### Theory and Applications of Belief Functions

Final Report  

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**Keywords:**  
Belief functions, probabilistic inference, reliability, risk assessment, expert systems, graphical models, network models, fusion and propagation algorithms.

**Abstract:**  
Graphical or network belief function models are defined and analyzed. Computational aspects are studied, including description and implementation of fusion and propagation algorithms. Applications to reliability of complex systems and to modeling dynamic systems are described.
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FINAL REPORT

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S-105, 9/86, A.P. Dempster, Toward computational theories of statistics. [unpublished]


S-107, 11/86, A.P. Dempster and A. Kong, Uncertain evidence and artificial analysis [rewritten and published, see S-120]

S-117, 11/87, A.P. Dempster, Bayes, Fisher, and belief functions [a revised version of this report will appear shortly in a Festschrift volume honoring George Barnard - exact citation not yet available]

S-118, 9/86, A. Kong, Construction of a tree of cliques from a triangular graph [A joint paper by Kong and Almond extending this work is under preparation]


S-121, 6/88, R. Almond, Fusion and propagation in graphical belief models [Ph.D. qualifying paper, incorporated in thesis S-130 below]


S-123, 2/88, A.P. Dempster, Comments on, "An inquiry into computer understanding", by Peter Cheeseman [Published in
S-124, 5/88, A.P. Dempster, Commentary on papers by Lane and Cooper [Published in Applied stochastic models and Data Analysis 5, 77-81 (1989)]*


S-128, 10/89, R. Almond, Operating instructions for the BELIEF package [accompanies S-129, 130]*

S-129, 10/89, R. Almond, The BELIEF package: a system for exploring graphical belief models [accompanies S-128, 130]*

S-130, 10/89, R. Almond, Fusion and propagation of graphical belief models: an implementation and an example [Ph.D. thesis accompanied by S-128, 129]*


Note: Items marked * will be furnished shortly to ARO, or when reprints become available.

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SUMMARY OF IMPORTANT RESULTS

1. Mathematical aspects of graphical models.

Over the past four years, important theory related to fully probabilistic (i.e., Bayesian) graphical models has been published by statisticians Lauritzen and Spiegelhalter in Europe, and computer scientist Pearl at UCLA. Almost coincidentally, generalizations of the theory to graphical models whose components may be partly logical and partly probabilistic have been developed by the ARO project at Harvard jointly with Glenn Shafer's group at the University of Kansas. A basic formulation and its properties are the major contribution of Kong's thesis S-106. Thoma's thesis S-127 provides further results on the nature of the mathematical structure, showing how to "lift" the theory from the purely logical case (used in the theory of relational data bases) to the mixed logical and probabilistic case that defines a core of potentially important belief function models.


Graphical belief functions represent uncertain knowledge that is dispersed throughout a network. Bringing the dispersed components to bear on a specific question requires a process of propagating and fusing the component belief functions through the network. Several reports (S-120, S-125) are devoted to deriving and illustrating the general process. Further details and a very extensive LISP program are provided in Almond's thesis (S-128, S-129, S-130).

3. Applications.

Almond's thesis applies the models with sensible results to assessing the reliability of a complex physical system where uncertainty about the failure rates of elements of the system comes partly from sample data and partly from ranges provided by experts. It is pointed out in S-125 that Kalman filter models are natural examples of graphical belief functions, whence a wide range of dynamical systems can be modelled with the benefit of insights and computational tools provided by the belief function theory. S-132, S-133, and S-135 provide preliminary reports of applications to genetic modelling, to pulsatile phenomena, and to groundwater flow problems.