Characteristics and Generation Mechanism of Holes in an Extended Electron Vortex

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Abstract. In nonneutral plasma, depleted regions (holes) in the vorticity often play a controlling role in the evolution of two-dimensional turbulence. We report experimental investigations of the generation process of ring holes (three-dimensionally annular depletion) surrounding a strongly-peaked clump of vorticity (point vortex) immersed in the background of a spatially extended vorticity distribution, and determine the empirical scaling for the generation time of the ring holes. Moreover, we report geometrical features of the ring hole, and report that ring holes play an important role for crystallization.

INTRODUCTION

Regions of excessive (depleted) density in the 2D plane are called clumps (holes) in the vorticity distribution. In nonneutral plasma, only density depressions (holes) can partially shield coulomb force because there are no counter-charge particles. In the early stage of our experimental studies of point vortex dynamics carried out in the presence of various shapes and different levels of a background vorticity distribution [1,2,3], we have observed the appearance of patched holes or ring holes. Such ring hole configurations have been introduced in a theoretical model for explaining crystallization of clumps in the framework of statistical mechanics [4]. According to this model, the ring hole acts to be as a barrier against local entropy transport. Huang et al., have experimentally examined the formation and dynamics of patched holes [5]. Schecter and Dubin theoretically analyzed the motion of a point hole in a background vortex [6]. It was pointed out that the holes have controlling effects on the speed of the evolution of turbulence because of their slow motion relative to clumps. Here we examine the time scale of generation and geometry of ring hole. And we report geometrical features of the ring hole. Finally, we discuss the effect of the ring hole for crystallization. The experimental data obtained using electron plasma confined in a Malmberg trap. The details of the configuration and diagnostics have been reported elsewhere [1,2,3,7].
OBSERVED PROCESS OF HOLE FORMATION

First, we suggest the experimental result of generation process of ring hole. Figure 1 shows the snapshots of the dynamics of a clump injected at $t=10\mu$sec in a low-density region of the background vortex. The vorticity strength is proportional to the darkness, and the black dot represents the clump. The peak value $\zeta_{\text{clump}}(x,y)$ of the vorticity $\zeta(x,y)$ of the clump is about 180 times higher than that of the local background vorticity $\zeta_{\text{bg}}(x,y)$, and its circulation $\Gamma_{\text{clump}}(x,y)=\iint dxdy\ zeta(x,y)$ is about 18% of the background circulation $\Gamma_{\text{bg}}(x,y)=\iint dxdy\ zeta_{\text{bg}}(x,y)$. The clump generates a rotational flow around its center, which has a direction same as that generated by the background vortex ($t=20\mu$sec). The interacting flows make the clump climb along the gradient $\nabla_{\text{bg}}(r)/\nabla_{r}$ of the background vortex, and consequently spiral streak of the vorticity perturbation is generated [3]. A part of the low vorticity area is dragged by the clump into the higher vorticity area of the background to generate a spiral streak winding around its center ($t=70\mu$sec). The differential flow around the clump tightens the fold of the spiral streak ($t=120\mu$sec). Two points along the spiral streak reconnect ($t=180\mu$sec), and the depleted-vorticity region involved in the background forms a ring that surrounds the clump ($t=5\mu$sec). Once generated, the ring hole has a long lifetime comparable to that of the clump ($t=500\mu$sec).

Now we attempt to make a quantity analysis of the generation of ring hole in terms of stream field. The vortex dynamics is strongly affected by the presence of the shear in a rotational background flow [3]. We examine the shear flow including the contribution of the clump by applying a field analysis to the observed data [8]. The potential distribution $\phi(x,y)$ is determined numerically from the experimentally determined density $n(x,y)\propto \zeta(x,y)$ of electrons [8]. Figure 2 shows the vorticity distribution and the equipotential surfaces that conform to the streamlines in the rest frame of the clump.

![Figure 1. Snapshots of the vorticity distribution generating a ring hole around a clump interacting with the background vortex.](image1)

![Figure 2. Vorticity distribution and the streamlines at $t=70\mu$sec in the stationary frame of the clump.](image2)
We can observe a set of closed lines around the clump and a stagnation point that distinguishes the background flow in the outer region. A part of the low vorticity area in the vicinity of the stagnation point is dragged into the higher vorticity region of the background vortex. It is also observed that the spiral streak extends along the separatrix.

We note that the starting point of the spiral streak is not at the edge of the distribution of the background vortex but it is close to the outer stagnation point lying well inside the background vortex. Our current understanding is that the density accumulates near the stagnation point and produces the electric field across the initial separatrix to drive electrons from the low-density area inward along the separatrix. The relative location of the streak of depletion in the vorticity distribution with respect to the contours of streamlines supports this interpretation. After the streak travels along the separatrix to form a full circle around the clump, there occurs a moment of reconnection to complete the topology of the ring hole.

**EMPIRICAL SCALING OF RECONNECTION TIME**

We have observed the formation process of the ring hole, by injecting a clump in the periphery of the background vorticity distribution that has various profiles as illustrated in Fig. 3(a). The arrow indicates the radial position of the injection. We define a reconnection time $T_R$ as the time that has elapsed from the injection of the clump to the completion of ring-shaped depletion circulating around it. The reconnection time intricately depends on $\Gamma_v$ and the local condition imposed by the distribution $\zeta_b(r)$ of the background vortex. After several trials of data analysis, we have reached a satisfactory expression for the entire data as described below.

Figure 3(b) plots the observed $T_R$ against the circulation $\Gamma_v$ of the clump for four different distributions of the background vortex. We have found empirically that a power law holds between $T_R$ and $\Gamma_v$, and that the power depends on $\beta = \|\zeta_b \partial \zeta_b(r)/\partial r\|$ at the initial location of the clump. The fitting lines for the data set in Fig. 3(b) provide an empirical relation, $T_R \propto (\Gamma_0/\Gamma_v)(\beta_0/\beta)^2$, where the fitting parameters are $\Gamma_0 = 15$ m$^2$sec$^{-1}$, and $\beta_0 = 8.9 \times 10^9$ m$^1$sec$^{-2}$. The coefficient of the empirical scaling is determined by plotting the observed values of $T_R (\Gamma_0/\Gamma_v)^{-2(\beta_0/\beta)^2}$ against $\beta$.

The final relation reads as,

$$T_{R-CAL} = b (\frac{\beta_0}{\beta})^{1/2} (\frac{\Gamma_0}{\Gamma_v})^{(\beta_0/\beta)^2}$$

where $b = 5.2 \times 10^3$ $\mu$sec. Figure 4 plots the observed values of the reconnection time $T_{R-EXP}$ against the associated values $T_{R-CAL}$ obtained from the empirical relation given by eq.(1).
Figure 3. (a) Radial profiles of the background vortex. (b) Reconnection time is plotted against \( \Gamma_v \). Each symbol corresponds to different profiles.

Figure 4. Observed reconnection time is plotted against the prediction of the empirical scaling (1). It reproduces a wide class of the observed data satisfactorily.

It demonstrates that eq.(1) reproduces a wide class of the observed data satisfactorily. We don't have a clear physical meaning of this empirical formula, but we guess reconnection time have some dependence on rotational shear \( A = -r \frac{d\Omega_b(r)}{dr} \), what may have more physical meaning to reconnection time than local parameter \( \beta \). Here, \( \Omega_b(r) \) is the angular frequency of rotation of the background vortex. We have examined our experimental data with shear and obtained similar empirical relation to eq.(1) using local parameter \( \beta \). We observe that new empirical relation can also reproduce the observed data within a reasonable width of scatter that is slightly larger than that from eq.(1). Detail discussion about these empirical relations has been reported elsewhere [9].

**GEOMETRY OF RING HOLE**

Now we report about the size of the ring holes. The radius \( R_{HO} \) of the outer edge of the ring hole depends on \( \Gamma_v \) and profile \( \zeta_b(r) \) of the background vorticity. As \( \Gamma_v \) increases, \( R_{HO} \) increases to the order of the radius of the stagnation zone \( l = (\Gamma_v / 2\pi \vert \Delta \vert)^{1/2} \) calculated by field analysis [8].

And the maximum depth in \( \zeta_{H}(x,y) \) of the ring hole decrement from the unperturbed level of the background vorticity \( \zeta_b(r) \) also depends on \( \Gamma_v \) and \( \zeta_b(r) \). Our observations indicate that the maximum \( \zeta_H \) is less than 20% of the local \( \zeta_b(r) \), and less than 1% of the peak vorticity \( \zeta_{vo} \) of the clump. We should note that \( \zeta_H \) is much smaller than the decrement in the open spiral streak generated in the wake of a clump that can reach almost 50-100% of \( \zeta_b \) [3].
Let us examine shielding effects that the ring hole may have by defining the circulation of the hole as \( \Gamma_H = \int dx \, dy \zeta_h(x,y) \). Figure 5 plots \( \Gamma_H \) against \( \Gamma_v \), and we can observe a positive correlation between \( \Gamma_H \) and \( \Gamma_v \). For all this state of our understanding, the experimental data in Fig. 5 clearly indicate that \( \Gamma_H \) lies from 5% to 20% of \( \Gamma_v \).

This experimental evidence might raise a question as to the fully-shielded vortex model in Ref. [4] and to a naive assumption of \( \Gamma_H = \Gamma_v \) introduced in Ref. [10].

Here let us note that the vortex-merging is not driven directly along the force between symmetric vortices but that merging requires transverse field associated with asymmetric deformations in the distribution of the vortices. The drive for merging is weak so that it may be modified by some redistribution in the hole vorticity \( \zeta_b(x,y) \) even if \( \Gamma_H \ll \Gamma_v \).

**RING HOLES IN LATTICE CONFIGURATION**

We now discuss the appearance of ring holes surrounding multiple clumps immersed in the background vorticity distribution. To simplify, we consider the evolution of three clumps required to test crystallization of discrete vortices. Figure 6 illustrates the dynamics of three clumps with uneven circulations (\( \Gamma_v \propto N_v/10^6 = 16, 9.3, 4.2 \)) placed at the vertices of a regular triangle in background vortex (\( \Gamma_v \propto N_v/10^8 = 1.1 \)). Evolution time is indicated at the upper left corner (clumps start free motion at \( t = 18 \mu \text{sec} \)). If clumps are in the absence of background vorticity distribution, they move chaotically due to mutual advection, and do not reach stable configuration. In the long run, they sometimes merge, or one of them goes away from the range of vision. On the contrary, clumps, immersed in a background vorticity, evolve into symmetric triangular array surrounded by ring holes. In the first process of dynamics, clumps move chaotically due to the interaction between the clumps and the background, and give rise to the fine structures in background vorticity. Deformation of background vorticity distribution is larger for the stronger clump. In the process of development, fine structures in background vorticity vanish and three clumps are arrested into a stable pattern. And we can observe ring holes surrounding clumps with different size corresponding to the circulation of the clumps.
This experimental result suggests that background vortex helps to generate symmetric lattice of multiple clumps with deformation of its distribution. The configuration of ring hole partially shield the interaction among clumps so that they find best positions that lead to the final stable state of equilibrium.

SUMMARY

In summary, we have experimentally studied the generation process of a ring hole around a clump that is placed in different initial distributions of the background vortex. The location of the hole generation is related to the separatrix produced in the shear flow of the background vortex. The time scale of the hole generation is empirically obtained; the scaling is related to the local gradient of the background vorticity. It is reported also that ring holes play an important role for crystallization of clumps, even though $\Gamma_0$ lies from 5% to 20% of $\Gamma_v$.

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