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MATHEMATICAL MODELS OF ELECTROMAGNETIC WAVE SCATTERING BY TWO-ELEMENT STRIP GRATING WITH A PERPENDICULARLY MAGNETIZED GYROTROPIC MEDIUM

Y.V. Gandel, V.V. Khoroshun
Kharkov National University
Pl. Svobody 4, Kharkov 61077, Ukraine
E-mail: gandel@ilt.kharkov.ua

ABSTRACT

Two mathematical models are proposed for analyzing a linearly polarized plane wave scattering by a strip grating placed on isotropic-gyrotropic media interface in the case of oblique incidence. The first mathematical model is based on reducing the original boundary value problem to the Riemann-Hilbert problem. The second model is based on reducing the same problem to a singular integral equation of the first kind with Cauchy kernel and its numerical solving by the discrete singularities method.

In this paper, the results obtained in [1-3] are generalized for the case of periodic structure consisting of two strips of different widths per period (two-element grating). This results in richer diffraction phenomena in comparison to simple grating because of additional control parameters. Moreover, unlike papers [1-3] here the case of oblique incidence of an H-polarized plane wave on a two-element grating is considered. The center of coordinate system is chosen in the middle of one of the strips.

The following set of dual series equations is mathematical model of a structure

\[ \sum_n A_n r_n e^{i\varphi} = \gamma_{01}, \quad \theta_m < \varphi < \theta_m^{(1)}, \]

\[ \sum_n A_n r_n e^{i\varphi} = -p_0, \quad \theta_m^{(1)} < \varphi < \theta_m^{(1)}, \]

where \( \gamma_{n1} = \sqrt{k_0^2 n_1^2 - h_n^2}, \quad n_1 = \sqrt{\varepsilon_1 \mu_1}, \quad h_n = k_0 n_1 \sin \zeta + \frac{2\pi}{l} n, \)

\[ r_n = 1 + \frac{\gamma_{n1}}{\varepsilon_1 (R \gamma_{n2} - i L h_n)}, \quad \gamma_{n2} = \sqrt{k_0^2 \varepsilon_\perp \mu_1 - h_n^2}, \quad \varepsilon_\perp = \frac{\varepsilon^2 - \varepsilon_a^2}{\varepsilon}, \]

\[ R = \varepsilon_\perp^{-1}, \quad L = -\frac{\varepsilon_a}{\varepsilon^2 - \varepsilon_a^2}, \quad k_0 = \frac{2\pi}{\lambda}, \]

\[ p_0 = 1 - \frac{\gamma_{01}}{\varepsilon_1 (R \gamma_{02} - i L h_0)}, \quad \phi = \frac{2\pi}{l} y, \quad \theta_m = \frac{2\pi}{l} \frac{y_m}{l}, \quad (m = 1, 2), \]

\( l \) is the grating period, \( d \) is the slot width, \( \lambda \) is the wavelength, \( \zeta \) is the incidence angle.

Denote \( \tilde{A}_n = A_n r_n + p_0 \delta_{0n} \), and \( \delta_{0n} \) for the Kronecker symbol.

Then initial set of dual series equations takes the form:

\[ \sum_n \tilde{A}_n e^{i\varphi} = 0, \quad \theta_m < \varphi < \theta_m^{(1)}, \]
The set of equations (3)-(5) can be reduced to non-homogeneous conjugation problem (Riemann-Hilbert’s problem) with a complex-valued coefficient in the case of account of dissipative losses.

To calculate matrix elements of final matrix equation, it is necessary to introduce polynomials $Q_n(u_m, \rho)$ [3], where $\rho = \frac{\ln|G|}{2\pi}$, $u_m = \cos \theta_{s_m}$, $(m = 1, 2)$.

In the case of multi-element gratings, an efficient numerical-analytical method for solving these dual series equations was suggested in [4]. The method consists in reducing them to a singular integral equation of the first kind with the Cauchy kernel on the set of segments, and its following solution by the method of discrete singularities [4, 5].

Integral equation is of the following form

$$\frac{1}{\pi} \int_{\xi} K(x, \xi) F(\xi) d\xi = f(x), \quad x \in L$$

where $L = \bigcup_{q=1}^{m} (a_q, b_q)$, $-\infty < a_1 < b_1 < ... < a_q < b_q < +\infty$;

$f(x), x \in L$; $K(x, \xi), x \in L, \xi \in L$ are known smooth functions, and function $F(\xi), \xi \in L$ is sought in the functional class whose restriction on interval $(a_q, b_q)$:

$$F_q(\xi) = F(\xi), \quad a_q < \xi < b_q, \quad q = 1, ..., m$$

can be represented in the form

$$F_q(\xi) = \frac{v_q(\xi)}{\sqrt{b_q - \xi}}, \quad a_q < \xi < b_q,$$

where $v_q(\xi), \xi \in [a_q, b_q]$ is a smooth function.

The sought function $F(\xi), \xi \in L$ satisfies additional conditions, which in general case are of the following form:

$$\frac{1}{\pi} \int_{\xi} S_p(\xi) F(\xi) d\xi = C_p, \quad p = 1, ..., m,$$

where $S_p(\xi), \xi \in [a_p, b_p]$ is a known smooth function, and $C_p$ is a known constant.

In conclusion we shall present the discrete mathematical model that is a set of linear algebraic equations for numerical solution of the integral equation (6) with additional condition (7).

Denote

$$t^n_i = \cos \frac{2i-1}{2n} \pi, \quad i = 1, ..., n; \quad t^n_{0j} = \cos \frac{j}{n} \pi, \quad j = 1, ..., n-1;$$
\[
\begin{align*}
 g_k(\tau) &= \frac{b_k - a_k \tau + b_k + a_k}{2}; \quad \xi_{qi} = g_q(t_{qi}^n), \quad i = 1, \ldots, n_q; q = 1, \ldots, m \\
 \chi_{pi}^{n_p} &= g_p(t_{pi}^n), \quad j = 1, \ldots, n_p - 1; p = 1, \ldots, m
\end{align*}
\]

To calculate approximate values \( \{v_{q_n}(\xi)\}_{q=1}^m \) of the desired functions \( v_q(\xi), q = 1, \ldots, m \) in principal points \( \{t_{pi}^n\}_{i=1}^{n_p} \), we have a set of linear algebraic equations (where \( R(x,\xi) = \frac{1}{\xi - x} + K(x,\xi) \))

\[
\sum_{q=1}^m \sum_{t_{pi}^n} R(\chi_{pi}^{n_p}, \xi_{qi}) v_{q_n}(\xi_{qi}^{n_p}) \frac{1}{n_q} = f(\chi_{pi}^{n_p}), \quad j = 1, \ldots, n_p - 1; \quad p = 1, \ldots, m,
\]

\[
\sum_{i=1}^{n_p} S_p(\xi_{pi}^{n_p}) v_{p_n}(\xi_{pi}^{n_p}) \frac{1}{n_p} = C_p, \quad (j = n_p), \quad p = 1, \ldots, m
\]

The values of the physical characteristic of scattered field,

\[
H = \int H(\xi) F(\xi) d\xi = \sum_{q=1}^m \int H_q(\xi) v_q(\xi) \frac{d\xi}{\sqrt{(\xi - a_q)(b_q - \xi)}}.
\]

are expressed in terms of the functions \( v_q(\xi), \xi \in [a_q, b_q], q = 1, \ldots, m \), where \( H_q(\xi), \xi \in [a_q, b_q] \) are known functions.

Approximate values of

\[
H_{\tilde{n}} = \sum_{q=1}^m \sum_{i=1}^{n_p} H_q(\xi_{qi}^{n_p}) v_{q_n}(\xi_{qi}^{n_p}) \frac{1}{n_q}, \quad \tilde{n} = (n_1, \ldots, n_m)
\]

are calculated in numerical experiments.

Obtained results can be applied in the design and elaboration of various devices containing periodic structures with ferrite substrates or in plasma.

REFERENCES


