Nonlinear Acoustic Techniques for Land Mine Detection

Murray S. Korman
Physics Department
United States Naval Academy
Annapolis, MD USA 20402

James M. Sabatier
National Center for Physical Acoustics
University of Mississippi
University, MS USA 38677

When airborne sound couples into the ground seismic waves can interact with a target buried in the soil and effect the vibration velocity of the surface. Acoustic-to-seismic coupling (using linear acoustic techniques) has proven to be an extremely accurate technology for locating buried land mines [J. M. Sabatier and N. Xiang, IEEE Trans. Geoscience and Remote Sensing 39, 1146-1154 (2001)]. Donskoy [SPIE Proceedings 3392, 211-217 (1998); 3710, 239-246 (1999)] has suggested a nonlinear technique that can detect an acoustically compliant buried mine that is insensitive to relatively noncompliant targets. (Utilizing both techniques could eliminate certain types of false alarms.) Airborne sound at two primary frequencies $f_1$ and $f_2$ undergo acoustic-to-seismic coupling and a superimposed seismic wave interacts with the compliant mine and soil to generate a difference frequency component that can effect the vibration velocity at the surface. Geophone measurements scanning the soil's surface at the difference frequency (chosen at a resonance) profile the mine with more relative sensitivity than the linear profiles - but off the mine some nonlinearity exists. Amplitude dependent frequency response curves for a harmonically driven mass-soil oscillator are used to find the nonlinearity of the soil acting as a "soft" spring. Donskoy's nonlinear mechanism (over the mine) involves a simple model of the top surface of the mine-soil planar surface separating two elastic surfaces. During the compression phase of the wave, the surfaces stay together and then separate under the tensile phase due to a relatively high compliance of the mine. This "bouncing" soil-mine interface is thought to be a bi-modular oscillator that is inherently nonlinear.

Introduction

Acoustic-to-seismic (A/S) coupling has been demonstrated by Sabatier and Xiang 1,2 to be an effective and extremely accurate technique for the detection of buried land mines in soils and in particular, roadways. A/S coupling requires that airborne sound induce vibrations in the soil, below the surface where a land mine might be buried. It is the porous nature of the ground (up to about 1 meter below the surface) that plays an important role for A/S coupling to be used successfully in the detection of buried land mines. The experimental technique requires that loudspeakers insonify broadband acoustical noise (or a swept tone) over the soil and a laser Doppler velocimeter (LDV) system (equipped with X-Y scanning mirrors) detect increased soil vibration across a scan region over the mine on the soil's surface.

In two benchmark papers, Donskoy 4,5 describes a nonlinear vibro-acoustic technique for buried landmine detection. Here, nonmetallic landmines are buried in soil and a single speaker driven with two tonal excitations $f_1$ and $f_2$ provides the airborne sound necessary to produce A/S coupling that ultimately interacts with the top surface of the mine. Profiles of the surface acceleration are measured at the difference frequency using a contact sensor (accelerometer) although Donskoy proposed using an LDV for future work. 6

The nonlinear mechanism, suggested by Donskoy, involves a simple one dimensional model where the top surface of the mine-soil planar surface separates two elastic surfaces. During the compression phase of the wave, the surfaces stay together and then separate under the tensile phase due to a relatively high compliance of the mine. It is this "bouncing" soil-mine interface which is thought to be a bi-modular oscillator that is inherently nonlinear.
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See Figure 1(a), (b) and (c) which illustrate (a) Donskoy's nonlinear experiment, (b) his dynamic one-dimensional model of a buried landmine and (c) the bi-modal oscillator restoring force. Donskoy points out that if the buried object is relatively noncompliant like roots, a piece of metal or a brick, for example, then false alarms would be eliminated as the nonlinear mechanism would vanish. His experiments are in support of these ideas.

Although a complete description of the nature of the nonlinear mechanism is probably more involved (and to the authors' knowledge not fully developed), it is in part for those reasons that lead to the experiments reported here. An initial investigation has been reported by Sabatier et. al. 7,8

**Experimental Setup**

Experiments are performed at the National Center for Physical Acoustics' mine lane field test site that is located in Oxford, Mississippi about 2 miles from the University of Mississippi. The mine lane is made up of a natural Mississippi loess soil that is comprised of a clay-silt mixture that is free from rocks or even small pebbles. The lane is kept free from grass or other vegetation growth. Figure 2, shows the experimental setup.

Individual loudspeakers (Peavey Impulse 200 Subwoofers) are decoupled from the soil and are placed at a height of 22cm above the soil's surface. The speakers are separated by 100 cm and centered 186 cm from the center spot of the buried landmine. A VS 1.6 inert anti-tank mine (23 cm diam) and a VS 2.2 inert anti-tank mine were buried months before the test in locations well removed from each other. Each mine was buried at a depth of 9 cm below the soil's surface. A model L-10A (100Hz) geophone (manufactured by Mark Products) was used to measure the velocity profile at the surface across the landmine.
The nonlinear vibro-acoustic detection of the buried land mines required a preliminary linear test. First the geophone is placed at the center of the landmine (on the soil's surface) and a swept sine wave is used to drive the speakers which are connected in parallel. A strong linear resonance at 145 Hz and 115 Hz was found for the buried VS 1.6 and VS 2.2, respectively. A trial and error study determined that using primary frequencies of $f_1 = 945$ Hz and $f_2 = 800$ Hz (for the VS 1.6 experiment) and $f_1 = 915$ Hz and $f_2 = 800$ Hz (for the VS 2.2 experiment) yielded the strongest difference frequency $f_1 - f_2$ component, respectively. Nonlinear experiments for each mine are performed separately but common drive voltages of 11.3 Vrms and 10.8 Vrms are maintained on the speakers labeled S1 and S2, respectively which generate separate primary tones at $f_1$ and $f_2$. The geophone had a linear gain of 500 (Tektronix 502 preamp) and was filtered in a 50 Hz bandwidth about the nonlinearly generated difference frequency component using a Stanford Research elliptical bandpass filter.

There is always the question in nonlinear experiments that the receiving electronics is generating the nonlinearities. A field-test check using two tones $f_1 = 945$ Hz and $f_2 = 800$ Hz (that excited the geophone suspended in the nearfield of the two speakers) demonstrated that there are no nonlinearites being generated (at the difference frequency) in the electronics at least above the noise floor of the spectrum analyzer.

**Experimental Results**

Figure 3 and 4 show the relative mean square velocity profile on the surface detected at the difference frequency across the center of the buried mine for the VS 1.6 and VS 2.2, respectively.

![Fig. 3. Mean squared velocity profile at the 145 Hz difference frequency. Horizontal scale is in inches.](image)

![Fig. 4. Mean squared velocity profile at the 115 Hz difference frequency. Horizontal scale is in inches.](image)

Here the geophone (whose diameter is 1 inch) is positioned at 1 inch intervals to generate the profile. Clearly, the nonlinear technique is able to detect the buried land mines but one could not determine off the mine if the skirts of the profile were due to the nonlinear effects of the mine, nonlinearites of the soil itself or the latter manifested by inhomogeneities of the soil.

These experiments, performed on June 20, 2001, were modified by planning similar experiments where the natural loess soil was excavated forming a 200 cm diam, 70 cm deep cavity and painstakingly refilled with very fine sifted - homogeneous and very dry - Mississippi loess soil. The VS 1.6 mine was then buried (at a depth of 3.8 cm) in this pool of fine loess soil on September 30, 2001 and the nonlinear experiments were continued.

Using the setup described earlier, it was determined, from a swept sine wave driving both speakers, that a strong linear resonance occurred at 92 Hz. It was suggested that a linear profile across the mine be taken, for later comparison with the corresponding profile. However, in order to disturb the soil's surface as little as possible, the velocity profile was determined every 4 inches over a 36 inch scan. Next, speakers S1 and S2 were driven at frequencies $f_1 = 300$ Hz and $f_2 = 392$ Hz and the geophone is positioned at 1 inch intervals over the 36 inch scan. Here the gain on the geophone is 1000 and the bandpass filter is set to pass from 80 to 100 Hz. Each data point corresponds to an average of 100 spectra that are measured on the Agilent spectrum analyzer which was set at a resolution of 2 Hz.

From the results of Figure 5 one can see that the relative mean square velocity profile from the geophone's response is more sharply peaked over the mine and displays more relative sensitivity for the
nonlinearly generated difference frequency component of 92 Hz, compared to the linear response at 92 Hz.

Fig. 5. Using primary frequencies \( f_1 = 300 \text{ Hz and } f_2 = 392 \text{ Hz} \) the nonlinearly detected profile (small data points) is obtained at the 92 Hz difference frequency; and compared with the linear detection profile (large data points) at a primary of 92 Hz - for the VS 1.6 mine. The horizontal scale is 1 inch.

A detailed velocity profile over the VS 1.6 mine using the nonlinearly generated 92 Hz difference component is shown in Figure 6.

Fig. 6. Using primary frequencies \( f_1 = 300 \text{ Hz and } f_2 = 392 \text{ Hz} \) the nonlinearly detected profile is obtained at the 92 Hz difference frequency - for the VS 1.6 mine - at finer increments in a second experimental trial. The horizontal scale is 1 inch.

One might conclude that some of the fluctuating nonlinearity "off the mine" has been eliminated by using a more homogenous soil (i.e. the baseline profile off the mine is less noisy).

Determining the elastic properties of a soil sample

An apparatus called the soil - mass oscillator, shown in Figure 7 was used to determine the longitudinal sound speed in a very finely sifted, dried, crushed and rolled and resifted sample of loess soil.

Fig. 7. Experimental setup to investigate the frequency response near resonance of the soil - mass oscillator.

Here a rigid rectangular frame of inside dimensions \( L_x = 33.0 \text{ cm, } L_y = 26.7 \text{ cm} \) is filled to a depth of \( L = 8.6 \text{ cm} \) with soil. A concrete plate of mass \( m_p = 9.272 \text{ Kg} \) rests on the surface of the soil. The density of the soil is \( \rho = 1220 \text{ Kg/m}^3 \) and the total soil mass is \( m_s = 9.272 \text{ Kg} \). In order to perform the experiment, the subwoofer driver, located 2m above the concrete slab is driven with a swept sinusoidal tone through the fundamental resonance of the system. A spectrum analyzer operating in the swept sine mode records the response of an accelerometer that is glued to the surface of the concrete plate. A family of resonance curves is obtained at a very low acoustic drive pressure from the subwoofer, by successively adding 100 gram weights to the top of the plate. At low frequencies (long wavelengths) the effective mass of the soil contributes a value of \( m_s / 3 \). Therefore the total mass of the oscillator is \( m = m_p + (m_s / 3) + m_a \), where \( m_a \) is the added mass.

Figure 8 shows a plot of the square of the peak (resonant) frequency vs the inverse of the total mass of the oscillator. For small damping the spring constant of the soil is given using the relation \( \omega = \sqrt{k/m} \) where \( \omega = 2 \pi f \) and \( k \) is the spring constant in N/m. It can be shown that for long wavelengths that the spring constant \( k \) can be expressed in the form \( k = \rho c^2 S / L \) where \( S \) is the crosssectional area of the soil and \( L \) is the soil's
thickness. The spring constant is determined to be \( k = 1.21 \times 10^7 \text{ N/m} \) and the longitudinal sound speed in the soil is \( c = 98.3 \text{ m/s} \).

![Graph of tuning curves](image)

When the soil-mass oscillator shown in Figure 7 is driven at sufficiently high pressure amplitudes (while the total mass \( m \) remains constant) the tuning curves shift their peak to lower frequencies. This suggests that the soil driven nonlinearly is acting as a soft spring. Figure 9 shows a family of these tuning curves.

![Graph of nonlinear tuning curves](image)

**Nonlinear soil-mass oscillator**

If one can model the nonlinear elastic behavior of a layer of soil, then perhaps one could modify Donskoy's nonlinear vibro-acoustic model of the soil-landmine oscillator to include the nonlinear behavior of the soil alone. Then, using a model more complex than a one-dimensional oscillator, one could model the nonlinear behavior on and off the mine. If this were possible, one could estimate the signal-to-noise ratio involving the detection of buried landmines using these nonlinear techniques.

The nonlinear behavior of the very fine granular loess soil is modeled to be a nonlinear spring for the experiments involving the apparatus shown in Figure 7. The nonlinear restoring force is modeled to have a quadratic nonlinearity which is given by
The damping force is
\[ f_{\text{damping}} = -k_3 \frac{dx}{dt} \]  (2)

and the externally applied force is
\[ f_{\text{external}} = F_c \cos(\omega t) + F_s \sin(\omega t) \]  (3)

Using Newton's 2nd Law the equation of motion is
\[ f_e + f_s + f_d = m \frac{d^2 x}{dt^2} \]  (4)

Using some abbreviations this can be written in the form
\[ \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + \alpha x + \beta x^2 = H\cos(\omega t) + G\sin(\omega t) \]  (5)

A trial solution of the form
\[ x = a \cos(\omega t) + b \sin(\omega t) + d \]  (6)

can be used to obtain information about the shape of the nonlinear tuning curve. The solution is just given below:
\[ S^2(\omega, a) + a^2 c^2 \omega^2 = H^2 + G^2 \]  (7)

where \( S \) is defined as
\[ S(\omega, a) = a(\alpha - \omega^2) - \frac{5}{6} \frac{a^3 \beta^2}{\alpha} \]  (8)

One can obtain horizontal tangents to the frequency response curves from \( \frac{da}{d\omega} = 0 \).

Using implicit differentiation one can show that the result is
\[ -\frac{5}{6} \frac{a^2 \beta^2}{\alpha} = \omega^2 - \frac{\alpha - \frac{c^2}{2}}{\omega} \]
\[ = \omega^2 - \omega_o^2 \]  (9)

where \( \omega_o \) is the infinitesimal amplitude angular frequency where a resonance peak has occurred. Here the actual linear resonant frequency is \( f_o = \frac{\omega_o}{2\pi} \).

If the model for the nonlinear behavior is correct then from Eq. (9) if one plots the experimental data in the form of the amplitude squared \( a^2 \) vs \( \omega^2 - \omega_o^2 \) then the slope should yield a value\[
\text{slope} = -\frac{6}{5} \frac{\alpha}{\beta^2}.
\]

**The parameter of nonlinearity of the soil**

We define a parameter of nonlinearity similar to the definition used by Beyer.\(^9\) The force of the spring
\[ f_{\text{spring}} = -k_1 x - k_2 x^2 \]

is now written in the form
\[ f_{\text{spring}} = -A \left( \frac{x}{L} \right) - B \left( \frac{x}{L} \right)^2 \]  (10)

Then, the ratio of \( B / A \) is given by
\[ \frac{B}{A} = \frac{k_2}{k_1} \]  (11)

The slope is given by
\[ \text{slope} = -\frac{6}{5} \frac{\alpha}{\beta^2} \]  (12)

where \( \alpha \equiv k_1 / m \) and \( \beta \equiv k_2 / m \). Since
\[ k_2^2 = -\frac{6}{5} \frac{1}{\text{slope}} \frac{k_1}{m} \]

the ratio of \( B / A \) can be expressed in the form

\[ \frac{B}{A} = \sqrt{-\frac{6}{5} \frac{1}{\text{slope}}} \frac{\omega_2}{\omega_0} \]

(14)

Figure 10 shows the experimental results using the tuning curves shown earlier. The data points represent experimental points corresponding to the square of the amplitude \( a^2 \) of the soil-mine oscillator vs the quantity \( \omega^2 - \omega_0^2 \).

It can be observed from the results of Fig. 10 that modeling the nonlinearity of the soil to have quadratic behavior alone is insufficient. However, from the slope of the linear fit, the soil's value for \( B / A \) is estimated to be 32.

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