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AIRCRAFT ARMAMENT FOR
AIR-TO-GROUND OPERATIONS (S)

PROJECT VISTA
CALIFORNIA INSTITUTE OF TECHNOLOGY

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FOREWORD

This report was prepared by R. M. Stevens of the Cornell Aeronautical Laboratories during his association with Project Vista during the summer and fall of 1951. It represents the opinions of the author and may not in detail reflect the viewpoints of Project Vista.

B. H. Sage
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AIRCRAFT ARMAMENT FOR AIR-TO-GROUND OPERATIONS

INTRODUCTION

Of late, extensive and valuable studies have been made of the aircraft weapons systems in their undisputed, but not necessarily most productive role, air-to-air combat. Comparatively little attention has been given to their other domains of usefulness, in particular, that of tactical air-to-ground operations.

A comprehensive and detailed study of this operation is beyond the scope of this project. However, it is believed necessary to identify sufficiently well the governing physical phenomena that substantial bases are provided for recommendations either for further studies or specific actions.

In particular, this report will be concerned with aircraft armament, the ammunition and propellant system carried aboard the airplane. However, the other parts of the system, the target, the tactical situation, the airplane, the fire control system, the navigation system, and the general logistics of the operation will be considered wherever the characteristics of the armament cannot be considered independently. Specifically, this report will

1. Submit a measure of aircraft armament effectiveness, $I_k$
2. Examine systems and tactical parameters of which $I_k$ is a function, and indicate methods of maximizing $I_k$
3. Compare $I_k$'s of various aircraft armament systems
4. Recommend specific actions toward obtaining the maximum aircraft armament effectiveness
A Measure of Aircraft Armament Effectiveness

It is submitted without argument that an adequate measure of aircraft armament effectiveness will be the ratio of the number of kills obtained against specified targets to the weight of the armament installation which must be carried by the aircraft to obtain those kills. Symbolically

\[ I_k = \frac{N_k T}{W_0} = \frac{N_k P_{(h/l)T}}{w_h} \]

where:

- \( w_h \) = Weight of armament system installation
- \( w_h \) = Weight of warhead per round
- \( W_0 \) = Total weight of armament in \( w_0 \)
- \( N_R \) = Number of rounds of ammunition in \( W_0 \)
- \( N_k \) = Number of kills obtained against target, \( T \), in \( W_0 \)
- \( P_{(h/l)T} \) = Probability of kill per round against target, \( T \)
- \( P_{(h/l)T} \) = Probability of kill per hit against target, \( T \)
- \( \Pr \) = Single-shot probability of hit per round against target, \( T \)

The effectiveness index equals the product of the probability of hit per round, the probability of kill per hit per pound of warhead, and the ratio of the total weight of warhead to the total weight of armament system installation.

In other reports, the inverse of the product of hit probability and the ratio of total warhead weight to ordnance system weight has been called the ordnance logistic factor, \( L \). We shall adopt the same terminology, but since in this case we are
considering in $W_o$ only part of the ordnance system weight (ignoring the airplane weight) we shall say that:

$$I_k = \frac{i}{L_w} \frac{\left| P_{(k/h)} \right|}{W_h}$$

where:

$L_w$ = The aircraft armament logistic factor

For the relationship between $L$ and $L_w$ see (9).

**Examination of Parameters**

$$\frac{P_{(k/h)T}}{W_h}$$

- Probability of kill per hit per unit warhead weight.

The parameter $P_{(k/h)T}$ is principally a function of the type of target, the type of warhead, the striking velocity, the striking attitude, and the type of fuzing. For simplicity, it will be assumed that comparatively this parameter is invariant among all aircraft armament propellant systems capable of delivering equal warhead weight, $W_h$.

$L_w$ = Aircraft armament logistic factor.

$L_w$ is a function of two parameters, which in turn are functions of many variables peculiar to the operation, as follows:

$$\left| P_{h(T)} \right| = \text{Probability of hit per round against target, } T,$$ is a function of:

$R = \text{Slant range from point of release to target}$

$T = T (A_T, x_T, y_T) = \text{Target (area, shape, normal to trajectory)}$
\( \mathcal{U}_1 \) = Dispersions of round due to all causes other than release error along the sight line

\( \mathcal{U}_g \) = Dispersion of round due to release errors along the sight line, in turn a function of:

- \( R \) = Slant range from point of release to target
- \( V_a \) = Airplane velocity at release
- \( V_r \) = Velocity of round
- \( \Theta \) = Airplane dive angle at release

\( \Delta \mathcal{E}_{si} \) = Release errors along sight line, in range, dive angle, and airplane speed

\( \frac{W_b}{W_o} \)

The ratio of total warhead weight to the ordnance system weight is a function of the ratios of round, gun, installation and control weights to the weight of the warhead per round \( (W_R, W_G, W_I, W_a) \)

\[ \frac{W_R}{W_h}, \frac{W_G}{W_h}, \frac{W_I}{W_h}, \frac{W_a}{W_h} \]

which in turn are functions of variables peculiar to the operation as follows:

\( \frac{W_R}{W_h} \) = Ratio of round weight to warhead weight, a function of:

- \( V_r \) = Velocity of round
- \( D_h \) = Caliber of round
- \( R_t \) = Type of round

\( \frac{W_G}{W_h} \) = Ratio of gun (or propelling system) weight to warhead weight, a function of:

-
\[ V_r = \text{Velocity of round} \]
\[ D_h = \text{Caliber of round} \]
\[ N_g = \text{Number of guns} \]
\[ N_R = \text{Number of rounds} \]
\[ r_g = \text{Rate of fire} \]
\[ G_i = \text{Type of guns} \]

\[ \frac{W_1}{W_h} = \text{Ratio of installation weight to warhead weight a function of:} \]
\[ \begin{align*} 
W_g &= \text{Weight of gun} \\
D_h &= \text{Caliber of round} \\
N_g &= \text{Number of guns} \\
N_R &= \text{Number of rounds} \\
r_g &= \text{Rate of fire} \\
G_i &= \text{Type of guns} \\
I_i &= \text{Type of Installation} 
\end{align*} \]

\[ \frac{W_c}{W_h} = \text{Ratio of control weight to warhead weight, a function of rates} \]
\[ \text{of change of the fundamental variables as well as of the} \]
\[ \text{variables.} \]

There are other fundamental variables and relationships than those
listed above. However, it is believed that enough have been recognized
for fairly accurate general comparison of aircraft armament systems,
yet the number has been kept sufficiently low that fairly simple analytic
expressions may be derived.

Thus we say:
\[ P_h = f (R, T, V_1, V_g (R, V_R, V_r, \Theta, \alpha E \beta I)) \]
It will be observed that \( f \) and \( g \) are functions of common variables. Therefore, \( P_h \) and \( \frac{W_h}{W_o} \) cannot be treated independently in maximizing \( L_w \).

An Expression for \( P_h \)

Assume a rectangular target of width, \( w \), and length, \( l \), (normal to the target). Assume that the dispersion of the weapons system may be represented by a linear standard deviation of \( \sigma^{-y} \) in \( y \) (width) and \( \sigma^{-x} \) in \( x \) (length), where \( \sigma^{-x} \) and \( \sigma^{-y} \) are measured in mils.

Then:

\[
P_h = P_{hx} P_{hy} = \frac{1}{\sqrt{2\pi} \sigma_x \sigma_y R} \int_{-l/2}^{l/2} e^{-\frac{1}{2} \left( \frac{x}{\sigma_x R} \right)^2} \frac{1}{\sqrt{2\pi} \sigma_x \sigma_y R} \int_{-w/2}^{w/2} e^{-\frac{1}{2} \left( \frac{y}{\sigma_y R} \right)^2} dy dx
\]

For values of \( \frac{h}{\sigma^{-y} R} \) up to 1.0, the following relation

\[
\int_{-h/2}^{h/2} e^{-\frac{1}{2} \left( \frac{z}{\sigma^{-y} R} \right)^2} dz = \sigma^{-y} R \int_{-h/2}^{h/2} e^{-\frac{1}{2} \left( \frac{y}{\sigma^{-y} R} \right)^2} dy \approx h
\]

does not introduce errors greater than 15%, the error decreasing with decreasing values of \( \frac{h}{\sigma^{-y} R} \).

Thus for \( \frac{h}{\sigma^{-y} R} \ll 1.0 \):

\[
P_h \approx \frac{2}{\pi} \frac{\sigma^{-x} R}{\sigma^{-y} R} \frac{w}{2 \sigma^{-x} R}
\]

with errors not greater than 35%.

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If it is assumed that the number of rounds required per hit is equal to the reciprocal of the single-shot hit probability, to the nearest higher integer,

\[ N_R = \frac{1}{P_h} \]

for values of \( \frac{h}{2 \sigma_x R} < 1.0 \), the maximum error will not be greater than 50%, and this error will be greatest for the least number of rounds required (1.5 required by approximation as against 3 required by exact expression for \( \frac{1}{2 \sigma_x R} = \frac{w}{2 \sigma_y R} = (0.99) \).

Inasmuch as for the targets and ranges with which we are most concerned, \( \frac{h}{2 \sigma_x R} \ll 1.0 \), it is considered that the following expression for hit probability is a satisfactory approximation.

\[ P_h = \frac{2}{\pi} \frac{A_t}{2 \sigma_x R^2 \sigma_y R} = \frac{A_t}{2 \pi R^2} \cdot \frac{1}{\sigma_x \sigma_y} \begin{cases} \frac{h}{2 \sigma_x R} < 1 \\ \frac{w}{2 \sigma_y R} < 1 \end{cases} \]

In the above expression, the variables which are not fundamental are \( \sigma_y \) and \( \sigma_x \). \( \sigma_y \) is a function of both \( \sigma_1 \) and \( \sigma_x \), while \( \sigma_y \) is primarily a function only of \( \sigma_1 \) as defined above. It will be assumed that for each type of aircraft armament \( \sigma_1 \) is a constant, the value averaged from firing tests. However, as defined above,

\[ \sigma_y = \sigma_y (R, V_a, V_R, \Theta, \Delta \varepsilon) \]

so that \( \sigma_y = \sigma_y (\sigma_x) = \text{const} = \sigma_x \)

\[ \sigma_x = \sigma_x (\sigma_x, \sigma_y) = \sigma_x (\sigma_x, \sigma_y (R, V_a, \Theta, V_R, \Delta \varepsilon)) = \sqrt{\sigma_x^2 + \sigma_y^2} \]

With the above assumption:

\[ P_h = \frac{A_t}{2 \pi R^2} \cdot \frac{1}{\sigma_x \sigma_y (\sigma_x + \sigma_y)^2} \]

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It may be shown that for a vacuum trajectory (see Figure 1) the trajectory drop may be expressed as:

$$E = \frac{\eta}{R} = \frac{g}{2} \frac{R}{(V_{av})^2} \cos \Theta$$

where:

- $E$ = Trajectory drop (ft./ft.)
- $\eta$ = Trajectory drop (ft.)
- $R$ = Slant range (ft.)
- $V_{av}$ = Average velocity of projectile (ft./sec.) over slant range, R.
- $\Theta$ = Angle of airplane flight path to horizontal at time of firing.
- $V_a$ = Airplane velocity at time of firing.

And

$$\frac{\partial E}{\partial R} = \frac{g}{2} \frac{R^2}{V_{av}^2} \cos \Theta \left[ \frac{1}{R} - \frac{2}{V_{av}} \frac{\partial V_{av}}{\partial R} \right]$$

$$\frac{\partial E}{\partial V_{av}} = -g \frac{R}{V_{av}^3} \cos \Theta \frac{\partial V_{av}}{\partial V_{av}}$$

$$\frac{\partial E}{\partial \Theta} = -\frac{g}{2} \frac{R^2}{V_{av}^2} \cos \Theta \left[ \tan \Theta + \frac{2}{V_{av}} \frac{\partial V_{av}}{\partial \Theta} \right]$$

If it is assumed that $\frac{\partial E_i}{\partial \rho} = \frac{\Delta E_i}{\Delta \rho} = \frac{\partial E}{\partial \rho}$

$$\sigma_R = \Delta E_R = \frac{\Delta R}{R} \frac{g}{2} \frac{R}{V_{av}^2} \left[ 1 - \frac{2}{V_{av}} \frac{\partial V_{av}}{\partial R} \right]$$

$$\sigma_{\rho} = \Delta E_{\rho} = \frac{\Delta \rho}{\rho} \frac{g}{2} \frac{R}{V_{av}^2} \left[ \frac{\partial V_{av}}{\partial \rho} \right]$$

$$\sigma_{\Theta} = \Delta E_{\Theta} = -\frac{\Delta \Theta}{\Theta} \frac{g}{2} \frac{R}{V_{av}^2} \left[ \Theta \tan \Theta + \frac{2}{V_{av}} \frac{\partial V_{av}}{\partial \Theta} \right]$$

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and that these release errors are the principal ones not included in \( \sigma \),

then:

\[
\sigma_y = \sqrt{\sigma^2 R^2 + \sigma_y^2 + \sigma_0^2} + \frac{g}{2} \frac{R \cos \theta}{V_{aw}} \sqrt{\left( \frac{\Delta R}{R} \right)^2 + \left( \frac{\Delta \theta \tan \theta}{\theta} \right)}^2 \left( \frac{2 \Delta V_a}{V_{aw}} \right) + \frac{2 \Delta V_a}{V_{aw}} \frac{V_a \delta V_{aw}}{\theta} \right] \]

Assume for a moment that \( V_{aw} \) may be chosen arbitrarily, independent of

\( R, V_a, \) and \( \theta, \) then:

\[
\frac{\sigma_y}{g} = \frac{g}{2} \frac{R \cos \theta}{V_{aw}} \sqrt{\left( \frac{\Delta R}{R} \right)^2 + \left( \frac{\Delta \theta \tan \theta}{\theta} \right)}^2 \]

and

\[
P_h = \frac{A_t}{\pi g R^3 \cos \theta} \left( \frac{1}{\sqrt{2} V_{aw}^2 \sigma_y} \right)^2 + \frac{1}{g R \cos \theta} \left( \frac{2 V_{aw}^2 \sigma_y}{g R \cos \theta} \right)^2 + \left( \frac{\Delta \theta \tan \theta}{\theta} \right)^2
\]

Here we observe that if \( \sigma_1 \) is small relative to \( \sigma_g \) (say \( \sigma_1 \gg \sigma_g \)),

and \( \frac{h}{2 \sigma_2 \sqrt{h}} < 1 \), \( P_h \) increases linearly with \( A_t \)

increases as the square of \( V_{aw} \)

increases as the reciprocal of the cube of the range

increases as the reciprocals of \( \sigma_1 \) and \( \sqrt{\Delta E} \)

increases as the reciprocal of \( \cos \theta \)

If, on the other hand \( \sigma_1 \) is large relative to \( \sigma_g \) (say \( \sigma_1 \gg \sigma_g \)),

and \( \frac{y}{2 \sigma_2 \sqrt{R}} \) \( < 1 \), \( P_h \) increases linearly with \( A_t \)

increases as the reciprocal of the square of the range

increases as the reciprocals of \( \sigma_1 \) and \( \sqrt{\Delta E} \)

Now, \( \frac{W_h}{W_o} \) is also a function of \( V_{aw} \) so without further exploration of their relationship, the variation of \( I_k \) with \( V_a \) cannot be stated. However, \( \frac{W_h}{W_o} \) is not a function of the other variables except as \( V_{aw} \) is a function of them, so that for our assumption of an arbitrary \( V_{aw} \), the
variations of $I_k$ with $A_T$, $R$, $\Theta$, $\sigma_1$ and $\sqrt{\Delta E_{\Delta t}}$ are generally the same as those of $P_h$.

It should be observed that range has a very significant effect on $P_h$ and $I_k$.

$$\frac{I_{k_1}}{I_{k_2}} = \left(\frac{R_2}{R_1}\right)^{2-3}$$

Therefore, training and equipment plans should consider the very considerable gains to be obtained by firing at short range. (For example, although aircraft armor to protect against small-arms antiaircraft fire and fragment damage would increase $W_o$, the reduction in $P_h$ sufficiently to result in higher $I_k$'s for the armored airplanes.)

For a particular weapon, where $V_{av}$ may not be independent of $R$, $V_a$, and $\Theta$, the same general trends follow, modified as indicated in the above expression for $\sigma_e$.

The above analysis has been based upon an approximation for the projectile trajectory. The errors introduced by this approximation should be explored. First, it is necessary to obtain expressions for the average velocities and their derivatives for the various weapons. These follow:

For bombs:

$$V_{av} = V_a (\text{within } 1.10 V_a > V_{av} > V_a \left\{ \begin{array}{l} R \leq 6000 \text{ ft.} \\
V_a \geq 500 \text{ f.p.s.} \end{array} \right.$$  

For rockets:

$$V_{av} = \frac{R \left( V_a + V_b \right)}{R + V_b t_b / 2}; \left( V_a + V_b / 2 \right) t_b < R \left( \text{within } R \leq 6000 \text{ ft.} \right)$$

For guns:

$$V_{av} = V_a + V_n (\text{within } \frac{R}{T_f} < V_{av} < 1.10 \frac{R}{T_f}; R \leq 6000 \text{ ft.})$$
Then:

For bombs:
\[
\sigma_R = \frac{\Delta R}{R} \frac{g}{2} \frac{R \cos \theta}{V_a^2},
\]
\[
\sigma_V = -2\Delta V_a \frac{1}{V_a} \frac{R \cos \theta}{V_a^2},
\]
\[
\sigma_\theta = -\frac{\Delta \theta}{\theta} \left( \theta \tan \theta \right) \frac{g}{2} \frac{R \cos \theta}{V_a^2}.
\]

For rockets:
\[
\sigma_R = \frac{\Delta R}{R} \left[ 1 - \left( \frac{V_{b+t}_b}{2R} \right)^2 \right] \frac{g}{2} \frac{R \cos \theta}{(V_a + V_b)^2} \left( \frac{V_{b+t}_b}{2R} < 1 - \frac{V_a}{R} \right),
\]
\[
\sigma_V = -2\frac{\Delta V_a}{V_a} \frac{V_a}{(V_a + V_b)^2} \frac{3}{2} \frac{g}{2} \frac{R \cos \theta}{(V_a + V_b)^2} \left( \frac{V_{b+t}_b}{2R} < 1 - \frac{V_a}{R} \right),
\]
\[
\sigma_\theta = -\frac{\Delta \theta}{\theta} \left( \theta \tan \theta \right) \frac{g}{2} \frac{R \cos \theta}{(V_a + V_b)^2}.
\]

For guns:
\[
\sigma_R = \frac{\Delta R}{R} \frac{g}{2} \frac{R \cos \theta}{(V_a + V_m)^2},
\]
\[
\sigma_V = 2\frac{\Delta V_a}{V_a} \frac{V_a}{(V_a + V_m)^2} \frac{3}{2} \frac{R \cos \theta}{(V_a + V_m)^2},
\]
\[
\sigma_\theta = -\frac{\Delta \theta}{\theta} \left( \theta \tan \theta \right) \frac{g}{2} \frac{R \cos \theta}{(V_a + V_m)^2}.
\]

Then, assuming that the errors are independent:

For bombs:
\[
\sigma_j = \frac{g}{2} \frac{R \cos \theta}{V_a^2} \frac{\Delta R}{R} \left[ 1 + 4 \left( \frac{\Delta V_a}{V_a} \frac{R}{\Delta R} \right)^2 + \theta^2 \tan^2 \theta \left( \frac{\Delta \theta}{\theta} \frac{R}{\Delta R} \right)^2 \right].
\]

For rockets:
\[
\sigma_j = \frac{g}{2} \frac{R \cos \theta}{(V_a + V_b)^2} \frac{\Delta R}{R} \left[ 1 - \left( \frac{V_{b+t}_b}{2R} \right)^2 \right] + \left( 1 + \frac{V_{b+t}_b}{2R} \right)^4 \left[ 4 \left( \frac{V_a}{V_a + V_b} \frac{\Delta V_a}{V_a} \frac{R}{\Delta R} \right)^2 + \theta^2 \tan^2 \theta \left( \frac{\Delta \theta}{\theta} \frac{R}{\Delta R} \right)^2 \right].
\]

For guns:
\[
\sigma_j = \frac{g}{2} \frac{R \cos \theta}{(V_a + V_m)^2} \frac{\Delta R}{R} \left[ 1 + 4 \left( \frac{\Delta V_a}{V_a} \frac{R}{\Delta R} \right)^2 + \theta^2 \tan^2 \theta \left( \frac{\Delta \theta}{\theta} \frac{R}{\Delta R} \right)^2 \right].
\]

If it is assumed that \( \frac{\Delta R}{R} = \frac{\Delta V}{V} = \frac{\Delta \theta}{\theta} \).
For bombs:
\[
\sigma_g = \frac{g}{2} \frac{R \cos \theta}{V_a^2} \sqrt{\frac{\Delta R}{R} \sqrt{5 + \theta^2 \tan^2 \theta}}
\]

For rockets:
\[
\sigma_g = \frac{g}{2} \frac{R \cos \theta}{(V_a + V_b)^2} \sqrt{\frac{\Delta R}{R} \left[ 1 - \left( \frac{V_b t_b}{2R} \right)^2 + \left( \frac{1 - V_b t_b}{2R} \right)^2 \right]} + \theta^2 \tan^2 \theta
\]

For guns:
\[
\sigma_g = \frac{g}{2} \frac{R \cos \theta}{(V_a + V_m)^2} \sqrt{\frac{\Delta R}{R} \left[ 1 + 4 \left( \frac{V_a}{V_a + V_m} \right)^2 + \theta^2 \tan^2 \theta \right]}
\]

Figure 4 plots \( \sigma_g \) versus \( R \) (\( \frac{\Delta R}{R} = \frac{\Delta V_a}{V_a} = \frac{\Delta \theta}{\theta} = 0.005 \)) for various values of \( \theta, V_b, \) and \( V_m \) corresponding to existing bombs, rockets, and guns.

For rockets and bombs, \( V_a \) was set equal 500 ft./sec.; for guns, \( V_a \) was set equal to zero. Actual values of \( \sigma_g \) as taken from trajectory Table I are shown for comparison. It will be observed that the calculated rocket deviations correspond quite well to the actual, the errors ranging from 2 to 7 percent high for the bomb, 2 to 12 percent low for the 5\(^{\text{th}}\) AR, 11 to 26 percent low for the 5\(^{\text{th}}\) HVAR, and 8 to 13 percent low for the gun.

These errors are, in most cases, no greater than those which must result from assumptions for \( \sigma_1 \). It appears then that trends shown by the approximate analytic expressions derived above will be generally correct and that the absolute values will not be greatly in error.

\( \sigma_1 \)

\( \sigma_1 \) has been defined as dispersions due to all causes other than release error along the sight line. These would include free flight ballistic dispersions, dispersions caused by mechanical and aerodynamic disturbances, sighting errors, alignment errors and azimuth errors.
It will be assumed that these errors are circular. The values given below are for the linear components \( \sigma_y = \sigma_x = 0.707 \sigma_1(\omega) \). The free flight ballistic dispersions have been given in other EngOrd reports (2,3,4,5) and are approximately:

- 3-4 mils - Bombs (Existing bombs with modified fin assemblies and proposed new family of bombs)
- 3-7 mils - Rockets (Air-fired, fin stabilized)
- 1-3 mils - Guns (Air-fired)

There have been no satisfactory isolations of the other dispersions contained in \( \sigma_1 \). Therefore, for the remainder of this analysis two assumptions as to its value will be made. The first (lower limit) will be that \( \sigma_1 \) is equal to the ballistic dispersion alone, value to be:

- 4 mils - Bombs
- 7 mils - Rockets
- 2 mils - Guns

The second (upper limit) will be that \( \sigma_1 \) is equal to dispersions generally found in firing tests corrected for \( \sigma_y \). These values are approximately (6):

- 9 mils - Bombs
- 9 mils - Rockets
- 5 mils - Guns

**Release Error Control Requirements**

With the aid of the above equations and numbers it is possible to approximate the values within which \( \Delta R, \Delta V, \text{ and } \Delta \Theta \) must be maintained in order that dispersion along the sight line, \( \sigma_x \), will approach its minimum practical limit. Since

\[
\sigma_x = \sqrt{\sigma_z^2 + \sigma_y^2}
\]

the minimum limit will be \( \sigma_1 \), with \( \Delta R, \Delta V, \Delta \Theta \) all equal to zero.
However, it will be assumed that a minimum limit below which further expenditure of effort to reduce release errors would be impractical will be:

\[
\sqrt[2]{\sigma_{x}^{2} + \sigma_{y}^{2}} \leq 1.2 \sigma_{x}
\]

\[
\sigma_{x}^{2} + \sigma_{y}^{2} \leq 1.44 \sigma_{x}^{2}
\]

\[
\sigma_{y} \leq 0.663 \sigma_{x}
\]

Then for bombs and rockets:

\[
\sigma_{y} = 6 \text{ mils (} \sigma_{x} = 9 \text{ mils) } \sigma_{x} = 11 \text{ mils}
\]

For guns:

\[
\sigma_{y} = 3.3 \text{ mils (} \sigma_{x} = 5 \text{ mils) } \sigma_{x} = 6 \text{ mils}
\]

Then using the approximate equation for \( \sigma_{y} \):

\[
.663 \sigma_{y} = \frac{1}{2} \frac{R \cos \theta}{V_{o}^2} \sqrt{\left( \frac{\Delta R}{R} \right)^2 + \left( \frac{\Delta V_{o}}{V_{o}} \right)^2 + (\Delta \theta \tan \theta)^2}
\]

\[
1.326 \sigma_{x} \frac{V_{o}^2}{\rho R \cos \theta} \geq \sqrt{\left( \frac{\Delta R}{R} \right)^2 + \left( \frac{\Delta V_{o}}{V_{o}} \right)^2 + (\Delta \theta \tan \theta)^2} = \Delta^2
\]

Using the above equation, the maximum allowable value of \( \Delta^2 \) may be approximated. Then it is necessary to assign maximum allowable value to

\[
\frac{\Delta R}{R}, \frac{\Delta V_o}{V_o}, \text{ and } \Delta \theta
\]

First, it will be assumed that there are maximum limits within which these errors can be controlled by even a very simple fire control system (fixed sight, standard release range, trained pilot). \( \Delta R \) is probably the most difficult error to estimate or control; \( \Delta V_o \) somewhat less difficult; and \( \Delta \theta \) is the least difficult. Experiences of trained gunnery pilots, firing with fixed sights indicate that these errors can be held within the following maximum limits:
\[ \Delta R = 0.5 R \quad \Delta V_a = 50 \text{ ft./sec.} \quad \Delta \theta = 5^\circ \]

It will be assumed that no errors greater than these will be permitted. Then the necessary reductions below these limits to satisfy the above equation will be determined. The distribution of errors will be such that when all the allowable maximum errors are below the simply controllable maximum limits,

\[
\frac{\delta \sigma_L}{\delta (\Delta R/R)} = \frac{\theta \sigma_L}{\theta (2 \Delta V_a/V_a)} = \frac{\theta \sigma_L}{\theta (\Delta \theta \tan \theta)}
\]

With these assumptions, the maximum allowable values have been computed for a range of \( V_{av} \)'s from 500 to 3000 ft./sec., \( R = 3000 \) and 6000 feet, \( \theta = 20^\circ \) and \( 60^\circ \), \( \sigma_o = 2, 5, \) and 9 mils and \( V_a = 500 \) ft./sec. The results are presented in Table I. A generalized summary of the results is given below, for \( \sigma_1 = 1 \) mils.

<table>
<thead>
<tr>
<th>Weapon</th>
<th>Range (ft.)</th>
<th>Allowable Errors</th>
<th>( \Delta H/R )</th>
<th>( \Delta V_a/V_a )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guns ((V_m = 1500 \text{ ft./sec.)})</td>
<td>6000</td>
<td>0.15</td>
<td>0.1</td>
<td>5°</td>
<td></td>
</tr>
<tr>
<td>Guns ((V_m = 1500 \text{ ft./sec.)})</td>
<td>3000</td>
<td>0.3</td>
<td>0.1</td>
<td>5°</td>
<td></td>
</tr>
<tr>
<td>Rockets</td>
<td>6000</td>
<td>0.05</td>
<td>0.1</td>
<td>5°</td>
<td></td>
</tr>
<tr>
<td>Rockets ((V_m = 1000 \text{ ft./sec.)})</td>
<td>3000</td>
<td>0.15</td>
<td>0.1</td>
<td>5°</td>
<td></td>
</tr>
<tr>
<td>Rockets ((500 \text{ V_m 1000 ft./sec.)})</td>
<td>6000</td>
<td>0.02</td>
<td>0.02</td>
<td>2°</td>
<td></td>
</tr>
<tr>
<td>Rockets ((500 \text{ V_m 1000 ft./sec.)})</td>
<td>3000</td>
<td>0.05</td>
<td>0.05</td>
<td>5°</td>
<td></td>
</tr>
<tr>
<td>Bombs</td>
<td>6000</td>
<td>0.005</td>
<td>0.003</td>
<td>1°</td>
<td></td>
</tr>
<tr>
<td>Bombs</td>
<td>3000</td>
<td>0.01</td>
<td>0.006</td>
<td>1°</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, if the effect of gravity drop is to be reduced to a minimum practical limit, range error must be controlled within limits from \( \frac{\Delta R}{R} < 0.005 \) for bombs to \( \frac{\Delta R}{R} < 0.15 \) for guns. Velocity and range error effects, for errors below those controllable by fairly simple systems, are negligible, except in the case of bombing.

Figures 4a through 4e show the effects of release error on \( \sigma_1 \) for
various weapons at various ranges and release angles.

Finally then, the following expressions are derived for \( P_h \).

For bombs:

\[
P_h = \frac{A_T V_m^2}{mg R^2 \cos \theta} \left[ \frac{1}{\sigma_r^2} \right] \frac{\Delta R}{R} \sqrt{1 + \left( \frac{2 V_m \sigma_r}{g R \cos \theta} \frac{\Delta R}{R} \right)^2 + 4 \left( \frac{\Delta V_m}{V_m} \right)^2} \left( \frac{\Delta \theta}{\theta} \right)^2 \]

\[
(4 < \sigma_r < 9 \text{ miles}) \quad (V_m \geq 500 \text{ fps}) \quad (R \leq 6000 \text{ ft}) \quad (P_h \leq 0.5)
\]

For rockets:

\[
P_h = \frac{A_T (V_a + V_b)^2}{mg R^2 \cos \theta} \left[ \frac{1}{\sigma_r^2} \right] \frac{\Delta R}{R} \sqrt{1 + \left( \frac{2 (V_a + V_b) \sigma_r}{g R \cos \theta} \frac{\Delta R}{R} \right)^2 + \left[ \frac{(V_a + V_b) \Delta \theta}{\theta} \right]^2} \left( \frac{\Delta R}{\theta} \right)^2 \]

\[
(\sigma_r < 9 \text{ miles}) \quad ((V_a + \frac{V_b}{2}) t_b < R) \quad (R \leq 6000 \text{ ft})
\]

For guns: \( (P_h \leq 0.5) \)

\[
P_h = \frac{A_T (V_a + V_m)^2}{mg R^2 \cos \theta} \left[ \frac{1}{\sigma_r^2} \right] \frac{\Delta R}{R} \sqrt{1 + \left( \frac{2 (V_a + V_m) \sigma_r}{g R \cos \theta} \frac{\Delta R}{R} \right)^2 + \left( \frac{\Delta V_m}{V_m} \right)^2 \left( \frac{\Delta \theta}{\theta} \right)^2 \left( \frac{\Delta V_a}{V_a} \right)^2 \left( \frac{\Delta \theta}{\theta} \right)^2}
\]

\[
(2 < \sigma_r < 5 \text{ miles}) \quad (R \leq 6000 \text{ ft}) \quad (P_h \leq 0.5)
\]

Figures 5a through 5d show the values of \( \frac{P_h}{A_T} \) for various weapons, dive angles, and the limiting values of \( \sigma_r \), plotted as a function of range. Figures 6a and 6b show \( P_h / A_T \) plotted as a function of \( V_g \) for rockets, and \( V_m \) for guns. The marked increase of hit probability with decreasing range will be noted in Figure 5. It will be noted that no significant improvements in rocket hit probabilities will be made until reductions in inherent dispersions are accomplished. However, improvements in bombing fire control above those assumed will result in significant improvements in bombing hit probabilities. Figure 6 indicates that as
long as fire control errors remain large, burnt or muzzle velocities should be kept high to improve hit probabilities. As fire control errors are reduced, velocities may be correspondingly reduced. It is obvious, that with perfect fire control, velocity will have no affect on hit probability. Similarly as long as inherent dispersions are high, burnt or muzzle velocities may remain relatively low. As inherent dispersions are reduced, velocities should be increased to improve hit probabilities.

In Figure 5, the hit probabilities obtained in Air Proving Ground tests with 5" HVAR rockets, using the A-ICM sight with varying degrees of sensitivity has been shown (7). Ranges were uncertain, between 2500 and 3500 feet. Results were in remarkable agreement with those hypothesized with the use of the foregoing approximations.

Based on the data of Figure 5, Figure 7 shows the number of rounds required per hit (subject to the errors inherent in use of the approximate formula) as a function of range and target area for guns, rockets, and bombs and for the combinations of maximum range errors and inherent dispersion and minimum range errors and inherent dispersions. The maxima may be considered as approximating present systems, the minimums as the limit of inherent improvement. Three target areas have been chosen, 200 sq. ft. (approximately that of a tank, side on), 2000 sq. ft. (a pillbox or artillery emplacement) and 20,000 sq. ft. (troop vehicles or supply concentration).

In analyzing these data, let us consider that a maximum of 8 rounds per hit are desired. Then the following table indicates the maximum ranges in feet at which the various weapons may be used.
<table>
<thead>
<tr>
<th>Target Area</th>
<th>Guns</th>
<th>Rockets</th>
<th>Bombs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present Errors</td>
<td>Improved</td>
<td>P.E.</td>
</tr>
<tr>
<td>200 sq. ft.</td>
<td>R &lt; 3000 ft.</td>
<td>R &lt; 6000</td>
<td>R &lt; 1500</td>
</tr>
<tr>
<td>2000 sq. ft.</td>
<td>R &lt; 6000</td>
<td>R &lt; 6000</td>
<td>R &lt; 4000</td>
</tr>
<tr>
<td>20,000 sq. ft.</td>
<td>R &lt; 6000</td>
<td>R &lt; 6000</td>
<td>R &lt; 6000</td>
</tr>
</tbody>
</table>

On the basis of hit probabilities above, with specifications as established, guns should be used for the 200 sq. ft. target, bombs should not be used, and extremely close ranges are necessary with rockets.

Either guns or rockets might be used against the intermediate area target, but close ranges are necessary with bombs. Guns, rockets, or bombs might be used against the 20,000 sq. ft. target.

\[
\frac{W_h}{W_o} = \text{Ratio of Total Warhead weight to Armament System weight}
\]

The total warhead weight carried by the airplane:

\[
W_h = N_R W_h
\]

The total armament system weight:

\[
W_o = W_R + W_j + W_i + W_c
\]

\[
W_R = N_R W_R
\]

\[
W_c = N_g W_c
\]

where:

- \( W_o \) = Armament system weight
- \( W_h \) = Total warhead weight
- \( W_R \) = Total round weight
- \( W_i \) = Total installation weight
- \( W_c \) = Total fire control weight
- \( W_g \) = Total gun weight
- \( N_R \) = Number of rounds
- \( N_g \) = Number of guns
\[ w_h = \text{Warhead weight per round} \]
\[ w_R = \text{Round weight} \]
\[ w_g = \text{Gun weight} \]

So:
\[
\frac{w_h}{w_o} = \frac{1}{N_RW_R + N_gw_g + W_L + W_e - N_RW_h} \\
= \frac{1}{w_h + N_gw_g + W_L + N_RW_h - N_RW_h}
\]

In other EngOrd reports (2, 3, 4, 5, 9) expressions have been derived for \( \frac{w_R}{w_h} \), \( \frac{w_g}{w_h} \), and \( W_L \) as functions of round diameter, rate of fire, velocity of round, and subsidiary parameters characteristic of the type of weapon. However, there are no such expressions derived for \( \frac{W_e}{N_RW_h} \). Therefore, in this report, it will be considered that comparable fire control systems are provided for all weapons (weapons will be compared for the same release errors), \( W_R \) will be included in the basic airplane weight, and the term eliminated from the above expression.

**Bombs**

For bombs, gun weight is zero, and the installation weight per round varies roughly linearly with the weight of the round. Thus
\[
\frac{w_g}{w_h} = 0
\]
and from (2),
\[
\frac{W_L}{N_RW_h} = 0.04
\]
\[
\frac{W_R}{w_h} = 1.05
\]
(considering warhead weight to be the total bomb weight minus the stabilizing system weight)

So that:
\[
\frac{w_h}{w_o} \approx \frac{1}{1.05 + 0.04} = 0.92
\]
Rockets

As for bombs, gun weight is zero, and the installation weight per round varies roughly linearly with the weight of the round. Thus

\[
\frac{W_g}{W_h} = 0
\]

and from (3),

\[
\frac{W_i}{N_{Ri}W_h} = \begin{cases} 
0.10 & \text{(pylon launcher)} \\
0.50 & \text{(packaged launcher)} 
\end{cases}
\]

\[
\frac{W_R}{W_h} = \frac{e^{V_b/6800}}{1 + \frac{W_m}{W_p} (1-e^{V_b/6800})}
\]

\[
\frac{W_m}{W_p} = \begin{cases} 
0.15 + 2.7/D_m & \text{(new rocket) (5" HPAG 2" 75 AAF FR)} \\
3.0 + 2.7/D_m & \text{(old rocket) (5 1/2" 6.5 AR, 3" 5 AR)}
\end{cases}
\]

\(D_m = \) Motor diameter, inches

\(V_b = \) Burnt velocity, relative to launcher, ft./sec.

\(W_m = \) Ratio of rocket motor weight (burnt) to propellant weight.

Then:

\[
\frac{W_h}{W_D} = \frac{1}{\frac{e^{V_b/6800}}{1 + \frac{W_m}{W_p} (1-e^{V_b/6800})}} + K \quad (500 < V_b < 2,500)
\]

\[
= \frac{1}{\frac{V_b/6800}{1 - \frac{W_m}{W_p} \frac{V_b}{6800}}} + \frac{6800 - \frac{W_m}{W_p} V_b}{K(6800 - \frac{W_m}{W_p} V_b) + (6800 + V_b)}
\]

For simplicity, say

\[
\frac{W_i}{N_{Ri}W_h} = K = 0.3
\]

And that

\[
\frac{W_m}{W_p} = \begin{cases} 
1.0 & \text{(new rocket)} \\
3.0 & \text{(older rocket)}
\end{cases}
\]

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Then:
\[
\frac{W_h}{W_o} = \frac{6800 - V_b}{8840 + 0.7 V_b} \quad (\text{new rocket})
\]
\[
= \frac{6800 - 3 V_b}{8840 + 0.1 V_b} \quad (\text{older rocket})
\]

These values are plotted in Figure 8. Actual values are shown for comparison. Good agreement is shown between the estimated and actual values for the older rockets but probably will be approached as development continues. It will be noted that \(\frac{W_h}{W_o}\) decreases markedly with increasing \(V_b\).

**Standard Guns**

From (45):

\[
\frac{W_g}{w_h} = \frac{V_m^2}{D_n \cdot 41} \left[ 19.1 + 0.455 r \right] \times 10^{-6}
\]

where

- \(V_m\) = Muzzle velocity, ft./sec.
- \(D_n\) = Projectile diameter, inches
- \(r\) = Rate of fire, rounds per minute

\[
\frac{W_k}{w_h} = \frac{.136 V_m^2 + 10^{-6}}{D_n \cdot 41} + 1.2
\]

\[
\frac{W_i}{\frac{N_h}{w_h}} = \frac{.9 w_g}{N_h w_h}
\]

so that:

\[
\frac{w_h}{W_o} = \left[ \frac{.136 V_m^2 x 10^{-6}}{D_n \cdot 41} \times 1.2 \right] + \frac{N_g}{N_h} \frac{V_m^2}{D_n \cdot 41} \left[ 19.1 + 0.455 r \right] \times 10^{-6} \left[ 1.9 \right]
\]

\[
= \frac{V_m^2 \left[ .136 + 1.9 \frac{N_g}{N_h} \left[ 19.1 + 0.455 r \right] \right] + 1.2 D_n \cdot 41 x 10^{-6}}{D_n \cdot 41 x 10^{-6}}
\]

For example, let us consider two classes of guns, a small caliber, high cyclic rate gun (say 20 mm., 650 rpm) and a large caliber, low cyclic rate gun (say 75 mm., 10 rpm). Then
Values are plotted in Figure 9, and compared against actual values. Good agreement is found. Again, it is found that $\frac{W_n}{W_0}$ decreases markedly with increasing muzzle velocity. It will be noted that a number of rounds per gun has a significant effect on $\frac{W_n}{W_0}$, the larger number of rounds per gun giving higher $\frac{W_n}{W_0}$ ratios. This would be expected until the total weight of the rounds are equal to or greater than the total weight of the gun. Also, inspection of the above equations reveals that for the same muzzle velocities and rounds per gun, increases in rates of fire decreases $\frac{W_n}{W_0}$.

**Recoilless Guns**

From (5 and 8):

\[
\frac{W_n}{W_0} = \frac{9.5 \times 10^{-6} V_m^2}{4.8 \times 10^{-6} V_m^2} \quad \text{(existing designs)}
\]

\[
\frac{W_n}{W_0} = \frac{w_h + w_c + w_p}{w_h} = 1 + \frac{w_c + w_h}{w_h}
\]

\[
w_p = 1.46 \times 10^{-4} \left( \frac{w_h}{2.9} V_m \right)^{-2} \cdot 795 W_h
\]

\[
w_p = 6.78 \times 10^{-6} \frac{V_m}{D_h} \cdot 1.59
\]

\[W_c = 0.7 w_p\]

For an automatic feed mechanism, we shall assume (based on the standard gun)

\[
\frac{W_c}{W_h} = 0.455 V_m \left( \frac{V_m}{D_h} \right)^4 \times 10^{-6} + 0.455 \frac{w_h}{V_m}
\]

\[= V_m^2 \left( \frac{V_m}{D_h} \right)^4 \left( 0.95 + \frac{0.455}{D_h} \right)\]

(existing design)
\[ V_m = 10^{-6} \left[ 0.48 + \frac{0.455 r}{D_h^0.6} \right] \] (proposed new design)

Then:
\[
\frac{W_g}{W_h} = V_m^2 \times 10^{-6} \left[ 0.45 + \frac{0.455 r}{D_h^0.6} \right]
\] (existing design)
\[
= V_m^2 \times 10^{-6} \left[ 5.28 + \frac{0.455 r}{D_h^0.6} \right]
\] (proposed new design)

\[
\frac{W_R}{W_h} = 1 + 11.5 \times 10^{-6} \frac{V_m}{D_h^0.6}
\]

where:

\[ w_f = \text{Weight of automatic feed mechanism} \]
\[ w_c = \text{Weight of care of round} \]

The installation weight should be between those for rockets and standard guns, say
\[
\frac{W_i}{N_R w_h} = 0.5 \frac{W_g}{N_R w_h}
\]

Then:
\[
\frac{W_h}{W_o} = \frac{1}{1 + V_m^2 \times 10^{-6} \left[ \frac{11.5}{D_h^0.6} \frac{W_g}{N_R} \right] + 1.5 \frac{N_g}{N_R} \left( \frac{0.455 + \frac{0.455 r}{D_h^0.6}}{5.28 + \frac{0.455 r}{D_h^0.6}} \right)}
\]

Let us now consider a large caliber, low cyclic rate gun (say 75 mm., 10 rpm). Then
\[
\frac{W_h}{W_o} = \frac{1}{1 + V_m^2 \times 10^{-6} \left[ \frac{6.25}{N_{RH}} + \frac{16 N_g}{N_R} \right]}
\]
\[
= \frac{1}{1 + V_m^2 \times 10^{-6} \left[ \frac{6.25}{N_{RH}} + 8.28 \frac{N_g}{N_R} \right]}
\]

Values are plotted in Figure 10. The same trend of marked decrease of \( \frac{W_h}{W_o} \) with increase in \( V_m \) is found as with guns. The
significance of the number of rounds per gun will again be noted.

**Gun-Launched Rockets**

Inasmuch as gun-launched rockets (or closed-breech rocket launchers) are still in early development stages, there are little statistical data against which empirical relationships describing the family can be checked. In general, it would appear that the weight of the launcher must be increased over that for the pure rocket as a function of the rate of fire and velocity upon leaving the launcher, as in the case of guns. The ratio of rocket warhead weight to round weight would be expected to be less than for the pure rocket because of the necessity for strengthening the case to withstand the higher initial accelerations. Therefore, the over-all \( \frac{W_h}{W_o} \) for the gun-launched rocket would be less than for the pure rocket. Counterbalancing this, however, is the increase in accuracy, the dispersions approaching those of guns.

For the T131 rocket, used with the T110E2 launcher

\[
\begin{align*}
W_h &= 5.2 \text{ lb.} \\
W_g &= 300 \text{ lb. (launcher, magazine, and feed weight)} \\
W_R &= 10.7 \text{ lb.} \\
V_b &= 2500 \text{ ft./sec.} \\
N_R &= 25 \\
W_{gr} &= 650 \text{ rpm}
\end{align*}
\]

Assuming an installation weight equal to 0.5 \( W_g \)

\[
\frac{W_h}{W_o} = 0.182 @ V_b = 2500 \text{ ft./sec.}
\]

This compares with the values of 0.44 found for the pure rockets, (light case), 0.038 for a standard gun with the same performance, or 0.095 for a recoilless gun with the same performance.

**Comparison of \( \frac{W_h}{W_o} \) for Various Weapons**

Of the weapons described, the bomb has the highest value of \( \frac{W_h}{W_o} \)
because no propellant, or structure to resist the supporting forces is necessary. The rocket (except for the closed breech launched rocket) requires a comparatively large amount of propellant, but relatively little structure. The recoilless gun requires less propellant than the rocket, but more structure. The gun requires even less propellant, but even more structure. This will be noted in the comparison of warhead weight to round weights where the rocket has the lowest ratio of the three, the recoilless gun an intermediate ratio, and the gun the lowest ratio. Therefore, at some number of rounds per gun, the \( \frac{W_h}{W_0} \) ratios of the three weapons should be the same, that weapon requiring the greatest structural weight requiring the most rounds per gun. These numbers are tabulated below, the numbers corresponding to the number of rounds per gun which must be carried to give an \( \frac{W_h}{W_0} \) equal to that of a rocket, the gun muzzle velocities being equal to the rocket's burnt velocity. An intermediate velocity of 1500 feet per second was chosen for this comparison.

<table>
<thead>
<tr>
<th>Projectile Diameter</th>
<th>Number of Rounds per Gun Required at 10 rpm</th>
<th>Number of Rounds per Gun Required at 600 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 inch</td>
<td>235</td>
<td>550</td>
</tr>
<tr>
<td>2 inch</td>
<td>145</td>
<td>345</td>
</tr>
<tr>
<td>5 inch</td>
<td>90</td>
<td>215</td>
</tr>
<tr>
<td>1 inch</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2 inch</td>
<td>2095</td>
<td>875</td>
</tr>
<tr>
<td>5 inch</td>
<td>50</td>
<td>145</td>
</tr>
</tbody>
</table>

A similar table based on the number of rounds per gun required to equal the \( \frac{W_h}{W_0} \) of the T131 rocket and launcher, 2475 projectile, 650 rpm, 25 rounds per launcher and 2500 feet per second burnt velocity is

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given below.

<table>
<thead>
<tr>
<th>Projectile Diameter</th>
<th>Number of rounds per gun required at 650 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.775</td>
<td>105 Standard Gun</td>
</tr>
<tr>
<td>2.775</td>
<td>70 Reconelss Gun</td>
</tr>
</tbody>
</table>

Values of $L_w$, The Aircraft Armament Logistic Factor for Various Weapons.

As previously defined, the aircraft armament logistic factor is the product of the reciprocal of the probability of hit and the ratio of total weight of armament system carried to total weight of warhead carried. Thus:

$$L_w = \frac{1}{P_h} \frac{W_0}{W_h}$$

or

$$\frac{1}{L_w} = P_h \frac{W_h}{W_0}$$

$P_h$ and $\frac{W_h}{W_0}$ are mutually related by their dependence on muzzle (gun) or burnt (rocket) velocities. Their variations as functions of velocity have been individually discussed in previous sections. The over-all variation will be discussed below.

Bombs

For bombs $\frac{W_h}{W_0}$ is essentially fixed. Therefore $1/L_w$ varies linearly with the probability of hit. Relative values of the product of $L_w$ and the target area are given in the following table:

<table>
<thead>
<tr>
<th>$R \rightarrow$</th>
<th>( \frac{W_h}{W_0} )</th>
<th>3000</th>
<th>9</th>
<th>4</th>
<th>6000</th>
<th>9</th>
<th>180,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{LW}{R} \rightarrow$</td>
<td>( \frac{W_h}{W_0} )</td>
<td>9500</td>
<td>21,500</td>
<td>74,000</td>
<td>180,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>22,000</td>
<td>49,000</td>
<td>175,000</td>
<td>390,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rockets

The variation of $1/L_w A_T$ for rockets as a function of range, $\sigma_1$, burnt velocity and range release error is plotted in Figure 11. It will be noted that $1/L_w$ reaches a maximum ($L_w$ reaches a minimum) within the velocity range of the rocket. As might be expected, the optimum velocity is somewhat lower for small release errors than for high; somewhat lower for large inherent dispersions than for small; and somewhat lower for short ranges than for long. The best compromise velocities (weighted toward firing at long ranges), appear to be approximately 1000 feet per second for the older (higher case weight) rockets, and 1600 feet per second for the newer (low case weight) rockets. Both values are somewhat lower than those found in existing designs. Relative values of the product of $L_w A_T$ are shown below:

<table>
<thead>
<tr>
<th>$\Delta R/R$</th>
<th>$\sigma_1$</th>
<th>3000</th>
<th>6000</th>
<th>feet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>0.1</td>
<td></td>
<td>2600</td>
<td>11,000</td>
<td>13,500</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>5350</td>
<td>15,500</td>
<td>13,500</td>
</tr>
<tr>
<td>0.1</td>
<td></td>
<td>1650</td>
<td>9100</td>
<td>8150</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>2650</td>
<td>9800</td>
<td>18,500</td>
</tr>
</tbody>
</table>

Older rockets

Newer rockets

Standard Guns

The variation of $1/L_w A_T$ for standard guns, as a function of range, $\sigma_1$, muzzle velocity, and range release error are plotted for two guns (a small caliber high cyclic rate, and a large caliber, low cyclic rate gun, for various numbers of rounds per gun) in Figure 12. There, optimum velocities are also shown to be lower than standard design, approximately
1700 feet per second for both types. The trends of variation of optimum velocity with release error, inherent dispersions, and range are the same as with rockets. Relative values of the product of $L_w A_T$ are shown below.

<table>
<thead>
<tr>
<th>$\Delta R/R$</th>
<th>$\varphi_i$</th>
<th>3000 feet</th>
<th>6000 feet</th>
<th>feet</th>
<th>miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2</td>
<td>785</td>
<td>4550</td>
<td>4000</td>
<td>20,000</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1700</td>
<td>6650</td>
<td>12,000</td>
<td>33,500</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>600</td>
<td>3500</td>
<td>3250</td>
<td>15,000</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1200</td>
<td>4750</td>
<td>9500</td>
<td>23,500</td>
</tr>
<tr>
<td>0.1</td>
<td>1550</td>
<td>11,000</td>
<td>11,500</td>
<td>30,000</td>
<td>10 rounds per gun</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1650</td>
<td>15,500</td>
<td>35,500</td>
<td>85,000</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1250</td>
<td>7050</td>
<td>6150</td>
<td>29,500</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>2550</td>
<td>9250</td>
<td>20,000</td>
<td>51,500</td>
</tr>
</tbody>
</table>

It will be noted that for corresponding inherent dispersions, in the ranges of rounds per gun shown, the logistic factor for guns is higher than for the newer rockets (effectiveness per pound of installation weight is lower). This was indicated in the comparison given in the section discussing $\frac{W_A}{W_D}$. The lower limiting values for guns than for rockets are due to the lower range of dispersions.

**Recoilless Guns**

The variation of $1/L_w A_T$ for recoilless guns, as a function of range, $\varphi_i$, muzzle velocity and range release error are plotted in Figure 13 for two gun designs, one corresponding to existing practice, and a
lighter one corresponding to proposed new designs, both taken as a large caliber, low cyclic rate weapon. The optimum velocity is shown to be approximately 1200 feet per second (weighted toward the longer range firing), for present design and approximately 1400 feet per second for the new design. The trends of variation of optimum velocity with release error, inherent dispersions, and range are the same as with rockets and standard guns. Relative values of the product of $L_w$ and the target area, for the above velocities are shown below:

<table>
<thead>
<tr>
<th>$R &gt;$</th>
<th>$\gamma_i$</th>
<th>3000</th>
<th>6000</th>
<th>feet</th>
<th>miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_R R$</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>1150</td>
<td>3330</td>
<td>7350</td>
<td>28,000</td>
<td>10 rounds per gun</td>
</tr>
<tr>
<td>0.5</td>
<td>3300</td>
<td>9100</td>
<td>21,500</td>
<td>111,000</td>
<td>Present Designs</td>
</tr>
<tr>
<td>0.1</td>
<td>800</td>
<td>2300</td>
<td>5100</td>
<td>18,000</td>
<td>20 rounds per gun</td>
</tr>
<tr>
<td>0.5</td>
<td>2300</td>
<td>6600</td>
<td>18,000</td>
<td>49,000</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>900</td>
<td>1750</td>
<td>5250</td>
<td>22,000</td>
<td>10 rounds per gun</td>
</tr>
<tr>
<td>0.5</td>
<td>2300</td>
<td>6650</td>
<td>18,000</td>
<td>45,500</td>
<td>New Designs</td>
</tr>
<tr>
<td>0.1</td>
<td>685</td>
<td>3600</td>
<td>3900</td>
<td>16,000</td>
<td>20 rounds per gun</td>
</tr>
<tr>
<td>0.5</td>
<td>1750</td>
<td>5400</td>
<td>13,500</td>
<td>26,500</td>
<td></td>
</tr>
</tbody>
</table>

It will be noted that for recoilless guns, as with standard guns, in the ranges of rounds per gun shown, for corresponding inherent dispersions, the logistic factor is higher than for the newer rockets, as was indicated in the $\frac{W_n}{W_o}$ comparison. The lower limiting values for recoilless guns than for rockets are due to the lower range of dispersions. However, the logistic factors for large caliber low cyclic rate recoilless guns are smaller than those for corresponding standard guns under...
comparable conditions.

**Gun-Launched Rockets**

The logistic factor for gun-launched rockets can be confidently determined only at the design velocity of the existing weapon, the T131 at 2500 feet per second. Assuming its dispersions to be in the same range as guns, the relative values of the product of $L_w A_T$ are given below.

<table>
<thead>
<tr>
<th>$R &gt;$</th>
<th>$R &gt;$</th>
<th>$R &gt;$</th>
<th>$R &gt;$</th>
<th>$R &gt;$</th>
<th>$R &gt;$</th>
<th>$R &gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1 &gt;$</td>
<td>$Q_1 &gt;$</td>
<td>$Q_1 &gt;$</td>
<td>$Q_1 &gt;$</td>
<td>$Q_1 &gt;$</td>
<td>$Q_1 &gt;$</td>
<td>$Q_1 &gt;$</td>
</tr>
<tr>
<td>$\Delta R^2/R$</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>1300</td>
<td>7800</td>
<td>5650</td>
<td>32,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1950</td>
<td>8550</td>
<td>12,500</td>
<td>42,500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These values are higher than for the recoilless gun (20 rounds per gun) at its best velocity. However, a reduction in the velocity of the T131 would reduce the logistic factors, until, at the same velocity the comparison would be approximately the same as given in the section on $\frac{W_h}{W_d}$, since the same inherent dispersions were assumed both for the recoilless gun and the gun-launched rocket.

**Comparison of Weapons on the Basis of $L_w$**

Figure 14 shows the values of $L_w$ versus range for the following weapons, against three target areas, 200, 2000, and 20,000 square feet.

1. Bomb
2. Rocket - Light case, 1600 ft./sec.
3. Recoilless Guns, Large caliber, 1400 ft./sec.
   a. 10 rpm, 10 rounds per gun
   b. 650 rpm, 20 rounds per gun
4. Standard Guns, 1700 ft./sec.
   a. Large caliber, 10 rpm, 10 rounds per gun
   b. Small caliber, 650 rpm, 600 rounds per gun

5. Gun-Launched Rocket T131, 2500 ft./sec., 650 rpm, 25 rounds per gun

No small caliber recoilless guns are included, since previous examination of logistic factors has indicated their relative inferiority to other weapons. No high cyclic rate large caliber standard guns are included because of the practical difficulties associated with their design and installation, and their relative inferiority in \(*_L_\). The projectile velocities are the optimum indicated in the previous sections, except for the T131, whose design velocity was taken.

The target areas, as stated previously, were selected to roughly correspond to the following tactical targets.

- 200 sq. ft. - Tank, or transport on rail or road
- 2000 sq. ft. - Pill box, artillery emplacement, or bridge abutment
- 20,000 sq. ft. - Troop vehicles or supply concentration

In general, the trends follow those shown in the hit probability section, emphasizing the greater weight of the hit probability term in the logistic factor. For the 200 sq. ft. targets at the longer ranges, the better guns have lower logistic factors than rockets, and rockets have lower logistic factors than bombs. For the 2000 sq. ft. targets, the rockets are generally comparable with guns, but better than bombs. For the 20,000 sq. ft. targets, rockets are better than most guns, but at short ranges or low errors and dispersions, bombs are better than all. Above 20,000 feet bombs will be best.

The effect of range is again emphasized, particularly against small
targets. Against tanks, effectiveness is gained only by firing at short ranges, no matter what the weapon.

There are some rather important supplements to the conclusions reached in the section on hit probabilities when the weight characteristics of the weapon are considered. The gain in effectiveness from the use of guns over rockets against small targets is not nearly as marked because of the greater weight ratios of the guns. However, the effectiveness of all weapons is so low against small targets that even the small gains in effectiveness made possible by the use of guns should be utilized, since it may make the difference between failure and success of a sortie.

Among the various guns or gun-launched rockets, the small caliber, high cyclic rate, large number of rounds per gun standard gun is superior against all areas. It also has a good logistic factor compared with rockets or bombs. The large caliber standard gun, however, is the least effective of the guns. Between these two lie the gun-launched rocket and the recoilless guns. The gun-launched rocket, for comparable rates of fire and rounds per gun, appears somewhat superior to the recoilless rifle at the longer ranges, higher dispersions, and smaller areas. The two are nearly equal at the shorter ranges, lower dispersions and larger targets.

Examination of Effectiveness Index of Various Weapons

The previous section compared weapons on the basis of logistic factor. However, the results must be modified by the influence of \((\frac{\gamma(h)}{\omega_n})^T\), the remaining term in the effectiveness index. There are not sufficient effectiveness data to quantize the effectiveness index in detail. However, the relative index of the different weapons may be examined qualitatively.
Against small targets, the small caliber high cyclic rate gun has the most generally favorable logistic factor. However, $\left(\frac{p(w/h)}{w_n}\right)$ for this weapon is essentially zero, when used against armored targets. The low cyclic rate recoilless rifle, or gun-launched rocket, which can deliver larger caliber and weight projectiles would have the best effectiveness index against tanks. The small caliber, high cyclic rate guns would have the best effectiveness index against convoys or trains of vehicles and troops.

Against the intermediate sized targets, unarmored or of light structure, the small caliber gun still shows the best effectiveness index. Against heavy structures, guns or rockets would show approximately equal effectiveness indices, but the lower accelerations of the rocket would enable the more efficient use of a greater number of warheads.

Against targets of 20,000 square foot area and greater, bombs have generally the highest effectiveness index, except where penetration must be accomplished by the kinetic energy of the round rather than by explosive effects.
REFERENCES


**TABLE I**

**RELEASE ERROR CONTROL REQUIREMENTS**

<table>
<thead>
<tr>
<th>$V_a$ (mph)</th>
<th>$AR/R$</th>
<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
<th>$AR/R$</th>
<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
<th>$AR/R$</th>
<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.0021</td>
<td>0.0011</td>
<td>0.33</td>
<td>0.0053</td>
<td>0.0027</td>
<td>0.84</td>
<td>0.0095</td>
<td>0.0048</td>
<td>1.5</td>
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<td>0.0084</td>
<td>0.0084</td>
<td>1.3</td>
<td>0.021</td>
<td>0.021</td>
<td>3.2</td>
<td>0.041</td>
<td>0.041</td>
<td>5.0</td>
</tr>
<tr>
<td>1500</td>
<td>0.019</td>
<td>0.029</td>
<td>3.0</td>
<td>0.054</td>
<td>0.081</td>
<td>5.0</td>
<td>0.13</td>
<td>0.1</td>
<td>5.0</td>
</tr>
<tr>
<td>2000</td>
<td>0.035</td>
<td>0.070</td>
<td>5.0</td>
<td>0.13</td>
<td>0.1</td>
<td>5.0</td>
<td>0.26</td>
<td>0.1</td>
<td>5.0</td>
</tr>
<tr>
<td>2500</td>
<td>0.076</td>
<td>0.1</td>
<td>5.0</td>
<td>0.22</td>
<td>0.1</td>
<td>5.0</td>
<td>0.41</td>
<td>0.1</td>
<td>5.0</td>
</tr>
<tr>
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<td>0.1</td>
<td>5.0</td>
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<td>5.0</td>
<td>0.5</td>
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<td>5.0</td>
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</table>

$\Theta = 60^\circ$

<table>
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<tr>
<th>$V_a$ (mph)</th>
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<th>$AV/V_a$</th>
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<th>$AR/R$</th>
<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
<th>$AR/R$</th>
<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.0040</td>
<td>0.0020</td>
<td>0.13</td>
<td>0.0099</td>
<td>0.0050</td>
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<td>0.018</td>
<td>0.009</td>
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<td>0.016</td>
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<td>0.040</td>
<td>0.040</td>
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<td>0.071</td>
<td>2.3</td>
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<tr>
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<td>0.048</td>
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<td>3.2</td>
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<td>0.40</td>
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<td>0.5</td>
<td>0.1</td>
<td>5.0</td>
</tr>
<tr>
<td>3000</td>
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<td>5.0</td>
<td>0.5</td>
<td>0.1</td>
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$R = 3000'$

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<th>$AR/R$</th>
<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
<th>$AR/R$</th>
<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
</tr>
</thead>
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<td>0.011</td>
<td>0.006</td>
<td>1.7</td>
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<td>0.017</td>
<td>2.7</td>
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<td>0.046</td>
<td>5.0</td>
<td>0.090</td>
<td>0.090</td>
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<tr>
<td>1500</td>
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<td>0.062</td>
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<td>5.0</td>
<td>0.28</td>
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<td>5.0</td>
<td>0.5</td>
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<td>0.1</td>
<td>5.0</td>
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<td>0.1</td>
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<td>0.5</td>
<td>0.1</td>
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</table>

$\Theta = 20^\circ$

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<th>$AV/V_a$</th>
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<th>$AV/V_a$</th>
<th>$\Delta \Theta (\degree)$</th>
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</thead>
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</tr>
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<td>0.031</td>
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<td>5.0</td>
<td>0.5</td>
<td>0.1</td>
<td>5.0</td>
<td>0.5</td>
<td>0.1</td>
<td>5.0</td>
</tr>
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<td>0.5</td>
<td>0.1</td>
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</tr>
<tr>
<td>3000</td>
<td>0.47</td>
<td>0.1</td>
<td>5.0</td>
<td>0.5</td>
<td>0.1</td>
<td>5.0</td>
<td>0.5</td>
<td>0.1</td>
<td>5.0</td>
</tr>
</tbody>
</table>

$\Theta = 60^\circ$

* Below $AR/R = 0.5, AV/V_a = 0.1, \Delta \Theta = 5.0$
$R = \frac{V_{AV} \cdot t}{\cos \theta} \quad V_{AV} \text{ DEFINED AS EQUAL TO} \left(\frac{R}{t}\right)$

$M = \frac{g \cdot t^2 \cdot \cos \theta}{2} = \frac{g \cdot R^2 \cdot \cos \theta}{2 \cdot V_{AV}^2} \quad \text{TRAJECTORY DROP (ft)}$

$E \quad \text{(MILS)} = \frac{M}{R} = \frac{g \cdot R \cdot \cos \theta}{2 \cdot V_{AV}^2} \quad \text{TRAJECTORY DROP (MILS)}$

$I \quad \text{TIME OF FLIGHT FROM RELEASE POINT TO GROUND (SEC.)}$

$\theta \quad \text{ANGLE OF RELEASE}$

$V_0 \quad \text{SPEED OF RELEASE (FPS)}$

**FIGURE 1. EFFECT OF GRAVITY UPON TRAJECTORY OF A PROJECTILE IN A VACUUM**
ASSUME SIGHT SETS DEFLECTION ANGLE $E_0$ AT $R_0$ BUT RANGE INFORMATION IS UNCERTAIN WITHIN $R_1 = R_0 + \Delta R = R_0 - R_0 - \Delta R = R_2$

TOTAL DISPERSION AT TARGET (MIL) $\Delta E_0 = \Delta E_0^2 - \Delta E_0^1 = \left( \frac{E_0 R_2 - \eta_2}{R_0} \right) - \left( \frac{E_0 R_1 - \eta_1}{R_0} \right)$

$\Delta E_0 = \frac{E_0 R_2 - E_0 R_2 - E_0 R_1 + E_1 R_1}{R_0} = \frac{R_0}{R_0} \left( \frac{E_0 - E_0}{R_0} \right) R_1 (E_1 - E_0)$

$\Delta E_0 = \frac{(R_0 - \Delta R)(E_0 - E_2) + (R_0 + \Delta R)(E_1 - E_0)}{R_0}$

$\Delta E_0 = E_1 - E_2 + \frac{\Delta R}{R_0} \left( (E_1 - E_0) - (E_0 - E_2) \right)$

BUT $(E_1 - E_0) = \text{CONST.} \times \Delta R = (E_0 - E_2)$

SO, $\Delta E_0 = E_1 - E_2$

THE LOCATION OF THE MEAN CENTER OF IMPACT (MIL) $\pm \text{CONST.} \left( \frac{\Delta R}{R_0} \right)$

ASSUME THIS CORRECTION IS INSERTED IN $E_0$, KNOWING RANGE OF $\Delta R$.

THEN INCREMENT OF LINEAR ERROR FROM MEAN (MIL) $\pm \frac{E_1 - E_2}{2}$.

ASSUME $\frac{\Delta R}{R_0} = \frac{E_1 - E_2}{2} = \text{CONST.} \left( \Delta R \right)$

**FIGURE 2. ASSUMPTIONS FOR RANGE ERROR**
FIGURE 3. COMPARISON OF APPROXIMATE AND EXACT $\Delta \Sigma y'$s
FIGURE 4A. EFFECT OF UNCERTAINTIES AS TO POINT OF RELEASE ON DISPERSION OF BOMBS
FIGURE 4B. EFFECT OF UNCERTAINTIES AS TO POINT OF RELEASE ON DISPERSION OF BOMBS
Figure 4c. Effect of Release Error on $\sigma_y$
FIGURE 5A. EFFECT OF RANGE UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET AREA
FIGURE 5B. PROBABILITY OF HIT PER SQUARE FOOT OF TARGET AREA AS A FUNCTION OF RANGE
FIGURE 5c—EFFECT OF RANGE UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET AREA.
SECRET

\[ \frac{\Delta R}{R} = 0.5 \]

\[ V_A = 500 \text{ FPS.} \]

\[ \circ = \theta = 0^\circ \text{DIVE ANGLE} \]

\[ \square = \theta = 60^\circ \text{DIVE ANGLE} \]

\[ P_H = 0.25 \]

\[ A_T = 200 \text{ SQ. FT.} \]

**Figure 5a:** EFFECT OF RANGE UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET AREA.

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FIGURE 6a—EFFECT OF VELOCITY OF PROJECTILE RELATIVE TO AIRCRAFT UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET.
Figure 6B. Effect of velocity of projectile relative to aircraft upon hit probability per square foot of target.
FIGURE 7. EFFECT OF RELEASE ERROR ON $\sigma_X$
FIGURE 7A. EFFECT OF SLANT RANGE UPON
ROUNDS REQUIRED PER HIT FOR
A SPECIFIED TARGET AREA
FIGURE 7B: EFFECT OF SLANT RANGE UPON ROUNDS REQUIRED PER HIT FOR A SPECIFIED TARGET AREA
FIGURE 7C. NUMBER OF ROUNDS REQUIRED PER HIT AS A FUNCTION OF RANGE - TARGET AREA = 20,000 SQUARE FEET
FIGURE 8—EFFECT OF BURNT VELOCITY OF ROCKET UPON RATIO OF WEIGHT OF WARHEAD TO THAT OF TOTAL ARMAMENT SYSTEM.
FIGURE 9—EFFECT OF MUZZLE VELOCITY OF GUN PROJECTILE UPON RATIO OF WEIGHT OF WARHEAD TO THAT OF TOTAL ARMAMENT SYSTEM.
FIGURE 10 - EFFECT OF MUZZLE VELOCITY OF PROJECTILES FROM RECOILLESS GUNS UPON RATIO OF WEIGHT OF WARHEAD TO THAT OF TOTAL ARMAMENT SYSTEM.
FIGURE II a– RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF ROCKET BURNT VELOCITY.
FIGURE II b - RECIPROCAL OF AIRCRAFT ORDNANCE LOGISTIC FACTOR AS A FUNCTION OF ROCKET BURNT VELOCITY.
FIGURE 12A. RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF GUN MUZZLE VELOCITY
SECRET

![Graph showing reciprocals of aircraft armament logistic factors as a function of gun muzzle velocity.](image)

**Figure 12B. Reciprocal of Aircraft Armament Logistic Factor as a Function of Gun Muzzle Velocity**

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FIGURE 13A. RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF GUN MUZZLE VELOCITY
LOW CYCLIC RATE — LARGE CALIBER
RECOILLESS GUNS — NEW DESIGNS.

FIGURE 13 B. RECIPROCAL OF AIRCRAFT ARMAMENT
LOGISTIC FACTOR AS A FUNCTION OF
MUZZLE VELOCITY OF PROJECTILE FROM
RECOILLESS GUN
FIGURE 14 EFFECT OF SLANT RANGE UPON AIRPLANE ARMAMENT LOGISTIC FACTOR.
FIGURE 14B. AIRPLANE ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF RANGE TARGET AREA = 2000 SQUARE FEET
FIGURE 14C. AIRPLANE ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF RANGE