Surface Polaritons in the System of Ideal Metal – Dielectric Plate – Vacuum in the Constant Electric Field

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Abstract

The system of an ideal metal or superconductor – dielectric plate – vacuum or unpolarizable substance is considered in presence of the constant electric field directed normal to the plane of the plate. Dispersion relations for surface phonon polaritons of this system are derived and analyzed. These polaritons are induced by a dynamical magnetoelectric effect. It has been shown that in the case, when the plate thickness essentially exceeds the penetration depth of the surface polaritons, the approximation of a half-infinite (bulk) insulator satisfactory describes the polariton spectrum and the "switching over" effect is possible for a magnetoelectric wave: the variation of the electric field on the opposite one leads to "switching on" or "switching off" of the corresponding frequency branches. The dependence of the spectrum of surface phonon polaritons from the plate thickness is calculated numerically. The corresponding criterion of an existence of magnetoelectric waves in the dielectric plate has been found.

1. Introduction

A number of publications devotes to the influence of a dynamical magnetoelectric (ME) effect on the spectrum of surface phonon polaritons in an insulator. In these publications the «switching over» effect and the «rectification» one of surface polaritons at the change of direction of applied constant electric or magnetic fields.

We consider the system of an ideal metal (or superconductor) – dielectric plate – vacuum (or unpolarizable substance) in presence of a constant electric field directed normal to the plate. Dispersion relations for the spectrum of surface phonon polaritons are derived taking into account the dynamical ME effect influence. Restrictions for the plate thickness, when the «switching over» effect takes place, are analyzed. It is shown that in the case when the plate thickness essentially exceeds the penetration depth of surface polaritons the approximation of a half-infinite (bulk) insulator describes quite exact the polariton spectrum. While the dielectric plate thickness decreases, the «switching over» effect disappears. However, this effect can be kept down to the sufficient small plate thickness (depending on the value of the constant electric field). The dependence of the spectrum of surface polaritons on the plate thickness is calculated numerically. Analytical expressions for the dispersion law and the penetration depth of surface polaritons are obtained for small thicknesses.
2. Dispersion relations for the system of an Ideal Metal-Insulator-Vacuum

Let us consider an uniaxial non-magnetic insulator (z is an easy axis). It occupies the region of the space \(0 < z < d\) and borders on ideal metal or superconductor (\(z < 0\)) and vacuum or unpolarizable substance (\(z > d\)), \(d\) is the insulator thickness. A constant electric field \(E_0\) is applied along the normal to the dielectric plate (i.e. along the z axis).

The linear response of an insulator in the field of an electromagnetic wave with electric and magnetic components \(\vec{E}, \vec{H}\) with taking into consideration of the dynamical ME effect has been obtained in [1]. Electric and magnetoelectric susceptibilities in the absence of damping and in the neglect of the space dispersion have been analyzed in works [2]-[6].

The electric (\(\vec{d}\)) induction of medium and the magnetic (\(\vec{b}\)) one are connected with electric and magnetic fields by means of the following relations

\[
d_i = \varepsilon_{ik} \epsilon_i + \gamma_{ik} h_k, \quad b_i = \mu_{ik} h_i + \gamma_{ik} e_k
\]

where \(\varepsilon_{ik}, \mu_{ik}\) are the tensors of dielectric and magnetic permeabilities correspondingly, \(\gamma_{ik}\) is the tensor of magnetoelectric permeability. Components of the electric polarization of medium can be presented in the form:

\[
P_i = \frac{1}{4\pi} (\varepsilon_{ik} - \delta_{ik}) \epsilon_i + \gamma_{ik} h_k
\]

(1)

where \(\delta_{ik}\) is Kroneker symbol. In our case \(\mu_{ik} = \delta_{ik}\) (non-magnetic insulator), and components of the electric and magnetoelectric permeabilities, which are not equal to zero, are determined by the following expressions:

\[
\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_1 = \frac{\Omega_1^2 - \omega^2}{\omega_0^2 - \omega^2}, \quad \varepsilon_{xx} = \varepsilon_2 = \frac{\Omega_2^2 - \omega^2}{\omega_0^2 - \omega^2}, \quad \gamma_{xx} = \gamma_{yy} = -i\gamma, \quad \gamma = \frac{4\pi \omega \sigma P_0}{\omega_0^2 - \omega^2}.
\]

(2)

Here \(P_0\) is an equilibrium value of polarization. If the external constant electric field \(E_0 = E_{0z}\) is applied to the insulator, then from the formula (2) we obtain \(P_0 = \frac{1}{4\pi} (\epsilon_{zz} - 1) E_0\). In the case of ferroelectric, \(P_0\) is spontaneous polarization; \(\omega_0\) is the excitation frequency of transverse components of the polarization \(P_x, P_y\); \(\omega_0\) is the excitation frequency of polarization along the easy axis z. Frequencies \(\Omega_0, \Omega_z\) are zeroes of dielectric permeabilities of transverse (\(\varepsilon_{xx}, \varepsilon_{yy}\)) and longitudinal (\(\varepsilon_{zz}\)) correspondingly. Besides, \(g = e/mc\) is gyromagnetic ratio (\(e\) is the particle charge, \(m\) is the particle mass, \(c\) is the speed of light). An ionic polarizability is excited effectively in the IR region of the spectrum. Therefore, \(g\) is gyromagnetic ratio for an anion-cation pairs. In the optic region of the spectrum where electronic polarizability prevails, \(g\) is gyromagnetic ratio for an electron.

The Maxwell's equations for a wave that propagates along the x axis can be presented in the following way:

\[
\frac{\partial}{\partial z} e_x + \frac{\omega}{c} \gamma e_x - i \left( k - \frac{\omega^2}{c^2} \right) e_z = 0, \quad i \frac{\omega}{c} \partial \frac{\partial}{\partial z} e_x + \frac{\omega}{c k} \frac{\partial}{\partial z} e_z - \frac{\omega^2}{c^2 k} \gamma e_z = 0, \quad h_y = -\frac{\omega}{c k} e_z
\]

(3)

The solution of equations (4) in the insulator is sought in the form:

\[
e_x = A e^{-ka} + B e^{ka}, \quad e_z = C e^{-ka} + D e^{ka}
\]

(4)

From the requirement of equality to zero of tangential components of an electric field at the boundary with an ideal metal (or superconductor) we obtain
\[ A + B = 0 \]  

For the rest of amplitudes in the equation (5) the following expressions can be obtained when amplitude \( C \) is considered as independent one:

\[
A = C \left( \frac{k_0 + \frac{\omega}{c} \gamma}{i \epsilon_k} \right), \quad B = -A, \quad D = C \left( \frac{\omega}{c} + \frac{i \omega}{\gamma} \right).
\]

The wave in vacuum is sought in the form

\[
\hat{\mathbf{A}} = F e^{-i k_0 x} e^{i \left( k_0 z - \omega t \right)}, \quad \hat{\mathbf{B}} = G e^{-i k_0 x} e^{i \left( k_0 z - \omega t \right)}
\]

Tangential components of an electric field and normal component of electric induction must be continuous at the boundary of the insulator with vacuum. Thus, such equalities must be implemented

\[
e_x = e_x|_{z=d}, \quad e_z = e_z|_{z=d}.
\]

Where \( \epsilon_0 \) is dielectric constant of vacuum (then \( \epsilon_0 = 1 \)) or any unpolarizable substance in the considered frequency region. For amplitudes from equations (5) it (7) we obtain:

\[
A \left( e^{-k_0 d} - e^{i k_0 d} \right) = F e^{-i k_0 d}, \quad C \epsilon_3 \left( e^{-k_0 d} - \frac{k_0 \gamma - \omega}{k_0 \gamma + \omega} e^{i k_0 d} \right) = G \epsilon_0 e^{-i k_0 d}
\]

Solving the system of equations (4)-(10) we obtain finally for boundary conditions:

\[
\left( k_0 + \frac{\omega}{c} \gamma \right) \left( e^{-2 k_0 d} - 1 \right) \frac{\epsilon_0}{\epsilon_1} = \left( e^{-2 k_0 d} - \frac{k_0 \gamma - \omega}{k_0 \gamma + \omega} \right) k_0
\]

Besides, the following relation for nonzero solutions in the insulator must be implemented

\[
\frac{\omega^2}{c^2} \epsilon_1 \epsilon_3 - k^2 \epsilon_1 + \epsilon_3 \left( k_0^2 - \frac{\omega^2}{c^2} \gamma^2 \right) = 0
\]

The analogous relation in vacuum

\[
\frac{\omega^2}{c^2} \epsilon_0 - k^2 + k_0^2 = 0
\]

must be realized when amplitudes \( F, G \) in (8) and (10) differ from zero.

The system of equations (11)-(13) determines the dispersion law of polaritons which propagate in the insulator parallel to the interface.

Let us analyze this system at \( k_0 d >> 1 \) i.e. when the insulator thickness essentially exceeds the value of the penetration depth of the wave into the insulator. Then the equation (11) can be realized in two cases: \( k_0^2 = -\omega \gamma \) and \( k_0^2 = -\omega \gamma \), \( \left( k_0^2 - \frac{\omega^2}{c^2} \gamma \right) \epsilon_0 = \bar{k}_0 \epsilon_1 \).

In the first case we obtain the solution in the form of a wave which exists due to the dynamical ME effect. It is easy to see that equation (12) in this case will be \( k^2 = \frac{\omega^2}{c^2} \epsilon_3 \). From equation (7) one can see that from all amplitudes in the insulator \( A, B, C, D \) only the amplitude \( C \) is not equal to zero. Thus, the wave propagates along the boundary of ideal metal with insulator, and its amplitude exponentially damps deep into the insulator. From the equation (10) it follows that the amplitude \( F \) in vacuum is equal to zero, and the amplitude \( G \rightarrow 0, k_0 d \rightarrow \infty \). Therefore, the equation (13) in this case does not exist. Further we will call this polariton wave as «magnetolectric» one [3].
In the second case \( \left( k_0 - \frac{\omega}{c} \right) e_0 = k_0 e_1 \) we have a common surface polariton wave at the boundary of the insulator with vacuum which exists in the frequency interval \( [\omega_0, \omega_3] \), where 
\[ \omega_0 = \left( \omega_1^2 + \omega_2^2 \right)/2 \] (see [7]). In [2] it has been shown that presence of the constant electric field (i.e. \( \gamma \neq 0 \)) in this case leads to interesting effects whose quantities, however, are small since the dynamical ME effect is small too.

Hence, we can indicate the application limits of result of work [3]–[6] if we find the qualitative condition of existence of the «magnetoelectric» wave in the case of the finite plate thickness. The plate thickness of the insulator \( d \) must essentially exceed the penetration depth into the insulator \( k_0^{-1} \). From \( k_0 d >> 1, \ c k_0 = -\omega \gamma \) and equality (3) we obtain:

\[ d >> k_0^{-1} = \delta_{ME} = \frac{\omega_0^2 - \omega^2}{4 \pi \omega_0^2 gP_0} c \]  

(14)

The order of penetration depth of the «magnetoelectric» wave at values of constant electric field which is typical for the spontaneous polarization in the ferroelectric crystal \( E_0 \sim P_0 \sim 10^4 \) CGSE and optical frequencies (\( \omega \sim 10^{14} \) sec\(^{-1} \), \( g \sim 10^7 \)) is \( \delta_{ME} \sim 10^{-2} \) sm. Thus, for the relation of the condition \( k_0 d >> 1 \) the plate thickness of insulator must be greater or of the order of magnitude 1 mm. The numerical calculations of dispersion relations (11)–(13) performed by us confirm these qualitative results.

3. Conclusion

In the system of an ideal metal (or superconductor) – dielectric plate – vacuum (or any unpolarizable substance) surface polaritons propagate along the interface. For plate thicknesses \( d \), which essentially exceed the characteristic penetration depth \( \delta_{ME} \) (see (14)), the «magnetoelectric» wave propagates along the boundary of insulator with ideal metal (or superconductor). Thus, the condition \( d >> \delta_{ME} \) shows the application limits of results of work [3]. At the optic frequencies and the value of the constant electric field \( E_0 \sim 10^4 \) CGSE the characteristic value is \( \delta_{ME} \sim 10^{-2} \) sm. Therefore, for the experimental verification of the work [3] at the given value of the constant electric field (or at such value of the spontaneous polarization in the case of the ferroelectric) we can use the insulator layer with the thickness greater or of the order of magnitude 1 mm.

While the thickness of the dielectric plate decreases, the «magnetoelectric» wave destroys.

References