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AD-P004 259 Development of a Soil-Wheel Interaction Model
AD-P004 260 Soil Compliance Influence on Tyre Performance
AD-P004 261 The Rolling Resistance and Sinkage of Towed Dual Wheel Combinations in Soil
AD-P004 262 Performance Prediction of Pneumatic Tyres on Sandy Soil
AD-P004 263 Effects of Slip on Energy Distribution between Tyre and Soil
AD-P004 264 Traction Forces of Drive Tyre on the Compacted Soil
AD-P004 265 Prediction of In-Sand Tire and Wheeled Vehicle Drawbar Performance
AD-P004 266 Dynamic Simulation of Track Laying Vehicles
AD-P004 267 Designing Off-Road Vehicles with Good Ride Behaviour
AD-P004 268 Theoretische Untersuchung Einer Aktiv-Federung fuer Rad-Schlepper (A Theoretical Investigation of an Active Suspension System for Wheeled Tractors)

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<table>
<thead>
<tr>
<th>AD#</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD-P004 269</td>
<td>Leistungsfähigerung und Verbesserung des Fahrkomforts bei Selbstfahrenden Baumaschinen durch Reduzierung Einsetzung Nick- und Hubschwingungen (Increase in Performance and Improvement of Ride Comfort of Self-Propelled Construction Machinery by Reducing Pitch and Vertical Vibration)</td>
</tr>
<tr>
<td>AD-P004 270</td>
<td>Stresses in situ generating by Bulldozers</td>
</tr>
<tr>
<td>AD-P004 271</td>
<td>Finite Element Analysis of Ground Deformation Beneath Moving Track Loads</td>
</tr>
<tr>
<td>AD-P004 271</td>
<td>A Rig for Testing the Soft Soil Performance of Track Systems</td>
</tr>
<tr>
<td>AD-P004 273</td>
<td>Die Abhängigkeit der Bodentragefähigkeit und der Zugkraft von der Abstandsgroesse der Bodenplatten (The Dependence of Soil Bearing Capacity and Drawbar Pull on the Spacing between Track Plates)</td>
</tr>
<tr>
<td>AD-P004 274</td>
<td>The Dynamic Interaction between Track and Soil</td>
</tr>
<tr>
<td>AD-P004 275</td>
<td>Analysis of Ground Pressure Distribution Beneath Tracked Hovel with Respect to External Loading</td>
</tr>
<tr>
<td>AD-P004 276</td>
<td>A Comparison between a Conventional Method and an Improved Method for Predicting Tracked Vehicle Performance</td>
</tr>
<tr>
<td>AD-P004 277</td>
<td>Effect of Hitch Positions on the Performance of Track/Grouser Systems</td>
</tr>
<tr>
<td>AD-P004 278</td>
<td>Grouser Effect Studies</td>
</tr>
<tr>
<td>AD-P004 279</td>
<td>Ride Comfort of Off-Road Vehicles</td>
</tr>
<tr>
<td>AD-P004 280</td>
<td>Further Development in Ride Quality Assessment</td>
</tr>
<tr>
<td>AD-P004 281</td>
<td>Comparison of Measured and Simulated Ride Comfort for an Agricultural Tractor and Influence of Travel Speed and Tyre-Inflation Pressure on Dynamic Response</td>
</tr>
<tr>
<td>AD-P004 282</td>
<td>Characteristics of Farm Field Profiles as Sources of Tractor Vibration</td>
</tr>
</tbody>
</table>
PREDICTION OF IN-SAND TIRE AND WHEELED VEHICLE DRAWBAR PERFORMANCE

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ABSTRACT

In an appendix of the ISTVS 6th International Conference Paper, "A Synopsis of Tire Design and Operational Considerations Aimed at Increasing In-Soil Tire Drawbar Performance," the author developed a procedure for defining $G^*_e$, effective sand penetration resistance gradient. $G^*_e$ was devised to approximate the value of $G$ that predominated during a given tire pass, normalized to one type of frictional soil (selected as Yuma sand). $G^*_e$ subsequently served as the soil strength term in sand-tire numeric $N^e$, which aimed at providing a normalized description of tire drawbar performance in different types of sands. Because the range of sand types included in the development of $G^*_e$ and $N^e$ was necessarily quite limited, the author suggested in the "Synopsis" paper that other investigators test the universality of relations involving $N^e$ and tire drawbar performance by using tire test results obtained in a variety of sands.

In their ISTVS 7th International Conference paper, "An Assessment of the Value of the Cone Penetrometer in Mobility Prediction," A. R. Reece and J. O. Peca applied the $N^e$ methodology for a quite different sand and obtained $N^e$ versus tire drawbar performance results that were not described well by those in the "Synopsis" paper. This prompted a reexamination by the author of information in the "Synopsis" paper, an analysis of data presented in Reece and Peca's "Assessment" paper, and a reanalysis of a sizeable body of U. S. Army Engineer Waterways Experiment Station (WES) field data on wheeled vehicle performance tests in a variety of sands (all supplemented by new laboratory sand test data). The primary result of this work is definition of a new $N^e$ methodology that accurately predicts tire and wheeled vehicle drawbar performance in a very broad range of sand types and conditions, including those of the "Assessment" paper.
INTRODUCTION

To obtain best wheeled vehicle performance in sandy soils requires implementation of a rational methodology for selecting the most appropriate tires and then using those tires to best advantage. In turn, such a methodology requires an ability to predict accurately in-sand tire and wheeled vehicle performance. Using laboratory and field test results, this paper describes a useful methodology for predicting the drawbar performance of tires and wheeled vehicles operating in a broad range of sand types and conditions.

BACKGROUND

Drawbar pull and drawbar efficiency were selected to describe in-sand tire performance herein because (a) the amount of pull a powered wheel can develop is often of major concern in vehicle in-sand operations, and (b) the efficiency with which a given amount of pull is developed determines the input energy required, a major concern in today's energy-conscious world.

It has proved useful to describe in-sand tire performance by relating dimensionless tire performance terms to a dimensionless sand-tire prediction term, or numeric. Tire drawbar performance is described herein by the following two terms:

\[
\text{Drawbar coefficient (} \mu \text{)} = \frac{D_P}{W} \tag{1}
\]

where

\(D_P\) = drawbar pull, the "force available for external work in a direction parallel to the horizontal surface over which the (tire) is moving"* \\
\(W\) = weight (load) on the tire

and

\[
\text{Drawbar efficiency (} \eta \text{)} = \frac{D_P \cdot v}{T \cdot w} \tag{2}
\]

where

\(D_P\) = drawbar pull \\
\(v\) = forward velocity of the wheel axle \\
\(T\) = torque input to the wheel \\
\(w\) = rotation velocity of the torque input shaft

* Each raised number in the main text refers to a reference of the same number at the end of the text.
The sand-tire numeric \( N_s \) is defined as:

\[
N_s = \frac{G(bd)^{3/2}}{W} \cdot \frac{\delta}{h}
\]

where

- \( G \) = sand penetration resistance gradient (described in the next paragraph)
- \( b \) = unloaded tire section width
- \( d \) = unloaded tire outside diameter
- \( h \) = unloaded tire section height
- \( \delta \) = tire deflection (the difference between unloaded tire section height \( h \) and loaded tire section height, with each height measured as the tire rests on a flat, level, unyielding surface)

\( G \) is the average slope of the curve of soil penetration resistance \( C \) versus cone penetration depth, with \( C \) and cone depth measured within a specified soil layer (ordinarily the 0- to 15-cm layer). \( C \) is the force per unit cone base area required to penetrate a soil normal to its surface at 3.0 cm/sec with a right circular 30-deg-apex-angle cone of 3.23-sq-cm base area. (The equivalent of \( C \) in English units is cone index, CI.) Figure 1 shows sample recordings of \( C \) versus cone penetration depth for a laboratory-prepared sand test bed. Note that zero cone penetration depth is defined as occurring when the base of the 3.23-sq-cm cone is flush with the initial sand surface.

For simplification, drawbar performance is analyzed herein only at 20 percent slip—i.e., only \( \mu_{20} \) and \( \eta_{20} \) data are considered. Use of this nominal slip value is meaningful because, as illustrated in Figure 2 (taken from Reference 2), 20 percent slip provides a reasonable balance of good in-sand tire \( \mu \) and \( \eta \) performance (with somewhat greater weight given to \( \mu \)) for a broad range of values of \( N_s \).

**EVOLOUTION OF DRAWBAR PERFORMANCE PREDICTIONS**

**BY \( N_s \) AND \( N_{mey} \)**

Early Development of \( N_s \)

\( N_s \) was first defined by Freitag almost 20 years ago by means of dimensional analysis of the results of laboratory dynamometer tests of single tires in air-dry Yuma sand (a desert sand taken from active dunes near Yuma, Arizona). Using data from Reference 4 for 10 tires tested in this sand, Figure 3 illustrates that \( N_s \) effectively consolidates \( \mu_{20} \) test data to one well-defined relation (Figure 3a) and \( \eta_{20} \) data to another (Figure 3b) for very broad ranges of values of \( G \), \( b \), \( d \), \( W \), and \( \delta/h \). Thus, Figure 3 strongly supports the conclusion that \( N_s \) describes in-sand tire \( \mu_{20} \) and \( \eta_{20} \) performance quite well—at least for air-dry Yuma sand.

In Reference 4, the relation of drawbar pull data to \( N_s \) was also examined for tests "conducted on coarse-grained soils in various parts of the world with a variety of military vehicles." In these field tests, sand "usually was moist or even wet; drawbar-pull tests usually were not run at a controlled slip but were made at several levels of pull with only the data relevant to the maximum pull recorded for each test; and no
special provisions were made to control differential wheel slip, dynamic weight transfer, or steering forces." Figure 4 illustrates, as would be expected, (a) that the \( u_{20} \) versus \( N_s \) relation defined by these field test data shows much smaller values of \( u_{20} \) at corresponding values of \( N_s \) than does the single-tire laboratory relation of Figure 3a, and (b) that the field relation exhibits substantially more data scatter than does the laboratory relation.* (Data from tests at sand moisture contents only up to about 7 percent are shown in Figure 4.) The field relation in Figure 4 was considered sufficiently well defined, however, to "offer the basis for a tentative performance prediction system . . . for vehicles operating in dry-to-moist sands."

For a number of years this "tentative performance prediction system" was accepted as workable, although it was recognized that the system had potential for further refinement. Such refinements were made piecemeal and in an evolutionary manner, primarily because of the lack of data for defining in detail a range of physical properties of the sands for which tire and wheeled vehicle drawbar performance data were available. Events of the past few years have caused a renewed interest, however, in refining and improving the drawbar performance versus sand-tire numeric methodology for wheeled vehicles. The remainder of this paper describes development of such a methodology, first taking into account some insights gained in earlier studies of the influence of sand type on single-tire drawbar performance.

First Considerations of Two Sand Types

In addition to single-tire tests in air-dry Yuma sand, WES also conducted a smaller, but significant, number of laboratory tests in air-dry mortar sand (a coarser-grained riverbed sand). Figure 5a uses data from tests of five tires in mortar sand, together with the \( u_{20} \) versus \( N_s \) curve from Figure 3a for Yuma sand, to demonstrate that these tires developed consistently smaller values of \( u_{20} \) in mortar than in Yuma sand at corresponding values of \( N_s \).

In 1972, Reference 5 attempted to account for this difference by using the relations of \( C \) to relative density (\( D_r \)) for the two sands, defined from Reference 6. For air-dry mortar sand:

\[
D_r = 75.0 \log C + 39.3
\]

and for air-dry Yuma sand:

\[
D_r = 71.1 \log C + 51.6
\]

where

\[
D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \times 100, \text{ percent}
\]

* For simplicity, all drawbar coefficient data considered herein are designated \( u_{20} \) data, although \( u \) in the wheeled vehicle field tests was sampled at the near-maximum-pull level, not necessarily 20 percent slip. Also, for brevity, tire drawbar performance in several subsequent figures is defined only in terms of \( u_{20} \). Performance is described in terms of both \( u_{20} \) and \( \eta_{20} \) in appropriate concluding figures.
$e_{\text{max}}$ and $e_{\text{min}}$ are void ratios for the loosest and densest sand states, respectively, and $e$ is void ratio for the before-tire-pass sand condition. A given value of mortar sand $G$ ($G_M$ in Figure 5) was converted to Yuma sand $G$ ($G_Y$) by first determining the mortar sand $D_r$ value in Equation 4 and then using that same $D_r$ value to solve for Yuma sand $G$ in Equation 5. This use of $D_r$ as the intermediate soil parameter in translating $G$ values between different sands appeared to produce the desired result, as evidenced in Figure 5b by the shift of the mortar sand tire test data to locations clustered about the $\mu_{20}$ versus $N_s$ curve for Yuma sand. Reference 5 recognised, however, that use of $D_r$ as described above must be considered tentative and recommended "that tests be conducted in several additional sands so that the relative density approach . . . can be further verified."

In 1975, Reference 7 reported drawbar performance results from a later series of tests in air-dry mortar sand, these conducted with four 9.00R20 radial ply tires (each different in terms of tread design and other construction features), plus two 9.00-20 bias-ply tires (one with nondirectional cross-country tread, the other with tread buffed smooth). Tests for each tire were conducted over a range of wheel loads and tire deflections, and at two levels of $G$, approximately 2.2 and 5.5 MPa/m. For tire deflections of 15 and 35 percent, Figure 6a shows that the relation of $\mu_{20}$ to $N_s$ separated as a function of $G$. Further, Figure 6b shows that the relation of $\mu_{20}$ to $(N_{20})_Y$ for these test data also separated by $G_Y$ (where $G_Y$ values in $(N_{20})_Y$ were obtained by Equations 4 and 5 and the process described in the previous paragraph). In attempting to account for this separation, Reference 7 noted that "ideally, the $G$ value to use in describing tire performance for a given (tire) pass is the value that predominated during that pass. For first pass, this value lies between the 0- and 1-pass values"—i.e., between the before- and after-first-pass values. Examination of mortar sand tire test data showed "that $G$ changes with tire traffic in a funnel-shaped pattern" like that shown in Figure 6c, and "indicated that the best $G$ value for describing first-pass, 20-percent-slip tire performance is $G$ at "pass number" 0.75 (hereafter termed $G_{0.75}$). That is, $G$ should be weighted 3:1 toward its after-first-pass value." Figure 6d shows the well-defined relation obtained for $\mu_{20}$ versus $N^'$, where $N^' = \frac{N}{N_s}$. While the $\mu_{20}$ versus $N_s$ relation collapsed the mortar sand test data for the six 9.00x20 tires quite well, the central curve in Figure 6d is notably different from the one in Figure 3a for Yuma sand. The thrust of the analysis in Reference 7 was not directed at accounting for the influence of sand type on tire drawbar performance. However, Reference 7 recognised that "clearly, more work is needed to develop techniques for describing sand strength that changes significantly with tire traffic."

A First-Cut, More General $N_s$ Methodology

In 1978, Reference 8 attempted to define a methodology to satisfy the two needs demonstrated in Figures 5 and 6—i.e., to define a means (a) for translating $G$ values between different sand types, and (b) for describing the effective (predominant) during-tire-pass value of $G$. This methodology was applied by means of the nomogram shown in Figure 7.

The aim of the nomogram was to define $G_{0.75}$, "effective sand penetration gradient, the value of $G$ that predominated during a given tire pass, normalised to one type of frictional soil (selected as Yuma sand)."
Use of the nomogram required known before-tire-pass values of \( G \) and of \( D_{rb} \) \( (G_b \) and \( D_{rb} \), respectively), and involved the following steps (identified by circled numbers in Figure 7):

**Step 1:** For the sand of interest (taken as mortar sand in Figure 7a), determine for \( G_b \) the corresponding value of \( D_{rb} \).

**Steps 2 and 3:** For the tire b/d value of concern (0.29 in Figure 7b, for example), translate the \( D_{rb} \) value of step 1 from Figure 7a to 7b (shown as a dot in the example). In Figure 7b, use the family of curves that relate \( D_{rb} \) to \( D_{re} \) (effective relative density) as a function of tire b/d to estimate \( D_{re} \).

**Steps 4 and 5:** Translate \( D_{re} \) of step 3 for the sand of interest to \( D_{re} \) for Yuma sand (step 4), and then to \( G_e \) for Yuma sand (step 5).

\( G_e \) from step 5 was then used as the soil strength term in

\[
N_{se} = \frac{N_{se}}{W} \times \frac{G_e}{G} \]

and \( N_{se} \) was related to tire drawbar performance terms \( \mu_{20} \tan \phi_{70} \) and \( \eta_{20} \tan \phi_{70} \). The rationale for using \( \tan \phi_{70} \) (tangent of sand internal friction angle from a direct shear test at 70 percent relative density) as a multiplier of \( \mu_{20} \) and \( \eta_{20} \) was that:

(a) \( D_{rb} \) tends toward a \( D_{re} \) value of about 70 percent for common tire shapes (b/d values from about 0.2 to 0.3), particularly with repeated transformations of \( D_{rb} \) to \( D_{re} \) to correspond to multiple tire passes, and

(b) the products \( \mu_{20} \tan \phi_{70} \) and \( \eta_{20} \tan \phi_{70} \) appeared, in conjunction with \( N_{se} \), to provide a normalized description of tire drawbar performance for the three frictional soils considered in Reference 8 (Yuma and mortar sands, plus a finely crushed basalt used as lunar soil simulant, LSS, described in Reference 9).

Figure 8 shows the relations (a) of \( \mu_{20} \tan \phi_{70} \) to \( N_{se} \) and (b) of \( \mu_{20} \tan \phi_{70} \) to \( N_{se} \) based on test results in Yuma sand for the same 10 tires as in Figure 3a, in mortar sand for the same 11 tires as in Figures 5 and 6, and in LSS for one tire-like wheel. (This wire-mesh wheel was evaluated for use on the lunar rover vehicle by testing the wheel in the rather exotic LSS. The two asterisks of Figure 7a define coordinates of \( D_{rb} \) and \( G \) for the two LSS test conditions.) For the test data considered, the relation of \( \mu_{20} \tan \phi_{70} \) to \( N_{se} \) in Figure 8b is considerably better defined than is that of \( \mu_{20} \) to \( N_{se} \) in Figure 8a. While the Figure 8a relation appeared promising, Reference 8 "recognized that the range of frictional soil types considered . . . is limited; thus it is hoped that other investigators will test the universality of the \( \mu_{20} \tan \phi_{70} \) and \( \eta_{20} \tan \phi_{70} \) versus \( N_{se} \) relations using tire test results obtained in a variety of frictional soils."

In 1981, References 10 and 11 applied the methodology described in Figures 7 and 8 to drawbar performance data obtained with a 6.00-16, 2-FK treadless (smooth) tire in air-dry Creaswell sand. For this tire-sand combination, Figure 9 shows that the \( \mu_{20} \tan \phi_{70} \) versus \( N_{se} \) relation obtained was very different from that obtained in Figure 8b. Clearly, the \( \mu_{20} \tan \phi_{70} \) and \( \eta_{20} \tan \phi_{70} \) versus \( N_{se} \) methodology was shown not to successfully treat all sand tire situations, and the need was established for analysing a broad range of sand types and conditions in one study. A description of that analysis follows.
A New Methodology for a Broad Range of Sand Types and Conditions

The Test Sands. Two major limitations in the WES analyses described to this point are that (a) only two ordinary test sands were considered (Yuma and mortar sand—the exotic LSS is hereafter not considered), and (b) these sands were each used only air-dry in single-tire testing. The new analysis considers 10 sands—the Yuma, mortar, and Cresswell sands, plus seven other sands for which vehicle field drawbar test data were available (six sands from References 4 and 12, one from Reference 13). A separate value of sand moisture content was reported in References 12 and 13 for each wheeled vehicle test; tests at moistures from 1 to 7 percent are considered herein.

A necessary first step in the analysis was to obtain samples of approximately 100 kg each for the 10 test sands. In this regard, particular thanks are extended to Dr. A. R. Reece for supplying the needed sample of Cresswell sand (the sand used in References 10 and 11), and to Dr. L. I. A. C. Crosjean, Etablissement Technique d'Angers, for supplying sand samples from beaches at La Turballe and at Suscinio, France (two of the test sites in References 4 and 12). Samples of the Yuma and mortar sands were obtained from large stockpiles at WES, and samples of the remaining five sands were obtained in re-visits to wheeled vehicle test sites in the United States.

A major concern in the new analysis was how closely the 10 sand samples matched the sands actually used in the tests of single tires or wheeled vehicles reported in References 4, 10, 11, 12, and 13. One means of evaluating this was to compare the original grain-size distribution curves shown in these references with the corresponding curves shown in Figure 10 for the sand samples that were used in 1983 WES soils laboratory testing. Results of this comparison are shown in Table 1 for grain-size diameters at the 10, 30, 50, 70, and 90 percent finer by weight levels. As expected, the original and the 1983 curves matched very closely for the Yuma and mortar sands (WES laboratory test sands). For Paw Paw Island sand, the original and 1983 curves are one and the same. For Cresswell sand, there is a noticeable difference between the original and 1983 curves.

The remaining six sands were tested in the field during 1958-1961, as reported in Reference 12 (1963). It was anticipated that the passage of some 20-25 years time, plus inability to locate precisely some of the original test sites, could cause substantial differences between the original and 1983 distribution curves, at least for some of the six Reference 12 sands. As it turned out, the original and the 1983 curves showed almost perfect agreement for the Padre Island site, very close agreement for the Mississippi River Bridge site, somewhat less agreement for the La Turballe and the two National Seashore Headquarters sites, and least agreement (by a considerable margin) for the Suscinio site. Implications of comparisons between the original and the 1983 sand grain diameters as described in Table 1 are discussed later in the analysis.

Relations Among C, D, and Sand Moisture Content. In analyzing data for the 10 sand samples, it was recognized, first, that D appeared not to be suitable for use as an intermediate soil parameter in translating between sand types. (Recall from Figures 6a and 6b that separation of data for the Yuma and mortar sands was not alleviated
by the use of $D_r$ in a translation role.) However, $D_r$ did appear promising for use in a standardized description of the change in sand strength that occurs during a given tire pass. (Note that Figure 7b uses $D_r$ in this role to describe the same process somewhat more crudely described in Figure 6c.) Further, $D_r$ has the advantageous characteristics (a) of increasing in value as $C$ increases, decreasing as $G$ decreases, and (b) of taking values within the same range (0 to 100 percent) for all sands.

To develop the desired standardized description, the relation between $G$ and $D_r$ was determined for each of the 10 sand samples at sand moisture conditions at least from air-dry to 7 percent moisture. Additionally, measurements of $G$ and $D_r$ were obtained for the Yuma and Cresswell sands at a fully saturated condition and for Cresswell sand at 0.1 percent moisture content.

Figure 11 shows relations among $G$, $D_r$, and sand moisture content representative of those obtained for the 10 sand samples. For Yuma sand, this figure illustrates that the $G$ versus $D_r$ relation is described by

$$D_r = a_1 \log G + a_2$$

where $a_1$ is a constant for a given sand, and $a_2$ changes value as a function of sand moisture content. Note in Figure 11 that $a_2$ decreases as sand moisture content increases from air-dry to about 7 percent (this same pattern was obtained for all the test sands), but $a_2$ increases markedly as moisture increases from about 7 percent to the fully saturated condition (this pattern was also obtained for Cresswell sand).

Figure 12 illustrates the relations (a) of $a_2$ to sand moisture content, and (b) of $G$ (at $D_r = 70$ percent) to sand moisture content that were obtained for the Yuma and Cresswell sands. For each of the 10 sand samples, the pattern of change in $a_2$ with change in moisture from air-dry to 7 percent was similar to that shown by the dashed curves in Figure 12—i.e., for each sand, $a_2$ decreased semilogarithmically as moisture increased from air-dry to about 2 percent, and then continued to decrease, but at a fast-diminishing rate until a minimum $a_2$ value was obtained at about 7 percent moisture. For the Yuma and Cresswell sands, $a_2$ increased rapidly as sand moisture content increased beyond about 7 percent.

The influence on $G$ caused by this pattern of change in $a_2$ with sand moisture content is seen by rearranging Equation 7 to

$$G = \text{antilog} \frac{D_r - a_2}{a_1}$$

Thus, for each of the 10 sand samples (constant $a_1$) and any constant level of $D_r$, $G$ attained a maximum value at minimum $a_2$—i.e., at about 7 percent moisture content. Further, based on data for two of the test sands (Yuma and Cresswell), it appears that, for a given sand and constant $D_r$, $G$ decreases rapidly as sand moisture content increases beyond about 7 percent.

Table 3 summarizes in columns 1-9 for each of the 10 sand samples the relation of $D_r$ to $G$ obtained in 1983 WSS laboratory testing at sand moisture contents from air-dry to 7 percent. (Values in other columns of Table 2 will be discussed subsequently.) Note in column 2 that each listed value of air-dry sand moisture content was obtained
in the WES soils laboratory after a given sand sample remained undisturbed for at least seven days. These at-WES air-dry moisture contents do not necessarily correspond to air-dry moisture contents at other sites.

**Prediction of During-Tire-Pass $G_e$**

To predict $G_e$ for the 10 test sands required implementation of both (a) the relations among $G$, $D_r$, and sand moisture content (summarized in columns 1-9 of Table 2), and (b) the relations among $G_0$, tire shape factor $b/d$, and $G_e$ shown in Figure 13. A three-step process is involved:

1. Use Equation 8 to estimate $D_{rb}$ (from known values of $G_0$, $a_1$, and $a_2$).
2. Obtain $D_{re}$ from Figure 13 (using $D_{rb}$ from step 1 and known $b/d$).
3. Compute $G - G_e = \text{antilog} \left( a_1 - \frac{a_2}{a_1} \right)$ (using the same values of $a_1$ and $a_2$ as in step 1).

Before applying the above process, it is useful to examine the relation in Figure 13. The shape of each curve in Figure 13a is the same as in Figure 7b for $b/d$ values of about 0.2 and larger. For smaller $b/d$ values, the curves in Figure 13a reflect recent analysis of single-tire drawbar test data in Yuma sand using the 1.75-26 bicycle and 4.00-20, 2-PR tires ($b/d$ values of 0.068 and 0.150, respectively) not considered in Reference 8.*

In agreement with Reference 7 and with Figure 6c herein, $D_{re}$ in Figure 13a reflects the condition obtained at tire pass number 0.75—i.e., first-pass $D_{re}$ is considered weighted 3:1 toward the after-first-pass condition. For two powered-wheel tire passes, the appropriate $D_{re}$ value is for tire pass 1.75; for three tire passes, 2.75; and for four tire passes, 3.75. The relation in Figure 13a was successively applied to obtain $D_{re}$ values for tire pass multiples of 0.75; $D_{re}$ values for tire passes 2, 3, and 4 were then obtained by interpolation as needed.

Detailed application of the $D_{rb}$, $b/d$, $D_{re}$ relation for a given all-axles-powered wheeled vehicle would require that a separate value of $G_e$ be determined for each axle, and that these $G_e$ values then be averaged to determine $G_e$ for the overall vehicle. Figures 13b, 13c, and 13d avoid this cumbersome process by reflecting averaged values of $D_{re}$ for tire passes 1 and 2, passes 1 through 3, and passes 1 through 4, respectively. For a given 4x4, 6x6, or 8x8 vehicle, then, use of the

* In Reference 8, $\mu_{20}$ (and $\eta_{20}$) values for all the tires considered reflected a mechanical/electrical correction to negate dynamometer carriage acceleration forces developed in the single-tire, programmed-increasing-slip tests. No such correction had been in use during tests of the 1.75-26 bicycle and 4.00-20, 2-PR tires reported in Reference 4. However, for a number of single tires tested over a broad range of $N_0$ values, acceleration-corrected $\mu_{20}$ from Reference 4 has been determined to be smaller than uncorrected $\mu_{20}$ by a near-constant 0.045. For both the 1.75-26 and the 4.00-20 tires, the acceleration-corrected $\mu_{20}$ values used herein were obtained by subtracting 0.045 from $\mu_{20}$ values previously uncorrected for carriage acceleration.
single appropriate relation in Figure 13b, 13c, or 13d produces a single
value of \( G_e \) very close to that obtained by

\[ G_e (bd)^{3/2} \]

soil strength term in sand-tire numeric as

\[ N_{se} = \frac{G_e}{W} \cdot \frac{A}{h}, \]

such that

\( N_{se} \) would collapse both single-tire and wheeled-vehicle drawbar data for

a broad range of sand conditions to a single relation for a given sand
type. The success of \( G_e \) in this role is illustrated, first, in Figures

14a and 14b which show for 10 single tires and for three 4x4 vehicles, all
tested in air-dry Yuma sand, that all the test data cluster closely about

the same central \( u_{20} \) versus \( N_{se} \) relation.

\( N_{se} \) was also determined to be more effective than \( N_e \) in

consolidating \( u_{20} \) data for each of the nine other test sands, in each
case producing (as expected) a separate \( u_{20} \) versus \( N_{se} \) relation.

Figure 15 shows representative results, using data (a) from tests of a

single 6.00-16, 2-PR tire in air-dry Creaswell sand and (b) from tests

of four wheeled vehicles in moist sand at the Padre Island site.

Normalization of \( G_e \) to \( G_{ey} \). Having developed a means to

predict \( G_e \), it remained to develop a means for normalizing \( G_e \) to one

sand type, selected as Yuma sand. Analyses were made involving a number

of parameters descriptive of physical properties of the 10 sand samples,

with best results obtained by application of the relations shown in Fig-

ure 16.

In Figure 16, three sand parameters are involved—penetration

resistance gradient (C), sand compactibility (\( D' \)), and sand grain median
diameter (\( d_{50} \)). Compactibility is defined as

\[ D' = \frac{C_{max} - C_{min}}{C_{min}} \times 100, \text{ percent} \]  \hspace{1cm} (9)

and \( d_{50} \) (sand grain diameter for which 50 percent of the sand sample

is finer by weight) is read directly from a sand's grain-size distribution curve. In Figure 16, subscript x denotes sand x, and subscript y
denotes Yuma sand. For a given sand x, known values of \( D'_x / D'_y \) and

\( (d_{50})_x / (d_{50})_y \) are used in Figure 16 to determine corresponding values of

\( (G_{ex} / G_{ey})_{D'} \) and \( (G_{ex} / G_{ey})_{d_{50}} \), respectively. A given value of \( G_e \) for

sand x (\( G_{ex} \)) is then normalized to the corresponding value for Yuma sand

(\( G_{ey} \)) by the relation

\[ G_{ey} = G_{ex} + (G_{ex} / G_{ey}) \]  \hspace{1cm} (10)

where

\[ G_{ex} / G_{ey} = (G_{ex} / G_{ey})_{D'} \cdot (G_{ex} / G_{ey})_{d_{50}} \]  \hspace{1cm} (11)
For use in normalizing $G_{ex}$ to $G_{ey}$, the curves in Figure 16 exhibit expected trends. A given sand of high compactibility requires less force for its displacement than does one of low compactibility at the same relative density. Thus, for $D_y/D_x > 1$ (all other conditions constant), $G_{ex}$ must be increased for normalization to $G_{ey}$. This is accomplished by taking the appropriate value of $(G_{ex}/G_{ey})_{D_y} > 1$ from Figure 16, applying this value in Equation 11, and then using Equation 10. (For $D_y/D_x < 1$, $(G_{ex}/G_{ey})_{D_y} < 1$ and $G_{ex}$ is decreased in normalization to $G_{ey}$.)

Note, also, that the penetration resistance of a sand with large-diameter grains is greater than that of one with smaller grains (all other conditions constant). Thus, for $(d_{50})_x/(d_{50})_y > 1$, $(G_{ex}/G_{ey})_{d_{50}} > 1$ and $G_{ex}$ is increased in normalization to $G_{ey}$. (For $(d_{50})_x/(d_{50})_y < 1$, $G_{ex}$ is increased in normalization to $G_{ey}$.)

Use of $G_{ey}$ in $N_{sey}$. Having determined the value of $G_{ey}$ for a particular sand $x$, the next step is to use $G_{ey}$ in $N_{sey} = G_{ey}(bd)^{3/2}/h$ to predict in-sand $\mu_{20}$ and $\eta_{20}$ tire and wheeled vehicle performance. The success of $G_{ey}$ in this role is illustrated in the following comparisons.

In Figure 17a, data for all of the single-tire tests considered herein for the Yuma, mortar, and Cresswell sands cluster about the same $\mu_{20}$ versus $N_{sey}$ curve obtained earlier for Yuma sand in Figures 14a and 14b. (Note that $N_{sey} = N_{se}$ for Yuma sand only.) In Figure 17b, single-tire test data for these three sands all cluster about the same $\eta_{20}$ versus $N_{sey}$ relation. Note further that, based on results from the same laboratory single-tire tests, data collapse about the two relations involving $N_{sey}$ in Figure 17 is considerably better than that about corresponding relations in Figure 18 involving $N_{e}$. In Figure 19, the wheeled-vehicle test data for six sandy field test sites show much less data scatter about the central $\mu_{20}$ versus $N_{sey}$ curve (the same curve as in Figures 14 and 17a) than do corresponding data for the same test sites in Figure 4 about the central curve of $\mu_{20}$ versus $N_{e}$. *

Based on Figures 17 and 19, $N_{sey}$ is demonstrated to be very effective in consolidating single-tire and wheeled-vehicle $\mu_{20}$ data to one relation, $\eta_{20}$ data to another. Remarks modifying this general conclusion need to be made, however, relative primarily to one of the laboratory test sands in Figure 17 (Cresswell sand) and to the one field test sand not shown in Figure 19 (Suscinio sand).

Some Strengths and Limitations of the $N_{sey}$ Methodology. First, regarding the Cresswell sand, determination of its $G_{ey}$ values in Figure 17 was made using as input data one set of $G$ values gleaned from References 10 and 11, plus values of $a_1$, $a_2$, $D'$, and $d_{50}$ from the 1983 WES laboratory tests of the Cresswell sand sample (using

* No $\eta_{20}$ versus $N_{sey}$ relation is shown in Figure 19 because measurements of $\eta_{20}$ were not obtained in any of the wheeled-vehicle tests considered herein.
132

at the WES air-dry condition). There was interest in determining how these predicted values of $G_{ey}$ (and of $N_{syy}$) compared with those obtained by using the same set of $G$ values, together with input values of $a_1$, $a_2$, $D'$, and $d_{50}$, obtained from References 10, 11, and 14. Table 3 summarizes this comparison.

For the 11 air-dry Crosswell sand test conditions considered, the major conclusion from Table 3 is that, although two quite different sets of input values of $a_1$, $a_2$, $D'$, and $d_{50}$ were used (see the two footnotes of Table 3), nearly identical values of $G_{ey}$ and of $N_{syy}$ were predicted (compare results in columns 9 and 10 with those in columns 14 and 15). This close agreement reflects that the overall process for translating values of $G$ to $G_{ey}$ (summarized in the first footnote of Table 3) is reasonably robust. That is, based on the comparison in Table 3, the $G$-to-$G_{ey}$ prediction process appears not to be unduly influenced by even fairly sizable variations in values of its required input parameters.

This tentative conclusion is supported by the wheeled vehicle field relations shown in Figures 4 and 19. For the first five sands in the legends of these two figures, the sand samples used in defining values of $a_1$, $a_2$, $D'$, and $d_{50}$ by 1983 WES laboratory testing were like the sands used in actual 1958 to 1961 field testing only to varying degrees—see Table 1. (For the sixth sand, from the Paw Paw Island site, the 1983 sand sample was taken from the precise location of field testing.) For the first five sands, taking this discontinuity between sample and test sands into account, the improvement in the relation of $\mu_{20}$ versus $N_{syy}$ in Figure 19 versus the $\mu_{20}$ versus $N_s$ relation in Figure 4 is rather remarkable, even with one significant caveat: the $\mu_{20}$ versus $N_{syy}$ relation obtained for Suacinio sand (not shown in Figure 19) is considerably different from that shown in Figure 19 for the six other field test sands (it is displaced far to the left).

There are two principal possibilities for explaining what at first seems to be the atypical $\mu_{20}$ versus $N_{syy}$ behavior of the Suacinio data. First, it is possible that one or more sand parameters needed in the process for translating $G$ to $G_{ey}$ have been omitted. The process described herein is the one that was determined to make the $G$-to-$G_{ey}$ translation best for the test data examined, based on analysis not only of the sand parameters now included in the process, but also of several other parameters initially considered potentially important (coefficient of uniformity $C_u = d_{50}/d_{10}$, angle of internal friction, etc.). Still, modifications might substantially improve the $G$-to-$G_{ey}$ translation process, and such modifications are welcomed.

The second, and much more likely, reason for the $\mu_{20}$ versus $N_{syy}$ behavior of the Suacinio sand relates to the fact that, of the 10 sand samples used in 1983 WES laboratory testing, Suacinio's grain diameter distribution showed least agreement with its corresponding original distribution, by a large margin—see Table 1. Thus, it was not surprising, when the $D_1 = 141.0 \log G + a_2$ laboratory relation for Suacinio sand was applied to Suacinio field values of $G$, that values of $D_1$ considerably larger than 100 percent were obtained in some cases. (This did not occur with the nine other sands.) Note, also, from Table 1 that the Suacinio field sand was considerably less coarse than the 1983 Suacinio sample sand (which included almost as much gravel as sand—see Figure 10). In fact, from Table 1, the Suacinio field sand's overall distribution of $d$ values is approximated just as well by the 1983 Le Turballe laboratory sample (from the low side) as it is by the Suacinio laboratory sample (from the high side). (Prediction of Suacinio $G_{ey}$ values by using for input
Suscinio field C values and La Turballe laboratory a₁, a₂, D', and d₅₀ values produced a Suscinio μ₂₀ versus Nₑₑₑₚ relation very closely approximated by the relation in Figure 17.) Finally, note that the good fit of the Suscinio data in the μ₂₀ versus Nₑₑₑₑ relation of Figure 4 further indicates that characteristics of the Suscinio sand, as encountered on-site and measured at least by Gₑₑₑₑ, were not foreign to those of the six other sands in Figure 4.

The above observations suggest that, for the Suscinio beach site, the discontinuity between 1983 sample sand and 1959 field sand was simply too large to overcome in using a₁, a₂, D', and d₅₀ values from the sample sand to describe drawbar performance in the field sand. These observations also lead to the caveat that it remains to be determined how coarse a sand must be for the Nₑₑₑₑ relations not to apply. (Sands at least as coarse as the La Turballe sand are successfully treated by Nₑₑₑₑ.) A second caveat is that a substantial amount of laboratory testing is necessary to define the input values of a₁, a₂, D', and d₅₀ required by the process for translating Cₑₑₑₑ to Gₑₑₑₑ for use in Nₑₑₑₑ (particularly to define a₁ and a₂ for the range of values of Dₑₑₑₑ and sand moisture content of possible concern). If the user is not restricted by these two caveats, the Nₑₑₑₑ relations of Figures 17 and 19 are useful now in predicting drawbar performance with better accuracy than do the Nₑₑₑₑ relations of Figures 18 and 4. If the above caveats negate use of the Nₑₑₑₑ relations, the μ₂₀ versus Nₑₑₑₑ relation of Figure 4 is still judged sufficiently well defined to offer the basis for a useful wheeled vehicle drawbar performance system.

SUMMARY AND CONCLUSIONS

To summarize, a five-step process was developed for predicting tire and wheeled vehicle μ₂₀ and n₂₀ performance for a given sand and sand moisture content, described as follows:

1. Use Equation 7 to estimate Dₑₑₑₑ (from known values of Gₑₑₑₑ, a₁, and a₂).
2. Obtain Dₑₑₑₑ from Figure 13.
3. Compute Gₑₑₑₑ = yₐₑₑₑₑ (Dₑₑₑₑ - a₂/a₁). For sand x, this is Gₑₑₑₑ.
4. Convert Gₑₑₑₑ to Gₑₑₑₑ by use of Figure 16 and Equations 11 and 10.
5. Use Gₑₑₑₑ in Nₑₑₑₑ and the relations in Figure 17 to predict μ₂₀ and n₂₀.

Relations of μ₂₀ and n₂₀ to Nₑₑₑₑ now offer better prediction accuracy than those of μ₂₀ and n₂₀ to Nₑₑₑₑ for a broad range of sand types and strengths, and for sand moisture contents up to about 7 percent. Implementation of the Nₑₑₑₑ relations is limited, however, by two caveats: (a) the exact range of sand types for which the Nₑₑₑₑ relations are applicable remains to be determined (sands from at least as fine as the Yuma sand to at least as coarse as the La Turballe sand considered herein are successfully treated by Nₑₑₑₑ), and (b) substantial laboratory testing is necessary to define values of a₁, a₂, D', and d₅₀, which are required as input by the process for defining Gₑₑₑₑ for use in Nₑₑₑₑ. Further work is needed to minimize or eliminate the influence of these two caveats. For now, with proper account taken of their...
limitations, either the $W_{p,s}$ or the $W_s$ methodology can be employed to predict in-sand tire and wheeled vehicle drawbar performance with useful accuracy.

REFERENCES


**NOTATION**

\( a_1, a_2 \)  
Constant and variable, respectively, in the equation \( D_r = a_1 \log G + a_2 \) for a given sand over a range of sand moisture contents

\( b \)  
Unloaded tire section width

\( C \)  
Soil penetration resistance

\( C_u \)  
Coefficient of uniformity

\( C_I \)  
Cone index

\( d \)  
Unloaded tire outside diameter

\( d_{50}, (d_{50})_x, (d_{50})_y \)  
Median diameter of sand grains, \( d_{50} \) of sand \( x \), \( d_{50} \) of Yuma sand

\( D', D'\), \( D'\)_x, \( D'\)_y \)  
Compactability, compactability of sand \( x \), compactability of Yuma sand

\( D_r, D_{rb}, D_{re} \)  
Relative density, before-tire-pass relative density, effective (predominant during-tire-pass) relative density

\( D_P, D_{P20} \)  
Drawbar pull, drawbar pull at 20 percent slip

\( e, e_{\text{max}}, e_{\text{min}} \)  
Before-tire-pass sand void ratio, maximum sand void ratio, minimum sand void ratio

\( G, G_b, G_e, G_{ex}, G_{ey} \)  
Sand penetration resistance gradient, before-tire-pass \( G \), effective (predominant during-tire-pass) \( G \), \( G_e \) for sand \( x \), \( G_{ey} \) for Yuma sand

\( h \)  
Unloaded tire section height

\( i \)  
Slip

\( N_s, N_{se}, N_{sey} \)  
Sand-tire numerics \( N_s = \frac{G(bd)^{3/2}}{W} \cdot \frac{\delta}{h} \), \( N_{se} = \frac{G_e(bd)^{3/2}}{W} \cdot \frac{\delta}{h} \), and \( N_{sey} = \frac{G_{sy}(bd)^{3/2}}{W} \cdot \frac{\delta}{h} \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Torque input to wheel</td>
</tr>
<tr>
<td>$v$</td>
<td>Forward velocity of wheel axle</td>
</tr>
<tr>
<td>$W$</td>
<td>Load on a single tire</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Tire deflection (under load)</td>
</tr>
<tr>
<td>$\mu$, $\mu_{20}$</td>
<td>Drawbar coefficient, drawbar coefficient at 20 percent slip</td>
</tr>
<tr>
<td>$\eta$, $\eta_{20}$</td>
<td>Drawbar efficiency, drawbar efficiency at 20 percent slip</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Rotation velocity of the torque input shaft</td>
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Figure 1. Sample recordings of cone penetration tests in a sand test bed

Figure 2. Relations of \( \mu \) and \( \eta \) to \( N_a \) and slip (\( i \)) for pneumatic tires operating in a frictional sand (adapted from Reference 2)

Figure 3. Relations of \( \nu_{20} \) and \( \eta_{20} \) to \( N_a \) for ten single pneumatic tires tested in air-dry Yuma sand (values of \( G \) from 0.62 to 7.52 MPa/m, \( W \) from 0.41 to 20.02 kN, and \( \delta/h \) from 0.15 to 0.35)
Figure 4. Relation of $\mu_{20}$ to $N_s$ for tests with a variety of wheeled vehicles at seven sandy field sites.

Figure 5. Effect of converting $G_w$ to $G_T$ in the relation of $\mu_{20}$ to $N_s$ for tires operating at 20 percent slip in mortar sand.
Figure 6. Relations of (a) \( \mu_{20} \) to \( N_h \), (b) \( \mu_{20} \) to \( (N_h)_y \), (c) \( G \) to tire pass number, and (d) \( \mu_{20} \) to \( N'_h \) for tests of six 9.00x20 tires in mortar sand
Figure 7: (a) Relations of \( D_t \) to \( C \) for three sands, and (b) from Reference 8, tentative relations among \( b/d \), \( D_t \), and \( D_\text{fa} \) for sands in general (subsequently replaced by relations developed herein)
Figure 8. Relations of (a) $\mu_{20}$ to $N$ and (b) $\mu_{20} \tan \phi_{70}$ to $N_{se}$ for the three frictional test soils considered in Reference 8.

Figure 9. Relation of $\mu_{20} \tan \phi_{70}$ to $N_{se}$ for 6.00-16, 2-PR tire tested in air-dry Creaswell sand (adapted from Reference 11).

Figure 10. Grain size distributions of ten sand samples analysed at WES in 1983.
Figure 11. Relations among $G$, $D_r$, and sand moisture content for Yuma sand

Figure 12. Relations of (a) $a_2$ to sand moisture content and (b) $G$ at $D_r = 70$ percent to sand moisture content for Yuma and Cresswell sands
Figure 13. Relations among $D_{rb}$, $b/d$, and $D_{re}$ for one, two, three, and four tire passes in sand (at 20 percent slip with tire deflection in the 15 to 35 percent range)
Figure 14. Single relation of $\nu_{20}$ to $N_{se}$ (a) for ten single tires and (b) for three 4x4 vehicles, each tested in air-dry Yuma sand.
Figure 15. Relations of $\mu_20$ to $N_a$ and $N_{m}$ to $N_{m2}$ (a) for the 6.00-16, 2-PR tire in al~dry Cresswell sand, and (b) for four wheeled vehicles in moist Padre Island sand.
Figure 16. Relations of \( \frac{D'_X}{D'_Y} \) to \( \left( \frac{G_{ex}}{G_{ey}} \right)_{D'} \) and \( \frac{(d_{50})_X}{(d_{50})_Y} \) to \( \left( \frac{G_{ex}}{G_{ey}} \right)_{d_{50}} \)

Figure 17. Relations of (a) \( \nu_{20} \) to \( N_{sey} \) and (b) \( \eta_{20} \) to \( N_{sey} \) for single-tire tests in air-dry Yuma, mortar, and Crosswell sands
Figure 18. Relations of (a) $u_{20}$ to $N_s$ and (b) $n_{20}$ to $N_s$ for single-tire tests in air-dry Yuma, mortar, and Cresswell sands.

Figure 19. Relation of $u_{20}$ to $N_{se}$ for tests with a variety of wheeled vehicles at six sandy field sites.
<table>
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<th>Sand</th>
<th>(d_{10}^{orig.})</th>
<th>(d_{30}^{orig.})</th>
<th>(d_{50}^{orig.})</th>
<th>(d_{70}^{orig.})</th>
<th>(d_{90}^{orig.})</th>
<th>Reference No. of Original Test Report</th>
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<td>1.75/1.80</td>
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<td>Mississippi River Bridge</td>
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<td>Yuma</td>
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<td>0.19/0.20</td>
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* All \(d\) values are in mm, and were obtained from plots such as those in Figure 10.

** For the La Turballe and the Suscinio test sites, each \(d\) value listed above from the original test report is the average of \(d\) values from similar grain-size distributions for two test areas at that site—a foreshore test area and a backshore test area.

† N.S.H. is National Seashore Headquarters, the agency in Massachusetts that presently occupies the test area occupied by Camp Wellfleet when the wheeled vehicle tests of Reference 12 were conducted.
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<th>Sand</th>
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<td>Void Ratio (e) at Moisture Content of (5) %</td>
<td>0.782</td>
<td>0.548</td>
<td>0.782</td>
<td>0.548</td>
<td>0.782</td>
<td>0.548</td>
<td>0.782</td>
<td>0.548</td>
<td>0.782</td>
<td>0.548</td>
<td>0.782</td>
<td>0.548</td>
<td>0.782</td>
<td>0.548</td>
<td>0.782</td>
</tr>
<tr>
<td>Void Ratio (e) at Moisture Content of (10) %</td>
<td>0.630</td>
<td>0.345</td>
<td>0.630</td>
<td>0.345</td>
<td>0.630</td>
<td>0.345</td>
<td>0.630</td>
<td>0.345</td>
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<td>0.630</td>
<td>0.345</td>
<td>0.630</td>
<td>0.345</td>
<td>0.630</td>
</tr>
<tr>
<td>Void Ratio (e) at Moisture Content of (20) %</td>
<td>0.732</td>
<td>0.332</td>
<td>0.732</td>
<td>0.332</td>
<td>0.732</td>
<td>0.332</td>
<td>0.732</td>
<td>0.332</td>
<td>0.732</td>
<td>0.332</td>
<td>0.732</td>
<td>0.332</td>
<td>0.732</td>
<td>0.332</td>
<td>0.732</td>
</tr>
<tr>
<td>Void Ratio (e) at Moisture Content of (30) %</td>
<td>0.908</td>
<td>0.572</td>
<td>0.908</td>
<td>0.572</td>
<td>0.908</td>
<td>0.572</td>
<td>0.908</td>
<td>0.572</td>
<td>0.908</td>
<td>0.572</td>
<td>0.908</td>
<td>0.572</td>
<td>0.908</td>
<td>0.572</td>
<td>0.908</td>
</tr>
<tr>
<td>Grain Diameter (d_{50}) mm</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
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<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Mean Grain Diameter (D_{x}) mm</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
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<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Median Grain Diameter (D_{y}) mm</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
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<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Median Grain Diameter (D_{z}) mm</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
</tbody>
</table>

* \(a_2\) values for the air-dry moisture contents of column 2 were obtained in tests at the particular moisture content values indicated. \(a_2\) values for moisture contents of 0.5, 1, 2, 3, and 7 percent were obtained from curves similar to those in Figure 12.
### Table 3
Comparison of $G_{et}$ Values Obtained for Air-Dry Creswell Sand from Two Sets of Input Values of $a_1$, $a_2$, $D'$, and $d_{50}$

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\nu_{20}$</th>
<th>$n_{20}$</th>
<th>$N_2$</th>
<th>$G_r$ [MPa/m]</th>
<th>$D_{rb}$ [m]</th>
<th>$D_{re}$ [m]</th>
<th>$G_{ex}'$ [MPa]</th>
<th>$G_{ey}'$ [MPa]</th>
<th>$D_{rb}''$ [m]</th>
<th>$N_{ey}$</th>
<th>$D_{re}$ [m]</th>
<th>$G_{ex}'$ [MPa]</th>
<th>$G_{ey}'$ [MPa]</th>
<th>$N_{ey}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.035</td>
<td>0.074</td>
<td>4.7</td>
<td>0.78</td>
<td>39.4</td>
<td>61</td>
<td>2.06</td>
<td>0.60</td>
<td>3.6</td>
<td>17.4</td>
<td>53</td>
<td>2.18</td>
<td>0.58</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>0.102</td>
<td>0.250</td>
<td>6.4</td>
<td>0.65</td>
<td>35.3</td>
<td>60</td>
<td>1.97</td>
<td>0.57</td>
<td>5.6</td>
<td>11.0</td>
<td>52</td>
<td>2.11</td>
<td>0.56</td>
<td>5.5</td>
</tr>
<tr>
<td>3</td>
<td>0.102</td>
<td>0.225</td>
<td>6.7</td>
<td>0.78</td>
<td>39.4</td>
<td>61</td>
<td>2.06</td>
<td>0.60</td>
<td>5.2</td>
<td>17.4</td>
<td>53</td>
<td>2.18</td>
<td>0.58</td>
<td>5.0</td>
</tr>
<tr>
<td>4</td>
<td>0.102</td>
<td>0.220</td>
<td>8.1</td>
<td>0.82</td>
<td>40.5</td>
<td>62</td>
<td>2.15</td>
<td>0.62</td>
<td>6.1</td>
<td>19.1</td>
<td>54</td>
<td>2.24</td>
<td>0.59</td>
<td>5.8</td>
</tr>
<tr>
<td>5</td>
<td>0.100</td>
<td>0.282</td>
<td>15.6</td>
<td>2.58</td>
<td>66.0</td>
<td>71</td>
<td>3.23</td>
<td>0.94</td>
<td>5.7</td>
<td>48.9</td>
<td>69</td>
<td>3.45</td>
<td>0.91</td>
<td>5.5</td>
</tr>
<tr>
<td>6</td>
<td>0.150</td>
<td>0.400</td>
<td>22.7</td>
<td>3.76</td>
<td>74.4</td>
<td>74</td>
<td>3.69</td>
<td>1.07</td>
<td>6.5</td>
<td>72.0</td>
<td>74</td>
<td>3.98</td>
<td>1.05</td>
<td>6.3</td>
</tr>
<tr>
<td>7</td>
<td>0.174</td>
<td>0.413</td>
<td>23.0</td>
<td>2.59</td>
<td>66.1</td>
<td>71</td>
<td>3.23</td>
<td>0.94</td>
<td>8.3</td>
<td>59.1</td>
<td>69</td>
<td>3.45</td>
<td>0.91</td>
<td>8.1</td>
</tr>
<tr>
<td>8</td>
<td>0.185</td>
<td>0.450</td>
<td>26.5</td>
<td>2.68</td>
<td>66.9</td>
<td>71</td>
<td>3.23</td>
<td>0.94</td>
<td>9.3</td>
<td>60.3</td>
<td>69</td>
<td>3.45</td>
<td>0.91</td>
<td>9.0</td>
</tr>
<tr>
<td>9</td>
<td>0.236</td>
<td>0.520</td>
<td>32.5</td>
<td>3.29</td>
<td>71.4</td>
<td>74</td>
<td>2.53</td>
<td>1.02</td>
<td>10.1</td>
<td>67.4</td>
<td>71</td>
<td>3.65</td>
<td>0.97</td>
<td>9.6</td>
</tr>
<tr>
<td>10</td>
<td>0.268</td>
<td>0.588</td>
<td>38.4</td>
<td>3.09</td>
<td>75.2</td>
<td>75</td>
<td>3.86</td>
<td>1.12</td>
<td>11.1</td>
<td>73.2</td>
<td>74</td>
<td>3.98</td>
<td>1.05</td>
<td>10.4</td>
</tr>
<tr>
<td>11</td>
<td>0.234</td>
<td>0.350</td>
<td>57.9</td>
<td>6.53</td>
<td>86.7</td>
<td>80</td>
<td>4.83</td>
<td>1.40</td>
<td>12.4</td>
<td>91.2</td>
<td>85</td>
<td>5.46</td>
<td>1.44</td>
<td>12.8</td>
</tr>
</tbody>
</table>

* Note that $D_{rb} = a_1 \log G + a_2$; $D_{re}$ is obtained from Figure 13; $G_{ex} = \text{antilog} \frac{D_{re} - a_2}{a_1}$; and $G_{ey} = G_{ex} \cdot \frac{C_{ex}}{C_{ey}}$. For columns 6-10, the following input data values were used: $a_1 = 51.3$, $a_2 = 44.9$, $d_{50} = 0.45$ mm (from References 10 and 11), and $D' = 47.6$ percent (from Reference 14). (These $D'$ and $d_{50}$ values produced a $\frac{C_{ex}}{C_{ey}}$ value from Figure 16 of 3.45.)

** For columns 11-15, the following input data values were used from 1983 WES soils laboratory testing: $a_1 = 80.0$, $a_2 = 26.0$, $D' = 42.7$ percent, and $d_{50} = 0.34$ mm. (These $D'$ and $d_{50}$ values produced a $\frac{C_{ex}}{C_{ey}}$ value of 0.93.)