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Numerical Analysis of Stochastic Dynamical Systems in the Medium-Frequency Range

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ABSTRACT

A stochastic finite element method (SFEM) has been applied to the mid-frequency vibration analysis of complex structural systems. A frequency domain model reduction strategy is devised using the energy operator approach. The dominant eigen-subspace of the energy operator adapted to a specified frequency band allows the construction of a reduced model using the Ritz-Galerkin procedure. Once an efficient reduced model is constructed, the system parameter uncertainty is performed using SFEM approach. The approach allows significant computational efficiency in performing system parameter uncertainty analysis. The SFEM approach utilized is based on an integration of the Karhunen-Loeve and the Polynomial Chaos expansions with the energy operator methodology. The approach presented avoids the combination of disparate approaches for the mid-frequency vibration analysis such as the statistical energy analysis (SEA), the traditional modal analysis (well-suited for high and low frequency vibration analysis respectively) and thus appears to provide a general framework for mid-frequency vibration analysis. The emphasis in the paper is on investigating the effect of system parameter uncertainty on the dynamical response. Firstly, a simple example involving a coupled uncertain rod system is studied to provide a better understanding of the formulation. Secondly, the examples of a capsule and a shell-plate assembly with random joint parameters are investigated using an existing FEM software.

INTRODUCTION

The traditional modal analysis technique in association with the finite element method (FEM) has been successfully applied for the linear dynamic analysis in the low frequency range. The success of the method relies on the fact that the first few structural normal modes primarily constitute the total response. In the higher frequency range, the statistical energy analysis (SEA) is a popular tool where a large number of resonant modes contributes to the total response. In this approach, the entire structural system is divided into a number of subsystems. In the state of dynamic equilibrium, the energy balance equations relate the responses in terms of total average energy for each subsystem. The parameter sensitivity of the high frequency vibration to minor deviations is accounted for by modeling the subsystem natural frequencies as Poisson random points along the frequency axis. However, the applicability of SEA is restricted only to the cases where each subsystem exhibits uniformly high modal density.

In the medium-frequency range, the situation is complicated by a number of facts: The modal superposition technique is computationally expensive as a large number of structural mode contributes to the total response, and various subsystems of the structure display high and low frequency behavior simultaneously not justifying the application of SEA.

The presence of imperfections and discontinuities significantly influences the response in the mid-frequency band due to short wavelength vibration features. Furthermore, uncertainty of the coupling parameters among the subsystems, lack of the precise model of the structure due to incomplete knowledge of the secondary systems complicate the analysis. As a result, the confidence level in the response prediction becomes questionable even with very detailed mathematical models based on nominal system parameters in the mid-frequency band.

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A number of recent studies have addressed the higher frequency vibration problem including the effect of system parameter uncertainty. Manohar & Adhikari (1998) and Adhikari & Manohar (1999) formulated a frequency-domain dynamic analysis procedure for a randomly-parametered beam assembly using a frequency dependent FEM shape functions (see, Fergusson & Pilkey 1993a,b). The random system parameters modeled as independent Gaussian stochastic processes result in a set of weighted integrals as random variables in the FEM formulations. Adhikari & Manohar described numerous approaches based on the random eigen-function expansion method. Vlahopoulos & Zhao (1999) presented a hybrid procedure based on FEM and SEA for the mid-frequency range vibration analysis. They used SEA to model the subsystems having high modal density (having several wavelengths of vibration in the frequency band of interest). The subsystems exhibiting only a few wavelengths of vibration in the excitation frequency band are analyzed using FEM. Langley & Bremner (1998) recently presented perhaps a similar hybrid methodology based on dividing the total degrees of freedom into a global set and a local set. In their approach, SEA is used to solve the local degrees of freedom accounting for the effect of system parameter uncertainty. The solutions of the global degrees of freedom are sought deterministically accounting for the presence of the local degrees of freedom in terms of an approach similar to fuzzy structure theory (Ohayon & Soize 1998). Soize (1998a,b) presented an approach to construct a reduced model for linear structural systems by describing the mid-frequency structural dynamics by an energy operator. Using only the dominant eigen-subspace of the energy operator (symmetric positive definite) adapted to a fixed frequency band, the reduced model in that specific frequency band is thus constructed. The modifications of the method to consider the case of a linear structure coupled with internal acoustic cavities are also reported by Soize (1999) when the structure and the internal acoustic cavity exhibit the mid-frequency and the low-frequency behavior respectively. The proposed method appears to offer a flexible framework of analysis by avoiding the hybridization of disparate approaches such as FEM, SEA or fuzzy structure theory for mid-frequency dynamic analysis. Soize (2000) also proposed a nonparametric model of random uncertainties for the reduced system using the principle of entropy optimization. Only the knowledge of the global mean mass, stiffness and damping matrices are considered to obtain the complete probabilistic information of these matrices using the maximum entropy principle.

The present paper develops a general purpose probabilistic framework that is applicable to large scale built-up systems. A frequency-domain model reduction strategy is adopted to minimize the computational effort in the mid-frequency band. First introduced by C. Soize, an energy operator adapted to a fixed medium frequency band whose dominant eigen-subspace allows the construction of a reduced model using a Ritz-Galerkin method. Consequently, the effect of parameter uncertainties is investigated on the reduced model in the framework of SFEM. The approach, essentially based on dynamic stiffness methodology, overcomes the need of determination of the joint statistics of natural frequencies and mode shapes by avoiding the modal superposition approach. The uncertain system parameters are described by Gaussian random fields. The well-known Karhunen-Loeve expansion is used to decompose the random fields by a set of Gaussian random variables. Consequently, a typical response quantity is expanded using a generalized random series known as Polynomial Chaos expansion. The deterministic matrix equations governing the unknown coefficients of the Polynomial Chaos expansion is derived and solved. The approach is exemplified through its application to complex structural systems with parameter uncertainties.

MID-FREQUENCY STRUCTURAL DYNAMICS: DETERMINISTIC CASE

A finite element approximation of the response of a linear time invariant distributed parameter system is given by,

$$u_n(\mathbf{x},\omega) = \sum_{i=1}^{n} q_i(\omega) N_i(\mathbf{x}) \qquad (1)$$

with $N_i(\mathbf{x})$ being the finite element shape functions and q_i is the i-th nodal response quantity. Consequently, the governing equation of motion can be expressed as

$$\mathbf{A}_n \mathbf{q} = \mathbf{F}_n \qquad (2)$$

where the dynamic stiffness matrix, $\mathbf{A}_n$ is given by

$$\mathbf{A}_n = -\omega^2 \mathbf{M}_n + i\omega \mathbf{D}_n + \mathbf{K}_n \qquad (3)$$

where $\mathbf{M}_n$, $\mathbf{D}_n$, $\mathbf{K}_n$ are the mass, damping, stiffness matrices and force vector respectively.

A brief description of the energy operator methodology for the mid-frequency vibration analysis is next presented (Soize, 1998a,b). The approach is adopted to construct a computationally efficient reduced model in the mid-frequency range. In this section, only the case of deterministic system parameters is considered. Subsequently, the extension of the procedure to consider the case of randomly parametered systems will be discussed.

ENERGY OPERATOR APPROACH

Definition of an Energy Operator

For the continuously parametered linear system under consideration, let **M** be the mass operator and the operator valued frequency response function **T** be defined as $\mathbf{A}^{-1}$. The energy operator over frequency band B, $\mathbf{E}_B$ is defined as

$$\mathbf{E}_B = \frac{1}{\pi} \int_B \omega^2 \operatorname{Re}[\mathbf{T}^*(\omega)\mathbf{M}\mathbf{T}(\omega)] d\omega \qquad (4)$$

Its eigenfunctions satisfies the following equation

$$\mathbf{E}_B \mathbf{e}_i = \lambda_i \mathbf{e}_i \qquad (5)$$

The energy operator is a positive-definite symmetric operator which possesses a countable set of decreasing positive eigenvalues. Consequently, its eigenfunctions form a complete basis by which a finite element displacement field can be approximated. Thus, the dominant eigen-subspace of this operator can be effectively used to construct a reduced model using a Ritz-Galerkin method.

Finite Dimensional Approximation

As the analytical expressions of the eigenfunctions are not generally available, $\mathbf{E}_B$ is approximated as $\mathbf{E}_{B,n}$ in n-dimensional subspace with its eigenfunctions represented by $\mathbf{e}^n_i$. Thus, $\mathbf{E}_{B,n}$ the projection on the n-dimensional subspace spanned by the shape functions $N_i(\mathbf{x})$, is expressed as (Soize, 1998)

$$\mathbf{E}_{B,n} = \sum_{i,j=1}^{n} [E_n]_{ij} (\cdot, N_j)_H N_i \qquad (6)$$

which means that for all **g**,

$$\mathbf{E}_{B,n}\mathbf{g} = \sum_{i,j=1}^{n} [E_n]_{ij} (\mathbf{g}, N_j)_H N_i \qquad (7)$$

with inner product operation defined over domain Ω as

$$(\mathbf{f},\mathbf{g})_H = \int_\Omega \mathbf{f}\mathbf{g} d\mathbf{x} \qquad (8)$$

where

$$E_n(\omega) = \frac{1}{\pi}\int_B \omega^2 \,\mathrm{Re}[\mathbf{A}_n^{*-1}\mathbf{M}_n\mathbf{A}_n^{-1}]d\omega \qquad (9)$$

In an FEM framework, these eigenfunctions are approximated as,

$$\mathbf{e}_i^n = \sum_{j=1}^{n} P_j^i N_j \qquad (10)$$

Consequently, the standard eigenvalue problem is shown to be transformed into a generalized eigenvalue problem as given by (Soize 1998a,b)

$$GE_nG\mathbf{P} = \lambda_n G\mathbf{P} \qquad (11)$$

where

$$G_{ij} = (N_i, N_j)_H \qquad (12)$$

Representation of the Solution in the Mid-Frequency Range

As previously mentioned, the eigenvalues of the energy operator span the functional space in which the solution exists. Furthermore, only the first few eigenvectors of the energy operator, adapted to the frequency band B can effectively represent the solution vectors in that frequency band. Thus, the solution vector can be approximated by using the first N such eigenvectors as

$$u^N{}_n(\mathbf{x},\omega) = \sum_{i=1}^{n} U_i(\omega)\mathbf{e}_i^n(\mathbf{x}) \qquad (13)$$

Consequently, equation (2) reduces to

$$\mathbf{A}_N\mathbf{U} = \mathbf{F}_N \qquad (14)$$

and

$$\mathbf{F}_N = P^T\mathbf{F}_n \qquad (15)$$

The new coordinates $\mathbf{U}$ relate to the coordinates $\mathbf{q}$ as

$$\mathbf{q} = P^T\mathbf{U} \qquad (16)$$

where $[P]$ is the $(n\times N)$ real matrix whose columns are the N eigenvectors corresponding to the N highest eigenvalues in equation (11). The operator $\mathbf{A}_n$ reduces to $\mathbf{A}_N$ as

$$\mathbf{A}_N(\omega) = P^T\mathbf{A}_n(\omega)P \qquad (17)$$

In general, N is much smaller than the original system dimension n, thus demonstrating the efficiency of the reduced model.

MID-FREQUENCY STRUCTURAL DYNAMICS: STOCHASTIC CASE

For randomly parametered system, the energy operator becomes a random quantity leading to a random eigenvalue problem stated in equation (5). In case of such systems, the eigensolution computed in equation (11) can be viewed as a first order approximation to the mean eigenproblem of the energy operator. As the energy operator is obtained by a frequency averaging process, it should show milder fluctuations than the underlying material properties. Furthermore, only the higher order eigenvalues and eigenvectors are expected to be susceptible to the system randomness. However, we are only interested in the dominant eigen-subspace of the energy operator. The higher order eigenvalues and eigenvectors are not of interest for the purpose of the present analysis. These facts justify the approximation made for the eigenvalue analysis. However, once these eigenvectors have been calculated, the coordinates of the solution with respect to this basis can be expressed as a random quantity. In the present study, the variability of the system parameters are modeled as stochastic processes.

Mathematical Characterization of Random Processes

A stochastic process, α, is a function of n +1 variables with n being the physical dimension and θ identifies its probabilistic dimension. Thus, a typical system parameter modeled as random process is characterized by a set of basis functions ξ in Hilbert space H_c. Consequently, the state of the system modeled also as random process represented by a set of basis ψ in Hilbert space H_L. Identification of the basis ξ and ψ is accomplished by the Karhunen-Loeve and Polynomial Chaos expansions, which are presented next.

Karhunen-Loeve Expansion

The Karhunen-Loeve expansion of a random process α takes the following form (Ghanem & Spanos 1991)

$$\alpha(\mathbf{x},\theta) = \overline{\alpha}(\mathbf{x}) + \sum_{i=1}^{\infty} \sqrt{\lambda_i}\,\xi_i(\theta)\phi_i(\mathbf{x}) \qquad (18)$$

Here $\overline{\alpha}$ denotes the mean process and $\{\xi_i\}$ is a set of orthogonal random variable. $\{\phi_i(\mathbf{x})\}$ and $\{\lambda_i\}$are the eigenvectors and eigenvalues of the covariance function $R_{\alpha\alpha}(\mathbf{x},\mathbf{y})$ of the random process and evaluated as the solution of the following integral equation (Ghanem & Spanos 1991)

$$\int_D R_{\alpha\alpha}(\mathbf{x},\mathbf{y})\phi_i(\mathbf{y})d\mathbf{y} = \lambda_i\phi_i(\mathbf{x}) \qquad (19)$$

where D defines the spatial dimension over which the process α is defined. If the random process α is Gaussian, then $\{\xi_i\}$ form an orthonormal Gaussian vector. The Karhunen-Loeve expansion is mean-square convergent irrespective of the probabilistic structure of the stochastic process provided it has finite variance.

Polynomial Chaos Expansion

As the covariance function of a solution process is not known *a-priori*, the Karhunen-Loeve expansion cannot be used to expand the solution quantity $S(\theta)$. Consequently, a typical solution quantity $S(\theta)$ is expressed by the Polynomial Chaos expansion (Ghanem & Spanos 1991)

$$S(\theta) = \sum_{j=0}^{\infty} S_j \Psi_j(\theta) \qquad (20)$$

where $\Psi_j(\theta)$ are the multidimensional Hermite polynomials of $\{\xi_i\}$, known as the Polynomial Chaos which form a complete basis in the space of second order random variables. Thus, the complete probabilistic description of the solution process $S(\theta)$ is obtained once the deterministic coefficients S_j are calculated. In

addition to the representation of the solution process, the Polynomial Chaos expansion can also be used to characterize non-Gaussian material properties.

Representation of the Stochastic System Equation

Expanding $\mathbf{A}_N$ in its Karhunen-Loeve expansion (obtained from the stiffness, mass and damping matrices, see Ghanem & Spanos 1991), the following expressions for the equilibrium equation is obtained,

$$[\sum_{k=0}^{nkl} \xi_k \mathbf{A}_N^k]\hat{\mathbf{U}} = \mathbf{F}_N \qquad (21)$$

where

$$\mathbf{A}_N^k = P^T \mathbf{A}_n^k(\omega) P \qquad (22)$$

Here *nkl* represents the number of terms retained in the Karhunen-Loeve expansions; ξ_i are the random variables obtained from the Karhunen-Loeve expansion, and are Gaussian whenever the material properties are modeled as Gaussian processes. Consequently, the solution process is approximated by the Polynomial Chaos expansion as

$$\hat{\mathbf{U}} = \sum_{j=0}^{npc} \Psi_j(\theta)\hat{U}_j(\theta) \qquad (23)$$

Here *npc* is the number of terms retained in the Polynomial Chaos expansions. Substituting the above expansion into the reduced model equation and performing a Galerkin error minimization procedure leads to,

$$\sum_{i=0}^{npc}\sum_{j=0}^{npc} < \xi_i \Psi_j \Psi_k > \mathbf{A}_N^i \hat{U}_j = < \Psi_k \mathbf{F}_N > \qquad (24)$$

where the deterministic coefficients U_j are computed as the solution of the aforementioned matrix equation. It is worthwhile to mention at this point that the frequency response curve for a fixed value of frequency is a random variable when the system parameters are modeled as random fields. Furthermore, it becomes a complex valued random process in its frequency evolution due to presence of damping. The presence of discrete natural frequencies induces strong non-stationary characteristics of the frequency response functions with the standard deviation peaking up significantly near the resonance points even with small random deviations of system parameters. This may potentially affect the extremes of the frequency response curves and thus the dynamic response.

NUMERICAL RESULTS

As an example of the foregoing analysis, firstly the dynamical response of a coupled rod system is investigated. The equation of motion of the rods is described by,

$$\frac{\partial}{\partial x}[EA(x)\frac{\partial u}{\partial x} + C_1(x)\frac{\partial^2 u}{\partial t \partial x}] = m(x)\frac{\partial^2 u}{\partial t^2} + C_2(x)\frac{\partial u}{\partial t} \qquad (25)$$

with appropriate initial and boundary conditions. Here EA, m, C_1 and C_2 are the stiffness, mass, strain-rate dependent and velocity dependent damping per unit length of a typical rod. The choice of this simple system is intended to provide a better understanding of the system behavior from the viewpoint of the energy operator approach. The schematic diagram of the system is shown in Figure 1. The free end of the bar B is excited externally. In this study, Young's moduli of the rods are assumed to be identical independent Gaussian random fields with a coefficient of variation equal to 0.05. The autocovariance function of each of the processes is taken to be of the following form: $R(x,y)=\exp^{-|x-y|/b}$; where b is the correlation length, assumed to be half of the

length of each rod. Only two terms in the Karhunen-Loeve expansion are used corresponding to the first Karhunen-Loeve mode for each rod. In the Polynomial Chaos expansion, the first six terms corresponding to a complete second order expansion, are considered. In the numerical investigation, the following parameters are assumed: for rod A, the mean EA_A = 405284.0 N; m_A=200.0 kg; C_{1A} = 30.0 Ns and C_{2A} =30.0 Ns/m^2, L_A = 100 m; for rod B, the mean $EA_B = EA_A/20$; $m_B = m_A$; $C_{1B} = C_{1A}/20$, $C_{2B} = C_{2A}/20$ and L_A = 100 m. The number of the finite elements is taken to be 350 for each rod. Figure 2(a) shows the eigenvalue spectrum of the energy operator for the frequency band 13-15 rad/s. Figure 2(b) presents the first few eigenvectors of the energy operator. Note that the eigenvectors fluctuate more rapidly in rod B than in rod A, identifying relatively higher frequency vibrations in rod B. Figure 2(c) presents the frequency response function at the free end of the rod. Only the eigenvectors corresponding to the first ten eigenvalues of the energy operator are used to construct the reduced model. In Figure 2 (c), the top and bottom solid lines represent the mean and standard deviation of the frequency response curve. The results of the direct Monte Carlo simulations with 100 realizations performed on the full-scale system is represented by point plots. The results from the Polynomial Chaos expansion performed on the reduced model are in very good agreement with the Monte Carlo simulations. Interestingly, only 10 degrees-of-freedom system can now effectively captures the original system behavior produced by a 700 degrees-of-freedom model.

Next, we consider the example of a capsule which includes two joints. The FEM mesh of the structure is shown in Figure 3. The total number of elements in the system is 128 with each joint having 16 elements. The cylindrical section of the capsule has a length of 60 m with radius of 7 m. The capsule and the joints are modeled as thin shell elements having six degrees-of-freedom in each node. The total number of degree-of-freedom of the system is 780. The effect of joint stiffness uncertainties on the frequency response function of the structure is investigated. The following properties of the shell elements are considered: Young's modulus E = 2×10^{11} N/m^2, Poisson ratio ν = 0.3, density ρ = 7500 kg/m^2, shell thickness t = 0.05 m. In all cases, a proportional damping model for the global matrix is adopted as: $\mathbf{D}_n = \alpha\mathbf{M}_n + \beta\mathbf{K}_n$. Young's moduli of both the joints are modeled as independent Gaussian random fields with exponential correlation function as: $R(x_1,x_2; y_1, y_2)=\exp^{-|x_1-x_2|/b_1-|y_1-y_2|/b_2}$ where b_1 and b_2 are the correlation lengths of the two dimensional random fields with coefficient of variation equal to .05. For both the joints, the correlation length in both circumferential and longitudinal directions are taken to be 20 m. The integral eigenvalue problem resulting from the Karhunen-Loeve expansion was solved using a standard eigen-analysis software package. Only two terms in the Karhunen-Loeve expansion and first six term Polynomial Chaos expansion are considered. The damping coefficients are arbitrarily taken as: $\alpha=10^{-2}$ Ns/Kg-m and $\beta= 10^{-5}$ s. Figures 4(a) shows the dominant eigen-subspace of the energy operator in the frequency band (260-330) rad/s. Figure 4(b) shows the direct frequency response function (along the outer normal direction) of the node on the junction of the cylinder and the hemispherical cap (referred to as Case A) as highlighted in Figure 3. In Figure 4(b), the top and bottom solid lines represent the mean and standard deviation of the frequency response curve obtained using SFEM performed on the reduced model. The results generated from the direct Monte Carlo simulations (with 100 realizations) performed on the full-scale system is represented by point plots. Again it should be noted that uncertainty analysis performed on the 10 degree-of-freedom system (being the dominant eigen-subspace) using SFEM shows excellent agreement with Monte Carlo simulations. The similar results for a node on the top joint, referred to as Case B (highlighted on Figure 3) is presented in Figures 4(c) and 4(d) for the frequency band (910-970) rad/s. From Figure 4(c), the dominant eigenspace of the energy operator is characterized by first 15 eigenvectors. In Figure 4(d), the top and bottom solid lines show the mean and standard deviation of the frequency response function generated using SFEM. The point plots represent the direct Monte Carlo simulation results for 100 realizations for the full-scale system. Note that the reduced model SFEM results show very good agreement with the Monte Carlo simulations.

Finally, the example of a shell-plate assembly with random connection properties at the junctions between the shell and plate is considered. Figure 5 shows the FEM mesh of the system. The total number of elements is 114 with each connection having 12 elements. The total assembly is modeled with thin shell elements having six-degrees of freedom for each node. The total number of degrees of freedom is 756. The cylindrical section has an outer radius of 2.4 m and a length of 10 m. Both the shell and plate are assumed to have a thickness of 0.05m. The system is assumed to have $E = 2\times10^{11}$ N/m^2, Poisson ratio ν = 0.3, density ρ = 7500 kg/m^2. The same proportional damping models as in the case of the capsule is considered. The uncertainty in the connections is modeled assuming a stochastic variation of Young's modulus as in the case of the previous example involving the capsule. Again, only two terms in the Karhunen-Loeve expansion and the first six terms

in the Polynomial Chaos expansion are considered for the SFEM analysis. In Figure 6(a), the dominant eigen-space of the energy operator for the frequency band (553-559) rad/s is shown for the case of damping coefficients: $\alpha=10^{-3}$ Ns/Kg-m and $\beta= 7\times10^{-7}$ s and a coefficients of variation of Young's modulus equal to 0.05. A typical frequency response function (for the response along the normal direction) for the node marked in Figure 5 is presented in Figure 6(b). Similar results are shown in Figure 6(c) and 6(d) for the case with $\alpha=2\times10^{-3}$ Ns/Kg-m and $\beta= 10^{-5}$ s and coefficients of variation of Young's modulus equal to 0.02. From both Figures 6(a) and 6(c), it is evident that the reduced model can be constructed using only the first 5 eigenvectors of the energy operator for the specified frequency band. In Figures 6(b) and 6(d), the top and bottom solid lines present the mean and standard deviation of the frequency response curve using SFEM. The results of Monte Carlo simulations performed on the full-scale system with 100 realizations are represented by point plots. Evidently, the Monte Carlo simulations show excellent agreement with the SFEM results.

CONCLUDING REMARKS

The paper integrates a SFEM approach based on the Karhunen-Loeve and the Polynomial Chaos expansions with an energy operator-based formalism for the mid-frequency vibration analysis of complex structural systems with parameter uncertainty. The uncertainties of the system parameters are modeled as Gaussian random fields. The well-known Karhunen-Loeve expansion is used to decompose the Gaussian random fields into a set of Gaussian random variables. Consequently, typical response quantities are expanded as Polynomial Chaos expansions. The use of the reduced model remarkably enhances the computational efficiency of the SFEM. The formulation presented has been implemented using existing finite element software. Thus, the approach is particularly useful for the mid-frequency vibration analysis of large scale structural systems in order to investigate the effect of parameter uncertainty on their responses.

REFERENCES

Adhikari, S. & Manohar C. S. 1999. Dynamic Analysis of Framed Structures with statistical uncertainties. *Int. J. Numerical Methods in Engineering*. 44: 1157-1178.

Fergusson, N. J. & Pilkey W. D. 1993a. Literature review of variants of dynamic stiffness method, Part 1: The dynamic element method. *The Shock and Vibration Digest*. 25 (2): 3-12.

Fergusson, N. J. & Pilkey W. D. 1993b. Literature review of variants of dynamic stiffness method, Part 2: Frequency dependent matrix and other corrective methods. *The Shock and Vibration Digest*. 25 (4): 3-10.

Ghanem, R. & Sarkar, A. 2000. The Medium-Frequency Range Vibration of Coupled Fluid-Structure Assembly with System Uncertainty, *The 14th ASCE Specialty Conference on Engineering Mechanics,* The University of Texas, Austin, May 22-24.

Ghanem, R. & Spanos, P. 1991. *Stochastic Finite Elements: A Spectral Approach*. Springer Verlag.

Langley, R. S. & Bremner P. 1998. A hybrid method for the vibration analysis of complex structural-acoustic systems. *Journal of the Acoustical Society of America*.105(3): 1657-1671.

Manohar, C. S. & Adhikari S. 1998. Dynamic stiffness of randomly parametered beams. *Probabilistic Engineering Mechanics*. 13 (1): 39-49.

Ohayon, R. & Soize C. 1998. *Structural Acoustics and Vibration*. Academic: San Diego.

Soize, C. 1998a. Reduced models in the medium frequency range for general dissipative structural-dynamics systems. *European Journal of Mechanics: A-Solids*. 17(4): 657-685.

Soize C. 1998b. Reduced models in the medium-frequency range for general external structural-acoustic systems. *Journal of the Acoustical Society of America*. 103 (6): 3393-3406.

Soize C. 1999. Reduced models for structures in the medium-frequency range coupled with internal acoustic cavities. *Journal of the Acoustical Society of America*. 106 (6): 3362-3374.

Soize C. 2000. A nonparametric model of random uncertainties for reduced matrix models in structural dynamics. *Probabilistic Engineering Mechanics*. 15: 277-294.

Vlahopoulos, N. & Zhao, X. 1999. Basic development of hybrid finite element method for midfrequency structural vibrations, *AIAA Journal*. 37 (11): 1495-1505.

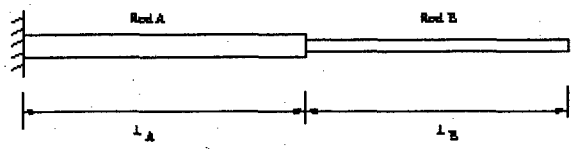

Figure 1. Schematic diagram for the coupled rod system.

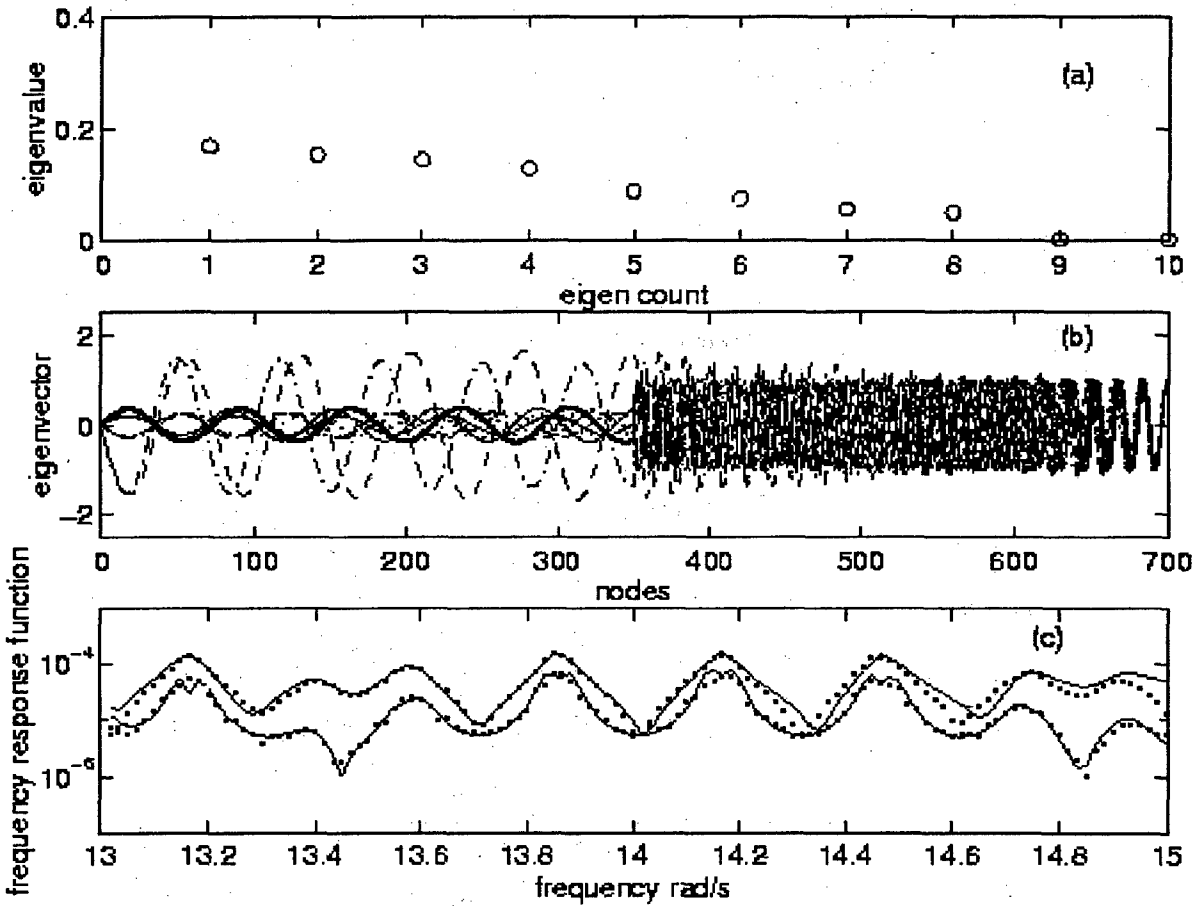

Figure 2. For the rod system: (a) The distribution of eigenvalues of the energy operator, (b) First few eigenvectors of the energy operator, (c) The frequency response function of the system at the free end.

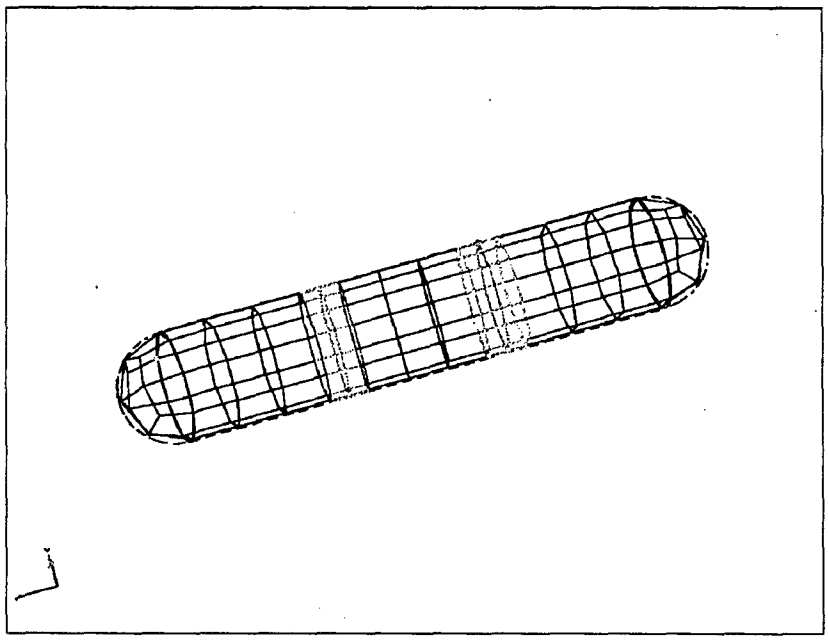

Figure 3. Finite element mesh for a capsule.

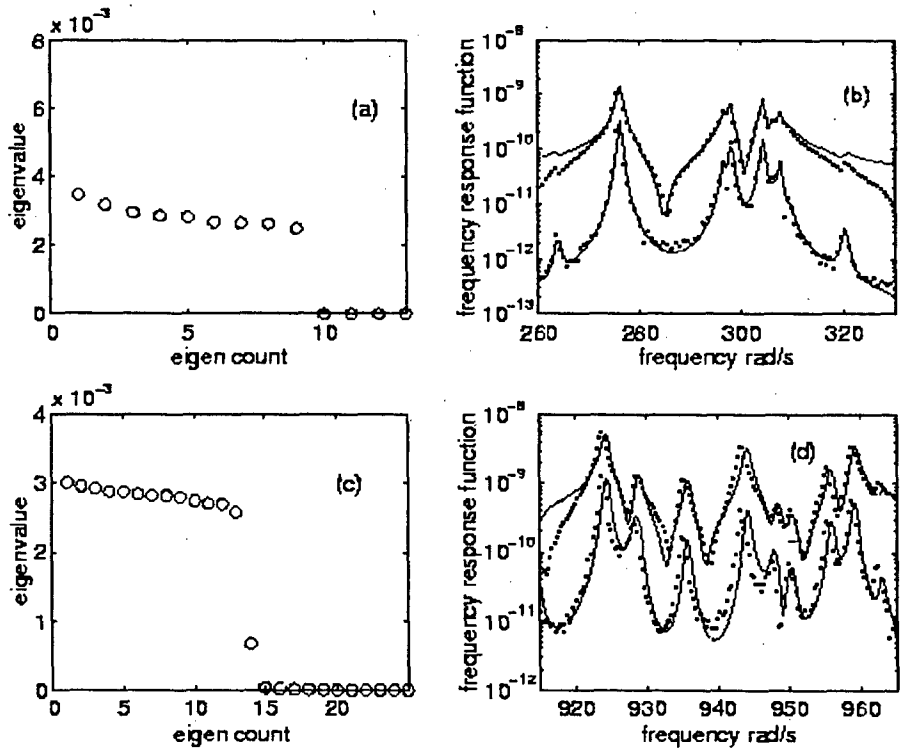

Figure 4. For the capsule: (a) The distribution of eigenvalues of energy operator in the frequency band (260-330) rad/s, (b) The frequency response function for Case A, (c) The distribution of eigenvalues of energy operator in the frequency band (910-970) rad/s, (d) The frequency response function for Case B.

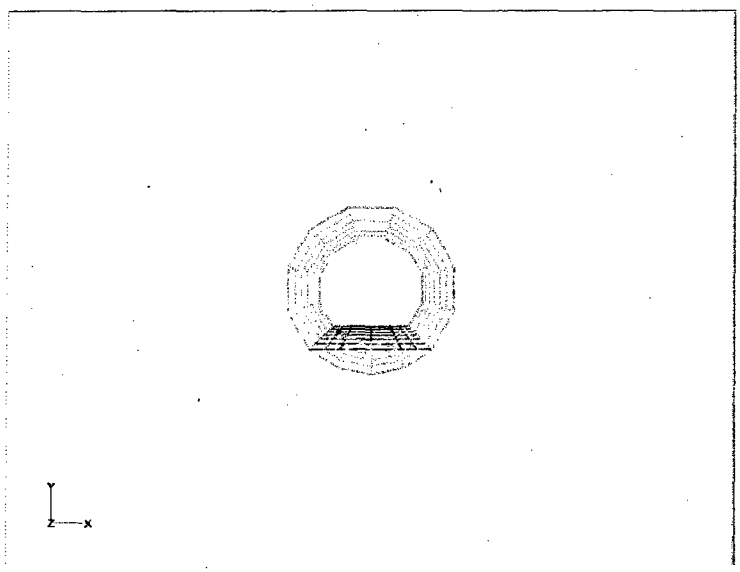

Figure 5. Finite element mesh for a shell-plate assembly.

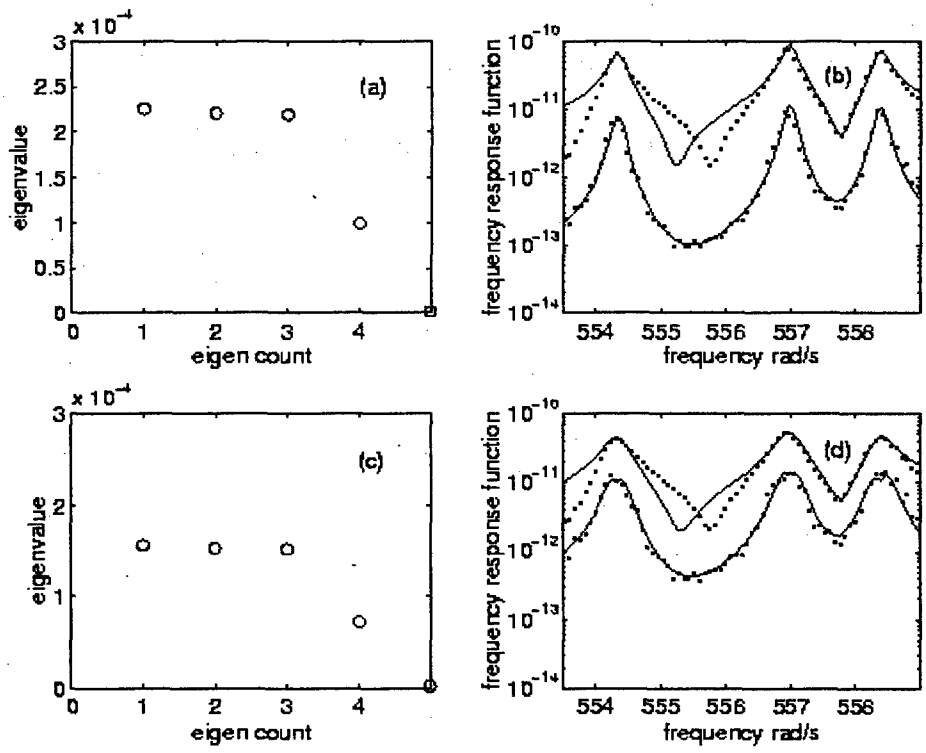

Figure 6. For the shell plate assembly: (a) The distribution of eigenvalues of energy operator for Case A, (b) The frequency response function for Case A, (c) The distribution of eigenvalues of energy operator for Case B, (d) The frequency response function for Case B.