**WAVELET SPECTRAL FINITE ELEMENTS FOR WAVE PROPAGATION IN COMPOSITE PLATES**

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The principal goal of this research is to advance the technology of structural health management (SHM) for composite structures. Wave propagation based methods have shown promise for SHM of composite structures. The spectral finite elements (SFE) method is highly suitable for wave propagation analysis due to its frequency domain approach which yields models that are many orders smaller than conventional FEM. Wavelet spectral finite element (WSFE) method overcomes the “wrap around” problem to accurately model 2-D plate structures of finite dimensions. The specific accomplishments of the research are: developed WSFE model for damaged composite plate with transverse crack; validated WSFE modeling of Lamb wave propagation in healthy composite plates through experimental measurements; implemented ‘baseline-free’ Damage Force Indicator method for delamination detection using dynamic stiffness matrix from WSFE model of healthy plate; developed Modified Time Reversal method for Lamb wave based ‘baseline-free’ damage diagnostics; and investigated effect of tone burst center frequency on instantaneous phase based delamination detection.

**Subject Terms**

structural health management, wavelet spectral finite element, composite structures, damage detection

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Wavelet Spectral Finite Elements for Wave Propagation in Composite Plates

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Final Technical Report

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The principal goal of this research is to advance the technology of structural health management for composite structures. The use of composites for aerospace structures is increasing rapidly; however, composite structures are susceptible to impact damage and delaminations and cracks may reach critical length before visual detection. Wave propagation based methods have shown promise for SHM of composite structures. The spectral finite elements (SFE) method is highly suitable for wave propagation analysis due to its frequency domain approach which yields models that are many orders smaller than conventional FEM. Also, the frequency domain formulation of WSFE enables direct relationship between output and input through system transfer function. Wavelet spectral finite element (WSFE) method overcomes the “wrap around” problem to accurately model 2-D plate structures of finite dimensions, unlike the existing Fourier transform based SFE. The specific accomplishments of the research are:

- Developed WSFE model for damaged composite plate with transverse crack;
- Validated WSFE modeling of Lamb wave propagation in healthy composite plates through experimental measurements;
- Implemented ‘baseline-free’ Damage Force Indicator method for delamination detection using dynamic stiffness matrix from WSFE model of healthy plate;
- Developed Modified Time Reversal method for Lamb wave based ‘baseline-free’ damage diagnostics; and
- Investigated effect of tone burst center frequency on instantaneous phase based delamination detection.

This award partially supported 2 MS students and 1 undergraduate honors student. The research resulted in the publication of 6 conference papers and 4 journal articles (in review/preparation).
1. BACKGROUND AND SIGNIFICANCE

The use of composites for aerospace structures is increasing rapidly due to several advantages such as lighter weight, fewer joints, improved fatigue life, and higher resistance to corrosion. However, composite structures have several forms of damage such as delamination between plies, fiber-matrix debonding, fiber breakage, and matrix cracking. These damages often occur below the surface due to fatigue, foreign object impact, etc., and may not be visible. The existing paradigm for structural safety leads to expensive inspections, unnecessary downtime and retirement, and sometimes catastrophic failures without any warning. Therefore, a structural health management (SHM) system capable of performing both diagnostics and prognostics for composite structures is urgently needed. A reliable SHM system will provide tremendous benefits in terms of life-cycle costs and allow realization of CBM+.

Lamb waves (fundamental symmetric and asymmetric modes) have shown promise for damage diagnosis of composite structures. The fundamental idea behind Lamb wave propagation based diagnostics is that different types of damages interact differently with waves. Therefore, based on the measured time history of the propagated wave, the traveling time, speed reduction, and wave attenuation parameters are extracted and used as the damage identification variables. Due to the dispersive nature of the Lamb waves, accurate identification of localized events is required to determine the location of the damage in the structure. Advanced signal processing using wavelet transform, Hilbert-Huang transform, etc. helps damage recognition.

Conventional finite element method (FEM) is not suited for wave propagation problems since the element size should be comparable to wavelength which is very small at high frequencies. This results in large system size and enormous computational cost. In addition, solving inverse problems (as required for diagnostics) is very difficult using FEM. Spectral finite element (SFE), which follows FE modeling procedure in the transformed frequency domain, is highly suitable for wave propagation analysis. SFE models are many orders smaller than FEM and especially suitable for on-board diagnostics. The frequency domain formulation of SFE enables direct relationship between output and input through system transfer function (or FRF).

Wave propagation analysis in composite plates using wavelet spectral finite element has been investigated in this research. The 2-D Wavelet based SFE (WSFE) can model 2-D plate structures of finite dimensions accurately. This is due to the use of localized Daubechies scaling functions as basis for approximation of the spatial dimension. Presence of elastic coupling in anisotropic materials results in coupled governing differential equations. The final reduced ODEs after spatial and temporal approximations are in the form of a set of coupled ODEs, for each temporal and spatial sampling point. The solution of these ODEs to derive the exact shape function involves determination of wave numbers and the amplitude ratio matrix. Unlike isotropic cases, the process of solution for composite materials is more complicated and is done by posing it as polynomial eigenvalue problem (PEP).
2. Wavelet Spectral Finite Element Modeling with Transverse Crack

The governing differential equations of a homogeneous, symmetric and balanced laminate based on first order shear deformation theory (FSDT) can be expressed in terms of displacements \((u, v, w, \phi, \psi)\) in Fig. 1 as,

\[
A_{11} \frac{\partial^2 u}{\partial x^2} + (A_{66} + A_{12}) \frac{\partial^2 v}{\partial y \partial x} + A_{66} \frac{\partial^2 u}{\partial y^2} = I_0 \frac{\partial^2 u}{\partial t^2} \tag{1}
\]

\[
A_{66} \frac{\partial^2 v}{\partial x^2} + (A_{66} + A_{12}) \frac{\partial^2 u}{\partial x \partial y} + A_{22} \frac{\partial^2 v}{\partial y^2} = I_0 \frac{\partial^2 v}{\partial t^2} \tag{2}
\]

\[
A_{55} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi}{\partial x} \right) + A_{44} \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial \psi}{\partial y} \right) = I_0 \frac{\partial^2 w}{\partial t^2} \tag{3}
\]

\[
D_{11} \frac{\partial^2 \phi}{\partial x^2} + (D_{66} + D_{12}) \frac{\partial^2 \psi}{\partial y \partial x} + D_{66} \frac{\partial^2 \phi}{\partial y^2} - A_{55} \left( \frac{\partial w}{\partial x} + \phi \right) = I_2 \frac{\partial^2 \phi}{\partial t^2} \tag{4}
\]

\[
D_{66} \frac{\partial^2 \psi}{\partial x^2} + (D_{66} + D_{12}) \frac{\partial^2 \phi}{\partial x \partial y} + D_{22} \frac{\partial^2 \psi}{\partial y^2} + A_{44} \left( \frac{\partial w}{\partial y} + \psi \right) = I_2 \frac{\partial^2 \psi}{\partial t^2} \tag{5}
\]

Fig. 1. The plate element with associated DOF

The associated boundary conditions are,

\[
\bar{N}_{xx} = N_x n_x + N_y n_y, \quad \bar{N}_{yy} = N_y n_x + N_y n_y, \quad \bar{V}_z = V_x n_x + V_y n_y \tag{6}
\]

\[
\bar{M}_{xx} = -M_{xx} n_x - M_{xy} n_y, \quad \bar{M}_{yy} = -M_{xy} n_x - M_{yy} n_y \tag{7}
\]
Where $N_{xx}$ and $N_{yy}$ are the applied normal forces in $X$ and $Y$ direction, $M_{xx}$ and $M_{yy}$ are the applied moments about $Y$ and $X$ axes and $V_i$ is the applied shear force in $Z$ direction. The terms $n_x$ and $n_y$ are unit normal into $X$ and $Y$ directions, respectively.

An assumption is made that the lamina is specially orthotropic which is true for many commercially available unidirectional laminae. Transverse motion is studied and therefore the in-plane displacements can be left out in the formulation hence forth. Further discussion of the WSFE formulation can be found in [1, 4-6] and is not reproduced here. Only the final transformed governing equations and boundary conditions are mentioned for completeness. The transformed governing equations that have transverse displacement components are,

$$A_{55} \left( \frac{d^2 \tilde{w}_{ij}}{dx^2} + \frac{d \tilde{\phi}_{ij}}{dx} \right) - A_{44} \beta_i^2 \tilde{w}_{ij} - A_{44} i \beta_i \tilde{\psi}_{ij} = -I_0 \gamma_j^2 \tilde{w}_{ij} \quad i = 0,1,...,m \quad j = 0,1,...,n$$

$$D_{11} \frac{d^2 \tilde{\phi}_{ij}}{dx^2} - (D_{66} + D_{12}) i \beta_i \frac{d \tilde{w}_{ij}}{dx} - D_{66} \beta_i^2 \tilde{\phi} - A_{55} \left( \frac{d \tilde{w}_{ij}}{dx} + \tilde{\phi}_{ij} \right) = -I_2 \gamma_j^2 \tilde{\phi}$$

$$D_{66} \frac{d^2 \tilde{\psi}_{ij}}{dx^2} - (D_{66} + D_{12}) \beta_i \frac{d \tilde{\phi}_{ij}}{dx} - D_{22} \beta_i^2 \tilde{\psi}_{ij} + A_{44} \left( -i \beta_i \tilde{w}_{ij} + \tilde{\psi}_{ij} \right) = -I_2 \gamma_j^2 \tilde{\psi}_{ij}$$

and the boundary conditions can be written as,

$$V_{xij} = A_{55} \left( \frac{d \tilde{w}_{ij}}{dx} + \tilde{\phi}_{ij} \right)$$

$$\tilde{M}_{xij} = D_{11} \frac{d \tilde{\phi}_{ij}}{dx} - i \beta_i D_{12} \tilde{\psi}_{ij}$$

$$\tilde{M}_{yij} = D_{11} \left( -i \beta_i \tilde{\phi}_{ij} + \frac{d \tilde{\psi}_{ij}}{dx} \right)$$

where $m$ and $n$ number of discretization of spatial and time windows.

The schematic diagram of spectral plate element (SPE) with a transverse open and non-propagating crack is shown in Fig. 2. The length of the element in the $X$ direction is $L_x$ and the plate has finite dimension of $L_y$ in the $Y$ direction. The crack is located at a distance of $L$ from the left edge of the plate and has a length $2c$ in the $Y$ direction.
Fig. 2. The plate element with transverse crack

Two sets of nodal displacement fields, \{w_1, \phi_1, \psi_1\} and \{w_2, \phi_2, \psi_2\}, are assumed on the left and right hand side of the crack whose wavenumber domain representations are \{\tilde{w}_1, \tilde{\phi}_1, \tilde{\psi}_1\} and \{\tilde{w}_2, \tilde{\phi}_2, \tilde{\psi}_2\} and they can be expressed as,

\[
\begin{bmatrix}
\tilde{w}_1 \\
\tilde{\phi}_1 \\
\tilde{\psi}_1
\end{bmatrix} = \begin{bmatrix} R \end{bmatrix} \begin{bmatrix} \Theta_1 \end{bmatrix}_{6 \times 6} \begin{bmatrix} C_1 \end{bmatrix}_{6 \times 1},

\begin{bmatrix}
\tilde{w}_2 \\
\tilde{\phi}_2 \\
\tilde{\psi}_2
\end{bmatrix} = \begin{bmatrix} R \end{bmatrix} \begin{bmatrix} \Theta_2 \end{bmatrix}_{6 \times 6} \begin{bmatrix} C_2 \end{bmatrix}_{6 \times 1} \tag{14}
\]

where \[\begin{bmatrix} \Theta_p \end{bmatrix}\] is a diagonal matrix with diagonal terms \[e^{-ik_Lx}, e^{-ik_Ly}, e^{-ik_L(L_L-x)}, e^{-ik_L(L_L-x)}, e^{-ik_L(L_L-x)}\]. \[\begin{bmatrix} R \end{bmatrix}\] is 3X6 amplitude ratio matrix and a total of 12 unknown constants from \{C_1\} and \{C_2\} to be determined using boundary conditions.

The unknown constants \{C_1\} and \{C_2\} can be calculated as a function of the nodal spectral displacements using the boundary conditions as follows.

1. At the left edge of the element \((x = 0)\)

\[
\tilde{w}_1 = \tilde{q}_1, \quad \tilde{\phi}_1 = \tilde{q}_2, \quad \tilde{\psi}_1 = \tilde{q}_3 \tag{15}
\]

2. At the crack location \((x = L_L\) for \(\tilde{w}_1\) and \(x = 0\) for \(\tilde{w}_2\))

\[
\tilde{w}_1 = \tilde{w}_2, \quad \tilde{\psi}_1 = \tilde{\psi}_2 \quad \text{(Continuity of transverse displacement and slope across crack)}
\]
\[
\tilde{\phi}_1 - \tilde{\phi}_2 = \theta M_{xx} \quad \text{(Discontinuity of slope across crack)}
\]
\[
M_{x1} = M_{xx1}, \quad M_{sy} = M_{sy2} \quad \text{(Continuity of bending moments)}
\]
\[
V_{x1} = V_{x2} \quad \text{(Continuity of shear force)} \tag{16}
\]
3. at the right edge of the element \((x = L - L_1)\) for \(\tilde{w}_2\)

\[
\tilde{w}_2 = \tilde{q}_4, \quad \tilde{\phi}_2 = \tilde{q}_5, \quad \tilde{\psi}_2 = \tilde{q}_6
\]  

\(\theta\) is the flexibility of the plate along the crack edge and the way of calculating it is presented in the next section. \(M_{xx1}, M_{yy1}\) and \(V_{s1}\) are the moments and shear force obtained in terms of \(\tilde{w}_2, \tilde{\phi}_2, \tilde{\psi}_2\) and similar for \(M_{xx2}, M_{yy2}\) and \(V_{s2}\). These boundary conditions can be further expressed in matrix form as,

\[
M \{ \mathcal{C} \} = \begin{bmatrix} M_1 & 0 \\ M_2 & M_3 \\ 0 & M_4 \end{bmatrix} \{ \mathcal{C} \} = \begin{bmatrix} u_1 \\ 0 \\ 0 \\ u_2 \end{bmatrix}
\]  

where \(u_1 = \{ \tilde{q}_1, \tilde{q}_2, \tilde{q}_3 \}^T\) and \(u_2 = \{ \tilde{q}_4, \tilde{q}_5, \tilde{q}_6 \}^T\), \(M_1, M_4 \in \mathbb{C}^{3 \times 6}\) and \(M_2, M_3 \in \mathbb{C}^{6 \times 6}\).

The shear force and bending moments at the left and right boundaries of the plate can be written as,

\[
\begin{bmatrix} V_{x1}(0) \\ M_{xx1}(0) \\ M_{yy1}(0) \\ V_{x2}(L) \\ M_{xx2}(L) \\ M_{yy2}(L) \end{bmatrix} = \begin{bmatrix} \tilde{V}_1 \\ \tilde{M}_{xx1} \\ \tilde{M}_{yy1} \\ \tilde{V}_2 \\ \tilde{M}_{xx2} \\ \tilde{M}_{yy2} \end{bmatrix} = \begin{bmatrix} P_1 \\ 0 \\ P_2 \end{bmatrix} \{ \mathcal{C} \}, \quad P_1, P_2 \in \mathbb{C}^{3 \times 6}
\]  

Combining eqns. (18) and (19),

\[
\begin{bmatrix} \tilde{V}_1 \\ \tilde{M}_{xx1} \\ \tilde{M}_{yy1} \\ \tilde{V}_2 \\ \tilde{M}_{xx2} \\ \tilde{M}_{yy2} \end{bmatrix} = \begin{bmatrix} P_1 \\ 0 \\ P_2 \end{bmatrix} \{ \mathcal{C} \} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\]  

\(\tilde{K}\) is the frequency dependent dynamic stiffness matrix of the SPE with transverse open and non-propagating crack.

The bending flexibility on both sides of the crack can be expressed in dimensionless form.

\[
\theta(\bar{y}) = \frac{6H}{L_y} \alpha_{bb}^0 f(\bar{y}) F(\bar{y}), \quad \bar{y} = \frac{y}{L_y}
\]  

where \(H\) is the total thickness of the plate, \(F(\bar{y})\) is a correction function and \(\alpha_{bb}^0\) is the dimensionless bending compliance coefficient at the crack center which can be expressed as,
\[
\alpha_{bb}^0 = \left( \frac{1}{H} \right) \int_0^h \xi (1.99 - 2.47 \xi + 12.97 \xi^2 - 23.117 \xi^3 + 24.80 \xi^4) \, dh, \quad \xi = \frac{h}{H} 
\]  

(22)

for the range, \(0 < \xi < 0.7\). Here \(h\) is the depth of crack at any given \(y\) and \(h_0\) is the central crack depth. \(f(\xi)\) represents the crack shape function defined as,

\[
f(\xi) = e^{-\left(\xi - \xi_0\right)^2 / 2\xi_0^2}, \quad \xi_0 = \frac{c}{L_y}
\]

(23)

where \(e\) is the base of natural logarithm. The correction factor is given as,

\[
F(\xi) = \frac{2c/H + 3(\nu + 3)(1 - \nu)\alpha_{bb}^0[1 - f(\xi)]}{2c/H + 3(\nu + 3)(1 - \nu)\alpha_{bb}^0}
\]

(24)

Fig. (4) presents the variation of \(\theta\) for few different values of crack parameters.

Fig. 4. Variation of \(\theta\) with \(y\) along the crack

The formulated 2-Element WSFE model with a transverse crack is used to study the wave scattering in graphite-epoxy AS4/3501 composite plate (Fig. 5). Time domain results (Fig. 6) show partial reflections of the excited wave due to the crack at different time with varying distances to the crack from the excitation point. The presence of crack does not change the peak amplitude and the group velocity at which the pulse is propagated. As expected, an additional reflection from the crack arrives before the reflection from the fixed boundary. The wave propagation in space at different time instants (with crack at \(L_1 = 0.50\) m away from the excitation point) shows partial wave reflection from crack (Figure 7). Experimental validation of these results is currently under progress.
3. Composite Delamination Detection Using WSFE and Damage Force Indicator Method

The Damage Force Indicator (DFI) Method was implemented for detection of delamination without the need for baseline data. The basic idea in the DFI method is that, using the dynamic stiffness of healthy structure and the measured damage response, the force necessary to create damage, called the ‘damage force,’ can be computed. These damage forces are then normalized and plotted at all sensor locations. The damage force indicator peaks only at those locations which are adjacent to the damage, which clearly helps to locate the damage. Some of the major advantage of the DFI method are that it does not need any baseline measurement, knowledge of input is not necessary and it is computationally very fast, making it an ideal tool for on-line health monitoring when the dynamic stiffness matrix is obtained from WSFE model.
The experimental setup (Fig. 8) and results for Lamb wave measurements are discussed in this section. The composite plate used in this study is fabricated from AS4/3501-6 carbon-epoxy prepreg using vacuum bagging and oven curing technique. The plate has dimensions of 254 x 127 x 1.25 mm (10 x 5 x 0.05 in) and consists of 8 cross-ply [0/90]_{2s}. A piezoelectric (PZT) actuator (diameter 13.5 mm and thickness 0.22 mm) is affixed onto the composite plate using epoxy. A National Instruments PXI 6339 and a BNC-2110 board are used to generate signals and a QuickPack® power amplifier is used to amplify the actuation signal. Tone burst actuates at 15 kHz with 3.5 cycles to generate A\textsubscript{0} Lamb waves (Fig. 9). A Scanning Laser Doppler Vibrometer (SLV) is employed to acquire the signals at designated locations. Based on Lamb wave dispersion curve, phase velocity calculations indicate that the acquired signals are Lamb wave A\textsubscript{0} mode.

Fig. 8. Experimental setup (L) and composite plate with attached PZT actuator (R)

Fig. 9. 15 kHz 3.5 cycle tone burst in time domain (left) and its frequency domain representation (right)
Experimental data was compared with WSFE predictions (Fig. 10) for a healthy composite plate to validate WSFE modeling. Damage is generally inferred by comparing healthy and damaged (delaminated) responses from a structure. At a sensor location close to delamination, there is a clear difference in the second wave packet (Fig. 11). For DFI method, we use only dynamic stiffness matrix from WSFE model (not healthy plate response data) and delaminated plate response. The DFI method correctly identifies damage since the dominant peak is located at degree of freedom 15 which belongs to the sensor located right after the delamination (Fig. 12). Further details are given in [1, 7-8].

Fig. 10. Comparison of velocity response from WSFE and experiments at two sensor locations

Fig. 11. Velocity responses from healthy and delaminated plates (sensor location close to delamination)
4. **Modified Time Reversal method for Lamb wave based ‘baseline-free’ damage diagnostics**

Time reversal method (TRM) is a new approach to mitigate Lamb wave dispersion effects and increase their applicability for SHM. We have developed modified time reversal method (Fig. 13), wherein some of the steps are modified (as denoted by bold font) compared to TRM.

1. A signal is sent from transducer A and recorded at transducer B
2. The received signal at B is reversed in time, that is \( V_B(t) \rightarrow V_B(-t) \)
3. **The time reversed signal is sent from A to B where it is recorded**
4. The received signal at B is time reversed and compared to the original signal actuated

![Modified Time Reversal Method Diagram]

Based on the assumption of linear reciprocity in the time reversal method, the signal due to the time reversal process and the original signal compare exactly; however, for nonlinearities
introduced into the system (damage), the linear reciprocity of the system breaks down and the irregularities between the two signals indicate the presence of damage. Unlike most other health monitoring techniques, this time reversal process introduces a baseline-free method for damage detection. In our MTRM, a major benefit is that only one actuator is necessary instead of two for any actuator/sensor pair. We have shown mathematically that the final result is same as the TRM process adopted by other researchers. Figure 14 shows the experimental set-up for composite plate with impact damages (left) and MTRM results for various damage levels. Results show that magnitudes of both damage indices are directly correlated to the severity of damage. Further details are given in [3, 9-10].

Figure 14. Experimental set-up for composite plate with impact damages (left) and effect of impact on damage index (right). Consistent increase in both damage indices with increasing damage is observed.

5. Effect of tone burst center frequency on instantaneous phase based delamination detection

Lamb waves in plates, known as guided waves, are elastic perturbations that can propagate for long distances in thin plate-like structures with little amplitude loss. But Lamb waves are highly dispersive, in other words wave propagation depends on the frequency-thickness relationship. Therefore, tone burst signals wherein most of the energy is concentrated at a central frequency, are generally used for Lamb wave excitation. Several parameters such as central frequency, number of cycles and amplitude of the incident Lamb wave may have significant impact on damage detection. Published papers discuss the effects of actuation pulse parameters in a general way, but specifics are seldom reported. In our research, frequency of tone burst signals is varied from 20 to 80 kHz with 5 kHz increments with fixed number of cycles (3.5 cycles) and actuator voltage input. Piezoelectric (PZT) transducers are used as actuators and both PZT and LDV perform sensing. LDV collects responses of flexural waves and PZTs collect responses of both flexural waves and axial waves. Those responses are evaluated with three different methods, namely comparisons of energy, frequency and phase between healthy and delaminated plates. A 10 mm wide delamination is created at the mid-plane along the width of 8-layer cross-ply plate. Wavelet transform (WT) is used for frequency evaluation and both WT and Hilbert-Huang transform (HHT) are used for phase comparisons.
The composite plate used in this study has dimensions of 432 x 324 x 1.25 mm which is fabricated from 8 plies of AS4/3501-6 pre-preg using vacuum bagging and oven curing technique. Hi-temp mold release wax from PARTALL was applied between 4th and 5th layers to create the delamination area (10 x 152 mm). Six circular PZTs (7 x 0.2 mm) are affixed onto the composite plate at 50 mm distance from each other using epoxy (Fig. 15). An Agilent 33220A 20 MHz function/Arbitrary waveform generator is used to generate tone burst signals and an AA Lab systems piezo-amplifier is used to amplify the excitation signals. Tone burst excitation signals (3.5 cycles) are sent to each PZT at 13 different frequencies from 20 to 80 kHz in 5 kHz increments. PZT sensors are connected to BK Precision 2540 60 MHz digital oscilloscope which sends stored signals to computer.

![Figure 15. AS4/3501-6 pre-preg cross ply composite plate with PZTs (left) and schematic diagram with delamination area (right)](image)

In HHT, empirical mode decomposition (EMD) is used to decompose original signals into N intrinsic mode functions (IMFs). The Hilbert transform of individual IMF’s gives instantaneous phase and the total instantaneous phase is obtained by a sum of the instantaneous phases for each IMFs. Phase difference calculation with HHT shows that the signal for Path 2 has significant phase change compared to other paths (Fig. 16). To quantify the magnitude of phase change, Figure 17 shows the mean of phase difference values for each path (with Path 5 as reference). Path 2 (delamination area) shows much larger phase difference and indicates damage very clearly. The phase difference is somewhat smaller at lower frequencies. Thus the experimental results for the selected frequency range (20 kHz to 80 kHz with 5 kHz increments) show significant effects of actuation frequency on both energy and phase difference angle [2, 11-12]. Further studies will include understanding the relationship between actuation frequencies and energy and phase difference observed and different size of delamination area.
6. Personnel  
This award partially supported 2 MS students (Dulip Widana-Gamage and Inho Kim) and 1 undergraduate honors student (Ryan Watkins). Dulip is continuing at Clarkson as PhD students while Ryan and Inho joined PhD programs at University of Michigan and Virginia Tech, respectively. Ryan won NSF Graduate Fellowship as well as DoD NDSEG Fellowship.
7. Publications (* indicates students supported by current project)

**Theses**


**Journal and Conference Papers**


