varying desired speed profiles**:
  - **Time** variables:
    \[ \eta_i : [0, 1] \rightarrow [0, t_d^*] \]

**In the case of spatially deconflicted paths**, there is no risk of collision; however, **mission-specific goals will not be satisfied**.
Consider a network of \( n \) integrator-agents:

\[
\dot{x}_i(t) = u_i(t) + d_i, \quad x_i(0) = x_{i0}, \quad i \in \mathcal{I}_n := \{1, \ldots, n\}
\]

with **dynamic information flow** \( \mathcal{G}_0(t) := (\mathcal{V}_0, \mathcal{E}_0(t)) \).

**Communications network:**

- Each agent can **only** exchange information with a set of **neighboring agents**;
- Communications between agents are **bidirectional** and information is transmitted **continuously with no delays**;
- **Connectivity** of the communications network at time \( t \) satisfies:

\[
\frac{1}{n} \frac{1}{T} \int_{t}^{t+T} Q_n L_0(\tau) Q_n^\top d\tau \geq \mu I_{n-1}, \quad \forall \ t \geq 0
\]

Arcak 2007

- Parameters \( T \) and \( \mu \) characterize the **QoS of the network**;
- Graph connected in an **integral sense**, not piecewise in time:
  - The graph may be disconnected during some interval of time...
  - ... or **may even fail to be connected at all times**.
Virtual Agents & Extended Network

- We add $n_{\ell}$ virtual agents with dynamics:
  \[ \dot{x}_{\ell}(t) = u_{\ell}(t), \quad x_{\ell}(0) = x_{\ell0}, \quad i \in I_{\ell} := \{1, \ldots, n_{\ell}\} \]

- To limit the amount of information transmitted over the network, both leaders and followers only exchange one coordination state with their neighbors.
  - **Followers** exchange their own actual states.
  - **Leaders** only exchange the state of their virtual agents.

**Having multiple leaders improves robustness to single-point failures**

Ren & Beard 2007

Adding virtual agents reduces the connectivity level of the extended network

\[ \frac{1}{N} \frac{1}{T} \int_{t}^{t+T} Q_N L(\tau)Q_N^T d\tau \geq \mu_{n_{\ell}} I_{N-1}, \quad \forall \ t \geq 0 \]

Xargay, Choe, Hovakimyan, & Kaminer 2012
PI Protocol & Collective Dynamics

- **Distributed protocol:**
  \[
  u_{\ell i} = k_P \sum_{j \in \mathcal{N}_{\ell i}} (x_j - x_{\ell i}) + \rho, \quad i \in \mathcal{I}_{\ell}
  \]
  \[
  u_i = k_P \sum_{j \in \mathcal{N}_i} (x_j - x_i) + \chi_i, \quad i \in \mathcal{I}_n
  \]
  \[
  \dot{\chi}_i = k_I \sum_{j \in \mathcal{N}_i} (x_j - x_i), \quad \chi_i(0) = \chi_{i0}, \quad i \in \mathcal{I}_n
  \]

  - Reference rate only available to the leaders;
  - Proportional-integral control structure:
    - Disturbance rejection capabilities;
    - Followers can learn the reference rate command;
  - Each vehicle exchanges only its coordination state \( x_{\bullet}(t) \) with its neighbors.

- **Closed-loop collective dynamics:**

  \[
  \dot{x}(t) = -k_P L(t)x(t) + \begin{bmatrix} \rho 1_{n_{\ell}} \\ \chi(t) + d \end{bmatrix}, \quad x(0) = x_0
  \]
  \[
  \dot{\chi}(t) = -k_I C^T L(t)x(t), \quad \chi(0) = \chi_0
  \]

  Switched LTI system
Convergence Properties

- Define the consensus error state $\zeta(t) := [\zeta_1(t)^T, \zeta_2(t)^T]^T$:
  
  $$\zeta_1(t) := Q_N x(t)$$
  $$\zeta_2(t) := x(t) - \rho 1_N + d$$

- Closed-loop collective dynamics:
  
  $$\dot{\zeta}(t) = A \zeta(t) \zeta(t), \quad \zeta(0) = \zeta_0$$

- There exist coordination control gains such that
  
  $$\|\zeta(t)\| \leq \alpha_{\zeta} \|\zeta(0)\| e^{\lambda_c t}$$

  where
  
  $$\lambda_c \geq \bar{\lambda}_c := \frac{k_p N \mu_{n\bar{\ell}}}{(1+k_p N \bar{\ell})^2} (1 + \beta_{\lambda})^{-1}, \quad \beta_{\lambda} \geq 2 \frac{N}{n_{\bar{\ell}}} \sqrt{\frac{N}{n_{\bar{\ell}}}}$$

- Moreover, the coordination states and their rates of change satisfy:
  
  $$\lim_{t \to \infty} |x_i(t) - x_j(t)| = 0, \quad i, j \in I_n$$
  $$\lim_{t \to \infty} \dot{x}_i(t) = \rho, \quad i \in I_n$$
Convergence under Quantization

- PI protocol under **quantized feedback**:

\[
\begin{align*}
\mathbf{u} &= -k_P \left( \tilde{D}(t) \mathbf{x} - \tilde{A}(t) q(\mathbf{x}) \right) + \left[ \rho \right] \\
\dot{\chi} &= -k_I \mathbf{C}^\top \left( \tilde{D}(t) \mathbf{x} - \tilde{A}(t) q(\mathbf{x}) \right), \quad \chi(0) = \chi_0
\end{align*}
\]

- Carathéodory solutions **might not exist**; need to consider solutions in the sense of *Krasovskiy*.  *(Ceragiolo, De Persis, & Frasca 2011)*

- Potential existence of **undesirable attractors/equilibria** if:

\[
\Delta \geq \frac{2 n_{\ell}}{n(n-1)} \frac{|\rho|}{k_P}
\]

- **Uniform ultimate boundedness** with ultimate bounds proportional to the step size of the (uniform) quantizers:

\[
\begin{align*}
|x_i(t) - x_j(t)| &\leq \alpha_1 \Delta \\
|\ddot{x}_i(t) - \rho| &\leq \alpha_2 \Delta , \quad \forall \ t \geq T_b
\end{align*}
\]