TITLE: Analytical and Numerical Studies of Non-Stationary Corona in Long Air Gaps

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Analytical and numerical studies of non-stationary corona in long air gaps


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The properties of a non-stationary corona in which the front of space charge has not bridged the gap are studied analytically and numerically for different one-dimensional geometries. It is shown that an analytical theory is applicable for describing the development of the non-stationary corona at any shape of applied voltage. The characteristics of a non-stationary corona differ greatly from those of a stationary corona. For the same applied voltage the current of non-stationary corona can be much higher than the current under steady conditions. The effect of aerosol ions on the properties of a corona near a grounded object is numerically studied under thunderstorm conditions.

1. Introduction

A non-stationary transient regime of a corona discharge (when the front of space charge has not bridged the gap) is initiated in any laboratory non-uniform gap after the voltage application. This regime is of particular importance under natural conditions when corona is ignited near the top of a high grounded object or near other extremities (tips of trees, bushes, grass, etc.) in the thundercloud electric field. It has been shown [1] that the corona space charge hinders the initiation of an upward leader from the object top and consequently is favourable to the protection of the object against lightning. The purpose of this work is to analytically and numerically study the main characteristics of a non-stationary corona for different one-dimensional electrode configurations (concentric spheres, coaxial cylinders and emitting plane).

2. System of equations

A corona discharge is described by the balance equations for the density of charge carriers and Poisson's equation. The boundary condition is that the electric field near the coronating electrode is equal to the corona onset field. In addition, we use the condition that the voltage drop along the discharge gap is equal to the applied voltage $U(t)$.

3. Results obtained

In a non-stationary case, there exist an exact analytical solution for one-dimensional electrode geometries and on assumption that the discharge current is time-independent [2]. (This is valid only for a certain voltage shape.) A generalization of this analytical approach to any voltage shape has been given in [1]. By using this approach to describe a non-stationary corona between concentric spheres with inner radius $r_0$ and outer radius $R_0$, we obtain the expression for the discharge current

$$i = 2\pi e_0 \frac{\mu U_m}{6\pi} \left( \frac{r_0}{\tau} - 1 + e^{-\tau/t} \right)^2$$

for the applied voltage shape

$$U = U_m \left( 1 - e^{-\tau/t} \right).$$

Here, $\mu$ is the ion mobility. Equation (1) is valid for $r_0 << R << R_0$, where $R$ is the radius of the space charge front.

Fig. 1. The ion cloud radius $R$ and corona current $i$ calculated (solid curves) numerically and (dashed curves) analytically, respectively. $r_0 = 1 \text{ cm, } R_0 = 10^3 \text{ m, } U_m = 1 \text{ MV and } \tau = 10 \text{ s.}$
The analytical approach used was verified by comparison with the results of a computer simulation. Figure 1 shows that the approximations used in the analytical theory introduce an accepted error, compared with the results of a computer simulation. The current was numerically calculated for various rise times of the voltage, \( t_f \). The shorter is \( t_f \), the higher is the peak current.

Fig. 2. The transient corona current in the gap between concentric spheres for \( r_0 = 1 \text{ cm} \) and \( R_0 = 5 \text{ m} \). The voltage rises linearly up to 300 kV at \( 0 < t < t_f \) and is constant at \( t > t_f \).

Expressions similar to (1) were obtained also for the case of coaxial cylinders and for the case of an emitting plane. (The latter one simulates a corona ignited in a thundercloud electric field near tips of trees, bushes or buildings distributed over a large area of the earth surface.)

The analysis of the results shows that the dependence of the current of a non-stationary corona upon the mobility \( \mu \) of charge carriers is weaker than that of the current of stationary corona. For a well developed non-stationary corona and at a given instant, we have \( i = \mu^2 \) for spherical electrodes, a weaker dependence \( (i \sim \ln(\mu)) \) for coaxial cylindrical electrodes and no \( \mu \) - dependence for plane electrodes.

For each electrode geometry there exists a critical shape of applied voltage (or that of the external field in the plane case) at which the corona current is time-independent. This occurs at \( U = i^2 \) in the spherical case and at \( E_0 = i \) in the plane case; the cylindrical case is intermediate between the spherical and plane cases. If \( U \) increases in time faster, the corona current also increases; if \( U \) increases slower, the current decreases in time.

The density of corona current from coronating earth’s surface is controlled by the evolution in time of a thundercloud electric field above the space charge layer rather than by electric field at ground level.

4. Effect of aerosol ions

The characteristics of a corona discharge initiated in a thundercloud electric field near grounded objects can be effected by the formation of aerosol ions [3]. In order to estimate the effect, we numerically simulated the properties of a non-stationary corona developed from an isolated spherical anode of radius \( r_0 = 10 \text{ cm} \) by taking into account aerosol ions. Our kinetic model for ions was similar to that used in [3]. Figure 3 shows the calculated radial distributions of the densities of light and aerosol ions at \( t = 60 \text{ s} \) when the ion cloud radius reached 120 m. Aerosol ions are important only at large distances from the electrode at which the total ion density drops down to the initial density of neutral aerosol particles, \( N_{ao} \sim 10^7 \text{ cm}^{-3} \).

Fig. 3. The radial distributions of the densities of light and aerosol ions around a solitary sphere of radius 10 cm at \( t = 60 \text{ s} \). The voltage rises linearly up to 4 MV for 30 s and is constant at \( t > 30 \text{ s} \).

5. References