Analysis of the Capability Portfolio Review (CPR)

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### Abstract

In February 2010, the Secretary of the Army directed the Under Secretary of the Army and the Vice Chief of Staff of the Army to implement a CPR pilot process to conduct an Army-wide, all components revalidation of the operational value of Army requirements within and across capability portfolios to existing joint and Army warfighting concepts. The CPR is designed to support the Army’s overarching and strategic level goals of fielding an effective, flexible, affordable, and modern force. This effort will describe and document the CPR process, but its main objective will focus on developing alternative trade space analysis methodologies using a combination of Bayesian Statistical methods and linear programming / optimization techniques in order to present an effective and informative portfolio trade space analysis to key decision makers.

### Subject Terms

Bayesian Statistical Methods, Subject Matter Expert Elicitation, Optimization

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ABSTRACT

In February 2010, the Secretary of the Army directed the Under Secretary of the Army and the Vice Chief of Staff of the Army to implement a CPR pilot process to conduct an Army-wide, all components revalidation of the operational value of Army requirements within and across capability portfolios to existing joint and Army warfighting concepts. The CPR is designed to support the Army’s overarching and strategic level goals of fielding an effective, flexible, affordable, and modern force. This effort will describe and document the CPR process, but its main objective will focus on developing alternative trade space analysis methodologies using a combination of Bayesian Statistical methods and linear programming / optimization techniques in order to present an effective and informative portfolio trade space analysis to key decision makers.
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LIST OF ACRONYMS AND ABBREVIATIONS

AAR    After Action Review
AoA    Analysis of Alternatives
ARCIC  Army Capabilities Integration Center
AFMS   Army Force Management School
AR     Army Regulation
AT & L  Acquisition, Technology, and Logistics
ATO    Army Technology Objective
CAA    Center for Army Analysis
CNA    Capability Needs Analysis
COA    Course of Action
CoE    Center of Excellence
CPR    Capabilities Portfolio Review
CSA    Chief of Staff of the Army
DOTMLPF Doctrine, Organization, Training, Material, Leadership and Education, Personnel, Facilities
EoA    Evaluation of Alternatives
EV     Expected Value
FDU    Force Design Update
FOC    Force Operating Capability
HQDA   Headquarters Department of the Army
IBCT   Infantry Brigade Combat Team
MCoE   Maneuver Center of Excellence
O & S  Operations and Support
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>OUSD</td>
<td>Office of the Under Secretary</td>
</tr>
<tr>
<td>PEG</td>
<td>Program Evaluation Group</td>
</tr>
<tr>
<td>PEO</td>
<td>Program Executive Office</td>
</tr>
<tr>
<td>PM</td>
<td>Program Manager</td>
</tr>
<tr>
<td>POM</td>
<td>Program Objective Memorandum</td>
</tr>
<tr>
<td>RC</td>
<td>Required Capability</td>
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<tr>
<td>R &amp; D</td>
<td>Research and Development</td>
</tr>
<tr>
<td>SA</td>
<td>Secretary of the Army</td>
</tr>
<tr>
<td>SME</td>
<td>Subject Matter Expert</td>
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<tr>
<td>S &amp; T</td>
<td>Science and Technology</td>
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<tr>
<td>TCM</td>
<td>TRADOC Capabilities Manager</td>
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<td>TRAC</td>
<td>Training and Doctrine Command Analysis Center</td>
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<td>TRADOC</td>
<td>Training and Doctrine Command</td>
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<tr>
<td>USA</td>
<td>Under Secretary of the Army</td>
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<td>VCSA</td>
<td>Vice Chief of Staff of the Army</td>
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<tr>
<td>WfF</td>
<td>Warfighting Function</td>
</tr>
<tr>
<td>WSMR</td>
<td>White Sands Missile Range</td>
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I would especially like to thank Mr Paul Works at the TRAC-MRO for making this project possible with his guidance and resourcing. Additionally, I would like to recognize one of the co-authors, MAJ Mike Teter, for his tireless efforts above and beyond what was asked of him in support of this effort.
SECTION 1. PROBLEM DEFINITION

1.1. BACKGROUND

The mounting economic challenges faced by the United States and its military have forced decision makers to develop a way in which to holistically examine those requirements that drive capability development, acquisition, and sustainment in order to determine if either current or proposed programs are aligned to meet critical national and defense strategies as well as Army plans (U.S. Army, 2012).

Early in 2010, then Secretary of Defense Robert M. Gates began his campaign to fix Department of Defense (DoD) spending habits through acquisition reform and overhead efficiency savings. His goal was to institute a department-wide “culture of savings” that better aligned expectations with results and resources. This vision was an extension of Secretary Gates’ program reductions in April of 2009, which eliminated several big-ticket, underperforming weapons projects. Specifically, the Secretary had directed each of the services to review its programs and administrative structure to “find” a collective $100 billion in cost reductions over the next five years (AUSA, 2010).

Secretary Gates outlined his position that he wanted to eliminate the extreme peak/valley characteristics of defense spending that are so disruptive and detrimental to both the planning and execution functions of programming. While valley-like cuts are obviously disruptive to long-term programs, high peaks are equally disruptive in that they can cause programmers to “throw money” at a problem rather than figure out ways to address scheduling issues, technology maturation problems, or general design faults. In exchange for a focus on good stewardship of taxpayer resources and overall spending reform, Secretary Gates hoped to win congressional support for stabilizing the defense budget with a sustainable, inflation-relevant growth rate. He indicated that his target was a 1 percent real growth rate with 2–3 percent internal savings for 3–4 percent total actionable growth in the next five years. The Army Capability Portfolio Review (CPR) process has been at the leading edge of this effort to increase efficiencies (AUSA, 2010).

On February 22, 2010, the Secretary of the Army (SA) directed the Under Secretary of the Army (USA) and the Vice Chief of Staff of the Army (VCSA) to implement a CPR pilot
process to conduct an Army-wide, all components revalidation of the operational value of Army requirements within and across capability portfolios (Keenan, 2013). The CPR then is designed to support the Army’s overarching and strategic level goals of fielding an effective, flexible, affordable, and modern force. The SA then implemented a CPR process with three distinct objectives: to revalidate Army-wide system requirements across all components; to align resources with warfighting and Soldier priorities; and to institutionalize resource and acquisitions along capabilities-based planning processes to provide flexibility for the future (AUSA, 2010).

“The starting point is determining what the requirement is. What documents the requirement? Is it a valid requirement?” said Theresa M. Sherman, Division Chief, Capability Portfolio Review and Integration, G-3/5/7. “And then you look across the DOTMLPF (Doctrine, Organization, Training, Materiel, Leadership and Education, Personnel, and Facilities) and see, have we accepted too much risk or can we take more risk; are we still in sync with the need, with the threat? Do we have any gaps or redundancies? … Where do we anticipate the threat will be, and where do we want to go over time? What’s the strategy?” Answers to these questions depend in large part on information provided by the U.S. Army Training and Doctrine Command (TRADOC) and Program Executive Offices (PEOs) (Roth & McCouch III, 2012).

As explicit dollar savings are directed from DoD, the other services are following the Army’s lead in reorganizing their acquisition processes and mechanisms to generate more affordability, oversight, and efficiency incentives across the full range of portfolios. In all likelihood, the CPR process is a permanent addition to both the Army’s and DoD’s acquisition models, since the affordability initiatives from the Secretary of Defense are meant to continue through the next 5 years. The CPR process will continue to grow and evolve to incorporate more areas and apply more expertise and analysis in order to provide the warfighters what they need, when they need it, and at the “best” price. It is designed to maximize fiscal resource expenditures as well as ensuring that Soldiers receive the very best capability at the best value to the taxpayer (AUSA, 2010).

1.2. PROJECT PURPOSE

To provide a literature review and document historical Capability Portfolio Reviews (CPRs) while using the work done by the Training and Doctrine Command (TRADOC) Analysis
Center – White Sands Missile Range (TRAC-WSMR) on the IBCT CPR as a case study for exploring alternative trades analysis methodologies, to include the use of Bayesian statistical methods, in order to better inform/influence decision makers.

1.3. CONSTRAINTS, LIMITATIONS, & ASSUMPTIONS

Constraints limit the study team’s options to conduct the study.

- The project will be completed by 30 April 2014.

Limitations are a study team’s inability to investigate issues within the sponsor’s bounds.

- Survey of TRAC support to a formational CPR case study will be limited to the IBCT as TRAC did not participate in the Rifle SQD CPR process.
- Lack of sufficient CPR documentation precludes an in-depth survey of historical functional CPR analysis techniques.

Assumptions are study specific statements that are taken as true in the absence of facts.

- CPR process input will be provided by the G-3/5/7 DAMO-CIP.
- Alternate statistical / Bayesian techniques applied to the IBCT CPR will be applicable to all types of CPR’s.

1.4. METHODOLOGY

While a description of the CPR process is certainly in order, and will be given later on in this technical report, this effort will focus less on the CPR process as a whole and more on an alternate trade space analysis methodology as well as a way to present decision makers with the correct information in order to make the most informed decisions with respect to future Army combat systems and formations.

This project will consist of three related but parallel efforts (see Figure 1). The first will focus on an in depth literature review of the CPR process as well as a summary of historical CPRs in order to give this effort the proper context, followed sequentially by documenting the best and worst decision analysis practices using those historical CPRs as use cases. At the same time, the project team will be reviewing both the IBCT CPR, as well as surveying alternate
decision analysis methodologies, to include Bayesian statistical methods. Using the goal programming work done by TRAC-WSMR on the IBCT CPR as a case study, we will then explore what alternate method(s) is (are) most appropriate, what additional data requirements there might be, and then apply said method in a small use case. We hope to find a better way in which we can both execute analysis and present information to decision makers. Lastly, the project team will document their efforts, as well as provide support for TRAC-WSMR’s effort in working on the ABCT CPR, as appropriate.

![Project Methodology](image)

**Figure 1: CPR Analysis Methodology**

### 1.5. STUDY TEAM

- LTC Thomas Deveans, Combat Analyst, TRAC-MTRY.
- Dr. Donald Gaver, Professor Emeritus, Naval Postgraduate School.
- Dr. Patricia Jacobs, Professor Emeritus, Naval Postgraduate School.
- MAJ Michael Teter, Combat Analyst, TRAC-MTRY.
SECTION 2. THE CAPABILITY PORTFOLIO REVIEW

2.1. PURPOSE

The overarching CPR process is designed to synchronize the planning, programming and budgeting with feedback from combatant commanders and lessons learned from Iraq and Afghanistan, all while leveraging emerging technologies with affordability. The CPR process is a critical element in restoring the balance to the Army and the equipping/fielding strategies (AUSA, 2010). The intent or purpose of the CPR process is to conduct an Army-wide (all components) revalidation of requirements while holistically examining, validating, modifying, or making termination recommendations for the requirements that drive capability development, acquisition, and sustainment. The process operates with four central goals in mind:

1. Establish the ability to examine and modify investment portfolios.
2. Develop an understanding of requirements driving investment, procurement, and sustainment.
3. Reconcile requirements across portfolios.
4. Validate, modify, and terminate investment and/or procurement strategy upon the reconciliation of requirements, with the goal of the reconciliation piece to ensure that funds are programmed, budgeted, and executed against validated requirements, cost, and risk-informed alternatives (AUSA, 2010).

2.2. DEFINITIONS & TYPES OF CPRS

Before we begin talking about the CPR process as a whole, let us first define a few key terms:

**Capability:** The ability to achieve a desired effect under specified standards and conditions through a combination of means and ways across DOTMLPF in order to perform a set of tasks to execute a specified course of action (Department of Defense, 2008).

In order to identify a capability then, we must determine:

1. What we need to accomplish, or the desired effect.
2. Why we need to accomplish it, or the driver.

3. Who, where, and when, or the requirements / standards and conditions.

4. How we will accomplish it, or the ways and means, usually through a DOTMLPF assessment.

**Portfolio:** A set of existing programs currently funded, plus any additional programs of initiatives, that provide capabilities within the domain under consideration.

**Capability Portfolio:** A collection of grouped capabilities as defined by joint capability areas and the associated DOTMLPF programs, initiatives, and activities.

**Trade Space:** The set of system parameters, attributes, and characteristics required to satisfy performance standards.

**Trade Space Analysis:** A process that identifies potential opportunities to allocate resources in order to optimize the balance between operational capability and financial efficiency.

There are 3 types of CPRs, a functional, formational, and cross-portfolio. The functional CPR follows the original guidance provided by GEN Chiarelli and focuses on prioritizing capabilities that are grouped together in terms of functionality. For example, Aviation, Mission Command, Combat Vehicles, Soldier Systems, and Mobility/Counter-mobility, are instances of functional CPRs and ones that have been completed by the Army in the 2011-2012 timeframe.

A formational CPR, on the other hand, focuses on the type of Army formation, rather than on functionality, and provide additional formation perspective by integrating across functional portfolios, war fighting functions, DOTMLPF analysis, and Program Evaluation Groups (PEGs). To date, there have only been two formational CPRs completed, Rifle Squad, and the IBCT. The ABCT CPR is nearing completion and should be completed by late summer of 2014.

The final type of CPR is the cross-portfolio CPR. In theory, a cross-portfolio CPR is an aggregation, at the Army level, of formational and functional CPRs designed to inform the entire Army’s portfolio and evaluate it against DoD and Army strategic guidance and operational plans.
As of completion of this document, none of this type has been attempted, much less completed, though the Center for Army Analysis (CAA) has been tasked to look into its feasibility.

2.3. PROCESS

Although each CPR is conducted differently, they all have their beginnings with the USA and VCSA. Running the CPR is the G-3/5/7, as the staff proponent for organizing and executing it. Contributing information on the portfolios are representatives from across the programmatic spectrum, including the TRADOC; the Program Executive Offices (PEOs); the Deputy Chief of Staff, G-8; and the testing community (Roth, 2012).

The G-3/5/7 runs the meetings, sets the schedule, gets the subject matter experts together, and identifies the major issues. After a series of meetings at the O-6 level, there are a few at the one and two star level, followed by (if necessary) a three-star meeting with the VCSA. At that point, most of the work is finished, namely a detailed review of the portfolio with particular attention to selected issues (Roth, 2012).

“We try to get from [TRADOC] what are the requirements, and how old; revalidate them; and try to find some form of strategic context so you could look at importance and redundancy to help stack them and there would be some form of strategy upfront,” said Dr. David Markowitz, Technical Advisor to the Deputy Chief of Staff, G-3/5/7. For instance, for a missile defense portfolio, the initial focus would be on the range of threats, then on the needs of the force, resourcing, and how much the systems cost. This information then helps in determining how many of each system the Army needs and how quickly the Army needs it, he said (Roth, 2012).

Finally, the DOTMLPF is applied to each of the systems, and is then refined in a series of meetings at successively higher levels until it gets to the three-star level, the Deputy Chief of Staff, G-3. “And then you see that final body of work and you say good, the doctrine is validated,” Markowitz said (Roth, 2012).

The G-3 then presents the information to the VCSA, who in the end gets a recommendation or status of the portfolio and its health, and is asked to validate and approve of what was done in terms of prioritization and overall portfolio strategy (Roth, 2012).
The VCSA, in turn, presents the recommendation to the USA, sometimes after several sessions with the staff. Generally, when the USA approves a recommendation, it will go to the CSA for approval. Ultimately there may be a need to modify the requirement, initiate additional review, or pursue DoD support to change the program (Roth, 2012).

The G-8 provides information on costing and affordability to support the CPRs, but that often comes directly from the PEOs. The CPR process is meant to be very open, and is centered around what the best way is to provide the most effective, capable, and cost-effective Army.

While the intent or purpose of the CPR process is clear, the execution is not. Currently there is no standardized, documented process for conducting a CPR, and there is no Army Regulation that specifies the ways in which a CPR should be accomplished. There was an effort by TRADOC and ARCIC in late 2013 in an attempt to codify the process. HQ TRADOC published a draft task order in December 2013 laying out the CY 2014 and 2015 CPR schedule for both formational and functional based CPRs, as well as providing some guidance on analysis and briefing content. A final, non-draft task order has yet to be published (as of publication of this report). ARCIC published a CPR SOP in early 2014, describing the purpose, goals, objectives, and components of a formational CPR, as well as portfolio assessment and strategy. As with the TRADOC draft task order, no final version of this SOP has been published. Additionally, in a report submitted to the Secretary of the Army in January of 2011 entitled, “Army Strong: Equipped, Trained and Ready, Final Report of the 2010 Army Acquisition Review”, the panel recommended, among other things, that:

Capability Portfolio Reviews (CPRs) are intended to conduct an Army-wide, all-components revalidation of requirements. The approach is to holistically examine, validate, modify or make recommendations to terminate requirements driving capability development, acquisition and sustainment across a series of portfolios. Having the VCSA and ASA(ALT) co-chair the first session of the materiel CPRs would further restore the traditional partnership discussed above. Codifying CPRs in an Army Regulation (AR) will give assurance that the process will be continued when leadership changes. The responsibilities and accountability of participants in a CPR should be clearly defined. The CPRs should be expanded in the future to review the interdependencies across portfolios (McHugh, 2011).

There is, however, a description of the process given by the Army Force Management School (AFMS) in their “Capabilities Development and System Acquisition Management Executive Primer (2013)”, though this is still a long way from inclusion in an AR.
The process has two phases, each chaired by different individuals, and each with a distinct and separate set of outputs that ultimately inform the Army FY Program Objective Memorandum (POM). HQDA, DCS G-3/5/7 is the lead agency for CPR coordination and synchronization. Each of the phases is described below:

(1) Phase 1: Chaired by the VCSA. The purpose is revalidation of the operational value of Army requirements to include cost, schedule, performance, life-cycle sustainability and the Army’s plan to manage the totality of the requirement. The output is actionable recommendations that can be addressed by Army senior leadership during phase 2.

(2) Phase 2: The USA, as the Army Chief Management Officer, chairs this phase. The purpose is to address follow-on analysis from phase 1 and the programmatic (cost, schedule, performance, life-cycle sustainment) implications of the recommendations presented. The product is actionable recommendations to the SA to validate, modify, or terminate research and development (R&D) investment, procurement, and/or life-cycle sustainment requirements within capability portfolio accounts for the current POM, in development, based on the results of the CPRs.

The analysis that has resulted from the CPRs conducted under the program has clearly highlighted the utility of this process in building an effective and affordable modernization strategy. The resulting recommendations will continue to assist the SA in establishing future priorities for investment, research, development, acquisition, and life-cycle sustainment. The SA will continue to rely on this process to help him make informed decisions on behalf of the Army.

CPR’s operate concurrently with, but do not supplant the authority of the Army Requirements Oversight Council (AROC), Army System Acquisition Review Council (ASARC), or Configuration Steering Board (CSB) forums, previously discussed (Keenan, 2013).

2.4. REVIEW OF CPR LITERATURE

This section is primarily focused on providing a brief survey of the portfolio analysis tools, theory, and trade space analysis methodologies developed both in academia or those used in the business world with potential applications to the Army.
2.4.1. Academia

The majority if the CPR literature found in academia has both its roots and applications in the finance world. This section will provide a short survey of some of the more well-known portfolio theory ideas and concepts that deal mainly with the notions of risk and return.

2.4.1.1. The Basics of Modern Portfolio Theory

Much of modern portfolio management has been motivated by the influential work of Harry Markowitz (Markowitz, 1952) and his well-known optimization approach. Markowitz demonstrated how stock investors could select an efficient set of portfolios that would minimize the standard deviation (risk), subject to a particular portfolio return (expected return). Markowitz (1956) showed through an optimization technique that investors could virtually eliminate their exposure to the unsystematic risk associated with individual securities, and is applied in the context of a fixed investment amount for the portfolio. The unsystematic risks are those risks specific to the business or industry. This ability to diversify away the unsystematic risk leaves the stock investor with a portfolio containing only the market-specific risks, such as inflation, purchasing power, etc... Markowitz demonstrated that with only a limited number of properly selected stocks, the investor could virtually eliminate all the unsystematic risk associated with individual stocks, leaving only the generally market risk (Walls, 2004).

The basic assumption of modern portfolio theory is that decisions are made on the basis of a tradeoff between risk and return. Return is measured by the expected value or mean of the probability distribution of payoffs for the stock under consideration. Risk is then measured by the variance or standard deviation associated with that payoff distribution. In addition, one can make the reasonable assumption that investors and decision makers prefer less risk to more risk, all other things being equal. Or, given a certain expected return rational investors will always prefer portfolios that have lower risk. Similarly, given a certain level of risk, those same investors will always prefer portfolios with higher expected returns (Walls, 2004).

In some portfolio optimization approaches, risk is defined as the standard deviation of returns (i.e., net present value) of the portfolio of assets. Indeed, standard deviation is a commonly used measure of risk in the financial markets where return distributions are generally normally distributed. Though standard deviation is utilized as a measure of risk in the
In finance literature, this measure is better defined as a statistical measure of uncertainty. It is, in effect, a measure of dispersion around the mean value. Unlike the decision maker who may characterize risk as that portion of the uncertainty that has “downside”, the standard deviation measure does not differentiate between “downside” and “upside” uncertainty. In this context, portfolio analysis based on standard deviation (as described by Markowitz) considers extremely high and low returns equally undesirable (Walls, 2004).

It is important to note that in some cases, returns on investments may not be normally distributed. In many cases the distribution of outcomes may have skewed value distributions with high probability of achieving low-value outcomes and small probability of achieving high-value outcomes. In certain cases, it may be more appropriate to utilize an alternative measure of risk such as semi-standard deviation. The semi-standard deviation measure concentrates on reducing losses where the loss point in the semi-standard deviation measure is defined by the decision maker. However, that there are practical complications associated with the application of the semi-standard deviation measure in portfolio optimization. For example, there are issues associated with selecting an appropriate “loss point” for a distribution of outcomes. Also, in an analysis based on standard deviation, only means, variances and covariances must be supplied as inputs to the analysis. In a semi-standard deviation analysis the entire joint distribution of outcomes is required in order to perform the analysis (Walls, 2004).

2.4.1.2. Risk Informed Trade Space Analysis

The RAND Corporation developed a tool for trade space analysis, and published it (only in draft version) in a paper titled “Developing a Methodology for Risk-Informed Trade Space Analysis in Acquisition”. However, at the time of the writing of this document, the report is proprietary and not cleared for open publication, circulation, or quotation. We mention it here only for completeness sake and the hope that it will be cleared for publication in the near future, as it appears to have great application to the topic of this effort.

2.4.2. Business

Capability Portfolio Reviews, if we can generalize, are not unique within either the Department of the Army, or the Department of Defense. Businesses with diverse interests also
conduct such reviews in an effort to streamline their companies. This section will highlight a few of the tools and methodologies used in industry for the purpose of reviewing a portfolio.

2.4.2.1. RAND’s Portfolio Management (PortMan) Method

Though originally designed to be applied to Naval Research and Development, this particular portfolio management technique developed by RAND is equally applicable to Army investment strategies. RAND’s PortMan Research and Development (R&D) decision framework computes the Expected Value (EV) of an R&D project as the product of three factors: military value of the capability sought, the extent to which the performance potential matches the level required to achieve the capability, and the project’s transition probability. A useful interpretation of this equation is that the performance potential scales the capability value that is based on achieving a required level of performance, which is then further reduced by the transition probability in order to obtain the expected value of the research (Silberglitt et al., 2004).

The purpose of PortMan is to evaluate a defined group of actual or proposed projects and to provide a means for creating a portfolio from them that maximizes the value of R&D investments. It does not generate an absolute score for the total portfolio that could be used to compare to portfolios of other projects or to proportionally allocate funds between portfolios of different projects. PortMan does not rely on the EV as a point solution, but rather includes an estimate of uncertainty, evaluating and comparing R&D projects based upon all three components of the EV, including uncertainty, plus their estimated direction of change over time. This requires evaluation based on best current information, and tracking over time as conditions change and the technology matures (Silberglitt et al., 2004).

An important feature of this approach is that it allows identification of those R&D projects seeking high-value capabilities, but may have difficult technical or fielding problems remaining to be solved, and for which management attention may have the greatest leverage (Silberglitt et al., 2004).

This framework can be used with a group of subject matter experts determining the values of capability, performance potential, and transition probability. It is capable of comparing and contrasting individual and groups of research projects as a function of key
management variables such as their potential value and status and quality of transition planning. It also provides a straightforward and logical set of repeatable steps in order to determine the expected value of research projects, together with a measure of uncertainty (Silberglitt et al., 2004).

### 2.4.2.2. RAND’s PortMan with a Linear Program

In this section, we describe a variant of PortMan that uses a linear programming model to select a portfolio that consists of Army Technology Objectives (ATOs), the highest priority Army S&T projects. This portfolio satisfies the Army’s Force Operating Capability (FOC) requirements designated for this group of ATOs to meet, while yielding the lowest total remaining lifecycle cost for all the systems developed from the selected ATOs. The model includes two classes of constraints. First, all individual FOC requirements must be fully met. Second, the total remaining S&T budget for the selected ATOs must not exceed a given budgeted amount. The linear programming model used here requires two inputs: (1) the EV of the ATOs and (2) the remaining lifecycle cost of the systems that will be developed from the ATOs (For specifics on the linear program model formulation, see Chow et al., 2009) (Chow, Silberglitt, & Hiromoto, 2009).

RAND has developed an approach called “gap space coverage” to make EV estimates of how well each ATO could meet FOC sub-requirements defined by TRADOC. This approach is based on the multiplication of three factors: how many situations encountered by warfighters to which the system derived from the ATO can make a contribution, the size of the gap space in an FOC that the system can mitigate, and the size of the contribution that the system can make to fill the gaps. The cost has three components: (1) acquisition, (2) upgrade, and (3) operating and maintenance (Chow et al., 2009).

Here is a listing of some of the applications of this method:

- Determination of the extent to which the FOC requirements would be met if all existing ATOs were completed and their systems fielded, thereby allowing the Army to trace the impact of ATOs on FOCs.
• Identification and introduction of new ATOs for which existing ATOs leave gaps in meeting FOC requirements.

• Determination of the subset of existing ATOs that can meet all individual FOC requirements at the lowest total remaining lifecycle cost.

• Determination of the extent to which each of the individual FOC requirements is exceeded.

• Determination of the optimal distribution of funds across FOC requirements.

• Determination of which set of ATOs should be ranked high and should protected from Army budgetary cuts. (Chow et al., 2009).

2.4.2.3. RAND Portfolio Analysis Tool (PAT)

PAT is not a model in the usual sense; rather, it is a cross-platform spreadsheet tool, built in Microsoft Excel, which facilitates planning by presenting information in a way that is useful to senior leaders. PAT can use a variety of separate or embedded models as sources of input data. PAT has many useful features (Davis & Dreyer, 2009):

• PAT generates stoplight charts, simple color-scorecard summaries of how options rate on a number of juxtaposed criteria, such as measures of capabilities, risks, upside potential, and costs.

• PAT generates its summaries from more detailed considerations, which can be viewed by drilling down to a level that provides assumptions, a terse logic, and a measure of rigor, even for qualitative assessments. Two levels of drilldown are available.

• PAT allows the analyst to quickly recognize key assumptions and to change them interactively. This may be done parameter-by-parameter or more broadly. These analyses are greatly facilitated by the MRM feature.

• PAT allows the analyst to quickly change how summary depictions are generated. Choices include; simple linear weighted sums, some nonlinear “weakest link” methods, linear weighted sums with threshold constraints, and rank ordering.
• PAT links to even more detailed information, such as that of an embedded or connected capabilities model, data generated separately from a capabilities model, empirical data, or structured judgments.

• Although PAT emphasizes multi-objective scorecards, it also generates scores of overall effectiveness or cost-effectiveness which can be used for marginal or chunky marginal analysis about how to spend (or cut) the next increment of funds.

   PAT takes a series of inputs and generates outputs in the form of portfolio-style tables and various charts and graphics. That is, viewed as a “black box,” it primarily generates displays to describe implications of input information in a structured way (Davis & Dreyer, 2009). Many of the inputs, such as the investment options to be compared, are what one might expect. This could include, e.g., separate expenditures in the budget categories of research and development (R&D), acquisition, and operations and support (O&S). Investment options may differ in what is to be developed and how fast, in what will be deployed operationally, and so on. Or they may differ because of alternative technical approaches or because of alternative strategies (Davis & Dreyer, 2009). PAT’s outputs include color-coded scoreboards, which compare options by different objectives or measures, with red indicating poor and green indicating good; tabular outputs on overall effectiveness and cost; and standard charts, such as charts of cost versus time (Davis & Dreyer, 2009).

2.4.2.4. The Mitre Corporation Portfolio Analysis Machine Tool (PALMA)

The Portfolio Analysis Machine Tool (PALMA®), from the Mitre Corporation, is “a computer program that helps decision-makers to select the best portfolio (combination) of investments from a set of potential investment options.” The tool is based on a mission tree structure, wherein a “strategy-to-task” tree is defined at several levels. Missions are decomposed into sub-missions, then further decomposed into subordinate missions, functions and tasks. At the lowest level (referred to as a “leaf node”) data for the forces and systems that provide relevant capabilities are defined in terms of effectiveness and cost. PALMA provides an assessment of which investment options should be funded to achieve the desired goals for the task. Like the RAND PAT, the displays are in terms of stoplight charts. The performance at each node in the hierarchy is specified by a number from 1 to 100, representing value judgments by
subject-matter experts made at the leaf nodes. Performance at higher levels are computed as weighted averages of the scores at the contributing lower levels based on heuristic “importance” percentage weights (again specified by subject-matter experts). The numerical ranges are translated into red-yellow-green-blue color codes for display purposes. By changing the mix of investment options, the performance at the leaf nodes will change, and those changes will “bubble up” to changes at the mid and top levels. Thus the user can determine a mix of investment options that provide a satisfactory assessment at the mission level. To assist the user, the tool can display efficient frontier charts of cost versus effectiveness to highlight which investment choices provide the most effectiveness benefit for a particular investment level. The least cost-effective systems will lie far away from the efficient frontier, which facilitates their identification for tradeoff analysis. The displays facilitate exploratory analysis by allowing the user to include or exclude an investment option with a simple mouse click. A special version of the model is also available allowing time-phasing of investment options and budgets across years. PALMA is similar to the RAND methodology, but there are differences. For example, PAT does not compute a weighted score for the top-level assessment from sub-tier assessment. Instead, it simply assigns the lowest score from the sub-tier to the next level. So if four sub-tiers are rated “green, green, yellow, red,” the next higher tier would be rated “red” (Porter, Bracken, Mandelbaum, & Kneece, 2008).

2.4.2.5. Mitre’s Matrix Mapping Tool (MMT)

The Matrix Mapping Tool (MMT) was developed for the Office of the Under Secretary for Acquisition, Technology, and Logistics, (OUSD AT&L) and Joint Staff/J-8 to facilitate cross organization coordination in support of capabilities based planning, analysis, and acquisition. The MMT is a database with supporting software that documents relationships between warfighting activities, the UJTL, systems, ACTDs, roadmaps, and capability areas. It allows for a common set of reusable data to support portfolio management (functional, operational), analysis of capability gaps, and other studies where it is necessary to understand the relationships across the dimensions listed above. This tool should be of particular value to EoA study groups because of its ability to display a wealth of the pertinent information about acquisition programs and how they link to capability areas. (Porter et al., 2008).
SECTION 3. IBCT CPR SUMMARY

The TRADOC (Training and Doctrine Command) Analysis Center’s (TRAC) role in the CPR process began only recently when in mid-February 2013 the Director, TRAC agreed to provide analytic support to the Infantry Brigade Combat Team (IBCT) CPR development effort. The IBCT CPR was the first of the so-called formational CPRs (which we defined in section 2.2) in contrast to all of the previous ones that had been done and classified as functional CPRs (see section 2.2. for definition). The function-based CPR structure was put in place several years ago by then-VCSA LTG Peter Chiarelli. Recently, however, LTG Keith Walker, the director of the Army Capabilities Integration Center (ARCIC), directed a shift toward formation-based CPRs, saying:

We don't fight as a portfolio -- we fight as formations. So, we're going to take a horizontal look across those portfolios and have a formation-based portfolio review. We have warfighting functions . . . [and] they are not the same as the capability portfolios. Then, in the acquisition world, we have our [program executive officers] and they don't exactly line up. Then, in the way we resource our Army with [program evaluation groups (PEG)], they don't exactly line up either. There are different bins and somehow we've got to integrate them all. They must inform our PEGs … because it provides better information to those folks who are in the business of resourcing our Army so they can adjust (Bertuca, 2013).

LTG Walker also said that he expected the formation-based CPRs to displace the function-based system in the near future.

The IBCT CPR was centered on eliminating capability redundancies in the formation. The IBCT remains the Army’s most numerous, versatile, and adaptive combat formation. Its ability to deploy rapidly and operate in complex terrain distinguishes it from other brigade combat teams, such an Armor or Stryker BCT, while maintaining its forcible entry capabilities. The inherent limitations of the IBCT pose a number of challenges for the formation. In order to enhance combat effectiveness, MCoE is focusing development of DOTMLPF solutions for the IBCT on the following priorities: Operational Depth and Tempo, Tactical Mobility and Agility, Firepower, Mission Command, and Sustainment & Protection (House, 2013). On 29 July 2013, MCoE briefed the results of the first formation-based CPR, to the Vice Chief of Staff of the Army (VCSA). The purpose of this briefing was to provide an assessment
across Doctrine, Organization, Training, Materiel, Leader Development, Personnel and Facilities (DOTMLPF) domains of IBCT capabilities and make recommendations for functional portfolio prioritization. With strong support from other CoEs, TRAC, ARCIC and HQDA, MCoE assessed current and projected strengths and weaknesses in the IBCT formation across all warfighting functions and identified opportunities to expand combat effectiveness through integrated DOTMLPF solutions (House, 2013).

TRAC became involved in this effort rather “late in the game”, with the Capability Needs Analysis (CNA) 16-20 already scheduled to begin in April of 2013. As a brief and relevant aside, the CNA process is a TRADOC-led assessment of the Army's ability to perform future organizational and functional missions as defined by joint and Army concepts, taking into account existing and programmed DOTMLPF solutions (ARCIC, 2013). TRAC did participate in the event with the intent of gaining insight into the CNA process and acquiring data to support IBCT CPR solution’s trade’s analysis. TRAC also agreed to provide feedback to ARCIC with recommendations for improving the CNA process. TRAC then leveraged the data captured during CNA 16-20, which in reality was an exercise in SME elicitation, and developed a list of potential trades that the TRADOC Capabilities Manager (TCM) IBCT might bring forward to senior Army leadership using a goal program that sought to maximize utility according to a set of prioritized task-scenario pairings subject to a series of postulated funding level constraints. The final brief that TRAC-WSMR provided can be found at this link: https://hq.tradoc.army.mil/sites/trac/Projects/060097/Shared%20Documents/TRAC%20Analysis%20Support%20to%20IBCT%20CPR%20Final%20Brief.pptx

TRAC-WSMR has continued working with MCoE and the formational CPR process this FY with developing another optimization program in support of the Armor Brigade Combat Team CPR, due to be completed late in the summer of 2014. The TRAC-MTRY project team not only shared our current work on CPR with TRAC-WSMR in order to possibly give them some ideas for future direction, but also provided an informal type peer review of their optimization model at several different times during its development, and will continue to do so until the work is complete.
SECTION 4. ALTERNATE METHODOLOGY

This section describes the main focus of this research project, the development of an alternative trade space analysis to support the CPR process. Our methodology can be divided into five phases as seen in Figure 2.

Figure 2: Trade Space Analysis Methodology

During the Subject Matter Expert (SME) elicitation process, we gather data from the SMEs, by warfighting function, as to the effectiveness of a solution in providing a pre-defined required capability through the lens of a specific scenario, as well as the overall risk of a solution to said required capability if it is not implemented. During this initial phase, SMEs will also weight the importance of each required capability / scenario pairing using a measure of consensus. We then calculate the effectiveness of a solution using the SME effectiveness scoring data, Bayes Law, and the weights from phase 1 in constructing an additive value model. Next we determine the risk while using SME risk scoring data to construct a distribution of risk scores across SMEs and
then find the mean of the super-quantile. Finally we display the results in terms of the 3 key elements that would enable decision making; cost, effectiveness, and risk.

4.1. SME ELICITATION

The SME elicitation process that the project team designed for this effort is similar too, though not identical, to the current method used by the Army, the Capability Needs Analysis (CNA), and can use some of the data already produced by the CNA. As such, it would be possible to only modify the CNA process in order to implement our alternative methodology, instead of completely supplanting the CNA with the “new” process presented here. In today’s fiscally constrained environment, this would be both more feasible and practical. The elicitation process can be divided into two distinct parts; the determination of weights, and the characterization of effectiveness and risk. Each part will be discussed below in turn.

4.1.1. Weight Determination

The process begins by dividing the SMEs into their respective “area of expertise” or warfighting function (WfF), which includes Mission Command, Movement and Maneuver, Intelligence, Fires, Sustainment, and Protection. Once this is done, each group of warfighting function SMEs will weigh the importance of each required capability / scenario pairing using a predetermined categorical scale (i.e., a Likert scale). The output of this process will be a single weight for each required capability / scenario pair, per WfF, utilizing a measure of consensus to combine all of the disparate WfF SMEs weights into a single one. This measure of consensus, whose form was inspired by the Shannon entropy, is simply a measure of the degree of attraction to a mean value. It is a measure that characterizes the entire set of SMEs or stakeholders, and thus a measure of the collective “feeling” (Tastle, Abdullat, & Wierman, 2010). Though generally requiring an ordinal scale, the consensus measure can be used with interval and ratio scales, and results in an easily interpretable number between zero and one (unlike variance or standard deviation) (Tastle et al., 2010). The equation for consensus is:

\[
Consensus(X) = 1 + \sum_{i=1}^{n} p_i \log_2 \left(1 - \frac{|X_i - \mu_x|}{d_x}\right)
\]
where $X$ represents the list of Likert categories, $X_i$ is an element of $X$, $\mu_X$ is the mean of $X$, and $d_x$ is the width of $X$, $d_x = X_{max} - X_{min}$. The mean of $X$ is given in the usual way by $\mu_X = \sum_{i=1}^n p_i X_i$. We provide a small visual example of what the SMEs would be asked to inform below in Figure 3 with 3 required capabilities and only 2 scenarios to help illustrate this idea.

![Figure 3: Example of SME Weight Elicitation for “Protection” WfF](image)

An additional requirement is that all of the weights, by WfF, must sum to one.

### 4.1.2. Characterization of Effectiveness and Risk

Once the weights are determined, we can move on to characterizing the effectiveness and risk. First, we ask each of the SMEs for each solution and required capability / scenario pairing, by WfF, to first determine the effectiveness of a solution by answering the following question: “How effective is this particular solution in providing this required capability, given this (a) particular scenario?” The answer is a categorical in nature, using a Likert scale type response. The actual number of scale levels as well as the exact wording of the responses is left up to a study lead and not critical to this effort.

For the risk determination of each solution (by required capability / scenario pairing, by WfF), we follow the well-known and accepted methodology laid out in the Risk Assessment Matrix, found in FM 5-19. We ask the SMEs to determine the two parts of risk, probability, “What is the likelihood of the scenario under consideration,” and severity, “What is the consequence to providing this required capability if this solution is not implemented.” These are then combined into an overall risk category as seen in the matrix below where 4 – Extremely High, 3 – High, 2 – Moderate, and 1 – Low.
Table 1: Risk Classifications

<table>
<thead>
<tr>
<th>Severity</th>
<th>Frequent</th>
<th>Likely</th>
<th>Occasional</th>
<th>Seldom</th>
<th>Unlikely</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catastrophic</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Critical</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Marginal</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Negligible</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

As in section 4.1.1., we provide a small visual example below in Figure 4 using 12 SMEs (2 per WfF), 3 solutions, 3 required capabilities, and 2 scenarios.

Figure 4: SME Elicitation Effectiveness and Risk Example Table

These effectiveness and risk values will be used in subsequent parts of this methodology.

4.2. CALCULATING EFFECTIVENESS USING BAYES LAW

We begin with a discussion on the theoretical underpinnings of using Bayes Law in conjunction with the SME data in order to calculate how effective a solution is in providing a specific required capability given a scenario, and then provide an example with actual SME elicited data for a more complete understanding of the process.
4.2.1. Theoretical Application of Bayes Law

Suppose a survey is conducted during which \( S \) subject matter experts (SMEs) assess the effectiveness of solutions to a required capability and scenario pair with a score which is an element of the set of ordered categories \( \{1, 2, \ldots, C\} \); 1 is the lowest category and \( C \) is the highest. We will subsequently refer to the required capability and scenario pair as simply a double when no confusion will result. We make the following initial assumptions:

1. All SMEs are statistically the same; that is, they are randomly drawn from the same population.

2. The SMEs score a solution to a double independently of the other SMEs, the other solutions, and the other doubles.

3. Each double, \( t \), has an unobservable random probability \( P_{t,c}(s) \) that a SME will assess score \( c \) to solution \( s \). The conditional distribution of the number of \( S \) SMEs that assess score \( c \) to solution \( s \) for the double \( t \) given \( P_t(s) = (P_{t,1}(s), \ldots, P_{t,c}(s)) \) is multinomial with \( S \) trials and probabilities \( P_t(s) = (P_{t,1}(s), \ldots, P_{t,c}(s)) \).

The distribution of \( P_t(s) \) is called the Bayesian prior distribution. The conditional distribution of \( P_t(s) \) given the result of the survey is called the Bayesian posterior distribution. The posterior distribution allows the evaluation of the conditional probability, given the results of the survey, of events such as the event that more than \( (p \times 100)\% \) of future SMEs will assess at least score \( c \) to solution \( s \) for double \( t \) for \( 0 < p < 1 \).

The posterior distribution depends upon the survey results and the prior distribution. If the prior distribution is Dirichlet, then the posterior distribution is also Dirichlet with parameters determined by the survey results and the parameters of the prior distribution (we use Dirichlet since it is the conjugate prior to the Multinomial distribution and will be discussed in more detail later on in the report) (DeGroot, 1970). When there is no a-priori information, a common Dirichlet prior is to set all the shape parameters equal to \( b > 0 \). The closer \( b \) is to 0, the greater the influence of the survey results on the posterior distribution.

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The Dirichlet distribution is a generalization of the beta distribution. Letting $\Gamma$ denote the gamma function, a random variable $\mathbf{P} = (P_1, ..., P_c)$ has a Dirichlet distribution with shape parameters $(\alpha_1, ..., \alpha_c) > 0$ if its density function is of the form,

$$f(p_1, ..., p_c) = \frac{\Gamma(\alpha_1 + \alpha_2 + \cdots + \alpha_c)}{\Gamma(\alpha_1)\Gamma(\alpha_2) \times \cdots \times \Gamma(\alpha_c)} p_1^{\alpha_1-1} p_2^{\alpha_2-1} \cdots p_c^{\alpha_c-1} \quad \text{for } 0 < p_c < 1$$

with $(p_1, ... p_c)$ such that $\sum_{c=1}^{C} p_c = 1$, and 0 otherwise.

A sequential procedure to allocate resources to solutions uses the following criteria for solutions and doubles: an assessment of the likelihood of the double occurring; a minimum acceptable category for the solution, $c_{\text{min}}$ (e.g. solution must be assessed at least category $c$); a minimum acceptable percentage of future SMEs that will assess that category or higher to the solution, (a measure of consensus among the SMEs); a minimum posterior probability that more than that percentage of SMEs will assess the minimum acceptable score or higher to the solution, (1 minus this posterior probability is the risk that a smaller percentage of future SMEs will assign category $c_{\text{min}}$ to the solution).

If $\mathbf{P} = (P_1, ..., P_c)$ has a Dirichlet distribution with shape parameters $(\alpha_1, ..., \alpha_c) > 0$, then for $c \in \{1, 2, ..., C\}$, $\sum_{i=c}^{C} P_i$, has a beta distribution with parameters $\left(\sum_{i=c}^{C} \alpha_i, \sum_{i=1}^{C-1} \alpha_i\right)$. In particular, the marginal distribution of $P_c$ is beta with parameters, $(\alpha_c, \sum_{i \neq c} \alpha_i)$. Thus, $[P_c] = \frac{\alpha_c}{\sum_{i=1}^{C} \alpha_i}$.

We will combine the Dirichlet with the Multinomial models for this effort. Assume that a SME will assess an effectiveness score $c$ to a solution to one required capability scenario pair with unobservable probability $P_c$ independently of the other SMEs. Also assume $\mathbf{P} = (P_1, ..., P_c)$ has a Dirichlet distribution with shape parameters $\alpha = (\alpha_1, ..., \alpha_c)$; this distribution is called the Bayesian prior distribution. Let $N_c$ be the number of the $S$ SMEs that assign effectiveness score $c$ to the solution. Put $N = (N_1, ..., N_c)$. Assume the conditional distribution of $N$ given $\mathbf{P} = (P_1, ..., P_c)$ is multinomial. Note that the distribution of $\mathbf{P}$ in general depends on the required capability and scenario.

The Dirichlet distribution is the conjugate prior for the multinomial distribution. The conditional distribution of $\mathbf{P} = (P_1, ..., P_c)$ given the results of the survey, $N = (N_1, ..., N_c) =$
is Dirichlet with shape parameters \((\alpha_1 + n_1, ..., \alpha_c + n_c)\); this conditional distribution is called the Bayesian posterior distribution. Note that

\[
E[P_c|N_1 = n_1, ..., N_c = n_c] = \frac{\alpha_c + n_c}{\sum_{i=1}^c \alpha_i + \sum_{i=1}^c n_i}
\]

\[
= \frac{\sum_{i=1}^c \alpha_i}{\sum_{i=1}^c \alpha_i + \sum_{i=1}^c n_i} \frac{\alpha_c}{\sum_{i=1}^c \alpha_i} + \frac{\sum_{i=1}^c n_i}{\sum_{i=1}^c \alpha_i + \sum_{i=1}^c n_i} \frac{n_c}{\sum_{i=1}^c n_i}
\]

Thus, the posterior expected value of \(P_c\) given the survey results is a weighted sum of the prior expected value of \(P_c\) and the maximum likelihood estimator which is the fraction of the \(S\) SMEs that assign score \(c\) to the solution.

When there is no a-priori information, a common Dirichlet prior is to set \(\alpha_i = \frac{b}{c}, i = 1, ..., C\) for some constant \(b > 0\). In this case,

\[
E[P_c|N_1 = n_1, ..., N_c = n_c] = \left[\frac{b}{b + \sum_{i=1}^c n_i}\right] \frac{1}{C} + \left[\frac{\sum_{i=1}^c n_i}{b + \sum_{i=1}^c n_i}\right] \frac{n_c}{\sum_{i=1}^c n_i}
\]

The constant \(b\) determines sensitivity of the posterior distribution to the survey data, and is sometimes referred to as the prior strength.

### 4.2.2. Example Application

We next provide an example of implementing Bayes as described above with some survey data captured by the project team at Ft. Lee during a Tactical Wheeled Vehicle workshop. Though this SME elicitation workshop was not set-up and executed as we describe our process in section 4.1, it will still provide the reader with an understanding of how we would apply Bayes.

Subject matter experts, (SMEs), participate in a number of surveys to consider possible solutions to 20 required capabilities. The same 9 possible solutions are considered for all the required capabilities. The number of SMEs that participate in the surveys is not constant.

A SME assigns a score of 1, 2, 3, or 4 to each solution for a required capability. The scores are: 1=low effectiveness; 2=medium effectiveness; 3=high effectiveness; and 4=very high effectiveness.
Let $P_{t,c}(s)$ be the (unobservable) probability a SME assigns score $c$ to solution $s$ for required capability $t$. Assume $P_t(s) = (P_{t,1}(s), P_{t,2}(s), P_{t,3}(s), P_{t,4}(s))$ has a Dirichlet Bayesian prior distribution with all shape parameters equal to 0.25. We initially assume that SMEs score the solutions for each required capability independently of each other and the scores given to other required capabilities and solutions. The Bayes posterior distribution of the probabilities a SME assigns score $c \in \{1, 2, 3, 4\}$ to solution $s$ for required capability $t$, $P_t(s|d)$, also has a Dirichlet distribution.

What is unique about this strategy is that it allows us to make statistical inferences about the entire SME population, not simply from the sample population. Additionally, this approach gives us the flexibility to eliminate or trim solutions that don’t meet a minimum criterion prior to using the optimization model, thereby reducing the amount of input data to the model. For instance, we can choose the following criteria for a solution, required capability/scenario pair:

- Choose a minimum acceptable SME score category, $c_{min}$ e.g. 3 = high effectiveness.
- Choose a minimum probability, $0 < p < 1$, that more than $(p \times 100\%)$ of new SMEs from the same population of SMEs will assign a score of $c_{min}$ or greater to a solution.
- Choose a minimum posterior probability that at least $(p \times 100\%)$ of new SMEs will assign a score of $c_{min}$ or greater.

Those solutions with posterior probability greater than or equal to the minimum posterior probability will be considered for the required capability/scenario pair.

We can perhaps illustrate this better in tabular format. Table 1 displays the maximum posterior probability over all solutions for each required capability $t$ that more than 70% of new SMEs will give a score greater than or equal to $c$ for $c = 2, 3, 4$. The choice of 70% is arbitrary; the posterior probabilities will decrease for 80% and will increase for 60%. In the third column of Table 1 only required capabilities 3, 6, 9, and 12 have at least one solution with posterior probabilities greater than 0.9 that more than 70% of SMEs will score the solution highly effective or very highly effective, (score $\geq 3$). In the second column of Table 1 there are no
solutions for required capabilities 1, 4, and 16 with posterior probability greater than 0.5 that more than 70% of future SMEs will score the solution with at least medium effectiveness, (score ≥ 2). In the fourth column there is at least one solution for required capability 12 for which the posterior probability is 0.75 that more than 70% of future SMEs will score the solution very highly effective, (score = 4); solution 9 has posterior probability 0.75 that more than 70% of new SMEs will score it very highly effective for required capability 12.

<table>
<thead>
<tr>
<th>Required Capability</th>
<th>Maximum Posterior Probability Over 9 Solutions More than 70% of Future SMEs will Score the Solution &gt;= 2</th>
<th>Maximum Posterior Probability Over 9 Solutions More than 70% of Future SMEs will Score a Solution &gt;= 3</th>
<th>Maximum Posterior Probability Over 9 Solutions More than 70% of Future SMEs will Score a Solution = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.28</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.87</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.99</td>
<td>0.98</td>
<td>0.43</td>
</tr>
<tr>
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Table 2: SME Elicitation Probabilities

This example illustrates how we can thus eliminate solutions, by warfighting function, which does not meet the minimum criteria that we choose.
4.2.3. Effectiveness Scoring

Once we have determined the probabilities associated with each solution, by required capability / scenario pair, by WfF, we will use an additive value model using the weights found during the SME elicitation process and the probabilities that we found in the previous section using Bayes Law.

First we define the following:

\[ i = \text{number of required capabilities}, \]
\[ j = \text{number of scenarios}, \]
\[ k = \text{number of solutions}, \]
\[ f = \text{number of warfighting functions}. \]

So, for each of the WfF’s, by solution, we calculate the effectiveness as:

\[
\sum_{1}^{i} \sum_{1}^{j} \text{weight}_{fij} \times \text{probability}_{fijk}, \forall_{f,k}
\]

In words, this means that we multiply each of the required capability / scenario weights by the corresponding probability and then sum over each solution, by WfF. Or, using our small visual example with only 3 solutions we see:

![Figure 5: Example Effectiveness Calculation](image)

And then for each solution, across all WfF, the final effectiveness score is given by:

\[
\sum_{f=1}^{6} \text{Effectiveness}_{fk}, \forall_{k}
\]
Once again, in words, this means that we sum the effectiveness values over all of the WfF, by solution, in order to obtain a single effectiveness score for each of the solutions. This effectiveness score will be what we are trying to maximize in our optimization model. And again, a small visual example yields:

![Effectiveness Scores Across all WfF](image)

### Figure 6: Effectiveness Scoring Example

#### 4.3. CALCULATING RISK

Our methodology for calculating risk consists of two parts. First, we estimate a single probability distribution for each solution that gives the frequency with which an arbitrary SME will respond with a specific risk category (as described in section 4.1.2.). Second, we use the estimated density to compute the superquantile, thereby arriving at a single risk score per solution. We briefly discuss each component in turn.

We determine a density that maximizes the likelihood of receiving the SME risk responses that were actually received during the SME elicitation process, and build a single and distinct density for each solution. In reality, the total number of densities per solution would equal: (the # of required capabilities) × (# of scenarios) × (# of WfF). We would aggregate all of these into one density per solution by simply combining all of the risk responses and correctly classifying their frequencies. The wavy line in Figure 7 is a notional estimate based on the given responses and the information that such a density should be unimodal.
The superquantile is a particularly useful way of numerically quantifying the inherent risk and variability of a particular distribution, in this case the risk density. A superquantile is the conditional average of a fraction of the most pessimistic SMEs. This risk superquantile will then serve as a constraint in our optimization model.

For example, Figure 8 illustrates a superquantile at the 80% level: look at the 100 - 80 = 20% most pessimistic SMEs according to the density in the shaded area. This is a number that then can be communicated to a decision maker as a reasonable conservative estimate, but not overly so. It is a number that takes into account variability in the SME responses as well as the importance of looking at asymmetric variability. We might not be as interested in variability on the low (optimistic) side as on the high (pessimistic) side. Consequently, the use of the standard deviation would not be appropriate.

**4.4. CAPITAL BUDGET OPTIMIZATION MODEL**

The mathematical model used for formulation is the basic capital planning structure (Brown, Dell, & Newman, 2004). In general, the solutions enter the optimal portfolio through maximizing their effect while meeting the risk demand and not exceeding the supply of capital.
As it is currently constructed, the model is for a single time period with the assumption of independent and mutually exclusive proposed solutions.

The model can be extended to consider variable and fixed costs through the introduction of additional budget constraints. Additionally, multi-time periods can be introduced through indexing. The multistage model would require additional constraints but would no longer need the assumption of independence or mutually exclusive solutions. The decisions to include these extensions would be based primarily on data availability.

4.4.1. Mathematical Formulation

Sets

\( k \in K \): set of all solutions possible for portfolio investment.

Parameters

\( c_k = \text{cost per solution } k \) [\$]

\( b = \text{acceptable budget for portfolio investment} \) [\$]

\( r_k = \text{risk associated with solution } k \)

\( d = \text{minimum acceptable risk} \)

\( y_k = \text{effectiveness of solution } k \)

Binary Variable

\( Y_k = \{1, \text{ if solution } k \text{ is purchased}\}

0, \text{ otherwise} \}

Problem (P)

Maximize effectiveness: The objective is to maximize effectiveness associated with optimal portfolio selection. The objective function includes the effectiveness coefficient (Score) calculated using SME input and Bayes Law for each solution.

\[
\max \sum_{k \in K} y_k Y_k
\]

Subject to:
**Acceptable risk:** This constraint dictates the minimum acceptable risk. Solutions must achieve a minimum cumulative risk score ensuring the optimal portfolio meets a determined risk level.

\[
\sum_{k \in K} r_k Y_k \geq d
\]

**Budget Limitations:** Limits the amount of cost to the budgeted amount.

\[
\sum_{k \in K} c_k Y_k \leq b
\]

**Integrality:** Ensures all of the variables are restricted to be binary.

\[X_k \text{ binary}\]

The output for this optimization model is a portfolio of solutions with a maximized effectiveness that achieves both a minimum level of risk as well as cost. The model can be run by maximizing the effectiveness of each solution individually, or grouping portfolio solutions into different Courses of Action (COAs) that would change by varying the acceptable risk score, budget considerations, or the number and identity of fixed solutions.

### 4.5. VISUALIZATION

In designing a visualization strategy, the project team first had to discern what the most important elements are that a decision maker must consider when evaluating the feasibility and desirability of a particular portfolio. For this effort, we identified effectiveness (which we again define as “How effective is a particular solution in providing a required capability, given a scenario?”), risk (based on probability, severity, and the superquantile of the overall risk distribution by solution), and cost. Figure 9 shows an example of visualizing the three key elements in two dimensions with the effectiveness score on the y-axis, risk on the x-axis, and cost portrayed as the relative diameter of the circle; the larger the circle, the more expensive the solution or COA. As Figure 9 is currently shown, we have only “Low” and “High” labels on the axes. This could easily be modified to show actual risk and effectiveness scores, or left as is in order to be more general for ease of use.
Figure 9: Visualization of the 3 Key elements

Ideally, we would like to see a solution or COA in the top left portion of the chart with a small diameter circle (the highlighted green portion), meaning that it is low risk, highly effective, and low cost solution. Conversely, using this visualization tool highlights the less desirable solutions, and allows a decision maker to eliminate those that meet specified criteria.
SECTION 5. SUMMARY

5.1. WAY AHEAD

The difficulty with this approach is in calculating the posterior distribution, i.e., the probabilities, while making the assumption, one that is not always true, that the prior is Dirichlet simply because the SME solution scores have a multinomial distribution. Additionally, updating the shape parameters for the distribution can be time consuming and somewhat troublesome.

There are numerous directions for follow-on work from this effort. Other statistical techniques, vice using Bayes, could be implemented in order to calculate an effectiveness score from SME input. While this effort did not explore additional alternate methodologies, certainly one could appreciate that any non-parametric or distribution free techniques that make no assumptions about the distribution of the SME solution scores would be useful in finding the overall effectiveness of a solution.

5.2. CONCLUSIONS AND RECOMMENDATIONS

At its core, this effort has sought to develop an alternative trade space analysis technique for use in future CPR work that TRAC may take part in. By using Bayes to calculate probabilities that future SMEs will assign certain effectiveness scores to certain solutions (given a required capability / scenario pair) and then weighting these scores according to importance (also derived from SMEs), we attempt to capture how well a solution provides a required capability in a given situation. We also endeavor to capture the inherent risk in a solution of not meeting a capability. We take these two factors, combined with available cost and budget information, and select a portfolio of solutions with a maximized effectiveness that achieves both a minimum risk and budget. Lastly, we visualize these three key elements on a single graph in a way that makes it obvious which solutions are the “best”, meaning those that have the lowest cost and risk but the highest effectiveness.
LIST OF REFERENCES

Davis, P. K., & Dreyer, P. (2009). RAND’s Portfolio Analysis Tool (PAT) (pp. 3–5).