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Deformation Criteria for the Direct Manipulation of Free Form Surfaces

Stéphane Guillet and Jean-Claude Léon

Abstract. The approach proposed here is based on coupling a mechanical model to the input surface geometry provided by the designer. The mechanical model is based on a set of bar networks subjected to tension forces only. Constraints are specified by the designer to express the deformation behaviour of the surface in the area of interest and define functional dimensioning objectives to perform a direct manipulation of the surface. Generally, this process leads to a globally underdetermined system of equations, i.e. the number of unknowns (external forces) is significantly greater than the number of equations generated by the designer's constraints. To this end, a minimization problem is formulated which expresses various deformation behaviours. In contrast to difference of deformation approaches based on mechanical models like membrane models, finite element models, which solely rely on strain energy minimization criteria, the approach proposed here provides the designer various criteria to help him/her create different deformation behaviours like an area minimizing criterion, expressing a minimum change of the shape in the deformed area, expressing a deformation with slowly varying curvature in the deformed area, providing a deformation behaviour which allows to approximately preserve the section of pipe-like surfaces subjected to bending deformations. As depicted, multiple criteria help the designer express various deformation behaviours which are required during a design process.

§1. Context of Surface Deformations

The shape modification of an object during a design process depends on the context of this process. In the field of mechanical engineering design, constraint requirements can be either aesthetic [3,7] or functional [1].

Without adequate 3D modification tools, the surface deformation leads the designer to tedious manipulations, i.e., displacements of numerous control polyhedron vertices, chain modifications of patches or surfaces, etc.

The basic aim of these deformation tools would be to provide the user an easy and intuitive control of the surface shape. Their parameters should be automatically related to the parameters governing the deformation process.

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Such approaches fit into a class [3,5,9] that helps a designer shape the overall object, but they are not suitable for generating free-form surfaces which accurately match geometric constraints involving functional parameters. A second class of 3D modification tools covers the approach of Celniker [2], Kondo [6], Light [8] and Welch [10]. These types of tools fall into the domain of parametric or variational design tools. Among these approaches, some [6,8] focus on parametric or variational models applied to 2D models. Others [2,10] perform a surface deformation subjected to constraints such as prescribed curvature or surface rectitude using a membrane model which cannot provide some deformation modes like bending.

The approach presented here fits into this last category. Similar to the approach of Celniker and Welch, the current one also uses a mechanical model. However, its formulation is simpler than Celniker and Welch's, and thus it is easier to manipulate and it allows generation of isotropic and anisotropic deformations. The approach introduced here is a new development around a free-form deformation method [4]. The scope of the present work focuses on the introduction of a set of deformation criteria which cannot be provided by membrane models or other mechanical models subjected to small displacements and linear behaviour material law hypotheses.

§2. Principle of the Parametric Deformation

Before studying the deformation criteria presented here, it is suitable to summarize the objectives and the constraints related to the parametric deformation process. The features of the current work are the following:

- the treatment of configurations involving multiple trimmed free-form surfaces based on a B-Spline model,
- the direct manipulation of the geometry through a small number of parameters to allow an easy and intuitive control of the surface shape,
- the possibility for the user to create local or global deformations of the geometry and to obtain different solutions with one set of geometric constraints,
- the fast computation which allows an easy integration of the parametric deformation tool into an integrated design process.

In the context of the approach introduced here, different constraints reduce the complexity of the problem:

- the surface patch decomposition is preserved, i.e., degrees, nodal sequences and topology are kept constant,
- C^0 continuity between patches is maintained. G^0 and G^1 continuities are approximated along the trimming lines. To this end, a discretization process is applied to these lines without modification of their degree,
- trimming lines on the surface are kept unchanged into their parametric space. A trimming line is defined as a set of connected trimming curves.

The aim of the parametric approach is to deform a set of trimmed free-form surfaces subject to geometric constraints. The geometric constraints are cur-

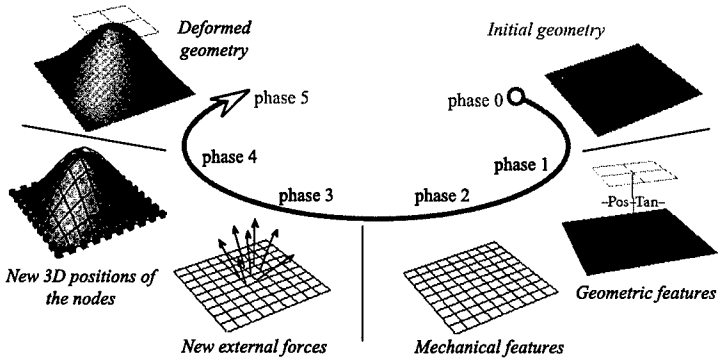


Fig. 1. Main steps of the parametric modelling process.

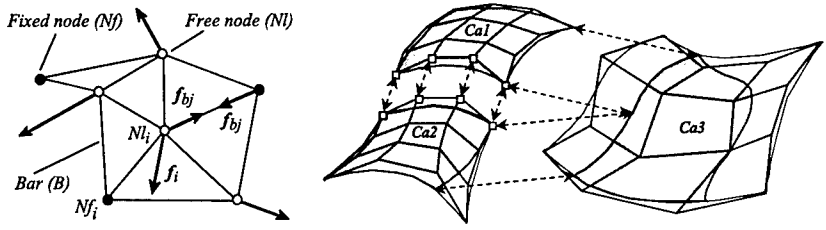
rently restricted to the control of the dimensions of an object though other categories of constraints can be set up to achieve other functions.

Prior to the description of the minimization criteria used here, it is suitable to describe how geometric and mechanical features fit together. The resulting surface geometry is obtained from the initial one through five steps (Figure 1):

- step one is devoted to the creation of the geometric features. These features help specify some target parameters of the surface shape, the deformed area and the continuity conditions between trimmed patches. In the case of Figure 1, one feature is generated by the user. Currently, the design constraints can be displacements of points, lines; tangency constraints with planes; contact with another free form surface; internal continuity constraints between patches,
- step two of this method involves mechanical features. These features are based on parameters of the mechanical model (topology, mobility and force density) used to obtain a deformed geometry and on the choice of a minimization criterion. Thus, the user can obtained different solutions with a unique set of geometric features.

These first two steps are devoted to the initialization of the process. The user can modify one or all these features if he/she does not accept the deformed geometry. The next two steps are transparent for the user, and focus on the computation of the deformed geometry:

- step three: a relationship between geometric and mechanical features contributes to the computation of new external forces through an optimization process. Different minimization functionals can be incorporated into this process,
- step four: these new forces influence the static equilibrium positions of bar networks (mechanical models). New 3D positions of the nodes of the bar networks are computed, i.e., new 3D positions of the vertices of the control polyhedrons of the trimmed surfaces.



(a) An example of bar network

(b) The control polyhedrons of Ca1 and Ca2 can be merged to form onebar network because they are connected along their entire boundary whereas Ca3 generates another bar network since Ca3 is connected to Ca1 and Ca2 along a trimming line.

Fig. 2. Bar networks used to control the shape of surfaces.

§3. Mechanical Deformation Features

Mechanical model of deformation

The bar network (Figure 2a) is built from bars B with pin joints which are assumed to rotate without friction [4,11]. All bars are under tension. The parameters governing the static equilibrium state of such a network are

- the *mobility* of the nodes N, i.e., fixed (N_f) or free (N_l) to move in 3D space,
- the *topology* of the bar network, i.e., the way the bars B are connected to the nodes N of the network,
- the *force density* q_j attached to each bar B_j of the network is defined as the ratio between the internal force $\mathbf{f}_{bj}$ into the bar and its length l_j ($q_j = \frac{\|\mathbf{f}_{bj}\|}{l_j}$, $q_j > 0$). The positivity constraint ensures the tension state in every bar B_j,
- the *external force* $\mathbf{f}_i$ which may be applied to the ith mobile node of the bar network.

Linear static equilibrium

Static equilibrium of a bar network is achieved when the sum of the external force $\mathbf{f}_i$ applied at the ith node equilibrate the forces applied by each bar meeting at that node. This statement becomes

$$\mathbf{f}_i + \sum_{j=0}^{n_{bi}} q_j.(\mathbf{x}_k - \mathbf{x}_{li}) = \mathbf{0}, \qquad i \in \{1, \ldots, n_l\}, \tag{1}$$

where $\mathbf{x}_{li}$ stands for the coordinates of the ith free node, $q_j.(\mathbf{x}_k - \mathbf{x}_{li})$ denotes the tension force into the jth bar meeting at node i, n_{bi} designates the number of bars meeting at node i and n_l the number of free nodes of the bar network.

Given the topology of the bar networks, the equilibrium equations are

$$\begin{cases} \mathbb{D}_l \,.\, \mathbf{x}_l + \mathbb{D}_f \,.\, \mathbf{x}_f = \mathbf{f}_x, \\ \mathbb{D}_l \,.\, \mathbf{y}_l + \mathbb{D}_f \,.\, \mathbf{y}_f = \mathbf{f}_y, \\ \mathbb{D}_l \,.\, \mathbf{z}_l + \mathbb{D}_f \,.\, \mathbf{z}_f = \mathbf{f}_z. \end{cases}$$

for one bar network, where $\mathbb{D}_l$ is a positive definite matrix which ensures the existence and unicity of either equilibrium positions ($\mathbf{x}_l$, $\mathbf{y}_l$, $\mathbf{z}_l$) or external forces ($\mathbf{f}_x$, $\mathbf{f}_y$, $\mathbf{f}_z$).

Generally, the objective is to calculate the 3D position of free nodes. These new positions are obtained with a linear equation system through changes of variables of the parametric process, i.e., the external forces $\mathbf{f}_i$.

Analogy used for surface deformation

The deformation method uses an analogy between the control polyhedron of a surface and the mechanical equilibrium position of a bar network. One bar network is associated with one or several surfaces as follows:

- the nodes of a bar network coincide with the entire set of vertices of the control polyhedron of a free-form surface (either trimmed or not),
- the C^0 continuities are directly incorporated into the mechanical model. In this case, several bar networks can be merged together.

§4. Deformation Criteria

The geometric constraints generated by the designer to prescribe dimensions combined with the equilibrium equations of the bar networks form the global set of constraints

$$\begin{aligned} \mathbf{G} &= \mathbf{G}(f_{1x}, \ldots, f_{n_l x}, f_{1y}, \ldots, f_{n_l y}, f_{1z}, \ldots, f_{n_l z}), \\ &= \mathbf{G}(\mathbf{F}) = \mathbf{0}, \;\; i \in \{1, \ldots, n_c\}, \end{aligned} \tag{2}$$

where each constraint is expressed in terms of the external forces applied to the bar networks. External forces have been chosen as unknowns rather than force densities q_j because they produced intermediate solutions which reflected a real deformation process of a surface whereas iterating with q_j produced oscillations around the solution. Then, the resolution has been conducted using an augmented Lagrangian method to provide robustness to the solving process.

Because the number n_c of constraints is usually significantly smaller than the number of unknowns, and assuming that there is no local configuration with an overconstrained subset of equations, a functional $\Phi(\mathbf{F})$ can be associated with (2) to obtain a solution which matches a specific designer's interest. Overconstrained subsets of equations can be encountered when a subset of (2) is such that there exists locally for a given surface more constraints then the

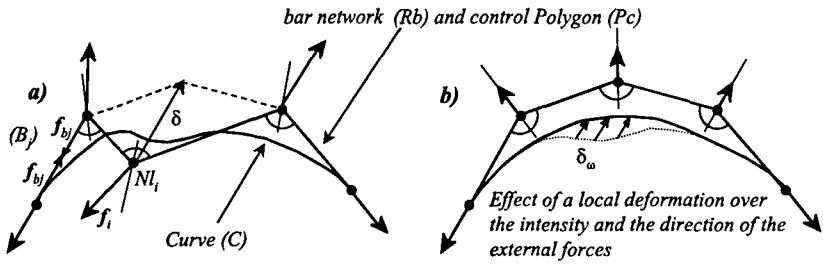


Fig. 3. Relationships between external forces at nodes and the shape of a curve.

number of free parameters x_l, y_l, z_l. When **G** contains nonlinear equations, their derivatives are related to the geometrical and mechanical parameters by

$$G_{ij}^{[k]} = \frac{\partial G_i}{\partial F_j} = \frac{\partial G_i}{\partial X_p} \cdot \frac{\partial X_p}{\partial x_{l_q}} \cdot \frac{\partial x_{l_q}}{\partial F_j} + \frac{\partial G_i}{\partial Y_p} \cdot \frac{\partial Y_p}{\partial y_{l_q}} \cdot \frac{\partial y_{l_q}}{\partial F_j} + \frac{\partial G_i}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial z_{l_q}} \cdot \frac{\partial z_{l_q}}{\partial F_j}, \quad \forall i.$$

where $\frac{\partial G_i}{\partial X_p}, \frac{\partial G_i}{\partial Y_p}, \frac{\partial G_i}{\partial Z_p}$ are related to the geometric constraints set by the designer, $\frac{\partial X_p}{\partial x_{l_q}}, \frac{\partial Y_p}{\partial y_{l_q}}, \frac{\partial Z_p}{\partial z_{l_q}}$ come from the relationship between the surface and the bar networks and $\frac{\partial x_{l_q}}{\partial F_j}, \frac{\partial y_{l_q}}{\partial F_j}, \frac{\partial z_{l_q}}{\partial F_j}$ are coefficients of $\mathbb{D}_l^{-1}$.

External forces at nodes and shape relationships

According to (1), external forces at nodes are governed by the length of the bars as well as the angle between the bars meeting at a node. Figure 3a illustrates such a configuration for a bar network which corresponds to the control polygon of a Bézier curve with uniform force densities in its bars.

Then, it can be stated that a regular bar network has *smaller external forces* than an irregular one, since regular control polygons resemble the shape of the curve, and therefore have smaller length sides and wider angles between bars (Fig. 3b).

With uniform force densities, the direction of the external forces is close to the bisecting line of two adjacent bars or, for a bar network attached to a surface patch, this direction is close to the average normal direction at the given node. However, this behaviour does not necessarily generate acceptable shapes (as depicted in Figure 4) and needs to be combined with the regularity criterion of the intensities.

A change in the intensity of an external force at a node N_k results in a change of position of the free nodes whose amplitude decrease from N_k. The direction of movement of the free nodes is similar to that of the external force which has been modified [7] at N_k. In turn, the displacement of the points on a curve or on a patch follows the geometric property of Bézier or B-Spline models, i.e., points move in the direction of the movement of a control point. Their displacement amplitude is therefore smaller than that of N_k.

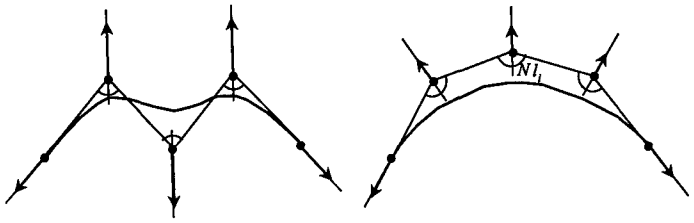


Fig. 4. Specific configuration where external forces coinciding with bisecting lines does not provide a smooth curve.

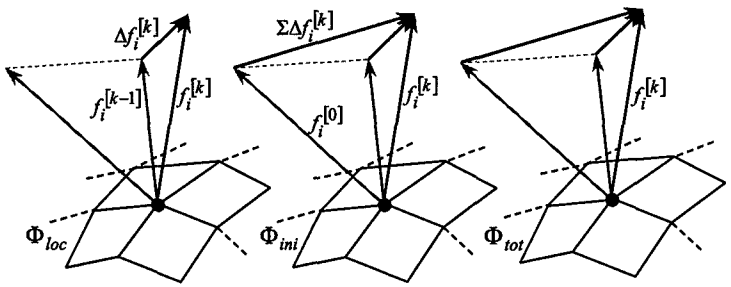


Fig. 5. Criteria related to the external force at a free node.

Various deformation criteria

Based on the previous relationships, three categories of criteria have been identified:

- the first one is related to the external forces at the free nodes of the bar networks,
- the second one acts simultaneously over the external forces and average normal direction at the free nodes,
- the third one acts over the average force around a free node, but is not described here because of lack of space).

The first category takes as input either the external force $\mathbf{f}_i^{[k]}$ or the variation $\Delta\mathbf{f}_i^{[k]}$of this force at the iteration $[k]$ when the constraints expressed are nonlinear (see Figure 5).

When considering the functional $\Phi_{loc}^{[k]} = \min\left({}^T\Delta\mathbf{F}^{[k]}.\Delta\mathbf{F}^{[k]}\right)$, the designer expresses the minimum change in the shape of the object at each iteration until the constraints $\mathbf{G}$ are satisfied.

When considering $\Phi_{ini}^{[k]} = \min\left({}^T(\mathbf{F}^{[k]} - \mathbf{F}^{[0]}).(\mathbf{F}^{[k]} - \mathbf{F}^{[0]})\right)$ as the functional associated with $\mathbf{G}$, the designer expresses the minimum change of the object shape between the input geometry and the output. In case of linear constraints, $\Phi_{loc}^{[k]}$ and $\Phi_{ini}^{[k]}$ are identical. This functional tends to preserve as much as possible the previous work of the designer, and therefore is of specific interest during a modeling process.

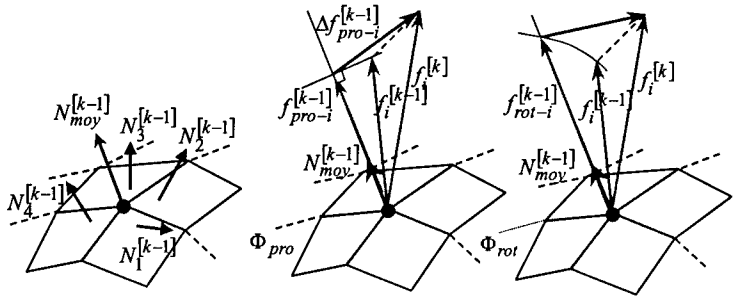


Fig. 6. Criteria acting over the intensity and the direction of the external force at a node.

When considering the functional $\Phi_{tot}^{[k]} = \min\left({}^T\mathbf{F}^{[k]}.\mathbf{F}^{[k]}\right)$, the designer expresses the fact that the resulting surface is not based on the input one since the initial external forces are not taken into account. Furthermore, the minimization of the intensity of the external forces at the free nodes expresses that the output control polyhedrons form an approximation of a minimal surface. Hence, the output surface attached to the bar networks represents an approximation of the minimum surface area satisfying the constraints. This approximation is even more effective when the control polyhedrons converge toward the surface itself.

The second category of criteria acts simultaneously over the intensity and direction of the external forces at free nodes so that the direction of the forces meet a given criterion. To this end, an average normal direction is built according to the position of the nodes around the target node. Figure 6 illustrates the planes surrounding the ith free node at iteration $[k-1]$ which participate to the definition of the average normal direction $\mathbf{N}_{moy-i}^{[k-1]}$. When considering the functional $\Phi_{pro}^{[k]} = \min\left({}^T\Delta\mathbf{F}_{pro}^{[k]}.\Delta\mathbf{F}_{pro}^{[k]}\right)$, the forces minimized correspond to the difference between the projection $\mathbf{f}_{pro-i}^{[k-1]}$ of the external force $\mathbf{f}_i^{[k-1]}$ onto $\mathbf{N}_{moy-i}^{[k-1]}$ and the force $\mathbf{f}_i^{[k]}$ at the kth iteration, i.e. $\Delta\mathbf{F}_{pro-i}^{[k]} = \mathbf{f}_i^{[k]} - \frac{\mathbf{f}_i^{[k-1]}.\mathbf{N}_{moy-i}^{[k-1]}}{\|\mathbf{f}_i^{[k-1]}.\mathbf{N}_{moy-i}^{[k-1]}\|}.\mathbf{f}_i^{[k-1]}$.

Using this criterion, the designer expresses that the output surface tends to minimize the area while being smooth since the intensity of the forces tend to decrease like $\Phi_{tot}^{[k]}$ as well as the direction of the forces tend to be more regular using $\mathbf{N}_{moy-i}^{[k-1]}$. This criterion takes into account the shape of the input geometry, but generates a surface which is 'smoother' than with $\Phi_{ini}^{[k]}$.

When considering the functional $\Phi_{rot}^{[k]} = \min\left({}^T\Delta\mathbf{F}_{rot}^{[k]}.\Delta\mathbf{F}_{rot}^{[k]}\right)$, the forces minimized correspond to the difference between the external force $\mathbf{f}_i^{[k-1]}$ rotated onto the direction of $\mathbf{N}_{moy-i}^{[k-1]}$ and the force $\mathbf{f}_i^{[k]}$ at the kth iteration, i.e. $\Delta\mathbf{F}_{rot-i}^{[k]} = \mathbf{f}_i^{[k]} - \|\mathbf{f}_i^{[k-1]}\|.\mathbf{N}_{moy-i}^{[k-1]}$. Such a functional tends to preserve

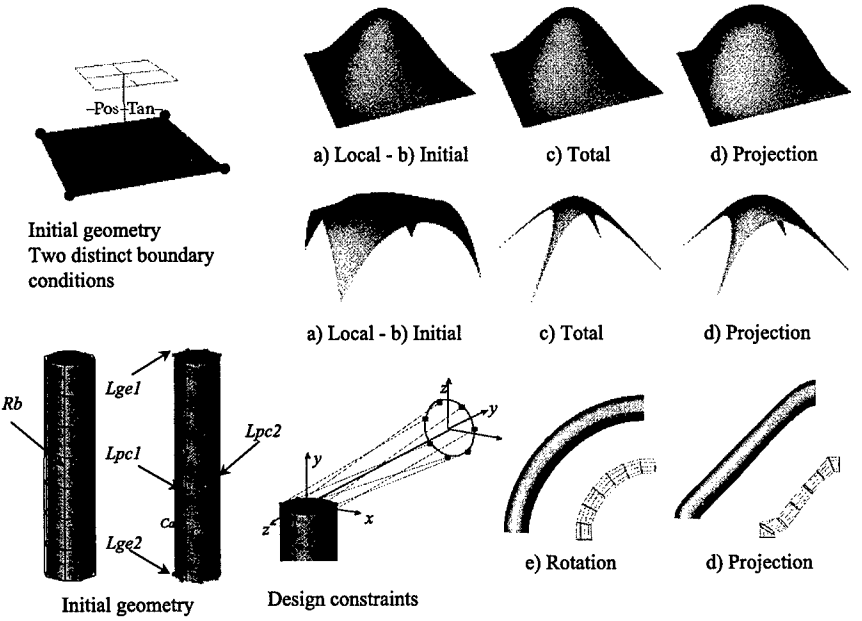


Fig. 7. Illustration of the influence of the deformation criteria.

the intensity of the external forces while modifying their direction in order to smooth the surface. Combined with anisotropic force densities in the bars of the networks, this criterion expresses the deformation behaviour of pipe-type objects when their are subjected to bending deformations. Though the previous criteria are nonlinear even if **G** is linear, their efficiency is strong enough to justify their use during a design process.

§5. Results and Examples

The above deformation criteria have been applied to different categories of surfaces to illustrate their typical behaviour according to the configurations described in the previous section. Figure 7a shows the effect of Φ_{loc}, 7b illustrates Φ_{ini}, 7c Φ_{tot}, 7d Φ_{pro} and 7e Φ_{rot} under various designer constraints.

Two distinct input geometries are used. The upper one is a one patch surface, and two types of boundary conditions were used, i.e. fixed boundary lines and fixed corner points. The design constraint is formed by a position and tangency constraint. The bottom one is a multipatch surface with G^1 continuity constraints, where the designer has specified position and tangency constraints along the extreme boundary line of the surface.

§6. Concluding Remarks

The deformation criteria presented provide a diversity of control of shape. They form an efficient complement to the geometric constraints set by the de-

signer to let him/her adapt the result to his/her needs. Such a diversity cannot be achieved using a mechanical approach solely based on a minimization of the strain energy of membrane type structures. Furthermore, the criteria set up are not bound by a small displacements hypothesis and can handle geometric constraints involving significant shape changes.

Future work will focus on the cross influence between the deformation criteria and the boundary conditions applied to the bar networks to provide more intuitive user interactions.

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Jean-Claude Léon and Stéphane Guillet
Laboratoire 3S, Domaine Universitaire, BP 53X
38041 GRENOBLE Cedex 9 FRANCE
`Jean-Claude.Leon@hmg.inpg.fr`
`Stephane.Guillet@hmg.inpg.fr`