Endogenous Split Awards as a Protest Management Tool: A Modeling & Computational Approach

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Bid Protests & Split Awards: Agenda

- Managing bid protests in DoD procurement
- Simple model of bidding & protest process
- Split awards as a protest management tool
- Key question: What is the right split?
- Bids & prices with fixed split awards
- Bids & prices with endogeneous split awards
- Conclusions
- Research agenda moving forward
"Managing" Bid Protests

- **Objective is not to minimize** number of bid protests
- **Protests intended to correct procurement mistakes**
  - **Honest mistake**: Limited information & bounded rationality
  - **Dishonest mistake**: Bias or fraud by procurement officials
- **Objective is to “right size”** number of protests
  - Encourage protests that correct (significant) mistakes
  - Discourage protests that don’t make significant corrections
- **Modeling** the process could help identify, compare, & characterize **levers of control** for managing protests
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Modeling Bid Protests

- As noted, the intended role of bid protests is, in the most general terms, to correct procurement mistakes.
- Such mistakes – whether honest or dishonest – result from some form of imperfect decision-making.
  - How best to model such imperfection?
- Consider a model driven by imperfect information.
  - Imperfect info $\rightarrow$ small mistake more likely than big mistake.
  - Bias $\rightarrow$ small injustice more likely than big injustice.
- Imperfect information consistent with empirical results.
  - “Agency mis-evaluation” is by far the most commonly cited reason for sustaining a DoD bid protest (Gansler, et al.).
Simple Model of Bid & Protest Process

Vendor 1 with cost $C_1$ submits bid $P_1$

Vendor 1 decides whether or not to file protest

Vendor 2 with cost $C_2$ submits bid $P_2$

Buyer compares bids with imperfect information

Buyer perceives $P_1 > P_2$

Vendor 1 decides whether or not to file protest

Protestor incurs cost $K_p$

Buyer incurs cost $K_B$

P_1 > P_2

Vendor 2 decides whether or not to file protest

P_1 < P_2

No

Vendor 1 profit = 0

Vendor 2 profit = $X(P_2 - C_2)$

Buyer cost = $XP_2$

Above excludes protest costs

Vendor 1 profit = $X(P_1 - C_1)$

Vendor 2 profit = 0

Buyer cost = $XP_1$

Above excludes protest costs

Yes

No

Vendor 1 with cost $C_1$ submits bid $P_1$

Vendor 2 with cost $C_2$ submits bid $P_2$

Buyer perceives $P_1 < P_2$
Managing Vendor Protest Incentives

- Losing vendor 1 protests iff $\text{Prob}(P_1<P_2) \times X - K_P > 0$

- Recall the two goals of protest management:
  1. Encourage/allow “good” or efficient protests
  2. Discourage “bad” or inefficient protests

- Levers of control?
  - $\text{Prob}(P_1<P_2)$ ➔ Influence initial assessment accuracy ➔ Change or shift burden of proof
  - $K_P$ ➔ Influence expected costs ➔ Different costs for successful vs. failed protests
  - $X$ ➔ Influence gain from successful protest ➔ Split awards
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Bidding with *Fixed Award Splits*

**Contract splits:**
- \( S_L = \) Share or split awarded *low-price* bidder
- \( S_H = \) Share or split awarded *high-price* bidder
- \( S_L + S_H = 1 \)
- \( 0 \leq S_H \leq \frac{1}{2} \) & \( \frac{1}{2} \leq S_L \leq 1 \)

**Award Determination:**
- If final decision is that \( P_1 < P_2 \):
  - Vendor 1 awarded contract to produce \( S_L X \) units
  - Vendor 2 awarded contract to produce \( S_H X \) units
- If final decision is that \( P_1 > P_2 \):
  - Vendor 1 awarded contract to produce \( S_H X \) units
  - Vendor 2 awarded contract to produce \( S_L X \) units
Bid & Protest Process with Split Awards

Vendor 1 with cost $C_1$ submits bid $P_1$

Buyer compares bids with imperfect information

Vendor 2 with cost $C_2$ submits bid $P_2$

Vendor 1 decides whether or not to file protest

Yes

Vendor 1 profit = $S_H (P_1 - C_1)$

Vendor 2 profit = $S_L (P_2 - C_2)$

Buyer cost = $X(S_H P_1 + S_L P_2)$

Above excludes protest costs

No

P_1 > P_2

Buyer perceives $P_1 > P_2$

Vendor 1 decides whether or not to file protest

Yes

Protestor incurs cost $K_p$

Vendor 1 profit = $S_L (P_1 - C_1)$

Vendor 2 profit = $S_H (P_2 - C_2)$

Buyer cost = $X(S_L P_1 + S_H P_2)$

Above excludes protest costs

No

P_1 < P_2

Buyer perceives $P_1 < P_2$

Vendor 2 decides whether or not to file protest

Yes

Vendor 2 profit = $S_H (P_2 - C_2)$

Vendor 1 profit = $S_L (P_1 - C_1)$

Buyer cost = $X(S_L P_1 + S_H P_2)$

Above excludes protest costs

No

P_1 > P_2

Vendor 1 decides whether or not to file protest

Yes

Protestor incurs cost $K_p$

Vendor 1 profit = $S_L (P_1 - C_1)$

Vendor 2 profit = $S_H (P_2 - C_2)$

Buyer cost = $X(S_L P_1 + S_H P_2)$

Above excludes protest costs

No

P_1 < P_2

Buyer perceives $P_1 < P_2$

Vendor 2 decides whether or not to file protest

Yes

Vendor 2 profit = $S_H (P_2 - C_2)$

Vendor 1 profit = $S_L (P_1 - C_1)$

Buyer cost = $X(S_L P_1 + S_H P_2)$

Above excludes protest costs
Revised Vendor Protest Incentives

- **Winner-take-all awards:** Losing vendor 1 protests
  \[ \text{iff } \text{Prob}(P_1<P_2) \times X - K_P > 0 \]

- **Split awards:** Losing vendor 1 protests
  \[ \text{iff } \text{Prob}(P_1<P_2) \times (S_L-S_H)X - K_P > 0 \]

- Split awards **raise the hurdle** for profitable protest
  - Is the hurdle high enough to limit “bad” protests?
  - Is the hurdle low enough to allow “good” protests?

- **Defacto split awards** already a *response* to protests
  - Alternative contracts, subcontracts, agency settlements, “Fed mail” buy-offs
  - Why not formalize this “under the table” process?
Bid Protests & Split Awards: Agenda

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- Research agenda moving forward
Key Question: What is the Right Split?

- Higher $S_H \rightarrow$ lower protest incentive
  - $\Pi_1(\text{protest}) = \text{Prob}(P_1<P_2) \times (1-2S_H)X - K$
  - $\frac{\delta\Pi_1(\text{protest})}{\delta S_H} = -2X \times \text{Prob}(P_1<P_2)$

- Higher $S_H \rightarrow$ higher total contract expense
  - Winner-take-all cost = $XP_L$
  - Split-award cost = $X(S_HP_H + (1-S_H)P_L)$
  - Difference = $XS_H(P_H - P_L)$

- Higher $S_H \rightarrow$ incentive to submit higher bid
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Focus on bid-stage only (for now):

- Ignore “continuation value” of protest stage
  - Effect of protest on bidding strategy ambiguous
- Also ignore buyer’s imperfect information
  - Assume buyer perfectly informed regarding $P_1$ & $P_2$
  - Symmetric imperfect info $\Rightarrow$ neutral impact

Expected profit function:

- $E\Pi_1(P_1) = X(P_1-C_1)[\text{Prob}(P_1>P_2)S_H+\text{Prob}(P_1<P_2)S_L]$
- $= X(P_1-C_1)[S_L-\text{Prob}(P_1>P_2)(S_L-S_H)]$
Equilibrium Bidding with Fixed Splits

Expected profit function:

- Assume $C_1$, $C_2$ identically & independently distributed over interval $[0,M]$
- Symmetric bidding strategy $\lambda(C)$
  - $\lambda: [0,M] \sim [0,M]$
  - $\lambda(M) = M$

Equilibrium bidding strategy:

- $(C_1) = \frac{S_H M + (S_L - S_H) (1 - F(C_1)) E(C_2|C_2 > C_1)}{S_L \left( S_L - S_H \right) F(C_1)}$
- Complete derivation included in appendix
Equilibrium Bidding with Fixed Splits

- Let $C_1, C_2 \sim U[0,100] \rightarrow$

$$(C_1) = \frac{S_{HM} + (S_L S_H) (1 - \text{F}(C_1)) \text{E}(C_2 \mid C_2 > C_1)}{S_L (S_L S_H) \text{F}(C_1)}$$

$$(C_1) = \frac{100S_H + (S_L S_H) (1 - \frac{1}{100} C_1) \frac{1}{2} (C_1 + 100)}{S_L \frac{1}{100} (S_L S_H) C_1}$$

$$(C_1) = \frac{20,000S_H + (S_L S_H) (100 - C_1) (C_1 + 100)}{200S_L 2(S_L S_H) C_1}$$

$$(C_1) = \frac{20,000S_H + (1 - 2S_H) (10,000 - C_1^2)}{200S_L 2C_1 (S_L S_H)}$$

$$(C_1) = \frac{20,000S_H + 10,000 - C_1^2}{200S_L 2C_1 (S_L S_H)}$$

$$(C_1) = \frac{20,000S_H + 2S_H C_1^2}{200S_L 2C_1 (S_L S_H)}$$

$$(C_1) = \frac{10,000 + (2S_H - 1) C_1^2}{200S_L 2C_1 (S_L S_H)} = \frac{10,000 \ (S_L S_H) C_1^2}{200S_L 2C_1 (S_L S_H)}$$
## Equilibrium Bidding with Fixed Splits

<table>
<thead>
<tr>
<th>$S_H$</th>
<th>$S_L$</th>
<th>$(C_1)$</th>
<th>Calculation</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td></td>
<td>( \frac{10,000 C_1^2}{200 \cdot 2C_1} )</td>
<td>( \frac{(100 - C_1)(100 + C_1)}{2((100 - C_1)(100 + C_1))} = 50 + \frac{1}{2} C_1 )</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>( \frac{10,000 \cdot 0.8C_1^2}{180 \cdot 1.6C_1} )</td>
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<tr>
<td>0.2</td>
<td>0.8</td>
<td>( \frac{10,000 \cdot 0.6C_1^2}{160 \cdot 1.2C_1} )</td>
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</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>( \frac{10,000 \cdot 0.4C_1^2}{140 \cdot 0.8C_1} )</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>( \frac{10,000 \cdot 0.2C_1^2}{120 \cdot 0.4C_1} )</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>( \frac{10,000 \cdot 0}{100 \cdot 0} )</td>
<td>( = 100 )</td>
</tr>
</tbody>
</table>
Equilibrium Bidding with Fixed Splits

Equilibrium Bid

Vendor Cost

$S_H = 50\%$
$S_H = 40\%$
$S_H = 30\%$
$S_H = 20\%$
$S_H = 10\%$
$S_H = 0\%$

(Winner-Take-All)

$C_1 \sim U[0,100]$
$C_2 \sim U[0,100]$
Average Price / Unit with Fixed Splits

Split Awarded to Higher-Priced Bidder

0% 10% 20% 30% 40% 50%

72 76 81 87 93 100
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Extension: Endogenous Split Awards

- Split awards **reduce frequency** of bid protest 😊
- **BUT 2 cost inflation effects** from split awards 😞
  - Direct additional cost = $XS_H(P_H - P_L)$
  - Indirect additional cost = bid inflation
- **Note:** Both inflation effects mitigated if size of $S_H$ is inversely related to $(P_H - P_L)$
- Potential solution: **Endogenous split awards**
  - Let $R_L = P_L / P_H$ (such that $0 \leq R_L \leq 1$)
  - Let $S_H = F(R_L)$
  - $0 \leq F(R_L) \leq \frac{1}{2}$
  - $F(R_L)$ increasing in $R_L$
Example Split Award Function

- Let $S_H = \alpha R_L^\beta$
  - $\alpha = \text{maximum share}$ to high-price bidder ($0 \leq \alpha \leq \frac{1}{2}$)
  - $\beta \geq 0$
  - $S_H$ is increasing in $\alpha$ & $R_L$
  - $S_H$ is decreasing in $\beta$
- Buyer decision: What are the best $\alpha$ & $\beta$?
## Split Award Scenarios with $S_H = \alpha R_L^\beta$

<table>
<thead>
<tr>
<th>$\alpha = 0$</th>
<th>$\beta = 0$</th>
<th>$0 &lt; \beta &lt; 1$</th>
<th>$\beta = 1$</th>
<th>$1 &lt; \beta &lt; \infty$</th>
<th>$\beta = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_H = 0$</td>
<td>$S_H = 0$</td>
<td>$S_H = 0$</td>
<td>$S_H = 0$</td>
<td>$S_H = 0$</td>
<td>$S_H = 0$</td>
</tr>
<tr>
<td>Winner-Take-All</td>
<td>Winner-Take-All</td>
<td>Winner-Take-All</td>
<td>Winner-Take-All</td>
<td>Winner-Take-All</td>
<td></td>
</tr>
<tr>
<td>$0 &lt; \alpha &lt; \frac{1}{2}$</td>
<td>$S_H = \alpha$</td>
<td>$0 \leq S_H \leq \alpha$</td>
<td>$0 \leq S_H \leq \alpha$</td>
<td>$0 \leq S_H \leq \alpha$</td>
<td>$S_H = 0$</td>
</tr>
<tr>
<td>Fixed Split</td>
<td>$S_H &gt; \alpha R_L$</td>
<td>$S_H = \alpha R_L$</td>
<td>$S_H &lt; \alpha R_L$</td>
<td>$S_H &lt; \alpha R_L$</td>
<td>$S_H = 0$</td>
</tr>
<tr>
<td>$\alpha = \frac{1}{2}$</td>
<td>$S_H = \frac{1}{2}$</td>
<td>$0 \leq S_H \leq \frac{1}{2}$</td>
<td>$0 \leq S_H \leq \frac{1}{2}$</td>
<td>$0 \leq S_H \leq \frac{1}{2}$</td>
<td>$S_H = 0$</td>
</tr>
<tr>
<td>Even Split</td>
<td>$S_H &gt; \frac{1}{2} R_L$</td>
<td>$S_H = \frac{1}{2} R_L$</td>
<td>$S_H &lt; \frac{1}{2} R_L$</td>
<td>$S_H &lt; \frac{1}{2} R_L$</td>
<td>$S_H = 0$</td>
</tr>
</tbody>
</table>

Better for High Bidder   Worse for Low Bidder

Worse for High Bidder   Better for Low Bidder
Split Award Scenarios with $S_H = \frac{1}{2}R_L^\beta$

![Graph showing share or split for high bidder ($S_H$) versus ratio of low bid to high bid ($R_L = P_L/P_H$). The graph includes different scenarios with $\beta = 0, 1/10, 1/2, 1, 2$. Each scenario is represented by a curve that shows how $S_H$ changes with $R_L$.

Acquisition Research Forum

Dr. Peter J. Coughlan
Split Award Scenarios with $S_H = \frac{2}{5}R_L^\beta$

- Share or Split for High Bidder ($S_H$)
- Ratio of Low Bid to High Bid: $R_L = \frac{P_L}{P_H}$

- $\beta$ values: 0, 1/10, 1/2, 1, 2, 10, Unknown
From Fixed Splits to Endogenous Splits

- Recall that the **equilibrium bidding strategy** under **fixed splits** of \( S_H = 0.4 \) & \( S_L = 0.6 \) with \( C_1, C_2 \sim U[0,100] \) was given by:

\[
P_j = (C_j) = \frac{10,000}{120} \frac{0.2C_j^2}{0.4C_j}
\]

- In equilibrium, this yielded an expected **price per unit** of **93**

- Now, consider the following **endogenous split award function**:
  - \( S_H = \alpha R_L^\beta \) with \( \alpha = \frac{1}{2} \) & \( \beta = 4 \)
  - \( S_H = \frac{1}{2}R_L^4 \)

- If both vendors continue to bid according to the above fixed-split equilibrium bidding strategy, we have:
  - Average split (average value of \( S_H = \frac{1}{2}R_L^4 \)) = 0.4
  - Median split (median value of \( S_H = \frac{1}{2}R_L^4 \)) = 0.4
  - Thus, “**apples-to-apples**” **comparison** to compare bidding under these two award rules (one fixed, one endogenous)
Split Award Scenarios with $S_H = \alpha R_L^\beta$

Note: If vendors bid according to fixed-price bidding strategy, average & median split will be the same under either split rule.
From Fixed Splits to Endogenous Splits

- If vendors follow fixed-split bidding strategy for \( S_H = 0.4 \), expected & median values of the endogenous split should still be \( S_H = 0.4 \)
  - But is this strategy still optimal when splits are endogenous?
- So, when contract splits are endogenous & given by \( S_H = \frac{1}{2} R_L^4 \):
  - What is the equilibrium bidding strategy?
  - What is the average price per unit paid by the buyer?
- We answered these questions computationally
  - Closed-form solution to equilibrium calculation is problematic
  - Thus, solve via “iterative best-response”
    1. Start: Assume vendor 1 follows given fixed-price bid strategy
    2. Compute: What is vendor 2’s best-response bidding strategy?
    3. Iterate: What is vendor 1’s best-response to 2’s best-response?
    4. Repeat: Until you reach a “fixed-point” solution
Equilibrium Bidding with Endogenous Splits

Equilibrium Bid

Vendor Cost

$S_H = \frac{1}{2}R_L^4$

$S_H = 40\%$

$S_H = 0\%$

(Winner-Take-All)

$C_1 \sim U[0,100]$

$C_2 \sim U[0,100]$
Average Price/Unit with Different Splits

Average Price / Unit Paid

- 0% Split Awarded to Higher-Priced Bidder: 72
- $\frac{1}{2}R_L^4$ Split Awarded to Higher-Priced Bidder: 81
- 40% Split Awarded to Higher-Priced Bidder: 93
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Conclusions

Research agenda moving forward
Bid Protests & Split Awards: Conclusions

- **Objective is to manage**, not minimize protests
  - Encourage protests that correct (significant) mistakes
  - Discourage protests that do not

- **Split awards** are lever for protest management
  - Raise the hurdle for profitable protest
  - Filters out unmerited protests more than merited

- **Challenge** is determining the **right split**
  - Higher split to 2nd-vendor reduces protest incentive
  - **BUT** higher 2nd-vendor split also increases costs
  - Higher fixed 2nd-vendor split induces bid inflation

- **Endogenous** split awards offer potential solution
  - Retains protest “filtering” benefits
  - Reduces inflation of bids & average price paid
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Research Agenda Moving Forward

- **Research questions:**
  - What is the optimal split award function?
    - Minimize expected and/or long-term buyer cost
    - Including cost of protests & corrective benefit of protests
    - Include impact of other benefits of split awards
  - What is the impact of changes in key variables?
    - Vendor & buyer information, costs of protest, etc.
  - What is the impact of repeated procurements?
    - Inter-temporal effects: Experience & innovation

- **Research methodology:**
  - Closed-form game-theoretic solutions & dynamics
  - Numerical computation & simulation
Endogenous Split Awards as a Protest Management Tool: A Modeling & Computational Approach

Appendix: Equilibrium Bid Strategy with Fixed-Splits

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Temporary Simplifying Assumptions

- For now, ignore “continuation value” of protest stage
  - Effect of protest on bidding strategy ambiguous
- For now, also ignore buyer’s imperfect information
  - Assume buyer perfectly informed regarding \( P_1 \) & \( P_2 \)
  - Symmetric imperfect info \( \rightarrow \) neutral impact
Expected Profit Function (Bid Stage)

- \( \pi_1(P_1) = X(P_1-C_1)[ \text{Prob}(P_1 > P_2)S_H + \text{Prob}(P_1 < P_2)S_L ] \)

- \( = X(P_1-C_1)[ \text{Prob}(P_1 > P_2)S_H + [1-\text{Pr}(P_1 > P_2)]S_L ] \)

- \( = X(P_1-C_1)[S_L + \text{Prob}(P_1 > P_2)(S_H - S_L)] \)

- \( = X(P_1-C_1)[S_L - \text{Prob}(P_1 > P_2)(S_L - S_H)] \)
Assume $C_1$, $C_2$ identically & independently distributed over interval $[0,M]$

- Distribution function $F$
- Density function $f = F'$

Symmetric bidding strategy $\lambda(C)$

- $\lambda: [0,M] \sim [0,M]$
- $\lambda(M) = M$
Equilibrium Bidding with Fixed Splits

- Calculate optimal bid $P_1$ for vendor 1 assuming:
  - Vendor 1 has cost $C_1$
  - Vendor 2 is bidding according to strategy $\lambda(C_2)$

- $\text{Prob}(P_2<P_1) = \text{Prob}[\lambda(C_2)<P_1] = \text{Prob}[C_2<\lambda^{-1}(P_1)]$
  $= F(\lambda^{-1}(P_1))$

- $\mathbb{E}\Pi_1(P_1) = X(P_1-C_1)[S_L-\text{Prob}(P_1>P_2)(S_L-S_H)]$
  $= X(P_1-C_1)[S_L-F(\lambda^{-1}(P_1))(S_L-S_H)]$

- Chain rule + inverse derivative theorem

\[
\frac{\delta \mathbb{E}\Pi_1}{\delta P_1} = X[S_L-F(\lambda^{-1}(P_1))(S_L-S_H)]
- X(P_1-C_1)(S_L-S_H)f(\lambda^{-1}(P_1))/\lambda'(\lambda^{-1}(P_1))
\]
Equilibrium Bidding with Fixed Splits

- First-order condition

\[S_L - F(\lambda^{-1}(P_1))(S_L - S_H) = (P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1))/\lambda'(\lambda^{-1}(P_1))\]

\[\lambda'(\lambda^{-1}(P_1))[S_L - F(\lambda^{-1}(P_1))(S_L - S_H)] = (P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1))\]

- At symmetric equilibrium, \(P_1 = \lambda(C_1)\) \(\Rightarrow\) \(\lambda^{-1}(P_1) = C_1\) \(\Rightarrow\)

\[\lambda'(C_1)[S_L - F(C_1)(S_L - S_H)] = (\lambda(C_1) - C_1)(S_L - S_H)f(C_1)\]

\[S_L\lambda'(C_1) = (S_L - S_H)[F(C_1)\lambda'(C_1) + \lambda(C_1)f(C_1) - C_1f(C_1)]\]

\[(S_L - S_H)[F(C_1)\lambda'(C_1) + f(C_1)\lambda(C_1)] = S_L\lambda'(C_1) + C_1(S_L - S_H)f(C_1)\]

\[(S_L - S_H)\frac{F(C_1)}{C_1} (C_1) = S_L (C_1) + C_1(S_L - S_H)f(C_1)\]
Equilibrium Bidding with Fixed Splits

\[(S_L \quad S_H) \frac{F(C_1)}{C_1} (C_1) = S_L (C_1) + C_1(S_L \quad S_H) f(C_1)\]

\[(S_L \quad S_H) \frac{F(C_1)}{C_1} (C_1) \, dC_1\]

\[= S_L \int_{C_1}^{M} F(C_1) (C_1) \, dC_1 + (S_L \quad S_H) \int_{C_1}^{M} C_1 f(C_1) \, dC_1\]

\[(S_L \quad S_H) F(M) (M) F(C_1) (C_1)\]

\[= S_L (M) (C_1) + (S_L \quad S_H) \int_{C_1}^{M} C_1 f(C_1) \, dC_1\]

\[(S_L \quad S_H) M \frac{F(C_1)}{C_1} (C_1) = S_L M (C_1) + (S_L \quad S_H) \int_{C_1}^{M} C_1 f(C_1) \, dC_1\]

\[S_H M (S_L \quad S_H) \int_{C_1}^{M} C_1 f(C_1) \, dC_1 = (S_L \quad S_H) F(C_1) (C_1) S_L (C_1)\]

\[S_H M + (S_L \quad S_H) \int_{C_1}^{M} C_1 f(C_1) \, dC_1 = (C_1) S_L 1 \quad F(C_1) + S_H F(C_1) (C_1)\]
Equilibrium Bidding with Fixed Splits

\[ S_{HM} + (S_L - S_H) \int_{C_1}^{M} C_1 f(C_1) dC_1 = (C_1) S_L \left(1 - F(C_1) \right) + (C_1) S_H F(C_1) \]

\[ S_{HM} + (S_L - S_H) \left(1 - F(C_1) \right) E(C_2 | C_2 > C_1) \]

\[ = (C_1) S_L \left(1 - F(C_1) \right) + S_H F(C_1) = (C_1) S_L \left( S_L - S_H \right) F(C_1) \]

\[ (C_1) = \frac{S_{HM} + (S_L - S_H) \left(1 - F(C_1) \right) E(C_2 | C_2 > C_1)}{S_L \left( S_L - S_H \right) F(C_1)} \]
Equilibrium Bidding with Fixed Splits

- Let $C_1, C_2 \sim U[0,100] \Rightarrow$

\[
(C_1) = \frac{S_H M + (S_L, S_H)(1 - F(C_1))E(C_2|C_2 > C_1)}{S_L (S_L, S_H)F(C_1)}
\]

\[
(C_1) = \frac{100S_H + (S_L, S_H)(1 - \frac{1}{100} C_1)^{1/2}(C_1 + 100)}{S_L^{1/100} (S_L, S_H)C_1}
\]

\[
(C_1) = \frac{20,000S_H + (S_L, S_H)(100 - C_1)(C_1 + 100)}{200S_L (S_L, S_H)C_1}
\]

\[
(C_1) = \frac{20,000S_H + (1 - 2S_H)(10,000 - C_1^2)}{200S_L 2C_1(S_L, S_H)}
\]

\[
(C_1) = \frac{20,000S_H + 10,000 - C_1^2}{200S_L 2C_1(S_L, S_H)} \cdot \frac{20,000S_H + 2S_H C_1^2}{200S_L 2C_1(S_L, S_H)}
\]

\[
(C_1) = \frac{10,000 + (2S_H - 1)C_1^2}{200S_L 2C_1(S_L, S_H)} \cdot \frac{10,000 (S_L, S_H)C_1^2}{200S_L 2C_1(S_L, S_H)}
\]
Equilibrium Bidding with Fixed Splits

\[ S_H = 0 \quad (C_1) = \frac{10,000}{200} \frac{C_1^2}{2C_1} = \frac{(100 + C_1)(100 + C_1)}{2(100 + C_1)} = 50 + \frac{1}{2}C_1 \]

\[ S_H = 0.1 \quad S_L = 0.9 \quad (C_1) = \frac{10,000}{180} \frac{0.8C_1^2}{1.6C_1} \]

\[ S_H = 0.2 \quad S_L = 0.8 \quad (C_1) = \frac{10,000}{160} \frac{0.6C_1^2}{1.2C_1} \]

\[ S_H = 0.3 \quad S_L = 0.7 \quad (C_1) = \frac{10,000}{140} \frac{0.4C_1^2}{0.8C_1} \]

\[ S_H = 0.4 \quad S_L = 0.6 \quad (C_1) = \frac{10,000}{120} \frac{0.2C_1^2}{0.4C_1} \]

\[ S_H = 0.5 \quad S_L = 0.5 \quad (C_1) = \frac{10,000}{100} \frac{0}{0} = 100 \]
Endogenous Split Awards as a Protest Management Tool: A Modeling & Computational Approach

Unused Back-Up Slides

Peter J. Coughlan
William Gates
Graduate School of Business & Public Policy
Naval Postgraduate School
The rising problem of bid protests
Why large contracts are being sidelined

When the General Services Administration announced 29 winners of the $50 billion Alliant contract in 2007, agency leaders heralded it as the government’s premier contract for information technology purchases. “With its expansive scope, access to the best in class in the private sector and ability to provide customized solutions tailored to agencies’ unique IT needs, we can again prove that GSA is at the forefront of serving the acquisition needs of the federal government,” GSA’s Federal Acquisition Service Commissioner James Williams declared at the time. But 18 months later, a lot has changed. The contract was held up because of bid protests from several firms that didn’t make the cut. A federal court ordered GSA to re-evaluate all bidders. And it wasn’t until two weeks ago that GSA got the giant Alliant contract back on track by awarding it to 50 companies. Alliant is one of a few high-profile, high-value procurements — another is the Air Force’s tanker contract — that have been waylaid by protests in recent years. While these large procurements get all the attention, most bid protests concern smaller contracts. Overall, the protests rose 44 percent since 2001 — in part because companies were recently allowed to protest not only awards, but also solicitations. See PROTESTS, Page 19.
DoD Bid Protest Trends

- % Merit
- % Sustain
- # of Protests

Acquisition Research Forum 49
Dr. Peter J. Coughlan
Vendor Protest Incentives

- **Expected Profit** from Protest
  = Expected Benefits – Expected Costs

- **Expected Costs** = $K$
  = Research + Legal + Reputation + Opportunity Costs

- **Expected Benefits**
  = Probability of Success × Gain if Successful

- **Gain** if Successful = Contract Revenue = $X$

- **Probability** of Success
  = \( \text{Prob}(P_1 < P_2) \ given \) that buyer perceived \( P_1 > P_2 \)

- **Expected Profit from Protest** = \( \text{Prob}(P_1 < P_2) \times X - K \)
Modeling Buyer Imperfect Information

- Let $R_1 = \frac{P_1}{(P_1+P_2)} \quad \& \quad R_2 = \frac{P_2}{(P_1+P_2)}$
  
  - $0 \leq R_1 \leq 1 \quad \& \quad 0 \leq R_2 \leq 1$
  
  - $R_1 + R_2 = 1$

- Let $r_1 =$ buyer’s estimate of $R_1$
  
  - $r_1 = \frac{r}{N}$ where $r \sim \text{Bin}(N,R_1)$
  
  - Binomial with $N$ draws & success probability $= R_1$
  
  - Higher $N \Rightarrow$ more accurate estimate of $R_1$

- Let $r_2 =$ buyer’s estimate of $R_2$
  
  - $r_2 = 1 - r_1$
Perceived Probability of Protest Success

- Assume buyer discloses estimate $r_1$
  - $r_1 < \frac{1}{2} \Rightarrow$ vendor 1 wins
  - $r_1 > \frac{1}{2} \Rightarrow$ vendor 2 wins
- If vendor 1 loses, his estimate of the probability of a successful protest is:
  - $\text{Prob}(P_1 < P_2)$ given that buyer perceives $P_1 > P_2$
  - $\text{Prob}(R_1 < \frac{1}{2})$ given that buyer estimates $R_1$ at $r_1$
  - $\text{Prob}(R_1 < \frac{1}{2})$ given $Nr_1$ successes from $\text{Bin}(N, R_1)$
  - $\text{Prob}(R_1 < \frac{1}{2} \mid Nr_1$ out of $N)$
Perceived Probability of Protest Success

\[
\text{Prob}(R_1 < \frac{1}{2} | \text{Nr}_1 \text{ out of } N) = \frac{\text{Prob}(R_1 < \frac{1}{2}) \text{Prob}(\text{Nr}_1 \text{ out of } N | R_1 < \frac{1}{2})}{\text{Prob}(\text{Nr}_1 \text{ out of } N)}
\]

\[
= \frac{\int_0^{\frac{1}{2}} \text{Prob}(z) \text{Prob}(\text{Nr}_1 \text{ out of } N | R_1 = z) dz}{\int_0^{1} \text{Prob}(z) \text{Prob}(\text{Nr}_1 \text{ out of } N | R_1 = z) dz}
\]

\[
= \frac{\int_0^{\frac{1}{2}} \text{Prob}(z) \binom{N}{\text{Nr}_1} z^{\text{Nr}_1} (1 - z)^{N(1 - \text{r}_1)} dz}{\int_0^{1} \text{Prob}(z) \binom{N}{\text{Nr}_1} z^{\text{Nr}_1} (1 - z)^{N(1 - \text{r}_1)} dz}
\]

where \(\text{Prob}(z)\) reflects vendor 1's prior probability distribution of \(R_1\).
Extension: Repeated Procurements

- What are the **other benefits** of split awards?
  - Why are split awards used currently?
- Split awards **preserve competition** for repeated or follow-on procurements

**Direct modeling implications:**
- Appropriate to model as **repeated** bidding game
- Implies presence of **learning/experience** effects

**Indirect modeling implications:**
- Incorporate **innovation** to avoid trivial outcomes
- Innovation driven by “**shocks**” or **investment**
Extension: Repeated Procurements

- Learning/Experience Effects
- Investment & Innovation
- Discounting Future Periods