Time-Dependent Modeling of Brillouin Scattering in Optical Fibers Excited by a Chirped Diode Laser

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Abstract—Numerical simulations are used to solve the coupled partial differential equations describing stimulated Brillouin scattering (SBS) built up from random thermal phonons as a function of time and the longitudinal spatial coordinate in an optical fiber. In the case of a passive fiber, a laser beam is incident with constant power, but its frequency is linearly ramped at 1.55 μm at a rate of up to 10^{16} Hz/s. High chirp rates lead to an increased Brillouin spectral bandwidth and decreased gain. The resulting SBS suppression is well described by an adiabatic model and agrees with experimental results. For an 18-m active fiber pumped at 1.06 μm and chirped at up to 2 × 10^{16} Hz/s, the suppression enables output laser powers in the kilowatt range while maintaining a narrow instantaneous linewidth.

Index Terms—Brillouin scattering, chirped lasers, fiber amplifiers, numerical simulation

I. INTRODUCTION

BRILLOUIN scattering is one of the lowest order nonlinear effects that arises in optical fibers and it thus limits the transmitted laser power $P_1$. Above a threshold incident laser power $P_{th}$, the Brillouin scattering becomes stimulated rather than spontaneous and the Stokes backscattered power $P_S$ rises dramatically. Various methods have been proposed to increase the threshold [1]–[5]. In the present paper, we analyze the effect of linearly sweeping the frequency of the pump laser at a fast chirp rate $\beta$. To be effective, the chirp must be large enough that the pump laser is swept out of the Brillouin gain bandwidth $\Delta \nu_B$ within the transit time of the fiber, $t_f = nL/c$, where $L$ is the length of the fiber, $n$ is the core refractive index, and $c$ is the speed of light. That is, one requires $\beta$ to be large enough that $t_f > t_c$ where $t_c = \Delta \nu_B / 2\beta$. (The factor of 2 comes from the fact that the laser and Stokes waves are counterpropagating in the fiber.) Here we analyze values of $\beta$ up to $2 \times 10^{16}$ Hz/s, close to the upper range of chirping that has been demonstrated to date for a diode laser by ramping its current [6], [7].

Section II describes numerical simulations of the Brillouin scattering in a passive fiber with an incident laser beam that is chirped. Section III then presents the simulation results for a 6-km fiber. Such long fibers have a low threshold because $P_{th} \approx 21A/g_0L$ in the unchirped case, where $A$ is the modal area and $g_0$ is the peak Brillouin gain coefficient. Consequently, low laser powers can be used to experimentally verify the simulations. Agreement is found between theory and measurements. Next, Sec IV presents results of the time-dependent simulations for a short fiber ($L = 17.5$ m) having characteristics similar to those used for high-power laser delivery. Correspondingly larger chirps are needed to suppress stimulated Brillouin scattering (SBS) in this case. The results agree with an adiabatic model that depends on frequency (instead of time) for chirps ranging up to $10^{16}$ Hz/s. Finally, Sec V adds laser gain to the simulations to predict the output power from a fiber amplifier. Chirping the input laser beam at $2 \times 10^{16}$ Hz/s enables one to reach the kilowatt range by increasing the SBS threshold.

II. THEORY FOR THE DYNAMIC SIMULATIONS

The Brillouin scattering is modeled by three complex coupled partial differential equations (PDEs) [8],

$$\frac{\partial E_L}{\partial z} + \frac{n \partial E_L}{c \partial t} = \frac{\zeta - \alpha}{2} E_L + i\kappa E_S\rho, \quad (1a)$$

$$-\frac{\partial E_S}{\partial z} + \frac{n \partial E_S}{c \partial t} = \frac{\zeta - \alpha}{2} E_S + i\kappa E_L\rho^*, \quad (1b)$$

$$\frac{\partial \rho}{\partial t} + \pi \Delta \nu_B \rho = i\Lambda E_L E_S^* + f. \quad (1c)$$

We checked that adding the two extra derivative terms from (8) in [5] to the left-hand side of (1c) did not noticeably change our simulation results; so for computational simplicity, those terms are omitted. Here the laser electric field $E_L(z,t)$, Brillouin Stokes-shifted field $E_S(z,t)$, and density variation $\rho(z,t)$ of the fiber from its mean value $\rho_0$ depend on time $t$ and longitudinal position $z$, varying from $z = 0$ at the front face of the fiber to $z = L$ at the rear face. The spectral fullwidth-at-half-maximum (FWHM) of the spontaneous Brillouin peak is $\Delta \nu_B$. For a silica fiber at an incident laser wavelength of $\lambda_L = 1.55$ μm, the refractive index is $n = 1.447$, the mean density is $\rho_0 = 2210$ kg/m$^3$, and the speed of sound...
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is \( v = 5960 \) m/s [8]. The loss coefficient in the fiber is \( \alpha = 0.2 \) dB/km = 0.0461 /km. The laser gain \( \zeta \) is taken to be independent of \( t \) and \( z \) for simplicity; \( \zeta = 0 \) for a passive fiber. The optic coupling parameter is [8]

\[
\kappa = \frac{\pi \gamma}{2 \eta \rho_0 \lambda L M}
\]  

(2)

where the electrostriction coefficient is \( \gamma = 0.902 \) [9] and the polarization is presumed to be completely scrambled in the fiber so that \( M = 1.5 \) [10]. The acoustic coupling parameter is [8]

\[
\Lambda = \frac{\pi n \varepsilon_0 \gamma}{\lambda_L v}
\]  

(3)

where \( \varepsilon_0 \) is the permittivity of free space. The Langevin noise source \( f(z, t) \) is delta-correlated in time and space such that [11]

\[
\langle f(z, t) f^* (z', t') \rangle = Q \delta(z - z') \delta(t - t')
\]  

(4)

where the thermal phonons are described by the strength parameter

\[
Q = \frac{4 \pi k T \rho_0 \Delta \nu_B}{\nu^2 A}.
\]  

(5)

Here \( k \) is Boltzmann’s constant, \( T = 293 \) K is room temperature, and \( A \) is the fiber modal area.

The three PDEs (1) are solved by iterated finite-difference approximations on a grid of time and space points [12]. The spatial step size \( dz \) is chosen to be small enough that a factor of two reduction does not change the final values in (8) below by more than 10%. The temporal step size is \( dt = n dz/c \). The boundary conditions are \( E_S(L, t) = 0 \) and

\[
E_L(0, t) = \sqrt{\frac{2 P_0}{n c \varepsilon_0 A}} \exp \left[ i \pi \beta (t + n L/c)^2 \right]
\]  

(6)

where \( P_0 \) is the constant laser power incident at \( z = 0 \). The argument of the exponential is the chirped phase

\[
\varphi(t, z) = 2 \pi \int_0^{t + n(L - z)/c} \beta t' dt'
\]  

(7)

evaluated at \( z = 0 \). The three complex fields are first calculated along the fiber for times up to \( t = 20 \tau_f \) to ensure that the initial relaxation oscillations [13] have decayed away. The equations are then iterated over 5 more transit times to acquire a statistical average. The transmitted laser power \( P_L \) and reflected Stokes power \( P_S \) are computed by averaging over those additional steps

\[
P_L = \frac{1}{2} \eta n \varepsilon_0 A \langle |E_L(L, t)|^2 \rangle \quad \text{and} \quad P_S = \frac{1}{2} \eta n \varepsilon_0 A \langle |E_S(0, t)|^2 \rangle.
\]  

(8)

The results were checked by verifying that \( P_L \approx (P_0 - P_S) \exp \left( -\alpha L \right) \) in a passive fiber, and agreement was found to within about 1% at zero chirp.

III. DYNAMIC RESULTS FOR A LONG PASSIVE FIBER COMPARED TO EXPERIMENT

We calculated \( P_S \) as a function of \( P_0 \) for a passive 6-km single-mode fiber. The FWHM of the spontaneous Brillouin peak was measured to be \( \Delta \nu_B = 39 \) MHz using a heterodyne technique with an RF spectrum analyzer [7]. The fiber modal area was measured to be \( A = \pi r^2 \) with \( r = 4.55 \) \( \mu \)m by a knife-edge scan in the far field.

The reflectivity \( P_S/P_0 \) is plotted in Fig. 1 for both an unchirped and a chirped laser. For incident powers well below threshold, the reflectivity levels off to a spontaneous value that is independent of chirp and is in good agreement with the experimental value of \( R_0 = (3.0 \pm 0.5) \times 10^{-6} \) measured using an optical spectrum analyzer (OSA). On the other hand, for large incident powers the reflectivity approaches 100%. The threshold (defined as the incident power at which the reflectivity is equal to 1%) is approximately two orders of magnitude larger in the chirped case than in the unchirped case.

To directly compare these results to the experimental data, the total backscattered power is next plotted in Fig. 2 as a function of \( P_0 \). This backscattered power is the sum of the Rayleigh power \( P_R = 2.3 \times 10^{-4} P_0 \) determined from low-power OSA measurements and the Brillouin Stokes power \( P_S \) from Fig. 1. The agreement is reasonable between experiment and theory with no free parameters.

The peak value of the Brillouin gain coefficient is

\[
g_0 = \frac{2 \pi \gamma^2}{n c \rho_0 \nu \lambda_L^2 \Delta \nu_B M}
\]  

(9)

which equals \( 6.4 \times 10^{-12} \) m/W for the parameters given above. This value is a factor of 2 to 4 times smaller than what is typically measured for fibers [14], possibly because of inhomogeneous broadening of the Brillouin peak. According to (9), it is only the product \( g_0 \Delta \nu_B \) that should be the same.
for all polarization-scrambling silica fibers pumped at 1.55 μm and not their individual two values.

The power spectral density (in dBm/Hz) of the reflected Stokes electric field $E_S(0, t)$ was plotted for each simulation run as a function of frequency up to the Nyquist limit. Lineshape functions were then fit to these spectra. The resulting FWHM are plotted as the dots in Fig. 3 for the long fiber at $P_0 = 1 \text{ mW}$, well below threshold, for four different chirps. At the two lower chirps $\beta$ of $10^{10}$ and $10^{12} \text{ Hz/s}$ the lineshapes were found to be Lorentzian, whereas at the two higher chirps of $5 \times 10^{12}$ and $10^{13} \text{ Hz/s}$ the lineshapes were Gaussian. The continuous curve is a plot of the broadening expected from a simple model, namely

$$\Delta v = \Delta v_B + nL\beta/c$$  \hspace{1cm} (10)$$

which fits the simulation results to within the error bars on the widths. Replacing $\Delta v_B$ by $\Delta v$ in (9), this increase in the Brillouin bandwidth implies a reduction in the peak gain and hence a higher threshold $P_{th}$ compared to the unchirped case.

**IV. DYNAMIC RESULTS FOR A SHORT PASSIVE FIBER COMPARED TO OTHER MODELS**

Consider a passive delivery fiber having a length of $L = 17.5 \text{ m}$, a core radius of $r = 13.75 \mu \text{m}$, and a spontaneous Brillouin FWHM of $\Delta v_B = 20 \text{ MHz}$ conservatively chosen to be on the low end of the range of linewidths measured for silica fibers [15] so that the peak Brillouin gain is as large as it might be (namely $g_0 = 1.2 \times 10^{-11} \text{ m/W}$). All other parameters are taken to be the same as those listed in Secs II and III. The reflectivity is plotted in Fig. 4 for seven different values of the chirp $\beta$.

Well below threshold, the reflectivity has a constant value of $R_0 \approx 5 \times 10^{-10}$ independent of chirp and pump power. A heuristic steady-state model [8] for the buildup of the spontaneous Stokes wave from thermal noise predicts that it should be

$$R_0 = \frac{2\pi^2 kT \gamma^2 L_a}{Mn^2 \rho_0 v^2 \lambda^2 A}$$  \hspace{1cm} (11)$$

where the effective absorption length of the fiber is $L_a \equiv [1 - \exp(-aL)]/a$. This model implies $R_0 \approx 3 \times 10^{-9}$ which is only a factor of 6 larger than our simulation value. Similar agreement is found for the long fiber in Sec III: (11) predicts $R_0 \approx 9 \times 10^{-6}$ whereas Fig. 1 has a low-power reflectivity of $R_0 \approx 2 \times 10^{-6}$.

The curves in Fig. 4 cross 1% reflectivity at the threshold incident laser powers, $P_{th}$. The interpolated crossing points are plotted in Fig. 5 as the squares. At the maximum chirp of $10^{16} \text{ Hz/s}$, the threshold has increased by a factor of 50 compared to an unchirped laser source.

In Fig. 5, the threshold power scales linearly with the chirp $\beta$ above $10^{14} \text{ Hz/s}$. These results agree with an adiabatic
model, according to which the threshold should increase as $t_f/t_c$, where the fiber transit time is $t_f = nL/c$ and the time required for the pump laser to chirp out of the Stokes bandwidth is $t_c = \Delta\nu_B/2$. Combining a resonant integration [7] of this increase with the familiar [16] factor of 21 for the unchirped threshold value $g_0 P_{th} L/A$, one obtains

$$P_{th} = \frac{21A}{g_0 L} \frac{t_f/t_c}{\tan^{-1}(t_f/t_c)}$$

plotted as the continuous line in Fig. 5. The agreement with the time-domain simulations is remarkable. Furthermore, one can verify graphically that

$$\frac{t_f/t_c}{\tan^{-1}(t_f/t_c)} \approx 1 + \frac{t_f}{2t_c} = \frac{1}{\Delta\nu_B} \left[ \Delta\nu_B + \frac{nL\beta}{c} \right]$$

thereby showing that (10) is also consistent with the adiabatic model.

V. PREDICTED OUTPUT LASER POWER FOR A SHORT ACTIVE FIBER

Chirping enables one to maintain gain in a fiber amplifier by suppressing the power lost to Brillouin backscattering. To demonstrate this effect, we simulated an ytterbium-cladding-doped fiber with length $L = 18$ m and modal radius $r = 13.75 \mu m$ at a laser wavelength of $\lambda_L = 1.064 \mu m$ with a spontaneous Brillouin FWHM of $\Delta\nu_B = 50$ MHz representing a round average of typical linewidths reported in the literature [16]. As a simple, conservative model of $10\times$ amplification, the gain coefficient for both the laser and Stokes-shifted Brillouin waves was set equal to $\zeta = \ln(10)/L$ in (1). (A more accurate model would simultaneously solve the rate equations for the populations of the ground and excited laser levels to compute the gain from a separate pump source [17].) In Fig. 6, values of $P_L$ calculated from (8) are plotted versus $P_0$ for chirps $\beta$ between $2 \times 10^{14}$ and $2 \times 10^{16}$ Hz/s. At low chirps, the transmitted power rolls off when it surpasses threshold. For example, at a chirp of $2 \times 10^{14}$ Hz/s and incident laser power of 100 W, which is well beyond threshold, in the absence of gain the laser power drops to and levels off at 87 W within the first few meters of the fiber, while the Stokes power correspondingly rises from 0 to 13 W over the same distance. In contrast, with the $10\times$ gain switched on, the laser power drops to a minimum after about 1 m of transit but is then amplified over the remainder of the fiber’s length to 215 W, as plotted in Fig. 6; meanwhile the Stokes beam begins to grow a few meters from the rear end of the fiber and attains about 400 W at the front face. At the highest chirp, in contrast, $10\times$ amplification is maintained for incident powers up to 200 W in Fig. 6, resulting in kilowatt output laser powers.

VI. CONCLUSION

Time-dependent numerical simulations show that SBS in optical fibers can be suppressed by linearly chirping the laser frequency. One significant advantage of this method compared to competing techniques that broaden the incident laser beam with white noise is that a narrow instantaneous linewidth is maintained, enabling coherent combination of the outputs of several fiber lasers for power scaling. Linear chirping allows a mismatch in the length of one amplifier relative to another to be compensated by acousto-optically shifting the frequencies at the input [18].

These dynamic simulations are in good agreement with experimental results for a 6-km fiber. The simulations should therefore also be valid for a short 17.5-m fiber having characteristics appropriate to high-power laser systems. The time-dependent numerical results for a short passive fiber are in good agreement with an analytic adiabatic model, thereby validating use of that model to predict the performance of chirped fiber lasers without requiring the demanding computer overhead of the full time-dependent calculations. The results for a short active fiber indicate that laboratory attainable
chirps enable amplification to kilowatt output powers while maintaining a narrow instantaneous laser linewidth.

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REFERENCES


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