COORDINATIVE AND STRATEGIC ASPECTS OF TRACKING DISCONTINUOUS INCLUDED
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COORDINATIVE AND STRATEGIC ASPECTS OF TRACKING DISCONTINUOUS INPUTS

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Two basic research projects were pursued. One project developed a finite-state model of how an individual tracker coordinates additively coupled position and velocity control sticks to capture a moving target. The model is multi-level. The top level consists of a discrete, probabilistic activity sequence generator closely related to the tracker's plan or general strategy for target capture. Lower levels of detail are more deterministic and indicate how the activities are conditioned by system
20. Error and by elapsed time. Marked individual differences were found among subjects at various levels of the performance model.

The second project developed a sensitive methodology for measuring a subject's extrapolated trajectory of a movement pattern that became hidden from view. The entire trajectory was mapped out over a series of trials, each of which measured a single point on the extrapolated trajectory. With prolonged practice and performance feedback the extrapolated trajectory closely approximated the objective movement pattern. However, withdrawal of feedback and the presence of irrelevant slower or faster movements shifted the entire extrapolated trajectory. These shifts can be described as parametric perturbations of an internal cognitive model of the movement pattern. There were marked individual differences in the lability of the internal model.
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Technical Information Officer
I. A Finite-State Description of Coordination in a
Two-Handed Target Acquisition Task*

R. A. Miller, R. J. Jagacinski, R. B. Nalavade, and W. W. Johnson

Research on the acquisition of moving targets with manual control systems has demonstrated that the control system dynamics can have large effects on target acquisition times. For example, Jagacinski, Repperger, Ward, and Moran (1980) showed that a velocity control system was considerably better than a position control system for capturing narrow, single dimensional, fast-moving targets. The present experiment investigated the acquisition of single dimensional moving targets with two control sticks, a position control stick and a velocity control stick, whose outputs were additively coupled. This particular control configuration was chosen for investigation for two reasons. First, this configuration allows for imitation of the control structure of the human eye. Poulton (1974) has commented that the major difference between visual and manual tracking performance is the superiority of the eye in target acquisition. While there are probably a number of factors such as the torque to inertia ratio that contribute to this superiority, one likely factor is the separate responses made to the position and velocity of a visual target. Contemporary control models of the eye (e.g., Young, Forster, and Van Houtte, 1968) consist of a saccadic

*The authors wish to thank Anant C. Misal and Samuel C. McNamee for their assistance in this project.
channel that responds to target position, and a parallel pursuit channel that responds to target velocity. The present manual control configuration thus allows for imitation of this structure.

The second reason for choosing this configuration is on the basis of stimulus-response compatibility arguments. Stimulus-response compatibility is a concept that has been introduced in reaction time research where, for example, it has been found that reaction times to a set of lights are faster and more accurate if the spatial arrangement of the lights corresponds in a simple manner to the spatial directions of the response motions (Fitts and Seeger, 1953). In terms of a process model of human performance, one might postulate a stage of processing in which the stimulus information is mapped into an appropriate response. The simpler this mapping process, the more compatible the sets of stimuli and responses are said to be. If one considers the two primary stimulus dimensions of a moving target to be its position and its velocity, then a parallel configuration of a position and a velocity controller permits a highly compatible set of responses. Namely, a step-response with the position controller could be used to match the target position, and a step response with the velocity controller could be used to match the target velocity.

In order to determine whether this conceptually simple control strategy or some more complicated method of coordinating the two control sticks would be used by experimental subjects, a finite-state modeling technique recently developed by Miller (1979) was employed. First, the movements of the control sticks were decomposed into sequences of broadly defined discrete maneuvers. Then a Markov description was developed of how the generation of these sequences was constrained by previous elements in the same sequence and by time and error dependencies.
This highly abstract description of movement processes can accommodate nonlinearities and non-stationarities, and does not necessitate long continuous time histories for parameter identification. Miller (1979) had previously used a similar technique to describe the coordination among three people in an anti-aircraft artillery team. The present study attempted to apply this approach to the coordination processes in the perceptual-motor performance of a single individual.

Method

Apparatus

The target acquisition system was simulated on an EAI Pace TR-48 analog computer. The target appeared as two 1.5 cm vertical lines moving horizontally across a 10 cm wide oscilloscope screen. A strip of yellow tape 1 mm wide by 20 mm long was positioned vertically at the center of the screen and served as the zero error reference marker. A chair was positioned such that the distance from the subjects' eyes to the screen was approximately 50 cm. At this distance the screen spanned 11.5° of visual angle, and the marker horizontally spanned 0.1° of visual angle. During each experimental session the subjects wore headphones over which they heard either a 390 Hz tone, white noise, or the experimenter's voice. The tone was used to alert subjects to upcoming trials and to provide feedback.

Two control sticks were mounted 30.5 cm apart on the surface of a table which was 76 cm high. Each control stick was pivot mounted, allowed approximately 30° of free excursion to the right or left, and required approximately 175 g of force to overcome a spring restraint. On the basis of pilot experimentation the gains of the position and velocity control sticks were respectively set at .38° and .76°/s of visual...
angle per 1° of control stick displacement. The velocity control system was sufficiently sensitive that subjects did not use the full limits of control stick excursion. Both control sticks were sufficiently sensitive that subjects could theoretically capture any of the moving targets using either stick singly.

Subjects

Thirty-six right-handed male college students performed 35 trials with a critical tracking task (Jex, McDonnell, and Phatak, 1966) that involved stabilizing a first order unstable system with a gradually decreasing time constant. The subjects were tested in three groups of twelve, and the six subjects in each group with the highest median score on the last 11 trials were randomly assigned to one of three experimental groups.

Procedure

Five different control configurations were of potential interest for comparison: a single position control stick (P), a single velocity control stick (V), a position control stick additively coupled with a velocity control stick (PV), a position control stick additively coupled with another position control stick (PP), and a velocity control stick additively coupled with another velocity control stick (VV). Pilot experiments indicated that for right-handed subjects the PV configuration was more effective with the position control assigned to the right hand, and that the PV configuration was superior to the PP configuration. Jagacinski, et al. (1980) had previously shown that for narrow fast-moving targets a single velocity control was superior to a single position control. Therefore, the present experiment restricted itself to a comparison of only three of the configurations: V, PV, and VV. Six subjects were randomly assigned to each of these three configurations.
Subjects in the V group used only their right hands, while subjects in
the PV and VV groups used both hands to control the two control sticks.
For these latter two groups, the outputs of the two independent
controllers were summated to form a single system output.

Subjects were instructed to manipulate their control sticks so as
to move the target to the center of the oscilloscope screen as quickly
as possible, and to hold it over the reference line at the center of
the screen for at least 400 ms. When this criterion was met, the target
disappeared from the screen. The subjects were further told that if
they failed to capture the target within four seconds, the trial would
be terminated. One second prior to each trial the warning tone was
sounded over the subject's earphones to alert him to the upcoming trial.
If the subject failed to capture the target within the four second time
limit, the tone was again sounded contiguous with the termination of the
trial to signal that the subject had failed on that trial. The inter-
trial interval within each set of ten trials was five seconds.

Three within-subject target variables were manipulated: 1) target
width, the gap between the two 1.5 cm lines, was either 2 or 4 mm
(.23° or .46° visual angle); 2) the initial target velocity was either
11.5 or 23.0 mm/s (1.32° or 2.64° visual angle/s); 3) the initial position/
direction of the cursor was such that the target either appeared 4.5 mm
(.52° visual angle) from center and moving away from the center of the
screen ("center targets"), or 50 mm (5.73° visual angle) from the center
of the screen and moving toward the center ("edge targets"). The
combinations of these last two variables with initial displacement to
the right or left of center are displayed as solid circles in Figure 1.
Figure 1. Solid circles indicate the various combinations of initial position and velocity for the targets used on Days 2-17. Solid squares indicate the targets used on Days 18 and 19. Target identification numbers are shown in parentheses next to the solid circles.
On the first day of target acquisition subjects received 8 blocks of 10 trials each to familiarize them with the control systems. Subjects with the PV and VV systems alternated across blocks using the right control stick alone for a block of trials and then the left control stick alone for a block of trials. Within each ten-trial block the target appeared randomly to the left or right of center with the constraint that there were five trials appearing to the left and five trials appearing to the right. Subjects received four blocks of trials with an initially stationary target and then four blocks of trials in which the initial target velocity was 11.5 mm/s. It was hoped that this procedure would allow the subjects to obtain an unambiguous understanding of how each of their control sticks affected the displayed position of the target.

On Days 2-17 subjects were permitted to use either or both of the two control sticks on all trials. They received 160 trials per session divided into 16 ten-trial blocks. Total capture time was summed over each of these 10-trial blocks. Again, the target appeared randomly to the left or right of center within each of these blocks with the constraint that there be five of each type. These 16 blocks were divided into eight sets of two blocks each. In each of these sets the subjects received one of the eight possible combinations of target width, initial velocity, and initial position/direction. These sets were randomly ordered within sessions, but the subjects were informed prior to each set about which type of target would be appearing next. At the end of each session, a subject was told his mean capture time for that day.

On Days 18 and 19 subjects were transferred to a new set of targets displayed as solid squares in Figure 1. These targets were
Results

Mean capture times minus the 400 ms capture criterion for subjects using each of the three control systems are shown in Figure 2. Instead of using both control sticks, one subject in the VV group used only a single velocity control stick, and this subject is excluded from Figure 2 and the subsequent statistical analyses. As can be seen in Figure 2, there were very large individual differences in mean capture times particularly for the subjects using the PV and VV control configurations. A Mann-Whitney U test was used to compare the PV group with each of the other groups. On asymptotic performance, the PV group was significantly better than the VV group ($U = 4, p = .026$, one-tailed), but the PV group was not significantly better than the V group ($U = 11, p = .155$, one-tailed). The same pattern occurred for transfer performance. The PV group was significantly better than the VV group ($U = 4, p = .026$, one-tailed), but the PV group was not significantly better than the V group ($U = 12, p = .197$, one-tailed). Sign-tests performed on the asymptotic performance times for all subjects indicated that there were significant effects of target width, target speed, and initial position-direction. Capture times were longer for narrow targets, fast moving targets, and edge targets ($p < .02$). For the transfer performance capture times were also longer for narrow targets ($p < .01$).

Discussion

The present experiment indicates that the VV system results in significantly longer capture times than the other two systems. Previous experimentation had shown that a system consisting of two independent position control systems was also inferior to the PV system. Therefore,
Figure 2. Mean capture times for individual subjects.
among the two-control-stick systems that were tested, the PV system provided the fastest target acquisition. This result is consistent with a stimulus-response compatibility hypothesis which argues that position and velocity are the two primary perceptual dimensions of a moving target, and that a well designed system for target acquisition should therefore have one degree of freedom corresponding to position control and one degree of freedom corresponding to velocity control. The present findings are subject to the experimental constraints that the velocity control sticks were sensitive enough that subjects did not use a bang-bang control mode, and that the same gain was used for all the velocity control sticks across the three control configurations. Pilot data suggests that subjects using a lower gain in the VV system may obtain superior capture times to those obtained in the present experiment by resorting to a bang-bang control strategy early in the trial. Whether this strategy would also improve performance with the PV system is not known, and this issue merits further investigation.

Comparison of the PV system with the V system did not reveal a significant difference in mean capture times. One interpretation of this result is that although the PV system does provide superior stimulus-response compatibility for two-control-stick systems, the difficulty in coordinating the movements of the two control sticks offsets any advantage over a good single-control-stick system. A second interpretation is that learning to coordinate the two control sticks of the PV system is a difficult task, which some but not all of the subjects may have accomplished over the course of the experiment. However, once appropriate coordination is learned, this system permits superior performance. Support for this second hypothesis comes from the finding that of the seventeen subjects analyzed in the present experiment,
the four subjects with the lowest capture times were all in the PV group (Figure 2). Stronger support for this hypothesis would require evidence that these four subjects used a different strategy for coordinating the two control sticks than the other two subjects in the PV group who had relatively longer capture times. In order to pursue this possibility a finite-state analysis of target acquisition behavior was conducted.

**Finite-State Analysis**

**Movement Categories and Target Categories**

Given the large individual differences in the capture times for the PV control system, the movement patterns of each of the six subjects in this group were analyzed in detail to determine how the two control sticks were coordinated. The time histories of the two control sticks were sampled at a rate of 100 Hz, filtered through the digital equivalent of two cascaded first-order 100 r/s low-pass filters, and then approximated as a sequence of straight line segments. Each stick was then coded as being either active or inactive at each sampling instant based on whether the slope of the corresponding line segment was greater or less than 2°/s. This criterion was derived from histograms of the line segment slopes. The joint state of the two control sticks was coded at each sampling instant into one of four categories: II, both control stick inactive; PI, position control stick active and velocity control stick inactive; IV, velocity control stick active and position control stick inactive; PV, both control sticks active.

In that individual subjects might use very different patterns of control for different targets, targets were grouped on the basis of similar degrees of position control stick activity relative to velocity
control stick activity for each subject. For the purposes of this analysis, the active and inactive states were further subdivided to form six movement categories. The inactive state was subdivided into "no response" and "offset" depending on whether the control stick position was respectively less than or more than .6° from its center position. The active state was subdivided into "medium" and "high" degrees of activity depending on whether the slope was less than or greater than 20°/s. Combining this distinction with whether the movement was to the right or left resulted in four sub-categories of the active state. The number of "events" occurring on a single control stick was defined as the number of transitions from one of these six states to another different state. The number of position stick events, N_p, and the number of velocity stick events, N_v, were summed across trials, and 

(N_p-N_v)/(N_p+N_v) was calculated for each target. This statistic can range from +1 for use of only the position stick, to -1 for use of only the velocity stick. As can be seen in Figure 3, subjects varied considerably in terms of which control stick exhibited the greater number of events. The subjects are ordered from 1 (best) to 6 (worst) on the basis of their mean capture time across all eight targets. Subjects 1, 4, 5, and 6 all exhibited differences between targets starting near the edge and near the center of the display. Subject 3 used only the position stick for the wide, slow-moving edge target (4W), and only the velocity stick for the remaining targets. Targets having approximately the same value of (N_p-N_v)/(N_p+N_v) were grouped together for the next stage of the analysis. Subject 2 had only one target group; Subjects 1, 3, and 5 had two target groups; and Subjects 4 and 6 had three target groups.
Figure 3. Relative usage of the position control stick and velocity control stick for each of eight targets. $N_p$ and $N_v$ are, respectively, the number of events on the position control stick and the velocity control stick. Target identification numbers 1-4 refer to Figure 1, and the accompanying letters, N and W, respectively refer to narrow and wide targets.
Activity Sequences

For each different group of targets, a first order markov description of the transitions among the joint stick states was constructed. The last state in each trial was labeled a "capture state" so that there were a total of eight different joint stick states (II, PI, IV, PV, II_c, PI_c, IV_c, and PV_c, where the subscript c indicates a capture state). The joint stick states were conditioned on the event number (the number of state transitions that had occurred up to that point in the trial), and the first order transitions among the eight states were tabulated across trials. Then multiple occurrences of the same type of joint stick state (e.g., a PI state that occurred early in the capture process and a PI state that occurred late in the capture process) were merged into a single state if: (1) each state was occupied on at least five percent of the trials; (2) a chi-square test indicated that they did not have significantly different probability distributions of transitions to immediately subsequent states (p > .05). In that there were no transitions out of the capture states, they were merged across trials without regard to these criteria. Two states were merged by summing the various types of transitions into and out of the two states.

Figures 4 and 5 illustrate the first order markov models for Subjects 1 and 6. Only those transitions which occurred on at least five percent of the trials appear in these figures. These transitions make up at least 80% of all the transitions which occurred for each of these markov structures. The circles in these diagrams represent the different states of control activity, and the arcs represent transitions between states. The numbers on the arcs are the probability of transition given that a state was entered.
Figure 4. Activity sequence generators for Subject 1 and associated mean capture times.
Figure 5. Activity sequence generators for Subject 6 and associated mean capture times.
At this level of abstraction any particular trial is characterized as a sequence of the control states shown in the above mentioned diagrams. These sequences may be referred to as "activity sequences." Each diagram itself is a representation of a process which generates such sequences and may be regarded as a discrete representation of each subject's abstract "activity sequence generator" or general strategy for capturing a particular class of targets. Each diagram shows the most frequently occurring transitions between control states without regard for detailed timing and without regard for the particular error state that accompanied these transitions. Even though Subjects 1 and 6 had very similar activity sequence generators for capturing the edge targets (Figures 4 and 5), they had very different mean capture times. In other words, the lower level details of how they implemented these processes must be quite different. On the other hand, for the capture of center targets, the activity sequence generators themselves are quite different. Subject 1 first transitions via one of three routes to a PV state that seems to segment the capture process into early and late stages. Subject 6's activity sequence generator for Targets 1N, 2N, and 1W lacks this simple symmetry and contains a considerable amount of transitioning back to previously occupied states. For Target 2W, this subject used only the velocity control stick. This comparison of Subjects 1 and 6 illustrates that individual differences may occur at different levels of description of this perceptual-motor skill, from the abstract activity sequence generator on down to lower levels of description.

A second aspect of the activity sequence generator is that it may suggest certain types of errors in the capture process. The target capture task is a time optimal control problem. An optimal control pattern is a step-ramp with the position control stick, or some combination
of movements such as simultaneous steps with the position and velocity control sticks. In theory such maneuvers can result in instantaneous capture of the constant velocity targets, though in practice even the step maneuvers would have some finite duration. Given this task structure, one might suspect that a capture strategy that involves activating one or more of the control sticks, deactivating both sticks, and then reactivating one or more of the sticks could result in poorer performance than a strategy that did not deactivate both control sticks until the cursor was over the target.

To test this hypothesis, the mean capture times of trials with and without II-noncapture states (other than at the start of a trial) were compared for each subject for each different group of targets. Of the thirteen such tests that were conducted, twelve indicated that capture times were significantly longer \((p < .01, \text{ one-tailed})\) when an II-noncapture state was present in a trial. The mean difference in capture times over these twelve target groups for trials with and without the II-noncapture state was 423 ms, which is relatively large in proportion to the mean capture times shown in Figure 2. The one exception to these findings was for Subject 6 capturing Target 2W, for which there was a small and not statistically significant reversal of this trend. This subject used only the velocity stick to capture Target 2W, and some of the II-noncapture states might correspond to constant velocity control episodes in which the cursor was nevertheless converging toward the target.

These results suggest that entering an II-noncapture state is indicative of some type of error, in that such trials have longer capture times. Whether the occurrence of an II-noncapture state represents a perceptual error in judging the target's position and velocity, an error
in extrapolating the target trajectory, or a deliberate pausing to plan
the next maneuver because of preceding errors of execution, cannot be
determined at this level of analysis.

A second suggestion of error can be seen in the activity sequence
generators of Subjects 1 and 6 for capturing edge targets. Most of
these trials begin with an II to PI transition followed by a PI to PV
transition (see Figures 4 and 5). However, occasionally each subject
bypasses the PV state. If the PV maneuver is a central element of the
overall process, then one might suspect that trials which lack this maneuver
might have longer capture times. To test this hypothesis, trials with and
without the PV state immediately following the initial PI state were
compared. For Subject 6 there was no significant difference in capture
times. However, for Subject 1 trials in which usual PI to PV transition
did not occur had capture times that were on average 219 ms longer
($p < .01$, one-tailed). This difference appears to be associated with a
higher proportion of trials containing an II-noncapture state when the
usual PI to PV transition is omitted.

Looking across the 13 activity sequence generators derived from the
six subjects, one may ask whether the frequency of entry into an
II-noncapture state is sufficient to distinguish efficient plans from
inefficient plans. On the basis of mean capture time, the thirteen
processes can be divided into one group of nine relatively efficient
processes having capture times from 731 to 1,013 ms, and a second group
of four relatively inefficient processes having capture times from
1,291 to 1,884 ms. Similarly, on the basis of the number of occupancies
of a noncapture II state represented in the diagrams, the thirteen
activity sequence generators can be divided into one group of seven
having 0 to .14 entries per trial, and a second group of six having .34
to .95 entries per trial. The four relatively inefficient activity sequence generators all belong to this latter group. The other two processes in this group are Subject 3 capturing Target 4W with a single position stick, and Subject 1 capturing edge targets. This latter activity sequence generator is particularly interesting because it so closely resembles the generator for Subject 6 capturing edge targets, and yet their captures times are so different. The lower level details of the target acquisition process must be examined to determine how the structure of these performances differ.

**Open-loop and Closed-loop Details**

The abstract activity sequence generators depict sequential constraints among the control actions in the capture process. However, the generators do not indicate whether the various activities associated with the different joint stick states depend on the error state (the discrepancy between cursor state and target state). Control activities guided by an error signal are typically termed "closed loop," and activities not guided by an error signal are termed "open-loop." However, it is quite possible for some aspects of an action to be open-loop and other aspects be closed-loop. Unless one explicitly introduces an exogenous perturbation into some aspect of an ongoing activity and notes whether or not a compensatory correction is made, it is often difficult to tell whether that aspect of the activity is open-loop or closed-loop.

For the present data, it was possible to construct a purely open-loop model that generated a distribution of capture times that was not statistically different from the distribution actually observed for each subject for each different class of targets. First order markov descriptions of the transitions among the eight joint stick states were constructed. Each successive 200 ms time interval from the beginning of
a trial was used as a conditioning variable. Across trials a tally was
made of what state transitions occurred within a given time interval and
how long a state occupancy lasted given that it started within that time
interval. These two statistics were taken to characterize a particular
"control mode" associated with that time interval. Two control modes
were merged by summing their corresponding state transition and state
occupancy distributions if two conditions were met: (1) the transition
probabilities out of each of the corresponding joint stick states were
not found to be significantly different by a chi-square test ($p > .05$);
(2) the distribution of state occupancy durations for each of the eight
states were not found to be significantly different in either mean
($t$-test, $p > .01$) or variance ($F$-test, $p > .01$).

The order in which the merging process was carried out was as
follows. First, the control modes corresponding to all of the time
intervals occurring toward the end of the capture process and having
transitions on less than five percent of the trials were merged without
regard for criteria 1 and 2. Then all pairwise comparisons among control
modes were conducted, and of those that passed both criteria 1 and 2,
the pair with the least significant chi-square value was merged. If
there was a tie in terms of the chi-square value, the two control modes
temporally closest were merged. This process was then repeated until
all the remaining control modes were significantly different from each
other either in the conditional transition probabilities or the state
occupancy distributions. The resulting characterization of the capture
process is a two-level hierarchical description. The upper level of the
hierarchy consists of the deterministic transitions among control modes.
The second level consists of transitions among joint stick states
occurring within each control mode.
The two-level description for Subject 1 capturing edge targets is shown in Figure 6. Only those joint stick state transitions which occurred on at least five percent of the trials are shown; however, these transitions comprise more than eighty percent of the total transitions which occurred. There are five different control modes, and four mode transitions occurring at 400, 600, 800, and 1,200 ms. The detailed joint stick state transitions associated with each mode correspond to different aspects of the overall activity sequence generator shown in Figure 4 that are activated at different times in the capture process. For example, the IV to IIc and PI to IIc transitions occurred in modes 3, 4, and 5, and their conditional probabilities of occurrence gradually increased as the capture process progressed. These time varying probabilities are approximated by a single average probability in the markov representation of the overall activity sequence generator. Similarly, the conditional probability of a PI to PV transition gradually decreased over control modes 2-4. Note that in control modes 2-4, the PI state may correspond to both PI states from the activity sequence generator. The distinction between these two states is lost in this open-loop representation. Although each mode shown in Figure 6 only occurred in a single contiguous time interval, other subjects had modes that repeated in noncontiguous intervals. Over the thirteen different target groups, the number of different control mode transitions ranged from 1 to 10 and was strongly correlated with subjects' mean capture times ($r = .86$).

To test the adequacy of these open-loop representations for predicting capture times, 1,600 trials were simulated and the resulting distribution was compared via a chi-square test to the experimentally observed capture time distribution. For each of the thirteen groups of
Figure 6. Open-loop (time conditioned) control structure for Subject 1 capturing edge targets. The numbers in the ovals are the mean duration of a state in milliseconds given that a transition into that state occurred in the indicated mode. The numbers on the arcs are the transition probabilities.
targets across the six subjects the experimentally observed and the simulated distributions were not found to be significantly different \((p > .05)\). Thus, it is possible to model the capture process in an open-loop fashion using time as a conditioning variable and reproduce the distributions of capture times. This does not imply, however, that information concerning the error state is not used by subjects performing this task. First, this simulation did not reproduce details of the movement trajectories, and it is doubtful that the trajectories could be described without reference to the error state. Secondly, to a large extent error and time may be correlated in this highly practiced task, thus making it difficult to determine to what extent the capture process is time driven (open-loop) or error driven (closed-loop). The simulation does argue for the plausibility of open-loop control as a major structural component of the capture process.

In order to assess the role of system error in conditioning transitions among the joint stick states, the loci of the beginnings and endings of various state occupancies were plotted in the phase plane (Figures 7 and 8). In order to overcome a slight lag induced by the segmentation of the time histories into constant velocity episodes, the error states depicted in these figures are those positions and velocities occurring 30 ms after the nominal time of the joint stick state transitions. Figure 7 shows that for Subject 1 capturing edge targets the beginnings of the PV state (when they occurred as the second transition in a trial) were spread over a considerable range. In contrast, the endings of the PV state were limited to a relatively small region about the origin.

Looking at the endings of the PV state in greater detail, one can ask whether the location in the phase plane determined whether the next state
Figure 7. The upper graphs show the error positions and velocities associated with the startings and endings of the PV state, when it occurred as the third state in a trial involving edge targets. The lower graphs are enlarged pictures of the PV endings from the upper graphs, with separate symbols to indicate transitions to PI and IV states.
Figure 8. Error positions and velocities associated with the startings of the II and IIc states when they occurred as the fifth and subsequent states in the capture of edge targets.
was a PI or an IV state. In other words, can the details of the error state lend greater determinism to transitions that are represented probabilistically at the level of the abstract activity sequence generator? A simple stimulus-response compatibility hypothesis is that if the velocity error is relatively large at the time of transition, an IV state is entered, and if the position error is relatively large at the time of transition, a PI state is entered.

Figure 7 shows that the beginnings of the PI and IV states can be dichotomized quite successfully, although not in the manner suggested by the stimulus-response compatibility hypothesis. It is possible to draw a rectangle enclosing the origin such that 92 percent of the transitions occurring outside the rectangle are from PV to PI, and 64 percent of the transitions inside the rectangle are from PV to IV. The termination of the P stick activity by a PV to IV transition (Figure 7) and by a PV to PI to II sequence of transitions (not shown) tends to be highly constrained in terms of error velocity. The termination of V stick activity by a PV to PI transition (Figure 7) and by a PV to IV to II sequence of transitions (not shown), is not as constrained in error velocity. However, termination of V stick activity is much more constrained in the time at which it occurs relative to the beginning of the PV state. This pattern suggests that termination of P activity may be predominately closed-loop, and the termination of V activity may be predominately open-loop at this point in the activity generation process for Subject 1. While illustrative of the kinds of control patterns that may be exhibited in this task, this particular pattern of error and time conditioning was not found for Subject 1 capturing center targets nor for Subject 6 capturing center or edge targets.
The final phase plane picture for Subject 1 (Figure 8) shows the distribution of the beginnings of II\textsubscript{C} states and the beginnings of II states when these states are entered on the fourth or subsequent state transition in a trial. The II states, which do not result in target capture, tend to begin farther from the target region than the II\textsubscript{C} states.

The overall picture of the target capture process that emerges from these analyses is a hierarchical one. The initial conditions of the target determine which activity sequence generator the subject will use. The activity sequence generator is a set of probabilistic constraints on the order in which the control sticks will be activated. Some joint stick state transitions are primarily time-determined, and other transitions are primarily error-determined. In that the errors that will arise on a given trial are not known beforehand, the structure of the activity sequence process is probabilistic. However, the overall process becomes more deterministic as the trial unfolds and specific errors develop. The next level of detail in the overall process would be a finer description of control stick movement and its relation to time and error state. These additional details are beyond the scope of the present study.

Given this hierarchical description, individual differences may arise at any of the levels. For example, Subjects 1 and 6 had highly similar activity sequence generators for edge targets, although even at this level one can observe that Subject 6 entered the II non-capture state more frequently (86 times vs. 57 times for Subject 1). At a lower level of detail, one can see in the phase plane pictures (Figure 7) that the velocities associated with PV state were much higher for Subject 1, and hence this maneuver was faster than for Subject 6. The
end of the PV maneuver was also more precisely delimited in the phase plane for Subject 1. Additionally, a comparison of the beginnings of IC states (Figure 8) shows that Subject 6 began such states farther from the origin. In that error velocities are very low in this state, it is not efficient to enter it very far from the target region. All of these factors probably contributed to Subject 6 having longer capture times. It is interesting to note that this analysis could form the basis for coaching Subject 6 toward improved performance.

Discussion

In their 1960 monograph, "Plans and the Structure of Behavior," Miller, Galanter, and Pribram define a **plan** as "any hierarchical process in the organism that can control the order in which a sequence of operations is to be performed (p. 16)." "Moreover, we shall also use the term 'Plan' to designate a rough sketch of some course of action, just the major topic headings in the outline, as well as the completely detailed specification of every detailed operation (p. 17)." Though much of the monograph deals with hierarchies of feedback loops called "TOTE units" (Test-Operate-Test-Exit), these authors do allude to the necessity of some open-loop control in motor skills at least at higher levels of organization. Furthermore, they suggest that while motor skills may be most conveniently represented in a continuous analog fashion at lower levels of organization, higher levels of organization of motor performance might be better represented in a discrete, digital language.

In the present analysis of the capture process, the markov descriptions of subjects' overall strategies seem to have some of the characteristics of Miller, Galanter, and Pribram's notion of a plan. Plans control the order of operation, and the activity sequence
generators in this representation describe probabilistically the sequential constraints that organize the capture process once the target is specified. However, the activity sequence generator is only an approximation to the plan because the activity sequence generator is corrupted by lower level deviations from the plan. That is, an activity sequence generator is identified from a set of activity sequences, which are in turn derived from observations of detailed control behavior. The observed control behaviors are not at the same level of abstraction as the plan. This estimation process therefore reflects both the plan and lower level deviations from the plan. For highly skilled subjects, it may be that the dominant activity sequence (the sequence obtained by selecting the most probable transitions at each successive state) provides a good indication of the plan.

Many details are missing at the level of description provided by the activity sequence generator. The precise movement trajectories cannot be generated by this structure without additional details of the movement processes. Further, the details of how and to what extent the events in an activity sequence are coupled to system error states remains to be more fully specified. The present work is incomplete in that it has not characterized these lower levels of organization in enough detail to permit full simulation of the capture movements. The contribution of the present work is that by providing some techniques for identifying the more abstract levels of skilled performance, it may hasten the time when it will be possible to provide fuller multi-level descriptions capable of generating such detailed simulation.

A final point concerns why the target acquisition task investigated in this study deserves to be called a "coordination task."
discussing motor skills, Bernstein (1967) argued that "the co-ordination of a movement is the process of mastering redundant degrees of freedom of the moving organ (p. 127)." He further argued that people solve the problem of coordination with different degrees of sophistication. "Fixation eliminating the redundant degrees of freedom mentioned above is employed only as the most primitive and inconvenient method, and then only at the beginning of the mastery of the motor skill, being later displaced by more flexible, expedient and economic methods of overcoming this redundancy through the organization of the process as a whole (p. 127)." In the present study, Subject 3 solved the problem of coordination by the former process, namely eliminating the redundant degree of freedom by using only one control stick for each target. The remaining five subjects attempted the latter approach of organizing all the available degrees of freedom, and these subjects had varying degrees of success. The activity sequence generators are one representation of this organization process. Constraints are placed on the use of the two control sticks selectively over the course of the capture process rather than by a simple all-or-none elimination process. In future work the analysis techniques developed in the present study should be used to measure how the organization process changes as a function of practice and what form it takes in more complex systems having more degrees of freedom.
References


II. Quantifying the Cognitive Trajectories of Extrapolated Movements*

R. J. Jagacinski, W. W. Johnson, and R. A. Miller

Research on the extrapolation of accelerating movement trajectories has been conducted using a number of different response modes and analysis techniques. For example, Gottsdanker (1952) used a pursuit tracking task in which subjects were instructed to continue tracking the target after it disappeared from view. Subjects' tracking after the target disappeared approximated constant velocity extrapolations that ignored acceleration exhibited by the target prior to its disappearance. Rosenbaum (1975) criticized Gottsdanker's use of pursuit tracking. Rosenbaum argued that the constant velocity extrapolation might arise from a motor limitation in executing the tracking response rather than a cognitive or perceptual limitation in appreciating the acceleration of the initial segment. To circumvent this problem he simply required subjects to press a button when a moving object that had disappeared from view would have reached a given point in space. Rosenbaum then conducted a correlational analysis of the times at which subjects pressed the button, and he concluded that subjects did not use a constant velocity extrapolation, but rather did take the acceleration of the initial trajectory into account.

*The authors wish to thank Dr. Harvey Shulman for making available his computer facility. The authors also wish to thank David Drucker, Tipp House, Diane Maute, Betsy Rader, and Steven Schwartz for their assistance in running and analyzing the present experiments.
The present research used a measurement technique that may be regarded as a hybrid of both the Gottsdanker and Rosenbaum approaches. As in the Rosenbaum experiment, subjects were simply required to press a button when a moving target that had disappeared from view would have reached a particular point in space. However, the point in space was varied from trial to trial so that the entire extrapolated trajectory could be mapped out in a manner analogous to the way Gottsdanker mapped out the extrapolated trajectory in a single trial. For example, Figure 1 shows a constant velocity trajectory (V) which starts on the far right, proceeds to the left, turns around, and returns to the far right again. A subject viewed the trajectory on its initial travel to the left. The target would disappear when it reached the turnaround point (El), and the subject would have to press a button when he believed the target would reach one of the nineteen locations indicated as P1-P19 in Figure 1. By recording over many trials when the subject believed the target would reach each of these points, it was possible to map out the subject's extrapolated trajectory. Furthermore, since the response was a simple button press, deviations from the actual trajectory could be ascribed to perceptual or cognitive limitations in the subject's performance rather than motor limitations. A series of three experiments was conducted to examine subjects' sensitivities to variations in the displayed visual trajectories and to variations in performance feedback.

Experiment 1

The first experiment compared subjects' extrapolated trajectories when they viewed either a constant velocity or a constant acceleration motion pattern. Individual subjects viewed only a single trajectory for over five-hundred experimental trials, and were not given any performance
Figure 1. Constant velocity trajectory (V) and constant acceleration trajectories (A1, A2, A3) used in Experiments 1-3.
feedback. This experiment was not designed to test whether a subject was sensitive to instantaneous acceleration, which would have required the use of many different trajectories. However, this task did require that the subject have an internal conceptualization or "internal model" of the constant velocity or constant acceleration trajectory.

For example, suppose the subject had an accurate conceptualization of the constant velocity trajectory analogous to the equation:

$$x(t) = |x(0) - vt|$$

where $v$ is the velocity of the trajectory, $t$ is time, and $x(t)$ is position as a function of time. $t$ is equal to zero when the subject first notices the moving display, $x(0)$ is the position of the moving display at this instant, and $x(t)$ is equal to zero at the turn-around point. Equation 1 corresponds to a straight-line trajectory such as Trajectory V in Figure 1. Knowledge of the value of $v$ and the ability to perceive $x(0)$ relative to the zero reference or turn-around point would be sufficient to initialize this model and perform the extrapolation task.

Similarly, suppose the subject had an accurate conceptualization of the constant acceleration trajectory analogous to the equation:

$$x(t) = \frac{1}{2}at^2 - \sqrt{2ax(0)}t + x(0)$$

$a$ is the acceleration of the moving display, and the other variables are defined in the same manner as in Equation 1. Equation 2 corresponds to a parabolic trajectory such as A2 in Figure 1 in which the moving display reverses direction when $x(t)$ equals zero. Knowledge of $a$ and the ability to perceive $x(0)$ relative to the turn-around point would be sufficient to initialize this model and perform the extrapolation task.

In summary, the present experiment (and those that follow) did not require that subjects be sensitive to instantaneous velocity or
acceleration. Rather, the present experiment mapped out subjects' internal models of the constant velocity and constant acceleration trajectories to ascertain how these cognitive trajectories differed from each other and from the veridical trajectories presented by the experimenter.

Method

Apparatus. The movement trajectories were displayed on a 13 X 10 cm oscilloscope screen positioned approximately 50 cm from the subjects' eyes. This screen had a display grain of approximately 79 points/cm in the horizontal dimension and 102 points/cm in the vertical dimension. The display was controlled by a Data General Nova 4 Computer with a Megatek BP-752 vector graphics interface that updated the display every 10 ms. Subjects wore headphones over which white noise was heard, and made their responses by pressing a pushbutton with their right index fingers.

Subjects. The subjects were four male and four female undergraduates who were recruited from Introductory Psychology classes or by means of a newspaper advertisement. Persons responding to the ad received $2.50 per day, while those recruited from the psychology class received course credit.

Design. Subjects viewed a trajectory which, when displayed on the screen, consisted of a target stimulus moving from right to left for a distance of 11.46 degrees of visual angle in a period of 2.08 s. This target would then immediately reverse its motion, moving the same distance in the same amount of time but in the opposite direction. However, in the experiment the target, which was a vertical line 1.58 degrees in length, was displayed only while moving in the right to left
direction; it disappeared when it reached the turnaround point (El, Figure 1). When the target line reached a point 2.75 degrees to the right of the point at which it would disappear (ONI, Figure 1), a stationary 11.46 degree vertical line called the "prediction line" would appear on the screen. The subjects' task was to extrapolate the motion of the target line on its reversed left to right journey, and they were asked to press a pushbutton when they believed that the target line would have crossed the displayed prediction line. The prediction lines were located at nineteen different locations (PI-PI9, Figure 1). In addition to the moving target line and the prediction line, five other stationary lines were also displayed as reference lines or "fenceposts." The fenceposts were 2.86 degrees apart in the horizontal dimension and were 1.15 degrees tall. The moving target first appeared at the right-most fencepost (11.46 degrees) and disappeared at the left-most fencepost (0 degrees) where the motion changed direction.

Subjects were split into two groups with each group containing two male and two female participants. One group viewed a constant velocity trajectory (Trajectory V, Figure 1), and one group viewed a constant acceleration trajectory (Trajectory A2, Figure 1). The equation for Trajectory V was:

\[ x(t) = 5.51t - 2.08 t^2 \quad -2.08 \leq t \leq 2.08 \quad (3) \]

where \( t \) is time measured in seconds and \( x(t) \) is position as a function of time, measured in degrees of visual angle. At the turnaround point, \( t \) is equal to 0. The equation for Trajectory A2 was:

\[ x(t) = \frac{1}{2}(5.27)t^2 \quad -2.08 \leq t \leq 2.08 \quad (4) \]
This equation describes an initial right to left deceleration reaching zero velocity at the turnaround point \( t = 0 \), followed by a left to right acceleration. Both trajectories cover the same total distance (11.46 degrees) in the same amount of time (2.08 s) with the left to right motion in both cases being the precise reversal of the right to left motion. The size and position of the fenceposts and the prediction lines were also the same in both conditions. For both trajectories the prediction line appeared when the target was 2.75 degrees away from the turnaround point. This constraint is equivalent to the prediction line being onset .5 s before the turnaround point for Trajectory V and 1.02 s before the turnaround point for Trajectory A2.

**Procedure.** Subjects received two sessions per day for two days. Each session consisted of 131 trials, with the initial five trials being considered practice while the remaining 126 trials were data trials. At the beginning of each trial the word "ready" appeared for .5 s. The screen was then blanked for .5 s, after which the five stationary fenceposts appeared. Simultaneously, the target appeared at the right-most fencepost beginning its right-to-left motion. The subject then proceeded with the task as outlined above, pressing the pushbutton when the extrapolated target reached the prediction line. Approximately 6.8 s after the response was made, the ready signal for the next trial would appear on the screen. The ordering of the 126 trials was randomized for each session. Each prediction line appeared six times in each session, with the exception of the prediction line located at the turnaround point, which appeared 18 times.

On Day 1 subjects were shown the entire trajectory several times without the target disappearing so that they would better understand their prediction task. Subjects were told nothing concerning the dynamics of the motions beyond the fact that the target "moved right to
left, changed direction, and moved left to right in the exact reverse motion." No feedback concerning their performance was given to the subjects.

**Results**

For each of the four sessions the median time at which each subject pressed the pushbutton was calculated for each of the 19 prediction positions. The mean of the four medians at each prediction position was calculated. The mean time at the turnaround position was then subtracted from each of the other 18 times so that the turnaround point could serve as a zero reference time \( x(0) = 0 \). The time corresponding to the turnaround point had been measured with a sample size three times larger than that at the other prediction positions.

Regression equations corresponding to a constant velocity function and a constant acceleration function, each passing through the turnaround point, were fit to each subject's mean prediction times, and the residual errors associated with these fits examined (Table 1). It was found that the prediction times of the four subjects observing the constant velocity trajectory and two of the subjects observing the constant acceleration trajectory were fit with numerically smaller residual error by the constant velocity function. Two one-tailed Mann Whitney U-tests were then conducted on the rank order of the proportion of variance accounted for by these functions. The constant velocity function accounted for a greater proportion of the variance in the prediction times of the subjects observing the constant velocity trajectory than for the subjects observing the constant acceleration trajectory \( p < .05 \), one-tailed. The opposite was true for the constant acceleration function, which accounted for a greater proportion of the
Table 1
Proportions of Variance Accounted for by Constant Velocity and Constant Acceleration Models in Experiment 1

<table>
<thead>
<tr>
<th>Subject</th>
<th>Trajectory</th>
<th>Constant Velocity Model</th>
<th>Constant Acceleration Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>V</td>
<td>.987</td>
<td>.870</td>
</tr>
<tr>
<td>2</td>
<td>V</td>
<td>.987</td>
<td>.827</td>
</tr>
<tr>
<td>3 (B in Figure 2)</td>
<td>V</td>
<td>.985</td>
<td>.882</td>
</tr>
<tr>
<td>4</td>
<td>V</td>
<td>.940</td>
<td>.898</td>
</tr>
<tr>
<td>5</td>
<td>A2</td>
<td>.960</td>
<td>.912</td>
</tr>
<tr>
<td>6</td>
<td>A2</td>
<td>.932</td>
<td>.927</td>
</tr>
<tr>
<td>7 (A in Figure 2)</td>
<td>A2</td>
<td>.916</td>
<td>.951</td>
</tr>
<tr>
<td>8</td>
<td>A2</td>
<td>.860</td>
<td>.959</td>
</tr>
</tbody>
</table>
variance in the prediction times of the subjects observing the constant acceleration trajectory than the subjects observing the constant velocity trajectory (p < .05, one-tailed).

Figure 2 shows the mean prediction times and the optimal trajectory for the subject in each condition whose data had the least mean absolute temporal deviation from the optimal trajectory. There appears to be less curvature in the data for the subject observing the constant acceleration trajectory (Subject A) than there is in the optimal trajectory. This trend was evident for all four subjects observing this trajectory. On the other hand there is evidence for a small initial curvature in the data for the subject in the constant velocity condition (Subject B). This trend was evident for three of the four subjects observing this trajectory.

**Experiment 2**

The second experiment was designed to determine how closely subjects could approximate the optimal trajectory given prolonged practice with performance feedback. An additional point of interest was how well subjects could maintain their extrapolation performance if feedback was withdrawn.

**Method**

**Subjects.** Seven male subjects and one female subject were recruited in the same manner as in Experiment 1. Due to the length of this experiment (7 days), some subjects received a combination of course credit and money while others received money only for their participation.

**Design and Procedure.** The design and procedure were the same as that used in Experiment 1, with the following exceptions. All subjects viewed Trajectory A2 for seven days (14 sessions). At the beginning of the first day the subjects were informed that the target stimulus would "slow
Figure 2. Average extrapolated trajectories over Sessions 1-4 in Experiment 1 for the subjects that most closely approached optimal performance without receiving any performance feedback.
to a stop, reverse direction, and speed up again in the opposite
direction with the exact reverse motion." However, subjects were given
no precise description of the dynamics of the motion. Two changes were
made in the stimulus display. First, the prediction line appeared when
the visible target line passed a point 1.70 degrees (or .8 s) before
the turnaround point (ON2, Figure 1). Secondly, the target line
disappeared at a point .1 degrees (or .2 s) away from the turnaround
point (E2, Figure 1). The visible portion of the trajectory covered
11.36 degrees in 1.88 s.

Another change was that subjects received both trial-by-trial feed-
back and daily feedback. On each trial, .15 s after the subject responded,
the word "early" or the word "late" would appear along with the time in
.01 s units by which the subject's response preceded or lagged the
correct response time. If the subject pressed the pushbutton within
.005 s of the optimal time, the words "right on" appeared on the screen.
The feedback remained on the screen for 1.85 s, and then the screen was
blanked for 4.8 s, until the ready signal for the next trial appeared.
Additionally, when subjects arrived each day they were told what their
mean absolute temporal deviation from the actual trajectory was for the
previous day's performance. Both of these types of feedback were given
for the first five days of the experiment (daily feedback being given
for the last time at the beginning of Day 6). The last two days of the
experiment were run without any feedback.

Results

For each subject asymptotic performance with feedback (Sessions
7-10) was compared with performance with feedback withdrawn (Sessions
11-14). The means of the median prediction times referenced to the
turnaround point \( (x(0) = 0) \) were calculated for each subject for each set of four sessions. In all cases a constant acceleration function accounted for a larger proportion of the variance in the subject's prediction times than did a constant velocity function. For asymptotic performance, the constant acceleration function always accounted for more than 98% of the variance, while the constant velocity function never accounted for more than 84% of the variance. For performance under feedback withdrawal, the constant acceleration function accounted for 97% or more of the variance for all subjects, while the constant velocity function accounted for less than 82% of the variance for all subjects.

For every subject the best fitting acceleration parameter was found to decrease when feedback was withdrawn. Table 2 shows the ratio of this parameter for each subject's feedback withdrawal performance to that subject's asymptotic performance with feedback. This decrease in the acceleration parameter corresponds to a slower extrapolation trajectory. A sign-test comparing the 18 post-turnaround prediction times in the feedback withdrawal condition with the corresponding times for asymptotic performance found this shift to be statistically significant for seven of the eight subjects \((p < .05)\). Figure 3 shows the mean prediction times for the subjects with the least (Subject D) and the greatest (Subject C) shifts. The asymptotic performance closely matched the optimal trajectory for all eight subjects, with the estimated acceleration being within 6% of the true acceleration for six of the eight subjects.

Experiment 3

The third experiment was designed to test how well subjects could ignore irrelevant aspects of the displayed trajectory. First, subjects
Table 2

Constant acceleration parameters, $a$, corresponding to the trajectory model $x(t) = \frac{1}{2}at^2$ for Experiment 2

$a_F$ corresponds to Sessions 7-10 with feedback.

$a_{FW}$ corresponds to Sessions 11-14 with feedback withdrawn.

<table>
<thead>
<tr>
<th>Subject</th>
<th>$a_{FW}/a_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (D in Figure 2)</td>
<td>.99</td>
</tr>
<tr>
<td>2</td>
<td>.92*</td>
</tr>
<tr>
<td>3</td>
<td>.86*</td>
</tr>
<tr>
<td>4</td>
<td>.82*</td>
</tr>
<tr>
<td>5</td>
<td>.77*</td>
</tr>
<tr>
<td>6</td>
<td>.75*</td>
</tr>
<tr>
<td>7</td>
<td>.73*</td>
</tr>
<tr>
<td>8 (C in Figure 2)</td>
<td>.39*</td>
</tr>
</tbody>
</table>

*Statistically significant shift in trajectory as assessed by a sign-test ($p < .05$).
Figure 3. Average extrapolated trajectories in Experiment 2 for Sessions 7-10 with performance feedback and for Sessions 11-14 when the feedback was withdrawn. Subjects C and D respectively exhibited the largest and smallest experimental effects.
were given prolonged practice with performance feedback in extrapolating the same trajectory used in Experiment 2 (Trajectory A2). Then feedback was withdrawn and at the beginning of different trials subjects viewed either the same trajectory (A2), or a speeded (A1) or slowed (A3) trajectory. The subjects' task was to extrapolate the same motion pattern that they had previously learned (A2) without allowing the speeding or slowing of the visible portion of the trajectory to influence their extrapolation performance.

Method

Subjects. Four male and four female subjects were recruited and compensated in the same manner as in Experiment 2.

Design and Procedure. The design and procedure were the same as that used in Experiment 2 for Days 1-5. However, on the last two days the subjects were divided into two groups of four subjects each. On half of the trials both groups viewed the same trajectory (Trajectory A2) that they had been viewing for the previous five days. For the other half of the trials the first group viewed Trajectory A1 (Figure 1), which is described by the equations:

\[
x(t) = \frac{1}{2}(21.08)(t + .1)^2 - 1.14 \leq t \leq -.2 \\
x(t) = \frac{1}{2}(5.27)t^2 \\
\]

-2 \leq t \leq 2.08

The second group viewed Trajectory A3 (Figure 1), which is described by the equations:

\[
x(t) = \frac{1}{2}(1.12)(t - .2)^2 - 3.96 \leq t \leq -.2 \\
x(t) = \frac{1}{2}(5.27)t^2 \\
\]

-2 \leq t \leq 2.08

Both Trajectories A1 and A3 are initially different from the Trajectory A2, but both become identical to Trajectory A2 at the point where the moving target disappears from view (E2, Figure 1). Trajectory A1 begins with an acceleration four times that of Trajectory A2, and Trajectory A3
begins with an acceleration one-fourth that of Trajectory A2.

At the beginning of Day 6 subjects were told that half of the trials would include a speeded (or slowed) visible motion. They were also told that after the target disappeared, it would revert to the motion with which they had been practicing for the previous five days, and that they should make their prediction on this basis. Subjects were urged not to let the speeding (or slowing) of the visible trajectory influence their extrapolation performance. They were also told that on the other half of the trials the normal or "reference" motion identical to the one they had been observing for the previous five days would be presented. Subjects were not given any feedback on their performance over the last two days.

Since the visible motion disappeared and reverted to Trajectory A2 at position C2, the time from the target's disappearance to the turnaround point remained .2 s. The prediction line also continued to appear when the target reached a position 1.70 degrees before the turnaround point, which was .5 s before turnaround for the speeded motion and 1.4 s before turnaround for the slowed motion. Another change during the last two days was that there were three different positions at which the moving target line appeared at the beginning of a trial. These three positions were 11.46, 8.94, and 6.42 degrees from the turnaround point (S1-S3 in Figure 1), and each occurred on one-third of the trials. In terms of the duration and distance traveled by the moving visible target, these starting points resulted in the following (distance, duration) pairings: (11.36 degrees, .94 s), (8.84 degrees, .82 s), and (6.32 degrees, .67 s) for Trajectory A1; (11.36 degrees, 1.88 s), (8.84 degrees, 1.64 s), and (6.32 degrees, 1.35 s) for Trajectory A2; (11.36 degrees, 3.76 s), (8.84...
degrees, 3.27 s), and (6.32 degrees, 2.70 s) for Trajectory A3. As in
the previous two experiments, there were 131 randomized trials per
session.

Results

For each subject asymptotic performance with feedback (Sessions 7-10)
and performance with feedback withdrawn (Sessions 11-14) were examined.
For these latter sessions, the data for trials with speeded (or slowed)
visible trajectories were analyzed separately from trials having the same
visible trajectory as in Sessions 1-10 (the reference trajectory). The
means of the median prediction times were calculated in the manner
described in Experiment 2. In all cases a constant acceleration function
accounted for a greater proportion of the variance in these means than
did a constant velocity function. For asymptotic performance, the
constant acceleration function always accounted for at least 97% of the
variance, while the constant velocity function never accounted for more
than 80% of the variance. For performance under feedback withdrawal
the constant acceleration function accounted for at least 96% of the
variance for all but two subjects, while the constant velocity function
accounted for less than 81% of the variance for all but one subject.

Each subject’s best fitting acceleration parameter was calculated
for extrapolation of the visible trajectory presented at the normal
speed with feedback (Sessions 7-10) and with feedback withdrawn (Sessions
11-14). The ratio of these parameters is shown in Table 3 (middle column).
As can be seen, the consistent decrease in the acceleration parameter
found in Experiment 2 is no longer present, with a two-tailed sign-test
showing an equal number of statistically significant (p < .05) upward
and downward shifts of the feedback withdrawal performances relative to
the asymptotic performances.
Table 3

Constant acceleration parameters, $a$, corresponding to the trajectory model $x(t) = \frac{1}{2}at^2$ for Experiment 3

$a_F$ corresponds to Sessions 7-10 with feedback.

$a_{FW}$ corresponds to Sessions 11-14 after viewing Trajectory A2 with feedback withdrawn.

$a_{FWC}$ corresponds to Sessions 11-14 after viewing one of the contrasting trajectories (A1 or A3) with feedback withdrawn.

<table>
<thead>
<tr>
<th>Subject</th>
<th>$a_{FW}/a_F$</th>
<th>$a_{FWC}/a_{FW}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (E in Figure 4)</td>
<td>.73*</td>
<td>1.19* (speeded trajectory, A1)</td>
</tr>
<tr>
<td>2</td>
<td>1.50*</td>
<td>1.18* (speeded trajectory, A1)</td>
</tr>
<tr>
<td>3</td>
<td>1.46*</td>
<td>1.08* (slowed trajectory, A3)</td>
</tr>
<tr>
<td>4</td>
<td>1.11</td>
<td>1.02 (speeded trajectory, A1)</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
<td>.97 (speeded trajectory, A1)</td>
</tr>
<tr>
<td>6</td>
<td>.85*</td>
<td>.92* (slowed trajectory, A3)</td>
</tr>
<tr>
<td>7</td>
<td>.59*</td>
<td>.86* (slowed trajectory, A3)</td>
</tr>
<tr>
<td>8 (F in Figure 4)</td>
<td>1.23*</td>
<td>.65* (slowed trajectory, A3)</td>
</tr>
</tbody>
</table>

*Statistically significant shift in the trajectory as assessed by a sign-test ($p < .05$).
The right column in Table 3 shows the ratio of each subject's best fitting acceleration parameters for performance with the speeded (or slowed) trajectory relative to performance with the unchanged reference trajectory in Sessions 11-14. A Mann-Whitney U test revealed a marginally significant (p = .057, one-tailed) correlation between the rank-order of these ratios and whether subjects viewed a speeded or slowed trajectory. The four subjects with the largest parametric changes (more than 10%) speeded or slowed their extrapolated trajectories depending on whether the visible trajectory was respectively speeded or slowed. A sign-test revealed that the changed speed of the visible trajectory resulted in statistically significant (p < .05) shifts in the extrapolated trajectory for six of the eight subjects. Figure 4 shows the mean prediction times for the two subjects with the largest shifts in performance.

General Discussion

One basic finding in the present series of experiments was the importance of performance feedback in both establishing and maintaining accurate extrapolation performance. Comparison of performance with Trajectory A2 in Experiment 1 with Sessions 1-4 of Experiments 2 and 3 revealed closer approximation to optimal performance in the latter experiments even at this early stage of practice. In Experiment 2 the withdrawal of feedback led to a pronounced drifting away from optimal performance. Whether other kinds of performance feedback might lead to more stable performance once feedback is withdrawn is an interesting topic for future research.

A second basic finding was the importance of the displayed trajectory even after prolonged practice. Experiment 1 demonstrated that early in practice subjects are sensitive to the displayed trajectory for
Figure 4. Average extrapolated trajectories in Experiment 3 for Sessions 11-14 in which feedback was withdrawn. Subjects E and F exhibited the largest effects of viewing the slowed or speeded trajectories.
determining the shape of their extrapolated trajectory, even in the absence of performance feedback. Experiment 3 demonstrated that after prolonged practice, the displayed trajectory affected extrapolation performance even when subjects were instructed not to let it do so. One possible explanation of this finding is that the differences in speed between Trajectory A2 and the speeded (A1) and slowed (A3) trajectories was so slight that subjects had difficulty in discriminating which type of trajectory was being presented. Subjects might then have greater difficulty in avoiding being influenced by the speeding or slowing. However, experimental tests in which subjects simply had to identify the trajectory as being speeded (A1) or being the reference trajectory (A2) revealed near perfect discrimination performance, and similar results would be expected for Trajectories A3 and A2. The experimental results therefore suggest that many subjects have difficulty ignoring aspects of the visible motion pattern that they know are irrelevant to their performance.

The present data also provide support for the construct of an internal cognitive model of the extrapolated trajectory (see Jagacinski and Miller, 1978; Kleinman, Baron, and Levison, 1971). In Experiment 2, the shifts that occurred in subjects' trajectories when feedback was withdrawn indicate that some parametric change occurred to the entire pattern of extrapolations. This result argues against the notion that subjects simply learn to associate an independent time-to-respond with each of the nineteen prediction positions (P1-P19). If the subjects' knowledge of the trajectory consisted of such a list-like structure, one would not expect the shifts at each position to be so strongly correlated. Furthermore, even as the trajectory shifts in the absence of feedback, it maintains its roughly parabolic shape consistent with a
constant acceleration pattern. The performance pattern is consistent
with the notion that subjects had an internal conceptualization or model
of the trajectory analogous to Equation 2, and that the parameter $a$
drifted to some value lower than the optimal value when feedback was
withdrawn. Why this parameter should drift to lower values rather than
higher ones is an interesting question that cannot be determined from
the present data. It is apparent from Experiment 3 that the consistent
direction of shift across subjects was disrupted when the reference
trajectory was mixed with other faster or slower visible trajectories.

The results of Experiment 3 provide additional evidence for para-
metric slowing down or speeding up of the entire extrapolated trajectory.
For those subjects who exhibited large (greater than 10%) shifts in the
extrapolated trajectory acceleration parameter after viewing the speeded
(or slowed) trajectory, the direction of the shift made the extra-
polated trajectory more closely resemble the trajectory that had
been viewed. Once again the parabolic shape of the extrapolated
trajectory was maintained despite this shift. This result suggests that
the internal model or conceptualization of the extrapolated movement
pattern should be regarded as a labile schema for which not only time and
position relative to turnaround must be initialized, but for which the
acceleration parameter, $a$, must also be initialized for a given trial.
Secondly, the initialization process is in some sense automatic, in that
it occurs even if subjects are instructed to maintain a constant $a$
parameter. A final point is that although these parametric shifts have
been discussed in algebraic terms, the internal model can be equally
well discussed in terms of geometric patterns. Namely, the family of
curves corresponding to variations in $a$ are a set of parabolas
passing through the point $x(0) = 0$ in Figure 1. The smaller the value
of \( a \), the wider the parabola is. Choosing the appropriate value of \( a \) is equivalent to choosing the appropriate parabola.

Studies of parametric shifts in extrapolation performance may be useful in understanding the limits of adaptivity for car drivers who must suddenly adjust from freeway speeds to the slower speeds of residential roads, or of baseball players who must adjust to the varying speeds of an opposing pitcher. In Experiment 3 it may be that subjects used an internal model to follow the visible motion. When the visible motion was speeded or slowed relative to the reference trajectory, the subjects would then have to shift to a new internal model or at least reinitialize its parameters to perform the extrapolation task. The observed shifts in the extrapolated trajectories could then be interpreted as failures to readjust the parameters of the internal model sufficiently. Driving skills and baseball skills may involve similar parametric adjustments.

Although the results in the present experiments have so far simply been described in terms of a single parameter constant acceleration model (not counting \( x(0) \)), additional work is underway to explore more detailed models such as a three parameter linear differential equation. Such models may lend greater insight into the nature of the parametric shifts that occurred in Experiments 2 and 3.

A final point concerns the large range of individual differences found across subjects in these tasks. If performance in these extrapolation tasks were found to correlate with performance in skilled tasks such as piloting an aircraft or some athletic skill necessitating extrapolation of movement trajectories, the present tasks might serve as a useful screening device.
References


