<table>
<thead>
<tr>
<th>UNCLASSIFIED</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD NUMBER</td>
</tr>
<tr>
<td>ADC062052</td>
</tr>
<tr>
<td>CLASSIFICATION CHANGES</td>
</tr>
<tr>
<td>TO:</td>
</tr>
<tr>
<td>FROM:</td>
</tr>
<tr>
<td>LIMITATION CHANGES</td>
</tr>
<tr>
<td>TO:</td>
</tr>
<tr>
<td>FROM:</td>
</tr>
<tr>
<td>AUTHORITY</td>
</tr>
<tr>
<td>Group-4 per DoDD 5200.10, 26 Jul 1962; NRL ltr, 23 Jan 2001</td>
</tr>
</tbody>
</table>

THIS PAGE IS UNCLASSIFIED
CHARACTERISTICS OF CAVITY RESONATORS
MADE OF FUSED QUARTZ
CHARACTERISTICS OF CAVITY RESONATORS
MADE OF FUSED QUARTZ ©
CHARACTERISTICS OF CAVITY RESONATORS
MADE OF FUSED QUARTZ

by

H. L. Wuerffel and L. Schlesinger

August 1947

Problem No. 39R09-14

Approved by:

L. A. Gebhard
Superintendent
Radio Division II

Commodore H. A. Schade, USN
Director
Naval Research Laboratory

DISTRIBUTION STATEMENT F: Further dissemination only as directed by
authority. JAN 99

NAVAL RESEARCH LABORATORY
Washington, D.C. 20375
### DISTRIBUTION

<table>
<thead>
<tr>
<th>Entity</th>
<th>Attn: Details</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>BuShips</td>
<td>Code 918</td>
<td>10</td>
</tr>
<tr>
<td>CNO</td>
<td>Op-413-B2</td>
<td>5</td>
</tr>
<tr>
<td>BuAer</td>
<td>Code TD-4</td>
<td>1</td>
</tr>
<tr>
<td>ONR</td>
<td>Code N482</td>
<td>2</td>
</tr>
<tr>
<td>Dir., USNEL</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>CO, ONR, Boston</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>JRDB</td>
<td>Library</td>
<td>2</td>
</tr>
<tr>
<td>JRDB</td>
<td>Navy Secretary</td>
<td>1</td>
</tr>
<tr>
<td>Chief of the Science and Technology Project</td>
<td>Mr. J. H. Heald</td>
<td>2</td>
</tr>
</tbody>
</table>
CONTENTS

Abstract iv
Problem Status iv
Authorization iv
INTRODUCTION 1

RESULTS OF THE INVESTIGATION

Physical and Electrical Characteristics of Fused Quartz 1
Design Considerations of Fused Quartz Cavity Resonators 2
The Effect of Fused Quartz on Q 3

CONCLUSIONS 6
ABSTRACT

Previous investigations have shown that the frequency dispersions of cavity resonators are caused by the expansivity anomalies common to all metals. In an investigation of non-metallic materials it was found that fused quartz also has an anomaly but that the magnitude of this anomaly is several orders smaller than the anomalies of metals. On this basis it is predicted that resonators made of fused quartz will be improved approximately 280 times over the equivalent metal cavity resonators.

It is shown that resonant cavities can be designed employing fused quartz as dielectric or as mechanical boundary. The fused quartz dielectric cavity resonators are approximately half the size of the equivalent air dielectric cavity resonators and are easier to fabricate but have a lower Q.

PROBLEM STATUS

Status of problem at present is in experimental stage.

AUTHORIZATION

The studies herein reported were requested by BuShips Ltr. (918Bb) R-918, 330000-12984, dated 3 Oct. '1946; Improvement of the Stability of Reference Cavities.
CHARACTERISTICS OF CAVITY RESONATORS MADE OF FUSED QUARTZ

INTRODUCTION

The causes for the frequency dispersions exhibited by invar cavity resonators have been investigated and summarized in three previously published NRL reports*. It has been found that the "stable invar" from which the resonators were fabricated has an expansitivity anomaly of \(8.7 \times 10^{-8}\) in/in/°C of temperature cycle. This anomaly causes a frequency dispersion of approximately 2.5 parts in \(10^5\) for a temperature range of 60 degrees Centigrade. The investigations also revealed that all metals have expansitivity anomalies which make them unsatisfactory for the construction of highly accurate cavity resonators.

In the investigation of other materials, suitable for the construction of cavity resonators of high accuracy, fused quartz was found to have some very desirable characteristics. The advantages of fused quartz for cavity resonators are discussed in this report.

RESULTS OF THE INVESTIGATION

Physical and Electrical Characteristics of Fused Quartz. - Fused quartz has a thermal expansion coefficient of \(0.54 \times 10^{-6}\) in/in/°C over the temperature range from -50°C to 200°C and increases only slightly outside this range. However, its primary advantage as applied to cavity resonators is its apparently low expansitivity anomaly. Although some experimenters have obtained values as great as \(15 \times 10^{-9} t\) (\(t = \) range of the temperature cycle in degrees centigrade) for the anomaly, most experimenters agree that a value of \(1.6 \times 10^{-9} t\) in the range from -50°C to 450°C is probably the true value.†

The frequency deviation for an uncompensated resonator constructed from fused quartz having a coefficient of thermal expansion of \(0.54 \times 10^{-6}\) in/in/°C and op-


† Sosman, Robert B. - The Properties of Silica, New York Chemical Catalog Company, 1927.
erating at a frequency of 9310 Mc is 0.30 Mc for the range from 0°C to 60°C, or 0.005 Mc/°C. This resonator would probably have a frequency dispersion of 14.9 cycles/°C of temperature cycle (1.6 parts in 10⁹), or 0.89 kc for a temperature cycle of 60°C (1 part in 10⁹). This is based upon the value of 1.6 x 10⁻⁹ t in/in anomaly agreed upon by most experimenters. Since the average frequency dispersion of the invar resonators for a 60°C temperature cycle was 0.25 Mc, the fused quartz resonator should exhibit an improvement by a factor of 280. A frequency dispersion as great as 8.38 kc for a temperature cycle of 60°C is highly improbable but cannot be entirely ignored since a few experimenters reported anomalies of 15 x 10⁻⁹ t in/in.

Fused quartz is more elastic and has a higher tensile and compressive strength than glass. It has a melting point of approximately 1700°C and can be drawn, etched, and plated by the same methods that are used for glass. Fused quartz at temperatures slightly higher than the melting temperature resembles a mass of wriggling threads arranged in a random manner. Slow cooling causes these threads to become aligned and transforms them into crystals. Conversely, rapid cooling fixes them in a random arrangement. If the fused quartz is maintained at a temperature between 1000°C and the melting temperature, crystalization will gradually take place.

The elasticity and transparency of fused quartz is determined by the elasticity and the degree of interlacing of the threads. In clear fused quartz the threads are of a finer texture and are more closely interwoven than they are in translucent fused quartz, thus making the clear material more elastic. When the temperature of the material is increased to a temperature approaching the melting point, and a rod or tube is then formed by the extrusion method, the threads tend to become oriented axially. This causes the material to have a transverse coefficient of expansion slightly different from the longitudinal coefficient. This condition is undesirable. Fused quartz which has been rapidly cooled to a temperature lower than the melting point and then extruded should be selected.

Unlike invar, the electrical and physical properties of fused quartz from different "melts" seem to be identical. Clear fused quartz and translucent fused quartz apparently have the same electrical and thermal properties.

Design Considerations for Fused Quartz Resonators. - Cavity resonators made of fused quartz can be of two types, one type using the material as the shell and the other type using the material as dielectric. One of each type is now being constructed for experimental purposes.

The cavity resonator illustrated by Figure 1 consists of a fused quartz cylinder ground precisely to a precalculated diameter and length and then silver-plated on all surfaces except two coupling ports of precalculated diameter placed diametrically opposite each other a precalculated distance from the ends of the cylinder. This resonator is designed to operate on the TE₀₁₁ mode of oscillation.

The dielectric constant of fused quartz has been measured to be 3.75 with a precision of 0.3 percent in the temperature range from 25°C to 85°C.† Since, for a

† Ibid.
given frequency, the radius and length of a fused quartz cavity resonator are proportional to the inverse square root of the dielectric constant, the resonator will be 0.515 times the size of an equivalent air dielectric resonator. The frequency variations in the resonator caused by variations in dielectric constant with changes in temperature cannot be predicted with an acceptable degree of accuracy, since the present day methods of measuring the dielectric constant are accurate to only 3 parts in $10^3$. For an acceptable degree of accuracy the dielectric constant must not vary more than 3 parts in $10^5$ throughout the temperature range from $0^\circ C$ to $60^\circ C$. Conversely, by comparing an air dielectric fused quartz resonator with a solid dielectric resonator the dielectric constant of the material may be determined with a precision of 3 parts in $10^6$. This appears to offer a new and improved method of measuring dielectric constant.

The construction of an air-dielectric fused quartz cavity resonator is considerably more complex and expensive than the construction of a fused-quartz-dielectric resonator. However, the air-dielectric cavity has the advantage of being tuneable. The inner surfaces of the resonator as well as the outer surfaces must be plated in order to avoid auxiliary modes of oscillation in the walls of the resonator. The inner and outer silvered surfaces must be connected together electrically. The silver plating must be sufficiently thick to permit the end pieces to be bonded to the cylindrical tube and thin enough to prevent separation of the silver plating from the inner surface of the tube. This type of resonator is shown in Figure 2.

The Effect of Fused Quartz on Q. - The Q of a cavity resonator is equal to $2\pi$ times the ratio of the maximum energy stored within the cavity resonator to the power lost in the dielectric, in the walls, and through the coupling holes of the resonator, during one electric cycle. For a fused quartz dielectric $TE_{011}$ mode resonator, the maximum energy stored is

Figure 2. The TE$_{011}$ Mode, Air Dielectric, Cavity and Holder

\[ W_e = \frac{k_e e_0}{2} \int_{\phi = 0}^{2\pi} \int_{r = 0}^{a} \int_{x = 0}^{x_0} E' \phi r \int r^2 \phi \int \text{dr dx} \]

\[ = \left(\frac{a}{r'_{01}}\right)^2 V A^2 \mu_0 k_e \left(\frac{\omega}{c}\right)^2 J_0^2 \left(r'_{01}\right) \]

where

- $k_e$ = dielectric constant of medium within the resonator
- $e_0$ = electric inductive capacity of free space
- $\mu_0$ = magnetic inductive capacity of free space
- $\omega$ = $2\pi f$
- $c$ = velocity of light
- $J_0$ = Bessel function of zero order of first kind
- $r'_{01}$ = root of the first rank of the derivative of $J_0$
- $V$ = volume of the resonator
\( A' \) = constant resulting from the general solution of the field equations for the propagation of waves in a cylindrically bounded medium

\( r \) = variable distance from the origin to the point of reference in the radial direction

\( a \) = radius of the resonator

\( x \) = variable distance from the origin to the point of reference in the longitudinal direction

\( x_0 \) = length of the resonator

\( \phi \) = variable angular distance from a ray chosen arbitrarily for the development to the point of reference

\( E' \) = amplitude of the electric intensity vector

Since for small power factors

\[
\sigma_1 = \rho e_0 \mu_0 k_e
\]

The power loss in the dielectric will be\(^\S\)

\[
P_d = \frac{\sigma_1}{2} \int_0^{2\pi} \int_0^a \int_0^{x_0} |E| r \, d\phi \, dr \, dx
\]

\[
= \mu_0 \epsilon_0 \frac{2}{a} \left( \frac{a}{r_{01}} \right)^2 V A' \left( \frac{\omega}{c} \right)^2 J_0^2 \left( r_{01} \right)
\]

and the power loss in the resonator walls will be\(^\dagger\)

\[
P_w = \eta \left[ \frac{1}{2} \int_{x=0}^{x_0} \int_{\phi=0}^{2\pi} H'_r \tan \phi \, dx + \int_{\phi=0}^{2\pi} \int_{r=0}^a H'_r \, d\phi \, dr \right]
\]

\[
= \eta \left\{ A'^2 A_r J_0 \left( r_{01} \right) + 4A'^2 \beta_{r01}^2 \left( \frac{a}{r_{01}} \right)^2 A_x J_0^2 \left( r_{01} \right) \right\}
\]

where

\( H' \) = magnetic intensity amplitude

\( \eta = (\pi f \mu_2 / \sigma_2)^{1/2} \)

\( \sigma_1 \) = conductivity of the dielectric material


\(^\dagger\) Ibid.
\[ \sigma_2 = \text{conductivity of the metal walls} = 6.1 \times 10^7 \text{ mhos per meter for silver} \]

\( p \) = power factor of the dielectric material = \( 8.93 \times 10^{-5} \) for fused quartz at a frequency of \( 10^4 \text{ Mc} \).

\( \mu_2 \) = magnetic inductive capacity of the metal walls

\[ A_r = \text{surface area of the metal cylinder} \]

\[ A_x = \text{surface area of the circular end disks} \]

\[ \beta_{01} = \left( \frac{\omega}{c} \right)^2 - \left( \frac{r_{01}}{a} \right)^2 = \left( \frac{\pi}{x_0} \right)^2 \]

If it is assumed that \( \mu_2 = \mu_0 \) and that \( x_0 = 2a \), the Q of the cavity will be

\[ Q = \frac{\omega W_E}{P_w + P_d} = \left( \frac{\mu_0}{\varepsilon_0} \right)^{3/4} \left[ \frac{\varepsilon_0}{2} \right]^{2} + \left( \frac{r_{01}}{a} \right)^{2} \right]^{5/4} \frac{1}{a^{3/4}} \]

where the power loss through the coupling parts has been neglected.

This yields an unloaded Q of 21,800 for the TE_{011} mode fused-quartz-dielectric resonator as compared with a Q of 30,600 for the equivalent air-dielectric resonator. When the cavity is coupled to an external circuit the Q will be somewhat less than the value given above, depending upon the degree of coupling.

**CONCLUSIONS**

Because of the low expansivity anomaly of fused quartz, it is predicted that the use of this material for the construction of cavity resonators will reduce the frequency dispersion to 1.6 parts in \( 10^9 \) per degree centigrade of temperature cycle or to 0.9 part in \( 10^7 \) for a temperature cycle of 60°C. It has been shown in previous reports 2, 3, and 4 that a frequency dispersion of 2.5 parts in \( 10^5 \) can be expected from invar cavity resonators for a temperature cycle of 60°C.

Cavity resonators constructed of fused quartz and not temperature compensated will have a frequency deviation of approximately 5 parts in \( 10^7 \) per °C.

Two types of cavity resonators are suggested, one type using fused quartz as the dielectric and the other using the same material for the shell. The solid dielectric resonators would be approximately half the size of the equivalent air dielectric resonators and should be much easier to fabricate but would have approximately 0.71 times the Q of the air dielectric resonators.
Naval Research Laboratory
Technical Library
Research Reports Section

DATE: January 23, 2001
FROM: Mary Templeman, Code 5227
TO: Code 5300 Paul Hughes
CC: Tina Smallwood, Code 1221.1
SUBJ: Review of NRL Reports

Dear Sir/Madam:

1. Please review NRL Report FR-3155 for:

- [ ] Possible Distribution Statement
- [ ] Possible Change in Classification

Thank you,

Mary Templeman
(202)767-3425
maryt@library.nrl.navy.mil

The subject report can be:

- [ ] Changed to Distribution A (Unlimited)
- [ ] Changed to Classification
- [ ] Other:

[Signature] [Date] 1/23/2001
CHARACTERISTICS OF CAVITY RESONATORS MADE OF FUZED QUARTZ, by H. L. Wuerffel and L. Schlesinger.
Aug 47, 8p, incl. illus. (Rept. no. R-3155)

NRL SUBJECT HEADINGS
DIV: Electronics & Electronic Cavities, Resonant Equipment (8)
Quartz, Fused
SECT: Components & Accessories (2)

DST: Copies obtainable from ASTIA-DSC
Tip no. C-326