Nonlinear transport in superlattices under quantizing magnetic fields

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Abstract. The transport in semiconductor superlattices subject to quantizing electric and magnetic fields is studied based on the double-time Green function method. Our rigorous quantum-mechanical approach, which goes beyond the Kadanoff-Baym ansatz, reveals the hopping nature of the high-field transport in narrow-miniband superlattices. In both, the electric- and magnetic-field dependence of the current, gaps appear.

1. Introduction

Coherent carrier dynamics has been studied in semiconductor superlattices (SLs). The application of a strong dc electric field parallel to the SL axis leads to a reduction of the electron coherence. The electric-field induced reduction of the electron coherence in a SL is due to Bloch oscillations of electrons confined to a region of the order of $\Delta/eE$ ($E$ is the electric field and $\Delta$ the miniband width). A different approach to Bloch oscillations is the formation of Wannier-Stark ladder states replacing the miniband energy spectrum, which is well defined in an unbiased SL. Nonlinear carrier transport along the SL axis is only possible via inelastic scattering.

When a strong magnetic field is applied parallel to the SL axis, the in-plane free electron motion quantizes into Landau levels. Consequently, the energy spectrum becomes completely discrete due to both WS and Landau quantization. The application of strong electric and magnetic fields creates a so-called quantum-box SL (QBSL), the characteristic properties of which are tunable by varying the electric and magnetic field.

From a theoretical point of view, the complete field-induced quantization of the electronic eigenstates poses some interesting problems related to the importance of lifetime broadening. Without any scattering, there is neither a collisional broadening of the energy spectrum nor carrier transport described by the nonequilibrium distribution function. To account for lifetime broadening, scattering has to be treated beyond perturbation theory. What is so fascinating about the quantum transport in QBSLs is the strong correlation between the particle spectrum and the statistical properties of the system. In the strongly biased nonequilibrium system, in which the field-dependent eigenstates are completely discrete, many-particle effects drastically change both the energy spectrum and the carrier statistics. The retroaction of the collisional broadening on the carrier statistics is accounted for by a specific, explicit time dependence of the distribution function. This so-called double-time nature of the problem emerges just beyond the Kadanoff-Baym (KB) ansatz [ ] or the hitherto published density-matrix approaches (see, e.g., Ref. [ ]). The nonlinear quantum transport in QBSLs provides an interesting example, for which the consideration of the double-time nature of the problem is inevitable.
2. Theoretical model

Our calculation of the SL current density is based on the double-time Kadanoff–Baym–Keldysh nonequilibrium Green function technique. The derivation of a general expression for the current density starts from the Dyson equation and takes into account the symmetry of a system subject both to electric and magnetic fields. In the related Wigner representation, the Dyson equation simplifies considerably. It is assumed that the nondegenerate electron gas remains essentially within the lowest miniband and that the lifetime broadening results mainly from elastic scattering on impurities. Inelastic scattering on polar-optical phonons has to be accounted for in the determination of the nonequilibrium distribution function. It is essential for our calculation that we go beyond the quasiclassical approach and do not rely on the Kadanoff–Baym ansatz. Our final result for the current density

\[ j_z = e n_s \frac{2\pi}{\hbar^2} V |M|^2 \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{f}_0^\omega (\omega) \tilde{f}_0^{\omega+1} (\omega + i\Omega - \omega_0) \]

\[ \times [((N_0 + 1) f(\omega) - N_0 f(\omega + i\Omega - \omega_0))] \]  

allows a clear physical interpretation within the hopping transport picture, in which the hopping length \( l_d \) (\( d \) is the SL period), the hopping probability \( F_1(\Delta /h\Omega) \) \( \Omega = eE_d/\hbar \) is the Bloch frequency, and the electric- and magnetic-field-dependent combined density-of-states (DOS) \( \tilde{f}_0^\omega (\omega) \tilde{f}_0^{\omega+1} (\omega + i\Omega - \omega_0) \) appear. The carrier transport proceeds by phonon-induced tunneling transitions governed by the distribution function \( f(\omega) \), which is the solution of a new integral equation obtained beyond the KB ansatz. In Eq. (1), \( n_s \) denotes the sheet density, \( N_0 \) the Bose–Einstein distribution function for polar-optical phonons, and \( M \) the energy-independent electron-phonon coupling constant. \( f(\omega) \) accounts for a possible non-Markovian behavior of the hopping transport. However, in the sequential tunneling limit (\( \Delta \rightarrow 0 \)), when the thermalization time is much shorter than a characteristic hopping time, we obtain

\[ f(\omega) = A \exp \left( -\frac{\hbar\omega}{k_BT} \right), \]

where the constant \( A \) is calculated from the normalization condition

\[ \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{f}_0^\omega (\omega) f(\omega) = 1. \]  

The lateral distribution function is calculated within the simple, self-consistent Born approximation. The Laplace transformed analytical solution has the form

\[ \tilde{f}_0^\omega (s) = \sum_n \left[ \frac{i\omega_n - s}{2u} + \sqrt{\frac{a^2}{2\pi^2 u^2}} + \left( \frac{i\omega_n - s}{2u} \right)^2 \right], \]  

where \( \omega_n = \omega_c(n + 1/2) \) and \( l_B^2 = h/m\omega_c \), with \( \omega_c \) denoting the cyclotron frequency. \( u \) refers to the dimensionless strength of the impurity scattering and \( a \) to the lattice constant of the SL layers. The hopping probability is given by

\[ F_1 \left( \frac{\Delta}{h\Omega} \right) = \frac{1}{\pi} \int_0^{\pi} dx J^2 \left( \frac{\Delta}{h\Omega} \sin x \right). \]  

With the approximations in Eqs. (2) and (4), we recognize that the simple model has not been chosen to give an accurate representation of real systems. Here, it is rather our intent to use a sufficiently simple model for extensive calculations to demonstrate qualitative features resulting from the double-time nature of the problem.
Fig. 1. (a) The electric field dependence of the relative current density \( j_z/j_0 \) \((j_0 = 2\pi \epsilon n \omega_0 / \hbar^2 \alpha^2 \) with \(|M|^2 = \omega_0^2 \Gamma^2 \) for \( \alpha = \Delta/\hbar \omega_0 = 0.5 \) (solid line) and \( \alpha = 1 \) (dashed line). The positions of electro-phonon resonances occurring at \( \Omega = \omega_0 \) are marked by thin vertical lines. Parameters used in the calculation are \( B = 15 \) Tesla, \( \beta = \hbar \omega_0 / k_B T = 1 \), and \( \delta = 0.02 \). (b) The magnetic field dependence of the relative current density \( j_z/j_0 \) for \( \alpha = \Delta/\hbar \omega_0 = 0.5 \), \( \beta = \hbar \omega_0 / k_B T = 5 \), \( \delta = 0.02 \), and \( E = 50 \) kV/cm (solid line). Combined cyclotron-Stark-phonon resonances occurring at \( \nu \omega_c = \Omega - \omega_0 \) are marked by thin vertical lines. The dashed line (multiplied by 23) has been calculated within the density-matrix approach \([\text{3}]\) by using a phenomenological damping parameter of \( \delta = 0.05 \).

3. Numerical results and discussion

Numerical results for the relative current density at high electric and magnetic fields calculated from Eqs. (1) to (5) are shown in Fig. 1. Two interesting properties appear in the electric-field dependence of the current density as displayed in Fig. 1(a) for \( \alpha = \Delta/\hbar \omega_0 = 0.5 \) and 1 by the solid and dashed lines, respectively. Electro-phonon resonances are marked by vertical lines. Firstly, real current gaps occur as long as scattering on acoustical phonons and the Coulomb interaction are not taken into account. This is a consequence of the fact that the DOS bands, which exhibit sharp edges, do not overlap. Secondly, at low and intermediate field strengths, a crossover occurs from the quasiclassical field dependence \((\sim 1/E)\) for wide minibands to an activated, hopping-like dependence for narrow minibands. These two features of the current-voltage characteristics are not reproduced by the density-matrix approach \([\text{3}]\).

In Fig. 1(b), the magnetic field dependence of the relative current density is depicted. Positions of combined cyclotron-Stark-phonon resonances at \( \nu \omega_c = \Omega - \omega_0 \) are marked by vertical lines. Again real current gaps occur at about 16 and 23 T for this particular electric field. As shown by the dashed line, this result is not reproduced by our former density-matrix approach \([\text{3}]\). The magnetic field dependence of the current density depends sensitively on the electric field strength.

References