New test statistics for signal detection with a multi-sensor array were developed. Bin grating was employed to exploit diversity in estimating noise covariance matrix (nuisance parameter). Work was performed on calibration of ULA's, and invariant tests developed for the validation of optimal array configuration.
Introduction and Background Work

The research conducted has focussed on problems of signal detection with a multi-sensor array. These have followed the paradigm of the paper on rapid convergence rate in adaptive arrays by Reed, Mallett and Brennan. In that case, the problem was to detect a signal in a particular snapshot called the primary in the presence of strong interference and noise with unknown statistics. For this purpose, additional snapshots sharing the same noise statistics were collected, possibly by exploiting diversity (range or bin gating, for example) to obtain an estimate of the noise covariance matrix. This was then used to derive a weight vector for nulling out the interference in the primary and devising a test for determining the presence of a desired signal. The resulting test statistic has come to be called the Adaptive Matched Filter (AMF) statistic and may be regarded as an adaptive beamformer followed by a threshold detector.

Kelly formulated this problem as a multivariate hypothesis testing problem thereby eliminating the two step procedure (adaptive beamforming followed by detection) inherent in the AMF test and laying open the ground for picking the best test from the hypothesis testing viewpoint. In this method, the noise was modelled as Gaussian with an unknown covariance matrix and was further assumed to be i.i.d. from snapshot to snapshot. The desired signal was modelled as a deterministic unknown. Therefore, this appeared as the mean in the distribution of the data and the problem of signal detection became one of determining if the mean was non-zero. Kelly used the Generalized Likelihood Ratio (GLR) procedure to derive the GLR test. This was a simple function of the AMF statistic and a quantity called the loss factor, which had cropped up in performance analysis of the AMF statistic with the assumption of Gaussian noise.

However, the GLR procedure turned out to be intractable when a subspace structure was imposed on the covariance matrix reflecting partial knowledge of the interference. This motivated the work outlined in this report which was conducted the past few years and led to a more comprehensive approach to detection problems with arrays.

Invariant Testing

Array detection problems are characterized by high dimensions both for the data and the parameters (covariance). Now for signal detection problems, involving the mean, the covariance matrix is a nuisance parameter which has to be estimated. Unfortunately this is a high dimensional parameter and so there is a considerable penalty for estimating it. Worse still, this nuisance parameter could affect the distribution of the test statistic considerably, complicating the performance specification of a test. Thus a test optimized for a certain set of parameters could be severely degraded for a different set of parameters. Consequently, simulations for a test do not have much predictive value in general. Therefore there is a
need to minimize the effect of these nuisance parameters.

This is achieved by invoking the principle of invariance. Briefly stated, this principle implies that transformations on the data which do not alter the parameter values significant to the hypothesis (thus for signal detection, we consider transformations that map zero mean data to zero mean data and leave the signal vector unchanged) leave the problem invariant and so a reasonable test statistic should also be invariant to these. This restriction reduces the class of tests to be considered and enables the search for an optimum test. In fact, all possible invariant tests are characterized by a low dimensional set of functions called the maximal invariant. This greatly reduces the size of problem. Furthermore the parameter set that affects the distribution of the maximal invariant is also a low dimensional set called the induced maximal invariant. This not only assures that the performance is less likely to degrade sharply, if at all, but also, in many cases, ensures the constant false alarm rate (CFAR) property of the test. This enables the pre-setting of a threshold for automatic testing at a given false alarm rate. It turns out that these ideas are efficacious in dealing with the high dimensional character of the adaptive array detection problem.

In our research work, we explored the application of these ideas to the problem scenario considered by Reed and Kelly under a diversity of signal and interference models.

One Dimensional Signal Model

The problem considered by Reed and Kelly is the case with a one-dimensional signal model and unstructured covariance matrix. For this case, the maximal invariant we obtained turns out to be a set of two functions which interestingly are the AMF and the Kelly statistics. This implies that not only are these two tests both invariant but that all invariant tests are functions of these two. Therefore, in looking for better tests we need only look at various combinations of these test statistics. Furthermore, the induced maximal invariant is simply the array SNR and so all invariant tests have a parameter free distribution under the null hypothesis (SNR = 0). This implies that these tests have the CFAR property. Further, the performance (probability of detection) of these tests is specified by the SNR alone. Moreover, the one dimensional nature of the parameter space permits the development of a test which is optimum in the limit of very low SNR (for weak signals). The low dimension of the maximal invariant also makes it possible to obtain the bound for the probability of detection of any invariant detector. We showed that the Kelly detector performs very close to this bound for the dimensional parameters we studied. The AMF detector was also endorsed as having the invariant property.

This work was submitted as a paper to the IEEE Transactions on Aerospace and Electronic Systems [BS93b].

There has been a lot of interest in imposing structure on the covariance matrix to improve performance. The most common one involves the dominant subspace structure
which could correspond to a strong low rank interference component added to the white sensor noise. In this case, assuming the interference to have a Gaussian distribution, it turns out that the GLR procedure is not tractable. Applying the maximal invariant framework to this problem, we obtain the maximal invariant as a set of four functions. With this reduced dimensional set, it becomes possible to derive an invariant test statistic heuristically. This is seen to perform almost as well as a clairvoyant matched filter test (based on the true noise statistics). Further, it was found that its distribution was parameterized by two parameters: the SNR and the INR (interference to noise ratio). Therefore, the tests will not be exactly CFAR, though simulations seem to indicate that the variation in the PFA is small.

This work was presented at ICASSP-92 [BS92b] and was subsequently submitted as a paper to the IEEE Transactions on Signal Processing [SS93].

Multi Dimensional Signal Models

The one dimensional assumption was relaxed to accommodate both uncertainties in the signal model as well as the case when the signal cannot be modelled as being one-dimensional. Once again the maximal invariant was obtained and from that, tests were derived analogous to the one-dimensional case. Thus, although the GLRT could be solved, the maximal invariant framework permitted the development of the analog of the AMF test, which is linear and can be implemented via a beamformer structure. The distribution of these tests was found to depend on SNR like quantities, the number of which was given by the rank of the signal model. These tests were therefore CFAR.

This work was presented at the 6th SSAP workshop at Victoria, BC [BS92a] and submitted as a paper to the IEEE Transactions on Signal Processing [BS93].

ULA Model Validation

Subsequently, work was done on model validation on arrays. Specifically, a number of algorithms assume that the array satisfies the uniform linear array (ULA) assumption: this would imply that plane waves impinging on the array would look like sinusoids. This requires an equi-spaced array with gain and phase balanced sensors. This is usually accomplished by costly calibration procedures. However, once in use, a number of factors can operate to degrade performance and invalidate this assumption. Therefore a test based on the received data itself which would check the above assumption would be a useful tool indicating if a dataset is to be discarded or if re-calibration is required. In order to be useful, though, these tests would have to be invariant to the actual configuration of the sources and the received waveform. The invariant tests fit this bill perfectly. The maximal invariant was obtained and tests were derived for both real and complex data. These were shown to have
a parameter free distribution under the null hypothesis, thereby enabling meaningful tests. Further, these distributions were obtained so as to permit setting the significance level (size) of the test.

This work was presented at ICASSP-93 [BS93a] and is also being written up incorporating the work in progress.

Work in progress

Currently, work is ongoing in applying this test to the MARS data collected at Lake Huron. Unfortunately, this data was collected in a strong multi-path environment and one of the weaknesses of this test is that it breaks down under multi-path. Therefore, work is on to apply this test to data coming from certain parts of the sky as well as to certain sensors only. In this way, a tool for identifying and localizing sensor errors as well as multi-path in the environment is being developed. Preliminary results on the data have been encouraging.

References


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