This report describes an extension of a mathematical model previously developed to study the forces to which an ejection seat occupant is exposed during high-speed ejections. The forces which tend to dislodge two limbs from one another, or from a restraining surface are calculated to exceed 600 pounds for Mach numbers above 0.7. It is concluded that, for high angles of attack, a pilot's musculo-skeletal system is not likely to withstand this tendency for dislodgment from a restraining surface and consequently, windblast and flail injuries are probable.
AERODYNAMIC FORCES EXERTED ON AN ARTICULATED HUMAN BODY SUBJECTED TO WINDBLAST.

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SECTION I
INTRODUCTION

Statistical data accumulated from five nations has brought to light the devastating effects of high-speed ejections from aircraft (Glaister, 1975). That is, the data show that windblast forces increase with aircraft speed to the point where an otherwise five or ten percent limb-flail injury rate rises to 40 percent or more. This is far from negligible, but control of the forces that produce excessive motion of the limbs of an ejection seat occupant can only be achieved if we increase our understanding of the aerodynamic loading to which a pilot is exposed during high-speed ejections. In a recent report (Schneck, 1976) it was shown that it is feasible to formulate a mathematical model which can predict such aerodynamic loading. Results from this model can then be incorporated into the Aerospace Medical Research Laboratory's modified Calspan Model of the Articulated Total Body (ATB) in order to assess the kinematics of limb motion under the action of specified aerodynamic forces. In the earlier report, potential flow solutions were presented for estimating the pressure distribution around the forearm of a human body subjected to windblast. Work completed under the present grant has expanded these preliminary results, and extended them in order that future studies may include some effects of flow separation.

The body model used here for the analysis of windblast forces is represented again by the 15-linkage system of spherical, circular-cylindrical, truncated-conical and flat-plate segments shown in Figure 1 (see Schneck, 1976, for a complete description). Moreover, specific attention is focussed on the events which take place where a body segment is either in contact with a restraining surface, such as the forearm in contact with an arm rest, or, equivalently, where two body segments are in contact with one another,
such as the upper arm pressing against the thorax. For this situation, the complex velocity potential for the aerodynamic cross-flow over the limbs (in the absence of flow separation) has been shown to be given by:

\[ w = \pi a V \coth \frac{a \pi}{\chi}, \]

which corresponds to the cross-flow streamline pattern depicted in Figure 2 for the cross-sectional geometric configuration shown, with \( \chi = y + iz \) and \( w = \phi + i\psi \). The streamlines illustrated are non-dimensionalized with respect to \( V \) and \( a \).

In the section which follows, the forces tending to dislodge the limbs from one another, or from a restraining surface, are calculated in the absence of flow separation, i.e., for the situation depicted in Figure 2. Section III then describes how the results can be modified to include the more realistic physical effects of separation of flow around the blunt body segments.

SECTION II

LIMB DISLODGING FORCES IN THE ABSENCE OF FLOW SEPARATION

Let \( p \) designate the aerodynamic pressure existing at a point along the cylindrical surfaces illustrated in Figures 2 and 3. If \( \gamma \) defines the azimuthal coordinate such that \( \tan \gamma = \frac{z-a}{y} \) (non-dimensionalized radius \( a = 1 \) in Figure 2), then the differential surface element \( a(d\gamma) \) having unit length along the axis of the limb has a force on it of magnitude \( df = p(a)(d\gamma) \) directed radially inward (see Figure 3). The component of this force tending to push the cylinders together (or pull them apart) is thus given by

\[ df_z = -(p \sin \gamma)(a \, d\gamma), \]

so that the net vertical force becomes:

\[ f_z = \int_{-\pi/2}^{3\pi/2} -ap \sin \gamma \, d\gamma \text{ per unit length} \]

(2)
Now, the pressure, \( p \), was obtained earlier (Schneck, 1976) in the form of a pressure coefficient which, for the cylindrical configuration illustrated in Figure 2, is given by:

\[
C_p = \frac{p - p_o}{\frac{1}{2} \rho U_o^2} = \sin^2 \alpha \left[ 1 - \frac{2^4}{a^2} \sin^2 \theta \right]
\]

Before substituting equation (3) into equation (2) and performing the indicated integration, it is convenient to make some transformations and redefine certain variables. Referring to Figure 3, we note that the equation of the circle is \( r^2 = 2az \), while \( r = 2a \sin \theta \) and \( y = r \cos \theta \). Thus,

\[
\frac{\pi v}{2z} = \frac{\pi (2a \sin \theta)(\cos \theta)}{4a^2 \sin^2 \theta} = \frac{\pi}{2} \cot \theta = \delta,
\]

And, \( d\delta = -\frac{\pi}{2} \cos^2 \theta \, d\theta = -\frac{\pi}{2} \cot \theta \, d\theta \).

Furthermore, \( \sin \gamma = \frac{z - a}{a} = \frac{z}{a} - 1 = \frac{r^2}{2a} - 1 = \frac{4a^2 \sin^2 \theta}{2a^2} - 1 = 2 \sin^2 \theta - 1 \)

\[
= \sin^2 \theta + (\sin^2 \theta - 1) = \sin^2 \theta - \cos^2 \theta = -\cos 2\theta
\]

so, \( \gamma = 2\theta + \frac{3\pi}{2} \), and \( d\gamma = 2 \, d\theta \).

Finally,

\[
\frac{a^2}{4z^2} = \frac{a^4}{16a \sin^2 \theta} = \frac{a^4}{16 \sin^2 \theta \sin^2 \theta} = \frac{3}{8} \frac{1}{\sin^2 \theta} \, d\delta
\]

And, \( \sin^2 \theta - \cos^2 \theta = 1 - \cot^2 \theta = 1 - \frac{4\delta^2}{\pi^2} \)

Putting equations (3) through (7) into (2) gives: (Limits are on \( \delta \))

\[
f_z = \int_{-\infty}^{\infty} \left\{ \frac{1}{2} \rho U_o^2 \sin^2 \alpha \left[ 1 - \left( -\frac{3}{8} \right) \frac{1}{\sin^2 \theta} \, \frac{d\delta}{d\theta} \right] \right\} \left[ -\cos 2\theta \right] d\theta
\]

Now, \( \int \left[ -\frac{a^2}{2} \rho U_o^2 \sin^2 \alpha + p_o \right] \left[ -\cos 2\theta \right] d\theta = 0 \), so, with \( q_o = \frac{1}{2} \rho U_o^2 \sin^2 \alpha \),
and the use of equation (7), equation (8) simplifies to:

$$
\int_{+\infty}^{\infty} 2(aq/\delta)^3 \sech^{4}\delta \left[\frac{4\delta^2}{\pi^2} - 1\right] d\delta = -aq \int_{+\infty}^{+\infty} \sech^{4}\delta \left[\delta^2 - \frac{\pi^2}{4}\right] d\delta
$$

(9)

Equation (9) contains integrals which are special cases of the general form (see Gradshteyn and Ryzhik, page 124):

$$
\int_{+\infty}^{+\infty} \frac{m \delta}{cosh^n \delta} d\delta = \frac{m-1}{(n-1)(n-2)cosh^{n-1} \delta} \int_{0}^{+\infty} \frac{m-2 \delta}{cosh^{n-2} \delta} d\delta
$$

(10)

In the first integral of equation (9), $m = 2$ and $n = 4$. In the second integral, $n$ is again equal to 4, but this time $m = 0$. Noting that the integrals are symmetric, we may write, for the case $m = 2$:

$$
\int_{0}^{+\infty} \delta^2 \sech^{4}\delta d\delta = \frac{1}{3} \int_{0}^{+\infty} \frac{2 \delta}{cosh^2 \delta} d\delta + \frac{2}{3} \int_{0}^{+\infty} \frac{2 \delta}{cosh^2 \delta} d\delta
$$

(11)

The first term on the right hand side of equation (11) evaluates to zero by application of L'Hospital's rule. The second term integrates to tanh $\delta$ evaluated at zero (see Gradshteyn and Ryzhik, page 99) and the third integral is of the form (Gradshteyn and Ryzhik, page 353):

$$
\int_{0}^{+\infty} \frac{2k \delta}{cosh^{2k} \delta} d\delta = \left(\frac{2k - 2}{2}\right) \frac{\pi^{2k}}{b(2b)^{2k}} |B_{2k}| ,
$$

for $k=1$ evaluated to $\pi^2/12$ with the Bernoulli number $|B_2| = 1/6$. Putting all of this together, the first integral of equation (9) may be evaluated in closed form to yield $(\pi^2 - 6)/9$. Now, for the case $m = 0$, equation (10) becomes:

$$
\int_{0}^{+\infty} \sech^{4}\delta d\delta = \frac{\sinh \delta}{3cosh^3 \delta} \left|B_{2k}\right| = \frac{2}{3} \int_{0}^{+\infty} \frac{d\delta}{cosh^2 \delta}
$$

(12)

As before, the first term on the right hand side of equation (12) evaluates to zero, and the second term integrates to unity. Thus, the second integral of equation (9) may be evaluated in closed form to yield $(4/3)(-\pi^2/4)$. We may then write:
Note the positive resultant sign of equation (13), indicating that the net force per unit length of limb is in the positive z-direction, or tending to cause dislodgment of the limb from a restraining surface (which could also be another limb).

In order to evaluate equation (13) further, we need to say something about \( q_0 \) and \( a \). As a reasonable approximation, we may begin by assuming that the radius of the human forearm is on the order of 1.5 inches (average) and that the length of this portion of the arm is about one foot. Thus, the total force acting on the forearm can be approximated by the relation

\[
f_t = \frac{9}{8} q_0.
\]

Now, consider an ejection taking place at an altitude of 10,000 feet. At this height, the density of air relative to a known standard is given by (Shames, page 540) \( \frac{\rho}{\rho_0} = 0.7385 \), where \( \rho_0 = 0.002378 \text{ slug/ft}^3 \), and the speed of sound, \( c = 1,078 \text{ feet per second} \). Thus,

\[
q_0 = \frac{1}{2} \frac{\rho}{\rho_0} \rho_0 [U_0^2/c^2]c^2 \sin^2 \alpha = 1020M^2 \sin^2 \alpha,
\]

where \( M \) corresponds to the Mach Number of the flow. Substituting this result into the equation for \( f_t \) gives the total limb-dislodging force as a function of Mach Number and Angle of Attack at the specified altitude of 10,000 feet. The final equation is:

\[
f_t = 1148 M^2 \sin^2 \alpha \quad (14)
\]

Figure 4 shows equation (14) plotted as limb dislodging force vs. Mach Number with the angle of attack appearing as a parameter. Although the figure illustrates force behavior in the supersonic range as well as the subsonic, it is recognized that the results past \( M = 1 \) are somewhat unrealistic in that the analysis has thus far neglected the presence of any shock waves in the flow. In any case, the subsonic values for \( f_t \) are, in themselves, very revealing. For example, at the higher angles of attack one observes a rapidly increasing
\[ F = 1148M^2 \sin^2 \alpha \]
limb-dislodging force. The significance of these large forces may be appreciated by examining them in relation to the super-imposed ordinate axis which is a measure of several published probability results. That is, in a study conducted by Horner and Hawker (1973) it was found that a pilot's average grip retention is such that the probability of his letting go is 100% if the dislodging force exceeds some 600 pounds. From Figure 4, we see that $f_t = 600$ pounds when $M = 0.72$ for $\alpha = 90^\circ$ and when $M = 0.83$ for $\alpha = 60^\circ$. Thus, at these higher angles of attack a pilot's musculo-skeletal system is not likely to withstand the tendency for dislodgment from a restraining surface if he is ejecting at Mach Numbers in excess of around 0.7. Similarly, in a study conducted by Payne (1975) it was found that the probability of major flail injury is around 100% if the ejection Mach Number exceeds 1.25. This would appear to be self-consistent with the predicted limb dislodging forces as interpreted above together with the results of Horner and Hawker. To this level of approximation, therefore, one would have to conclude that the generation of stagnation points in the flow produces forces that can cause limb-dislodgment (with subsequent flail and possible serious injury). Moreover, these forces are sensitive to the angle at which the limb intercepts the flow, such that the higher the angle, the greater the tendency for dislodgment. And finally, the forces increase rapidly with speed of ejection, which correlates well with the finding that windblast injuries increase dramatically as a function of airspeed (Glaister, 1975).

Figure 5, taken from Payne, Hawker and Euler (1975) is a photograph of an ejection seat occupant sitting in the ACES-II Seat at $-15^\circ$ Yaw and $-15^\circ$ Pitch, during a wind-tunnel simulation of an ejection from the F-105. The arrows point out several critical regions of the pilot-ejection-seat configuration where flow stagnation, with its consequent limb-dislodging force distribution, is likely to occur. Observe, in particular, those regions where the
Fig. 5
pilot is gripping the seat, where his upper arms are in contact with his torso, where the lower legs are in contact with the seat pan and where his head is in contact with the back of the seat. All of these represent sites of potentially serious windblast and flail injury. Moreover, Based on these results, one is therefore encouraged to pursue the mathematical analysis with a certain degree of assurance that the approach is faithfully describing events as they have been experimentally observed to take place. With this in mind, we now proceed to modify the theory to include flow separation effects.

SECTION III

VORTEX MOTION IN A REGION OF SEPARATION

In aerodynamic flow theory, two simplifications have been conveniently and commonly employed to investigate analytically separated flows over arbitrary three dimensional bodies of revolution (Marshall and Deffenbaugh, 1975). The first asserts that there is a direct analogy between three-dimensional steady flow and two-dimensional unsteady flow (Allen and Perkins, 1951) such that a three-dimensional steady separated flow problem can be analyzed as a two-dimensional unsteady separated flow problem. The second is based upon the observation (verified experimentally) that a two-dimensional unsteady wake can be described by a distribution of inviscid vortices, originating from the separation of shear layers and superimposed on the unseparated potential flow solution (Mello, 1959, Sarpkaya, 1968 and Marshall and Deffenbaugh, 1975). These vortices are modified by diffusion. It is the second of these two assumptions that we intend to pursue here, since arguments have already been presented for considering flow over the forearm in contact with a restraining surface to be two-dimensional (Schneck, 1976).
Thus, to examine the separated flow patterns around the geometric configuration illustrated in Figure 2, consider the situation depicted in Figure 6. A vortex pair has been placed on the lee or downstream side of the two cylinders as shown. Neither the location, $y_o \pm z_o$, nor the strength, $\kappa$, of these two vortices can be specified at this time. In fact, the location of the vortex pair can not be determined uniquely from potential flow theory alone. One must either consider the variation of pressure across the boundary layer, as described by Sarpkaya (1968) and Bar-Lev and Yang (1975), or, one must perform a series of experiments to determine empirical values for $y_o$ and $z_o$. The intent of this work has been, and is to exploit both of these techniques and we have begun with the latter.

Depicted in Figure 7 is a Scott-Engineering Sciences Model 9093 Hydrodynamic System for Flow Visualization Studies. The system is designed to utilize the hydrogen-bubble method for flow visualization, but we have performed experiments thus far with a Pliolite brand of solid particles. The Model 9093 provides a self-contained light source and a simple, but effective, light guide which enables the user to illuminate almost any part of the flow channel.

In addition to providing means for observing many features of incompressible flow, Model 9093 can also be used to model mathematically a wide range of compressible fluid flow behavior. To do this, the well known analogy between compressible and open channel flows is employed. This analogy has inevitable limitations which we are in the process of evaluating, but, nevertheless, there are experiments, both quantitative and qualitative, which are worth performing. Shown in Figure 8 are some early results which we have obtained using the Model 9093 water table, into which was placed a double-cylinder configuration such as that depicted in Figure 2. Note the vortices forming on the downstream side of this geometrical configuration as the flow moves.
Fig. 6

17
Fig. 8

19
from left to right. Such experimental visualization techniques can readily
yield information relevant to the further development of the mathematical
theory and we hope to be able to continue the work with future AFOSR support.

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V. Publications Arising From This Research Grant:
