ABSTRACT

Variable capacitance electrostatic generators are capable of rugged efficient operation and high power output. An exposition is given of a variable capacitance electrostatic energy conversion system, employing self-contained excitation, for electrical pulsed power generation, including mathematical analysis and comment on practical realization.

Electrostatic generators have long been used for production of high voltages at relatively low power; however, the variable capacitor generator, as opposed to the friction or influence type machine, is capable of efficient operation at high power output. For a variable capacitance electrostatic generator, the electrical power input to the capacitor, $P_e$ (watts), is equal to the increase in capacitor electrostatic field energy, $W_{ef}$ (joules), per unit time, $t$ (seconds), and the mechanical power output, $P_M$:

$$P_e = \frac{dW_{ef}}{dt} + P_M$$

With

$$P_e = V \frac{dq}{dt}; \quad W_{ef} = \frac{CV^2}{2}; \quad q = CV; \quad V = \text{capacitor potential (volts)}; \quad q = \text{capacitor charge (coulombs)}; \quad C = \text{capacitor capacitance (farads)}.$$ 

Therefore, the time rate of conversion of mechanical to electrical energy, $\frac{dW_{ME}}{dt}$, equal to $-P_M$, is given as follows:

$$\frac{dW_{ME}}{dt} = -\frac{V^2}{2} \frac{dC}{dt}.$$ 

Consider a variable capacitance electrostatic energy conversion system employing self-contained excitation used for generation of electrical pulsed power as shown schematically below. Energy conversion is obtained by operation of two variable capacitors in complementary opposition to one another across the load. The capacitors each essentially possess a charge $Q_0$, with charge excursion $\pm q$ from $Q_0$; there exists a contained or "trapped" charge $2Q$ for the system. For the pulse generator cycle, initially Capacitor No. 1 has high capacitance $C_H$ and a high charge $Q_H = (Q_0 + q_0)$; Capacitor No. 2 has
Variable Capacitance Electrostatic Electrical Pulse Generator


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low capacitance \( C \), and a low charge \( q_1 = (Q_0 - q_o) \). The switch is open; the switch, for the purpose of this exposition, shall be considered as an ideal switch (that is, it has negligible inductance, infinitely small capacitance relative to other circuit elements, infinite resistance when open, and zero resistance when closed). The potentials of the two variable capacitors are equal:

\[
\frac{Q_0 + q_0}{C_H} = \frac{Q_0 - q_0}{C_L} = V_M;
\]

with the total electrical energy of the circuit given as follows:

\[
W_A = \frac{(Q_0 + q_0)^2}{2C_H} + \frac{(Q_0 - q_0)^2}{2C_L}.
\]

As the drive shaft rotates, Capacitor No. 1 goes to capacitance \( C_L \) and high potential \( V_H = \frac{(Q_0 + q_0)}{C_L} \), with high energy \( W_H = C_H V_H^2/2 \); Capacitor No. 2 goes to capacitance \( C_H \) and low potential \( V_L = \frac{(Q_0 - q_0)}{C_H} \), with low energy \( W_L = C_H V_L^2/2 \). Mechanical energy is converted to electrical energy in Capacitor No. 1 and electrical energy is converted to mechanical energy in Capacitor No. 2; however, since Capacitor No. 2 is drive shaft coupled to Capacitor No. 1, this mechanical energy may be considered as simultaneously reconverted to electrical energy, reducing the external energy input required for the conversion in Capacitor No. 1. The total electrical energy of the circuit is now given as follows:

\[
W_B = \frac{(Q_0 + q_0)^2}{2C_L} + \frac{(Q_0 - q_0)^2}{2C_H}.
\]

When the switch is closed, the potential rise across Capacitor No. 1 is now equal to the potential drop across the load, \( V_{Load} \), and the potential drop across Capacitor No. 2. It is assumed, for the purpose of this exposition, that the transient response time of the electric circuit is much faster than the time required for one complete cycle of the variable capacitors. With \( q \) now representing the charge being transferred, the equation for the electrical circuit response is given as follows:

\[
\frac{Q_0 + q_0 - q}{C_L} = V_{Load} + \frac{(Q_0 - q_0 + q)}{C_H};
\]
when \( t = 0 \), \( q = 0 \); when \( t = \infty \), \( q = 2q_0 \). Rearranging the above equation, obtain:

\[
Q_0(1/C_L - 1/C_H) + q_0(1/C_L + 1/C_H) = V_{\text{Load}} + q(1/C_L + 1/C_H).
\]

It is thus seen that the pulse is equivalent to a transient resulting from switching of a constant source of potential (generated by the contained charge) onto a circuit consisting of the load and the total variable capacitor capacitance. The pulse energy to the load, \( W_p \), is given as follows:

\[
W_p = W_B - W_A = 2q_0Q_0(1/C_L - 1/C_H);
\]

with pulse potential \( V_p = V_H - V_L \), and with pulse charge \( q_p = 2q_0 \).

At the end of the pulse, the variable capacitors are at the same potential and the switch is then opened. As the drive shaft continues to rotate, the variable capacitors will interchange their roles in the second half of the cycle, pulse energy \( W_p \) will again be delivered to the load, and the variable capacitors will return to their initial states as the cycle is completed; thus energy \( 2W_p \) is delivered to the load per cycle.

For the choice of \( V_H, C_H \), and \((C_L/C_H)\) as the independent operating parameters, the dependent operating parameters are expressed as follows in terms of the independent parameters:

\[
q_0 = (1 - C_L/C_H)(C_L/C_H)(C_HV_H/2) \quad Q_0 = (1 + C_L/C_H)(C_L/C_H)(C_HV_H/2)
\]

\[
Q_H = (C_L/C_H)C_HV_H; \quad Q_L = (C_L/C_H)^2C_HV_H
\]

\[
W_H = (C_L/C_H)^2(C_HV_H^2/2); \quad W_L = (C_L/C_H)^4(C_HV_H^2/2)
\]

\[
C_L = (C_L/C_H)C_H; \quad V_L = (C_L/C_H)^2V_H; \quad V_M = (C_L/C_H)V_H
\]

\[
V_p = (1 - (C_L/C_H)^2)V_H; \quad q_p = (1 - C_L/C_H)(C_L/C_H)C_HV_H
\]

\[
W_p = (1/2)(C_L/C_H)(1 - C_L/C_H)(1 - (C_L/C_H)^2)C_HV_H^2;
\]

and for

\[
F = (1/2)(C_L/C_H)(1 - C_L/C_H)(1 - (C_L/C_H)^2), \quad W_p = FC_HV_H^2.
\]

The pulse energy \( W_p \) has no maximum or minimum for the independent parameters \( C_H \) and \( V_H \); the physical range of operation of \( C_H \) is greater than zero to infinity, the physical range of operation of \( V_H \) is zero to infinity. Within the physical range of operation of the independent parameter \((C_L/C_H)\), greater than zero to one, there exists
a maximum for its function F, and hence for $W_p$, for its following value

$$\left(\frac{C_L}{C_H}\right) = \left(\frac{C_L}{C_H}\right)^* = \frac{1}{4} + \frac{1}{2} \left(\frac{11}{3}\right)^2 \sin\left[\frac{\arcsin\left(\frac{3^5}{11^3}\right)}{3}\right]$$

$$\left(\frac{C_L}{C_H}\right)^* = .390388 , \text{ to six places, with}$$

$$F^* = .100858 , \text{ and hence}$$

$$W_p^* = .100858 C_H^2 V_H^2 .$$

Any dependent parameter with asterisk indicates that parameter with the value of $\left(\frac{C_L}{C_H}\right)$ that maximizes $W_p$. A plot of $W_p$ as a function of $C_H$ and $V_H$ is given below.

Selected relations among the parameters are given below:

$$\frac{V_M^*}{V_H} = \left(\frac{C_L}{C_H}\right)^* = .390388$$

$$\frac{V_L^*}{V_H} = \left(\frac{C_L}{C_H}\right)^*^2 = .152403$$

$$\frac{V_p^*}{V_H} = (1 - \left(\frac{C_L}{C_H}\right)^*^2) = .847597$$

$$\frac{W_L^*}{W_H^*} = \left(\frac{C_L}{C_H}\right)^*^3 = .0594963$$
\[ q_0^*/q_0^* = (1 - (C_L/C_H)^*)/(1 + (C_L/C_H)^*) = 0.438447 \]

\[ W_p = q_p V_p/2 \quad ; \quad W_p^* = q_p^* V_p^*/2 \]

\[ q_p^* = (1 - (C_L/C_H)^*)(C_L/C_H)^* C_V H = 0.237985 C_V H \]

\[ V_p^* = (1 - (C_L/C_H)^*2)V_H = 0.847597 V_H \]

A choice of pulse potential \( V_p^* \) determines the operating value of \( V_H \), together with a choice of pulse charge \( q_p^* \) determines the operating value of \( C_H \) and also \( W_p^* \).

For the variable capacitors, operating values of \( V_H \) are limited by the breakdown voltage 3,4 of the capacitor plate gaps; practical values, for variable capacitor generators5-7, may be on the order of hundreds of kilovolts for gap insulation being either compressed gases (such as air or sulfur hexafluoride) or vacuum. Although compressed gases have specific resistance values approaching infinity, vacuum does have infinite specific resistance and, in addition, the use of vacuum as the dielectric eliminates the main loss associated with gas insulation, the windage of the moving parts. Although breakdown gradients for vacuum gaps decrease as gap spacing increases, field strengths in the range of 40 to 80 kilovolts per multimeter may be achievable, with corresponding power densities of 7 to 20 millijoules per cubic centimeter. Practical rotational speeds up to at least 400 revolutions per second are attainable, with the number of poles in the range of ten to twenty. Rotary seals have been developed8 which have the capability for containment of compressed gases and vacuum (very high vacuum, to \( 10^{-8} \) Torr and above) for rotational speeds of 400 revolutions per second and much higher (to 2000 revolutions per second); vacuum can be easily maintained within the working chamber to the rotary seal capability by use of vacuum appendage pumps of the sputter-ion type9.

Although the variable capacitor electrical pulse generator having self-contained excitation is basically a bipolar output device, unipolar output may be achieved through use of full wave bridge rectification; however, bipolar output may be required, for example, for corona discharges10 and other plasma discharges employing dielectric electrodes. The drive shaft is always positively loaded (that is, is inputting mechanical energy to the pulse generator) and the capacitor plates may be designed for constant power loading of the shaft. Although the pulse generator as presented essentially involves the use of contained charge, the generator could be charged or discharged in operation as load conditions dictated. A monitor may be used for the pulse generator, as example, in another schematic for the generator shown below. The load pulse is monitored and changes may be effected by change in the trapped charge, by charge input from the low potential supply through the rectifier into Variable Capacitor No. 1 or by charge output from Variable Capacitor No. 2 through the rectifier into the high potential supply; values for the low and high potential could be controlled by the monitor; in addition, energy could be bled off the load by the monitor.
to power both the monitor and the supplies. A basic use of the low potential supply would be to replenish charge lost by leakage. With the low potential supply set at \( V_l \), if Capacitor No. 1 low potential would attempt to fall below this \( V_l \)-value because of charge leakage, then charge would be bled in from the supply through the rectifier to the capacitor to reestablish the correct operating value.

The electrical pulse generator, a device (as indicated by the analysis and comment given above) which may have potential for simple, rugged, efficient flexible operation, may be considered as one possible application for variable capacitance electrostatic energy conversion employing self-contained excitation.


R. Hawley, Vacuum, 10, 310(1960).


R. Moskowitz, American Society of Lubrication Engineers Transactions, 18, 135(1975).
