Entropy: A Unifying Path for Understanding Complexity in Natural, Artificial and Social Systems *

Constantino Tsallis

Centro Brasileiro de Pesquisas Fisicas, and National Institute for Science and Technology for Complex Systems
Rua Xavier Sigaud 150, Rio de Janeiro, RJ 22290-180, Brazil [tsallis@cbpf.br]

and

Santa Fe Institute
1399 Hyde Park Road, Santa Fe, NM 87501, USA

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1- Brief History and Epistemological Motivation

Energy and *entropy* are basic concepts in thermodynamics and elsewhere. The concept of energy emerged in mechanics, the branch of physics which studies the motion of bodies and their causes. In contemporary physics, it appears in classical, relativistic and quantum mechanics. The concept of entropy emerged quite later than that of energy. It was introduced by Rudolf Julius Emmanuel Clausius around 1865 in order to further understand the roles of heat and work in thermodynamics. A decade later, Ludwig Eduard Boltzmann gave an interpretation of entropy in terms of the microscopic world – atoms, molecules, and their motion --, whose existence was at the time very controversial – in January 1897, at the Viennese Academy of Sciences, Mach brazenly lambasted Boltzmann with his sadly famous, and ignorant, “I do not believe that atoms exist!” --.

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* Chapter of the book *Complex Socio-Technical Systems – Understanding and Influencing Causality of Change*, edited by William B. Rouse, Kenneth R. Boff and Penelope Sanderson (2011).

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14. ABSTRACT
This report summarizes some of the points that were learned from various applications of nonadditive entropy and its associated nonextensive statistical mechanics. The focus of the effort is on understanding and influencing causality of change of complex socio-technical systems. The conclusions include that we can understand that almost uncorrelated N elements yield exponentially increasing (with N) possible collective configurations and that the role of memory emerges as mostly relevant.

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In great contrast, Josiah Willard Gibbs [1] supported and enriched Boltzmann ideas. The Boltzmann-Gibbs (BG) (logarithmic) expression for the entropy (written here for the discrete case, as used by Claude Elwood Shannon) is

$$S_{BG} = -k\sum_{i=1}^{W} p_i \ln p_i \ , \qquad \left(\sum_{i=1}^{W} p_i = 1\right), \tag{1}$$

where W is the total number of configurations whose probabilities are $\{p_i\}$. The conventional constant k is usually taken equal to the Boltzmann constant k_B, or just to unity. If all probabilities are equal, hence equal to $1/W$, we have that

$$S_{BG} = k \ln W, \tag{2}$$

the celebrated expression carved on Boltzmann´s gravestone in Vienna. We may check in this expression one of the most distinctive properties of entropy, namely that it characterizes the lack of information on the system. Indeed, when W increases, we loose information (about the precise configuration, or microscopic state, where the system is) and S_{BG} increases. When W=1, the entropy vanishes, reflecting the fact that we exactly know the state of the system. For fixed W, the BG entropy (1) increases from zero (corresponding to certainty about the microscopic state of the system) to its maximum value (2) (corresponding to full uncertainty, i.e., equal probabilities for all admissible microstates).

The BG entropy stands at the foundation of *statistical mechanics*, one of the theoretical pillars of contemporary physics. This remarkable theory connects the laws of the microscopic world with those of the macroscopic one, i.e., thermodynamics. The BG entropy and statistical mechanics enable us to quite deeply understand a wide class of interesting and relevant systems, that we will from now on refer to as *simple systems* (although they can mathematically be extremely complicated!) By skipping a long and fascinating history, let us address now the so called *complex systems*. The typical characterizations of simple versus complex systems will be mentioned later on. The fact is that a possible generalization of the BG statistical mechanics was proposed in 1988 [2] on the basis of the following entropy:

$$S_q = k\,\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} = k\sum_{i=1}^{W} p_i \ln_q \frac{1}{p_q} \ , \qquad \left(q \in \Re;\ \sum_{i=1}^{W} p_i = 1\right), \tag{3}$$

where

$$\ln_q z \equiv \frac{z^{1-q}-1}{1-q} \qquad (z>0;\ \ \ln_1 z = \ln z). \tag{4}$$

We can verify that this entropy recovers the BG one as its $q \rightarrow 1$ limit (i.e., $S_1 = S_{BG}$). Therefore, we are talking of a generalization, *not* of an alternative to the BG theory. If all probabilities are equal, hence equal to 1/*W*, we obtain

$$S_q = k\frac{W^{1-q}-1}{1-q} = k\ln_q W. \tag{5}$$

If we consider two probabilistically independent systems *A* and *B* (i.e., such that $p_{i,j}^{A+B} = p_i^A p_j^B, \forall(i,j)$), we straightforwardly verify that

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k}. \tag{6}$$

In other words this *q*-entropy, as is sometimes referred to, is *nonadditive* unless *q*=1, in which case it is *additive*. Several other details on this *nonadditive entropy*, and the so called *nonextensive statistical mechanics* (which recovers the standard BG statistical mechanics for *q*=1) associated with it, can be found in [3,4].

Let us address now the typical features of the so called simple ($q = 1$) and complex ($q \neq 1$) systems.

The *q*=1 systems typically (but not necessarily) exhibit the following properties:

Short-range space-time correlations

Markovian processes (short memory)

Additive noise in Langevin-like mesoscopic equations

Strong chaos (i.e., positive maximal Lyapunov exponent)

Ergodic

Euclidean geometry

Short-range interactions in many-body systems

Weakly quantum-entangled subsystems

Linear and homogeneous Fokker-Planck mesoscopic equations

Gaussian distributions

Probability exponentially dependent on energy at thermal equilibrium (i.e., the BG weight)

The $q \neq 1$ systems typically (but not necessarily) exhibit the following properties:

Long-range space-time correlations

Non-Markovian processes (long memory)

Both additive and multiplicative noises in Langevin-like mesoscopic equations

Weak chaos (i.e., vanishing maximal Lyapunov exponent)

Nonergodic

Multifractal (or similar hierarchical) geometry

Long-range interactions in many-body systems

Strongly quantum-entangled subsystems

Nonlinear and/or inhomogeneous Fokker-Planck mesoscopic equations

q-Gaussian distributions (i.e., asymptotic power-laws)

Probability *q*-exponentially dependent on energy at stationary (or quasi-stationary) states (i.e., asymptotic power-laws)

All these features can be, and frequently are, used to characterize practically the degree of complexity of a system. Their deep cause is, however, quite simple in its essence. It has to do with the absence or presence of strong correlations between the (many) elements of the system. There are two basically different cases, which we address now. We focus on how the total number *W* of microscopic configurations increases with the total number *N* of elements of the macroscopic system.

First possibility:

$$W(N) \sim \mu^N \quad (W(1) \simeq \mu > 1;\ N >> 1), \tag{7}$$

which implies that $W(N+1) \sim \mu\ W(N)$, which means that any new element which is added to the system makes the number *W(N)* of pre-existing possibilities to be multiplied by its own number μ of individual possibilities. In other words, each of the pre-existing configurations is still possible and accommodates practically without change with each of the configurations of the newcomer. This is the sign of a *q*=1 system.

Second possibility:

$$W(N) \sim N^{\rho} \quad (\rho > 0;\ N >> 1), \tag{8}$$

which implies that $W(N+1) << \mu\ W(N)$, which means that any new element which is added to the system makes the number *W(N)* of pre-existing possibilities to very mildly increase, sensibly less than multiplying by its own number μ of individual possibilities. In other words, the pre-existing configurations are not possible any more, and new collective configurations emerge in the presence of the newcomer. This is the sign of a $q \neq 1$ system.

Other possibilities can of course exist in principle [for instance, $W(N) \sim (\ln N)^{\delta} \quad (\delta > 0;\ N >> 1)$ or $W(N) \sim \mu^{N^{\gamma}} \quad (\mu > 1;\ 0 < \gamma < 1;\ N >> 1)$)], but the above two are the most simple and paradigmatic ones.

How the entropy – which, as we said, characterizes the lack of information of the observer on the system -- enters into this discussion? As an unifying concept! Amazingly enough, it enters in order to satisfy the classical thermodynamical demand of *extensivity*, i.e., that $S(N) \propto N \quad (N >> 1)$! Let us be more explicit.

If we are facing the case $W(N) \sim \mu^N$, it follows from Eq. (2) that it is the BG entropy (q =1) which is extensive, since in this case we have that $S_{BG}(N) \propto N$. See Figure 1.

If we are facing the case $W(N) \sim N^{\rho}$, it follows from Eq. (5) that it is $S_q \quad (\text{with } q = 1 - 1/\rho < 1)$ the entropy which is extensive, since in this case we have that $S_{1-1/\rho}(N) \propto N$, as can be straightforwardly verified. See Figure 2.

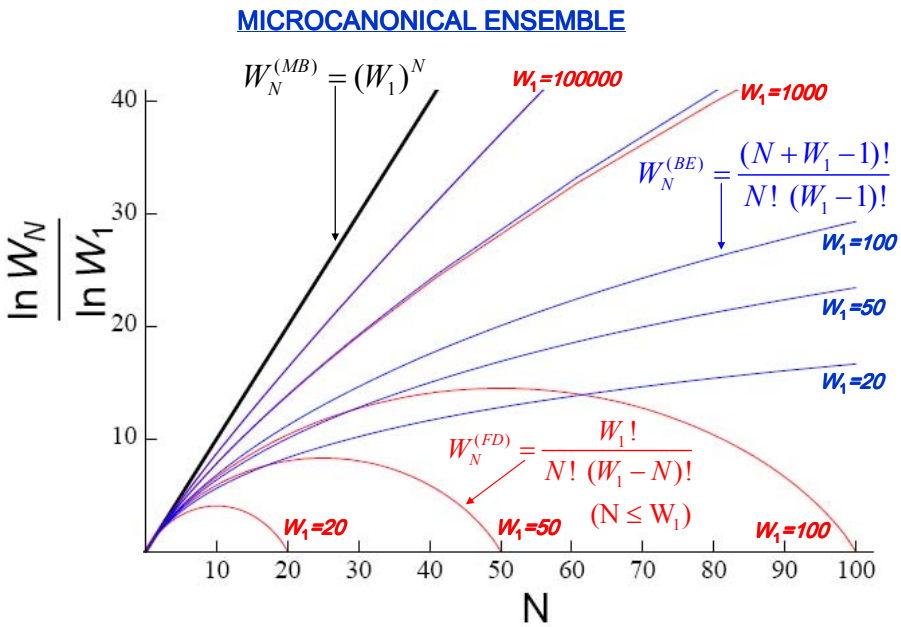


Figure 1 – BG entropy as a function of the total number *N* of particles for Maxwell-Boltzmann (MB) statistics (i.e., all *N* particles are probabilistically independent, hence no quantum effects are present), Fermi-Dirac (FD) statistics (i.e., every single quantum state can be occupied by either zero or one particle, no more; consequently $0 \le N \le W_1$), and Bose-Einstein (BE) statistics (i.e., every single quantum state can be occupied by an arbitrarily large number of particles; consequently $N = 0,\ 1,\ 2,\ ...,\ W_1 - 1,\ W_1,\ W_1 + 1,\ ...$). In the limit $W_1 \to \infty$, the BG entropy is *extensive* in all cases. All these illustrations correspond to hypothesis (7). From [4], where further details can be seen.

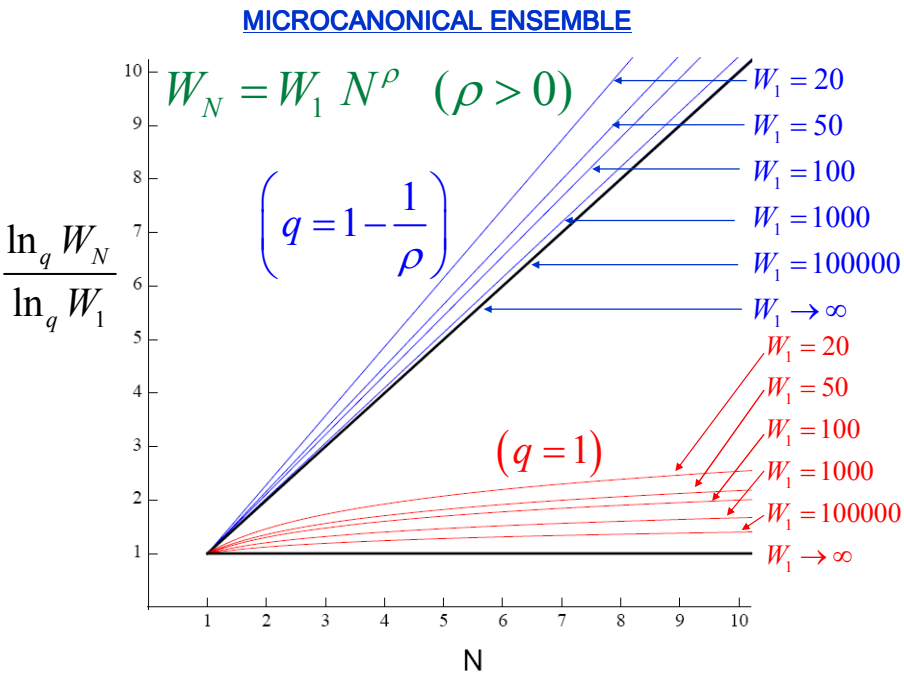


Figure 2 - BG entropy and q -entropy as functions of the total number N of particles. The BG entropy is not extensive, whereas the q -entropy is so. This illustration corresponds to hypothesis (8). From [4], where further details can be seen.

We should emphasize that, in spite of their innocuous aspect, the above statements carry together a sort of new paradigm, quite in the sense pointed by Thomas Samuel Kuhn [5]. Indeed, the concept of entropy has been taught during more than one century as being unique. More precisely, that the connection between Clausius thermodynamic entropy and the microscopic world is uniquely given by the BG entropy functional. We are here assuming that it is not so! *We are assuming that it is the system which determines the entropic functional form to be used to make the bridge with the macroscopic world.* This might seem strange at first sight. However, this uniqueness does *not* resist deeper analysis, and -- more important -- it does *not* resist confrontation with experimental data in what concerns its consequences. Definitively the BG entropy can only be understood nowadays as a first, most important, step, but not as the ultimate and unique scientific truth in what concerns entropy. The change of paradigm that the present approach involves might explain the curious fact that, although thousands of papers have been published by thousands of scientists (see the Bibliography in [6]) providing support, there are still some establishment scientists who apparently are against it.

The whole thing is in fact quite simple and very analogous to the following problem. Let us consider the surface of a glass covering a table, assuming the surface to seen a simple plane. *What is its volume?* Clearly zero. *What is its length?* Clearly infinity. *What is its area?* A finite number, with physical units!, say square meters. It is the system which, through its geometry, which determines the useful question to be asked! We may ask about Lebesgue measures of all kinds, but the only one

which is useful (and finite) is the area! Suppose that we have now not a glassy surface but a fractal-like object. *What is the correct measure to ask?* Clearly, the measure must be asked in terms of its Hausdorff (or fractal) dimension, again determined by the geometry of the system! As before, this is the only one which leads to a *finite* answer.

This is precisely the idea behind the entropic index *q*. It is the system (through its microscopic probabilistic-dynamic nature) which determines what specific *q*-entropy to be fruitfully used. For a classical Hamiltonian system, if the corresponding microscopic nonlinear dynamics is ergodic, we must use the BG entropy to establish its connection with thermodynamics, in other words we must use the BG statistical mechanics. Indeed, the phase space region occupied during the time evolution of the system will have a *finite* Lebesgue measure. If, however, the system is nonmixing/nonergodic, we might be led to a *zero* Lebesgue measure occupancy of phase space, and consistently to a *q*-entropy with a specific value of *q*, characterizing in fact not only that particular system, but an entire universality class of systems to which the specific system belongs. Probabilistic illustrations [7] as well as physical ones [8,9] are available in the literature which explicitly show this fact, namely that for special classes of systems, special values of *q* are to be used in order for the entropy to be extensive in the thermodynamic sense previously defined. If we were to use the BG entropy for these anomalous systems, whose elements are strongly correlated, we would obtain $S_{BG}(N) \propto \ln N$, which is in heavy contradiction with the thermodynamical requirement that $S(N) \propto N$. This contradiction satisfactorily disappears as soon as we use instead the *q*-entropy with the appropriate value of *q* (see Figure 3).

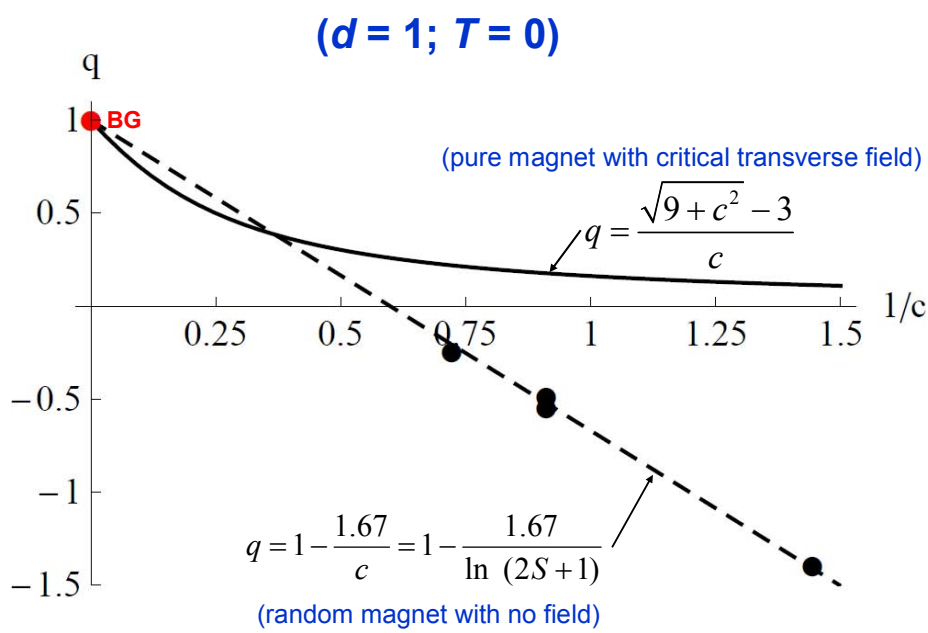


Figure 3 – *q* as a function of the inverse central charge 1/c, where *q* is such that $S_q(L)$ is extensive (i.e., $S_q(L) \propto L$), as required by thermodynamics for *d* =1 systems. See details in [8] for the fully

entangled (temperature T =0) pure magnetic chain with critical transverse field (it is $q \in (0,1]$), and in [9] for the random magnetic chain with no field (it numerically appears to be $q \in (-\infty,1]$). In both examples, the entropy which is extensive approaches the BG one in the limit $c \to \infty$ (red dot).

The present approach is summarized in Table 1, where we easily verify that entropic additivity and entropic extensivity are *different* properties, the former depending only on the mathematical functional form of the entropy, the latter depending on that form as well as on the specific system (more precisely on the nature of the correlations between its elements). Additivity and extensivity are different concepts, but the words are still used (wrongly) as synonyms by many scientists, because the systems that they have (inadvertently in most cases) in mind basically are those for which the confusion has no serious consequences. These are the so called simple systems. The distinction becomes, however, crucial when we focus on the so called complex systems. Such mistakes are recurrent in the history of Humanity: see an example in Figure 4.

SYSTEMS	**ENTROPY S_{BG}** **(additive)**	**ENTROPY S_q (q<1)** **(nonadditive)**
Short-range interactions, weakly entangled blocks, etc	**EXTENSIVE**	**NONEXTENSIVE**
Long-range interactions (QSS), strongly entangled blocks, etc	**NONEXTENSIVE**	**EXTENSIVE**

quarks-gluons, plasma, curved space ...?

Table 1 – The BG entropy is additive, and the q-entropy is nonadditive. Whether they are extensive, as thermodynamically required, or not depends on the nature of the correlations between the elements of the system. Both notions coincide *only* for the standard systems that have been approached within the BG theory for more than one century. This is the cause of the current confusion apparently still present in the mind of many contemporary physicists. Further details can be seen in [4].

Figure 4 – At the time of Thutmosis III, the Egyptian scientists referred to the North as “along the stream”, transparently meaning “along the stream of the (sacred) Nile”, which flows from South to North into the Mediterranean sea. But then the Pharaoh and its army invaded Mesopotamia, where they found the Euphrates, which flows more like North to South into the Persian Gulf. The motion of the stars did of course not show any sensible modification. This created a big confusion in the mind of the Egyptian scientists. Back to Egypt they included in the corresponding honoring obelisk the phrase “That strange river that, when you along the stream, you go against the stream”! The cause of the big confusion clearly was the fact that two totally different concepts – namely, the motion of the stars and the flows of rivers – were (wrongly) merged into a single concept. The annoying scientific confusion dissipated when they gradually encountered rivers other than the Nile. A change of paradigm in the sense of Kuhn had occurred!

2 – Illustrative Applications to Complex Natural and Artificial Systems

2.1 – Optimal Distribution of Probabilities

Once we have a specific expression for the entropy, we can look for the probability distribution which extremizes it under appropriate constraints. This typically corresponds to a relevant stationary state (for example, thermal equilibrium if q =1).

Let us illustrate this (variational) method with the continuous form of the q -entropy, namely

$$S_q = k \frac{1 - \int dx \left[p(x) \right]^q}{q-1}. \tag{9}$$

If we extremize this functional by imposing the norm $\int dx\, p(x) = 1$ as well as a constraint such as $\int dx\, x\, p(x) = C_1$ (or something analogous), C_1 being a constant, we straightforwardly obtain

$$p_q(x) = \frac{e_q^{-\beta x}}{\int dy\, e_q^{-\beta y}}, \tag{10}$$

where β is determined by the constant C_1, and the q –exponential function (inverse of the previously defined q –logarithmic function) is given by

$$e_q^z \equiv \left[1 + (1-q)\, z \right]_+^{\frac{1}{1-q}} \qquad (e_1^z = e^z), \tag{11}$$

with $\left[u \right]_+ = u \;\; \text{if}\, u > 0,$ and zero otherwise. The admissible values of q must satisfy q < 2, so that the probability distribution (10) is normalizable.

If instead of imposing a constraint on the first moment of *p(x),* we do it on the second moment, i.e., if we impose $\int dx\, x^2\, p(x) = C_2$ (or something analogous), we obtain

$$p_q(x) = \frac{e_q^{-\beta x^2}}{\int dy\, e_q^{-\beta y^2}}, \tag{12}$$

Where β is now determined by the positive constant C_2. This distribution is currently referred to as q –Gaussian since it recovers, for q =1, the celebrated Gaussian distribution. For q =2 it recovers the Cauchy-Lorentz distribution. The admissible values of q must satisfy q < 3, so that the probability distribution (12) is normalizable. These distributions constitute attractors in the sense of the Central Limit Theorem. If the large number of variables that are being summed are independent (or quasi-independent in some sense), the attractor is a Gaussian. If the variables are strongly correlated in some specific sense (see [4]), then q >1, and the attractors are q -Gaussians.

Distributions (10) and (12) exhibit power-law fat tails for q >1, and compact support for q <1. They both emerge very frequently in complex systems, as we illustrate in what follows.

2.2 – Applications in Natural and Artificial Systems

The motion of several micro-organisms and their cells has naturally evolved, along millennia, in such a way as being non-Gaussian, clearly in order to better achieve a satisfactory feeding and

reproduction. The distribution of velocities consistently exhibits tails that are neatly fatter than those of a Maxwellian distribution, typical of molecules in the air. See in Figures 5 and 6 some illustrative examples, respectively *Hydra viridissima* [10] and *Dictyostelium discoideum* [11].

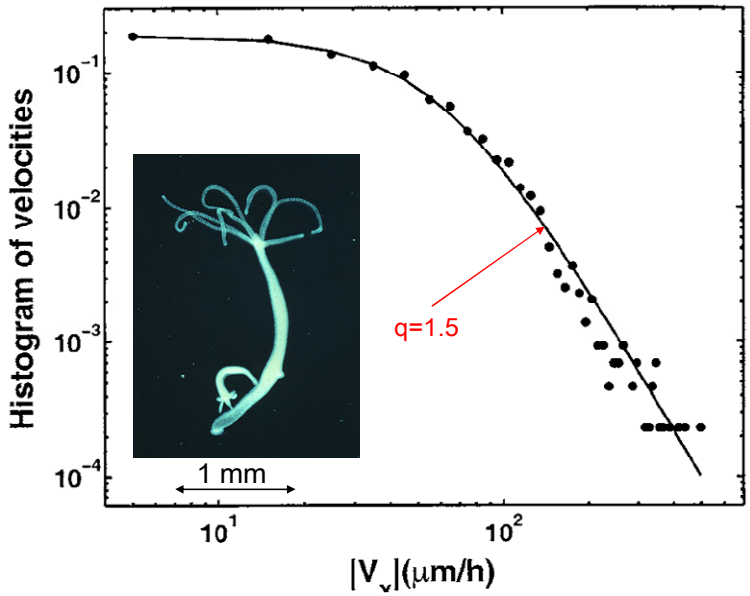


Figure 5 – The distribution of velocities of cells of *Hydra viridissima* is well fitted by a *q* –Gaussian with *q* =1.5. See details in [10].

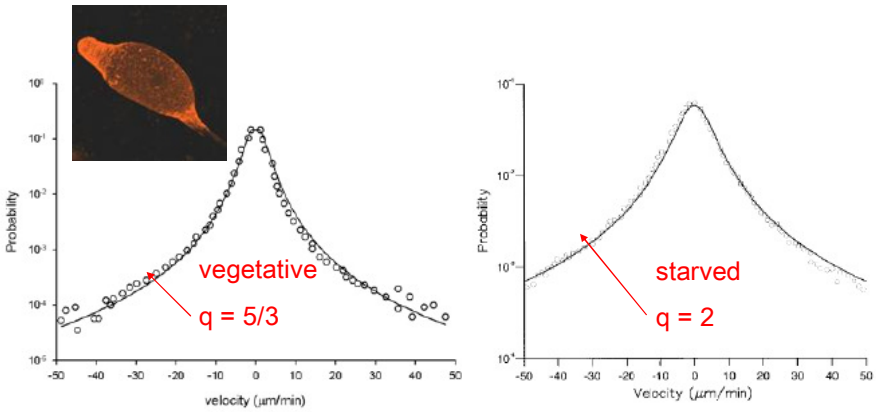


Figure 6 – The distribution of velocities of cells of *Dictyostelium discoideum* is well fitted by a *q* –Gaussian with *q* =5/3 in the vegetative state and *q* =2 in the starved state. See details in [11].

Many phenomena occur in outer space whose complexity appears to be of the type addressed herein. Consequently functions such as the *q*-exponential and its extensions frequently emerge in such obervations. This is the case of the flux of cosmic rays, along an extremely wide range of energies and fluxes: see Figure 7.

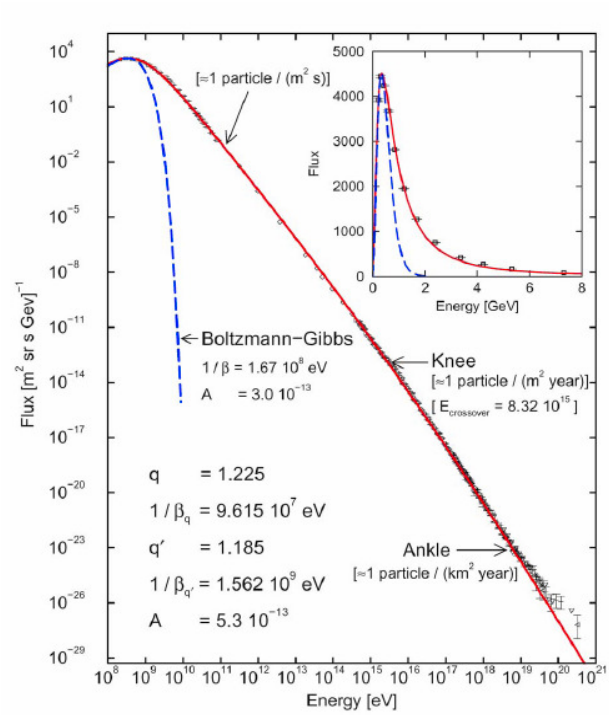


Figure 7 – The distribution of fluxes of cosmic rays for a remarkably wide range of energies (along 13 decades of energies, corresponding to 33 decades of flux!). The analytical (red) curve is a combination of hypergeometric functions and shows a crossover from a *q*-exponential with *q* =1.225 to one with *q* =1.185. The Boltzmann curve (*q* =1; in blue) is shown for comparison. See details in [12].

Another example of nonextensive behavior is the fluctuations of the magnetic field in the plasma of the solar wind, as detected by Voyager 1 and Voyager 2. Let us briefly describe this empirical evidence. It was expected, due to theoretical considerations, that each nonextensive system would exhibit at least three different values for the index *q*, respectively corresponding to sensitivity to the initial conditions (q_{sen}), to relaxation (q_{rel}), and to the stationary state (q_{stat}). This is nowadays referred in the literature as the *q*-triplet or the *q*-triangle. It was first found in the data arriving to NASA from Voyager 1 [13], and since then it has been repeatedly verified and extended. The values found in that occasion were $(q_{sen}, q_{rel}, q_{stat}) = (-0.6 \pm 0.2,\ 3.8 \pm 0.3,\ 1.75 \pm 0.06)$. On the basis of some dual transformations (*q*→2-*q*, and *q*→1/*q*), they were conjecturally suggested [7] to be $(q_{sen}, q_{rel}, q_{stat}) = (-1/2,\ 4,\ 7/4)$. See Figure 8.

The presence of a *q*-triplet may be considered as a strong indication that the system is nonextensive in the sense herein, and that could therefore benefit from the available theoretical body to study such systems. Another example is constituted by the ozone layer around the Earth (see Figure 9). Indeed

the width of this layer along the vertical above Buenos Aires has been recently addressed [14], and it was found that

$$(q_{sen}, q_{rel}, q_{stat}) = (-8.1 \pm 0.2,\ 1.89 \pm 0.2,\ 1.32 \pm 0.06)$$

The present examples, and some others, suggest what might be, for a wide class of systems, a generic property, namely $q_{sen} \leq 1 \leq q_{stat} \leq q_{rel}$.

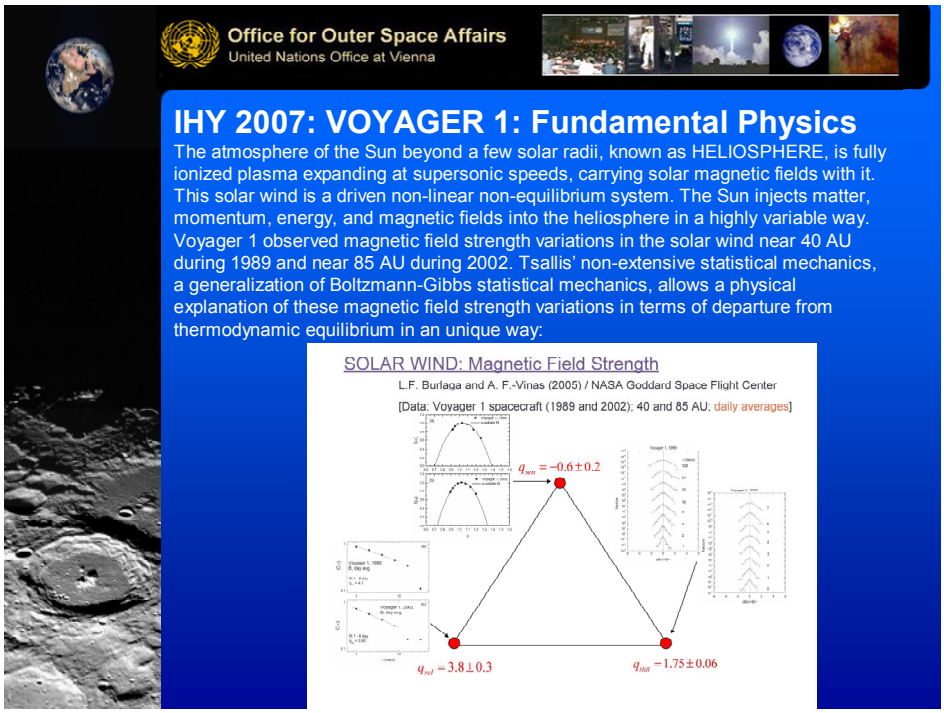


Figure 8 – The *q*-triplet detected in the solar wind through the analysis of the fluctuations of its magnetic field. See details in [13]. This is a Poster prepared by United Nations and exhibited in Vienna at the launching ceremony for the International Heliophysical Year 2007.

OZONE LAYER HOLE

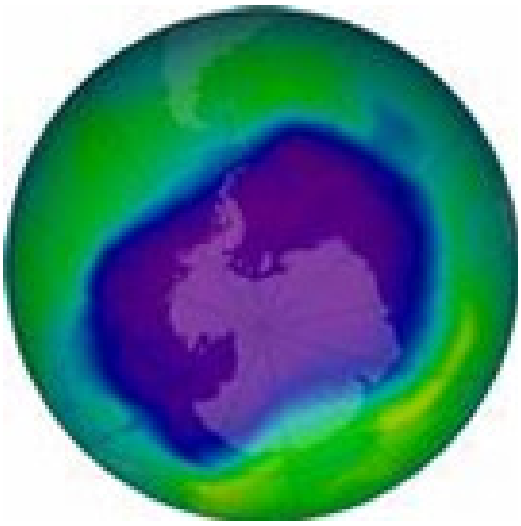


Figure 9 – The ozone layer is located 10-50 Km above the Earth. It absorbs 93-99% of the Sun´s high-frequency ultraviolet light. Image from Google.

The CMS detector at the Large Hadron Collider (LHC) at CERN has recently produced the first results in physics. Proton-proton collisions at energies up to 7 TeV (the highest energy up to now produced by humankind for controlled collisions between elementary particles) produce hadronic jets whose transverse momentum distributions have been measured. They systematically are well fitted by q-exponentials with q close to 1.1 [15] (see Figure 10), and the same happens for collision experiments done at the Brookhaven Laboratories [16] (see Figure 11). The reasons for these facts remain elusive nowadays, possibly to be clarified in terms of quantum chromodynamics (QCD), or some other similar theory.

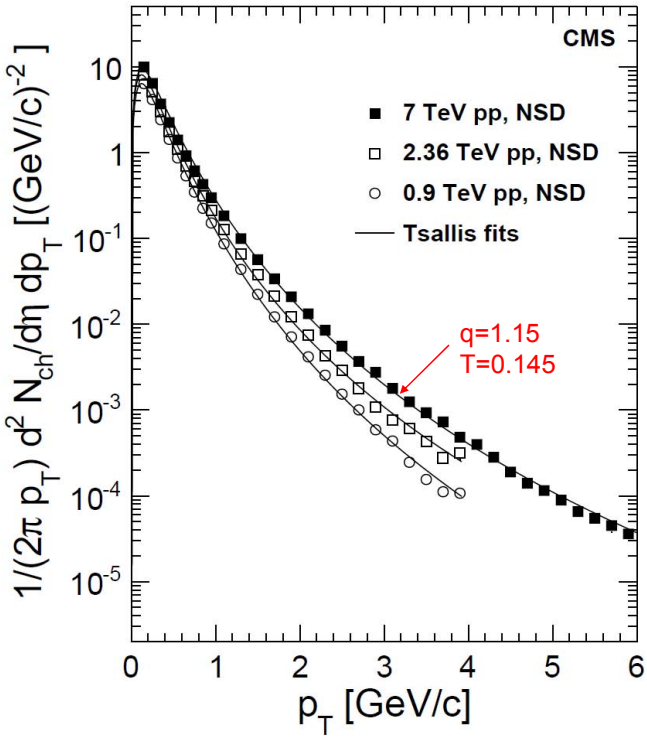


Figure 10 – Distributions of transverse momenta of the hadronic jets produced by proton-proton collisions in the LHC at energies of 0.9, 2.36, and 7 TeV, and detected by the CMS. From [15].

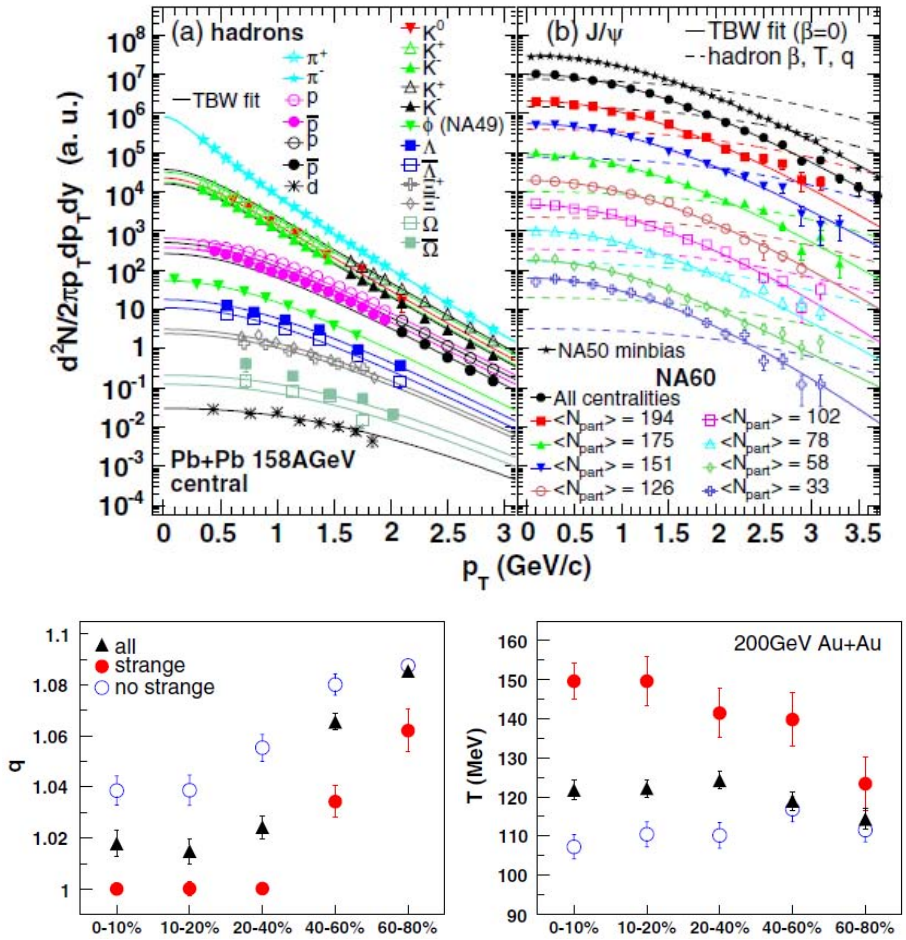


Figure 11 – Distributions of transverse momenta of the hadronic jets produced by various collisions in Brookhaven, and the corresponding values for the index *q* and the temperature *T*. From [16].

It was predicted in 2003 [17] that the distribution of velocities of cold atoms in dissipative optical lattices should possibly be *q*-Gaussians with $q = 1 + 44\frac{E_R}{U_0}$, where E_R and U_0 are parameters of the (mesoscopic) model. The prediction was computationally and experimentally verified in 2006 [18] (see Figure 12).

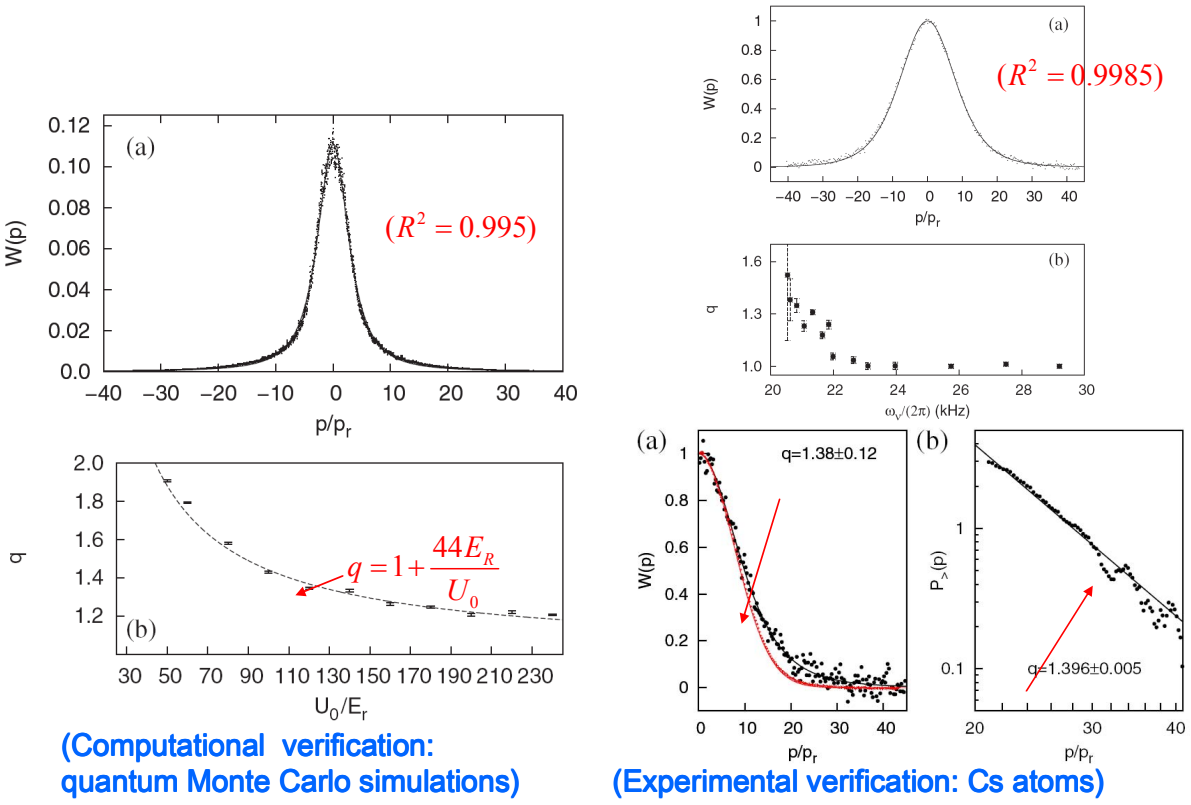


Figure 12 – Computational (left; quantum Monte Carlo) and experimental (right; Cs atoms) verifications of the *q*-Gaussian distributions of cold atoms velocities suggested in [17]. From [18].

As well known, the cells of biological tissues are very affected by radiation (e.g., the dangerous human melanoma). The survival fraction strongly depends on the radiation dose. A systematic study has recently been carried out [19], and the remarkable results can be seen in Figure 13. The authors conclude that their model can be used in hypofractionation radiotherapy treatments where current models cannot be applied.

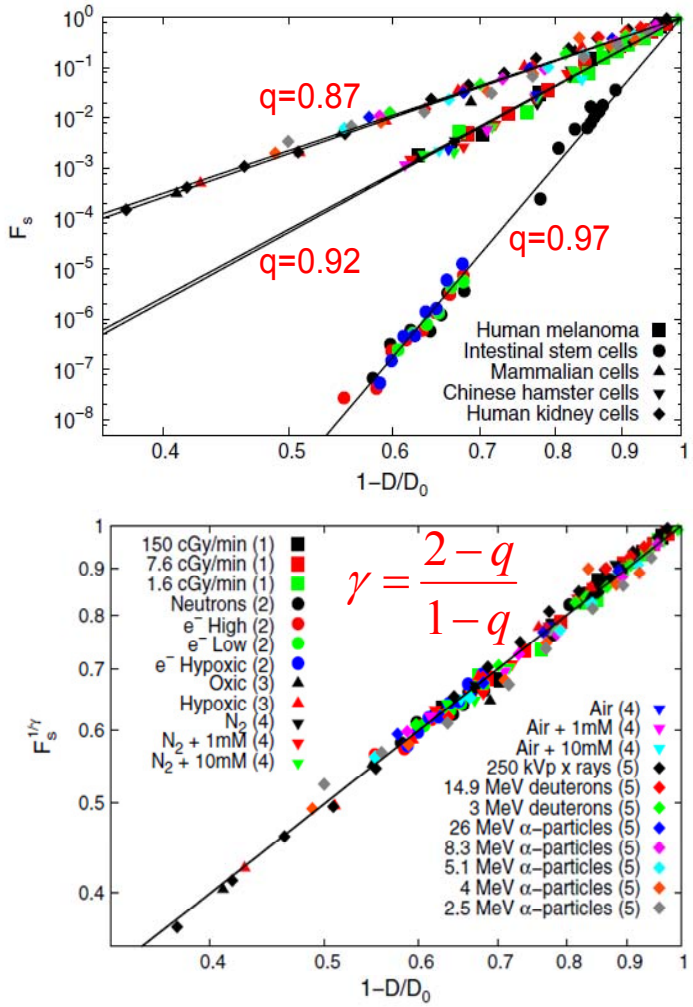


Figure 13 – Survival fraction as a function of the dose D (relative to a reference dose). The data collapse (for different cells and different types of radiation) into a universal straight line exhibited in the lower panel is quite remarkable. From [19], where further details can be seen.

Let us finally present some performant procedures for image processing [20,21,22]. They all improve on pre-existing methods: see Figures 14, 15 and 16.

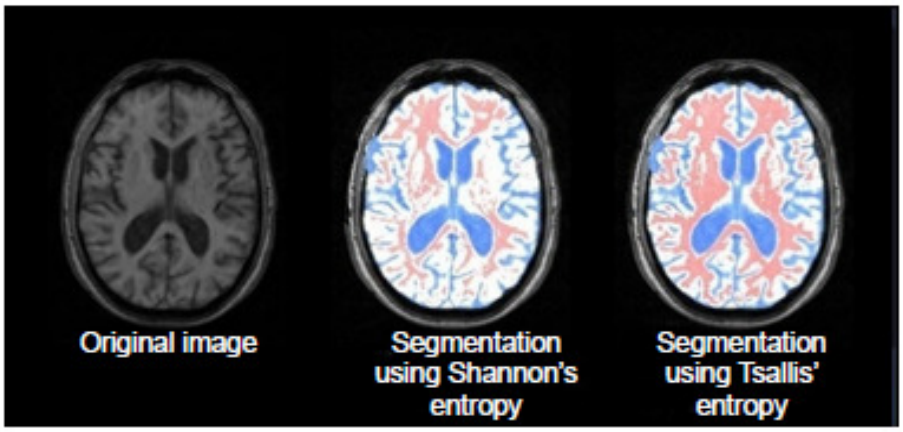


Figure 14 – Using the *q*-entropy to improve segmentation in multiple sclerosis magnetic resonance images. The authors conclude that this procedure could be applied in clinical routine. From [20], where all details can be seen.

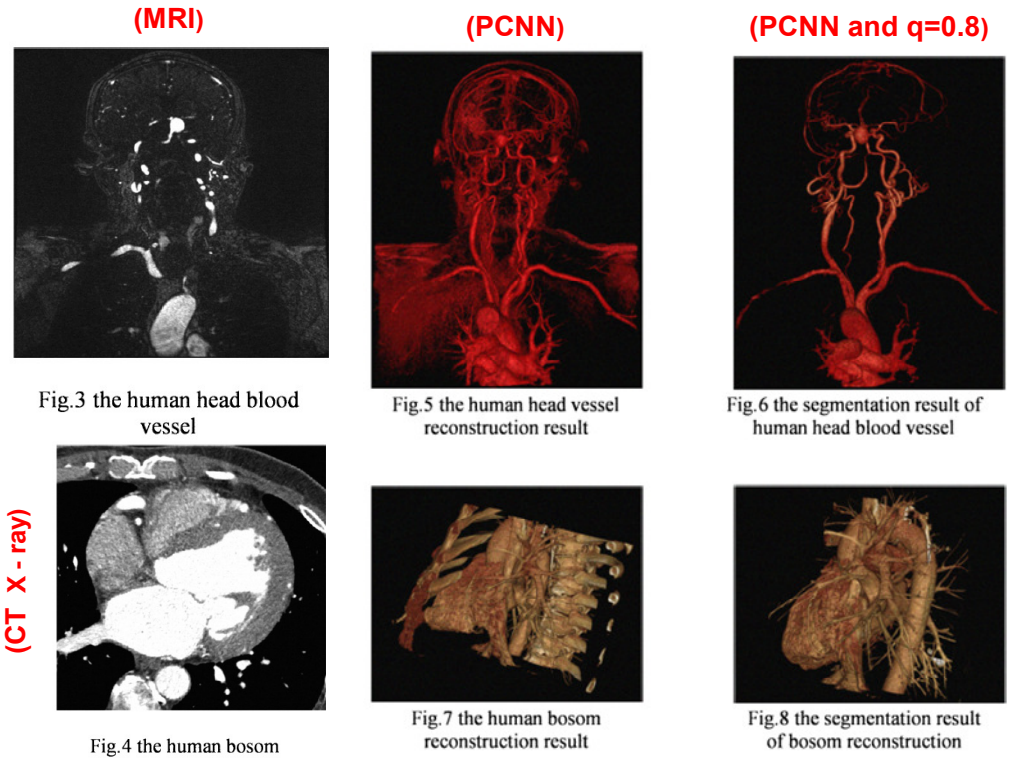


Figure 15 – Segmented medical images improved through use of the q-entropy. From [21], where further details can be seen.

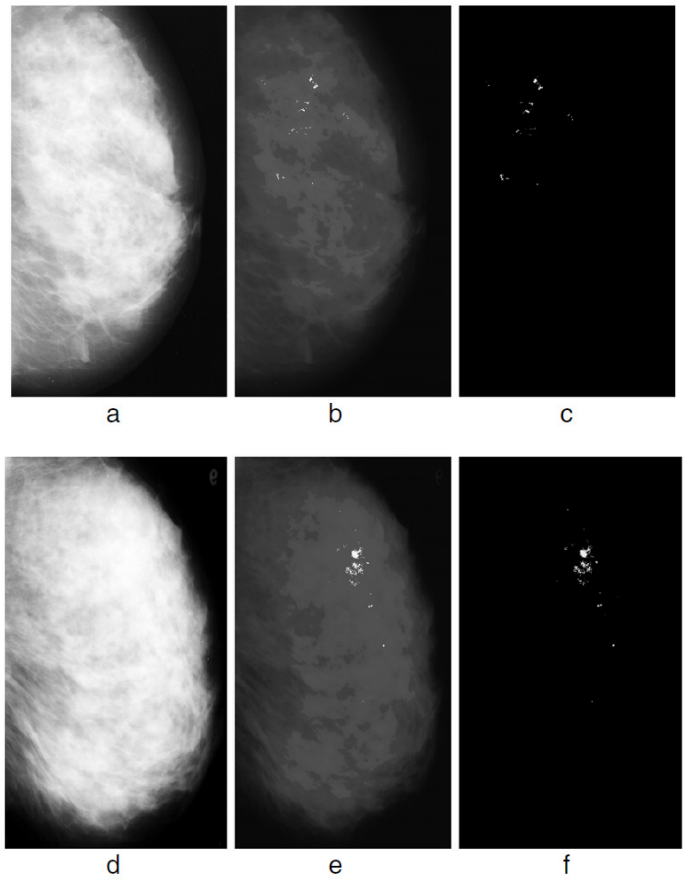


Figure 16 – Microcalcification detection technique applied to mammograms. The use of the *q*-entropy improves the detection of true positives from 80.21% to 96.55%, and decreases the detection of false positives from 8.1% to 0.4%. From [22], where further details can be seen.

Many other applications to natural systems (trapped ions, spin-glass, dusty plasma, earthquakes, turbulence, astrophysical objects, cosmology, black holes, etc) and to artificial systems (signal processing, global optimization, computational algorithms, internetquakes, etc) are available in the literature (see [3,4]). We hope however that the present selection provides some intuition and knowledge concerning the applicability and potentialities of the concepts that we have been handling, on a unifying (entropic) background.

3 – Illustrative Applications to Complex Socio-Technical Systems

Let us now address complex systems which include a substantial social component. We may start with economics and theory of finance. Given the long memory effects, and strong correlations, that characterize this area, it will be no surprise that *q*-statistics will be helpful in the discussion of many properties, such as distributions of price returns, volumes, wealth, land prices, risk function related with extreme values, volatility smile, among others. Some of these are illustrated in Figures 17, 18 and 19.

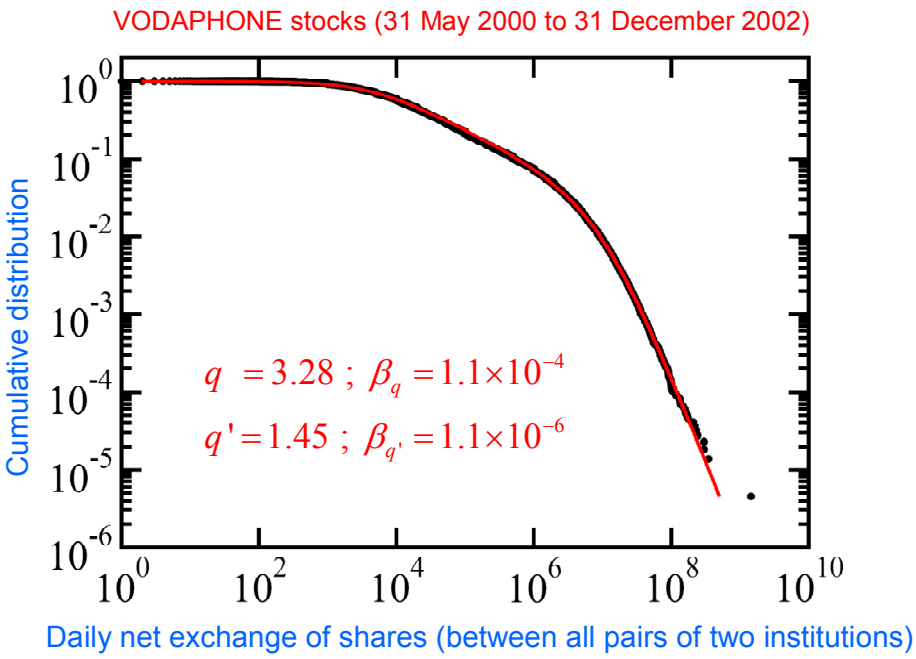


Figure 17 – Cumulative distribution of traded volumes of VODAPHONE at the London Stock Exchange (Block market). The real data (black dots) are well fitted by the same analytical combination of hypergeometric functions (red curve) that was used for cosmic rays [12]. From [23].

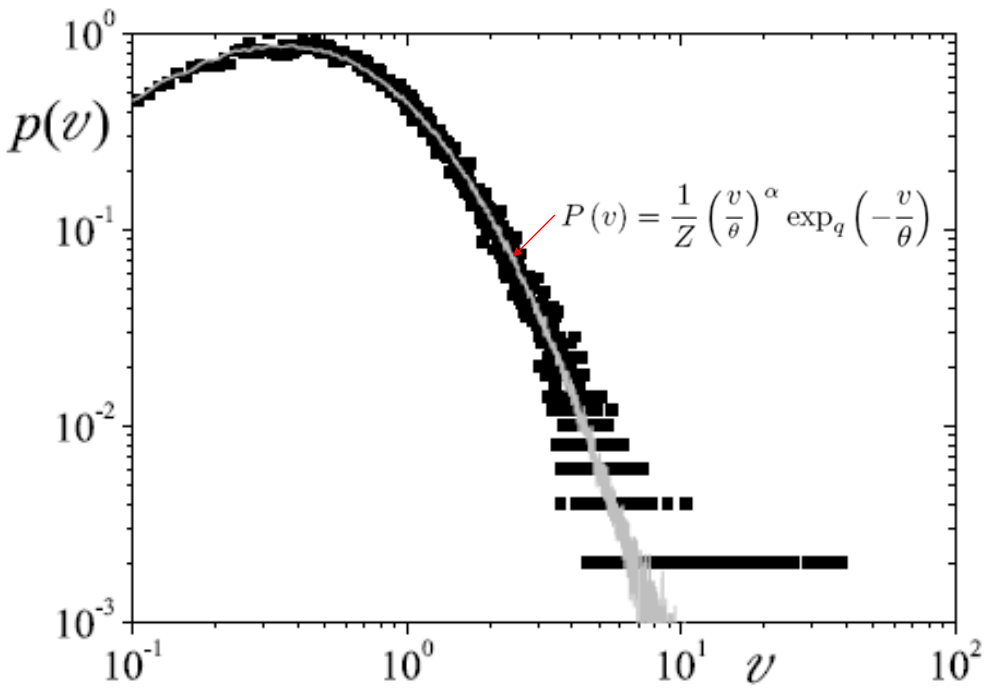
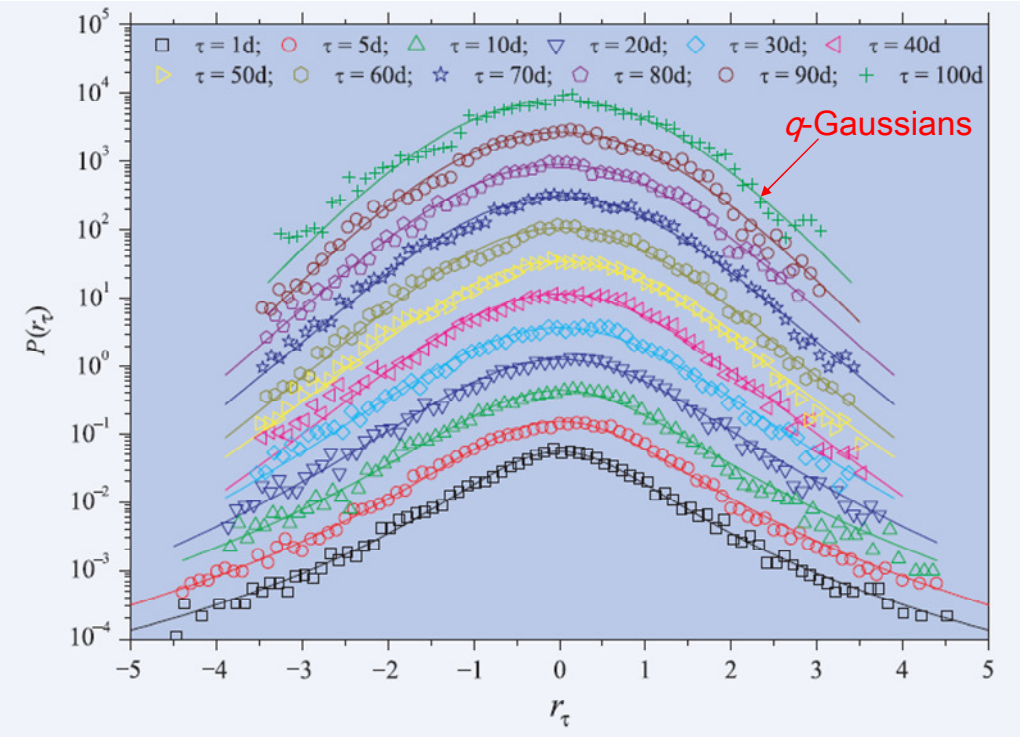


Figure 18 – Distribution of one minute traded volumes of the Citygroup stocks at the New York Stock Exchange. From [24], where further details can be seen.



τ	q
1	1.488
5	1.419
10	1.369
20	1.287
30	1.233
40	1.192
50	1.175
60	1.151
70	1.135
80	1.114
90	1.085
100	1.073

Figure 19 – Distributions of price returns at the New York Stock Exchange for typical lag times. From short to long lag times *q* varies from close to 1.5 to 1. From [25], where further details can be seen.

Let us now address a different area, namely that of (static or growing) networks made by *nodes* and *links* between them. The nodes can be people, computers, airports, and many other kind of elements. The links can be directed or not, all equal or not. Some of the nodes can have a large number of links, and those are referred to as *hubs*. A huge class of them is constituted by the so called *scale-invariant* networks (strictly speaking, they are only asymptotically scale invariant). The probability of a node to have *k* links (with other nodes) is called the degree distribution. The number of links plays a role very analogous to the microscopic energy of a many-body physical system. Consequently, its degree distribution is in many cases given by the *q*-exponential function, where *q* depends on ingredients such as the range of interactions between nodes. A typical model is the Natal one [26]. This is a geographical preferential attachment growing model. Once a newcomer node is spatially fixed somewhere, its probability to (permanently) attach with the pre-existing site *i* is given by $p_A \propto \frac{k_i}{r_i^{\alpha_A}} \quad (\alpha_A \geq 0)$, where k_i is the degree of site *i*, and r_i is the geographical distance of the new comer to site *i* . A typical cluster realization is shown in Figure 20. The resulting degree distribution is

shown in Figures 21 and 22. It is worthy emphasizing that the distinctive feature of this model is that it might be more adequate to attach to somebody less *powerful* (i.e., with less links) but which is closer.

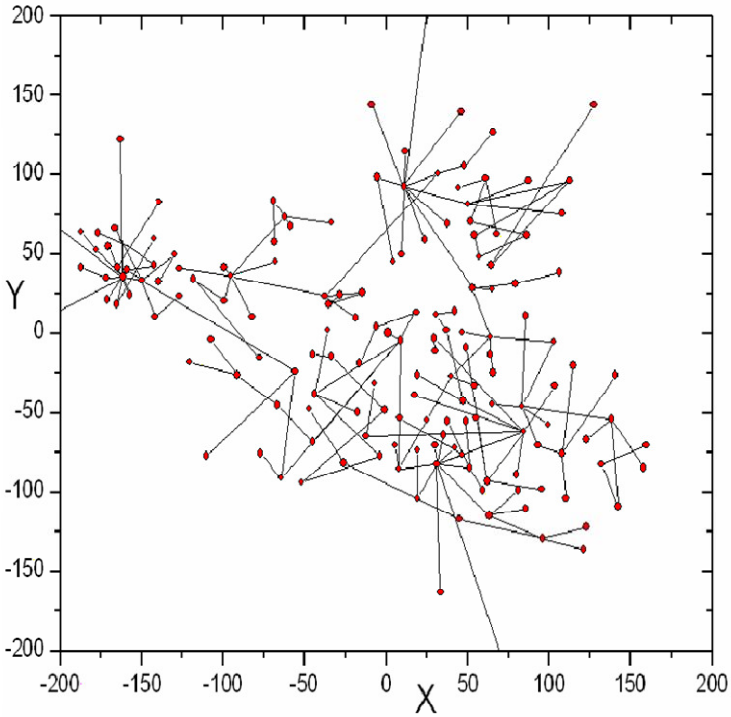


Figure 20 – A typical cluster with N=250 nodes, with $\alpha_A = 1$. From [26], where further details can be seen.

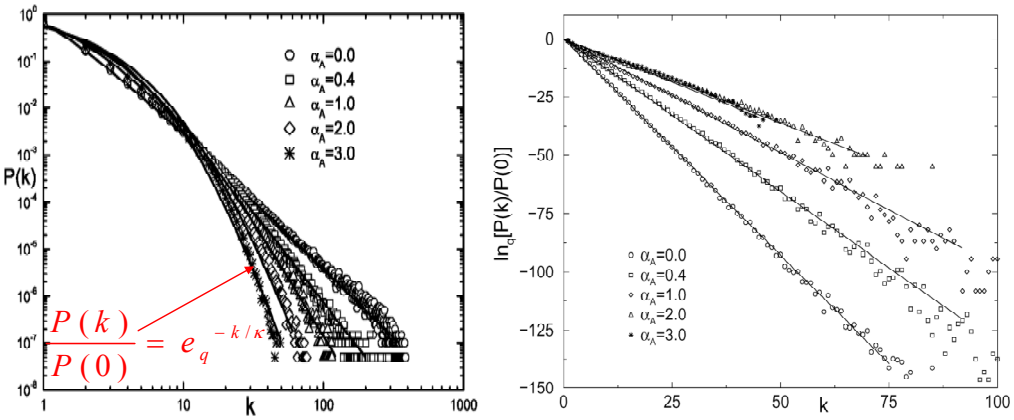


Figure 21 – The resulting degree distribution *P(k)* (in log versus log at the left; *q*-log versus linear at the right). From [26], where further details can be seen.

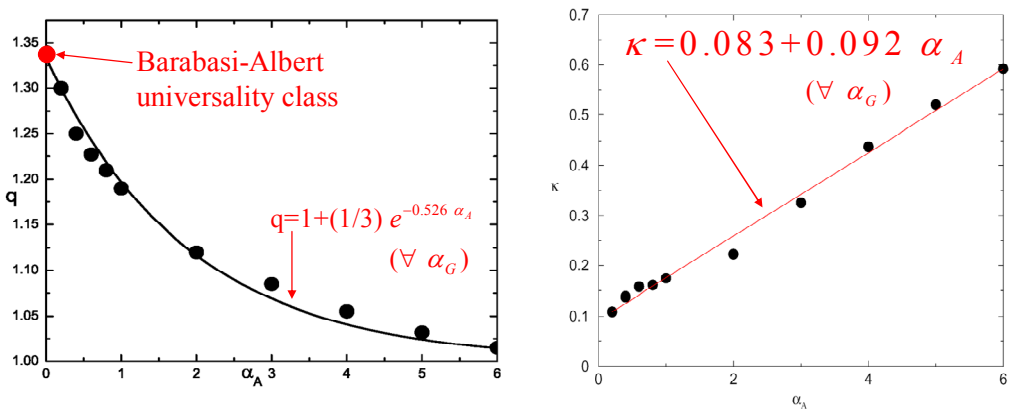


Figure 22 – The parameters (q,κ) of Figure 21. From [26], where further details can be seen.

Let us now exhibit a connection of *q*-statistics to linguistics. We briefly present here *Zipf's law* and its generalizations. If we rank the words of a book, or of various books, or of similar sets (e.g., spoken words in TV or analogous media), from the most frequent (*rank s*=1) to the less frequent (maximal value of *s*, coincident in fact with the size of the vocabulary) we roughly find Zipf's law for the frequency of appearance *f(s)*, namely $f(s) = A/s \quad (A > 0)$, later generalized by Mandelbrot into what is sometimes referred to as *Zipf-Mandelbrot* law:

$$f(s) = \frac{A}{(s_0 + s)^{\nu}} \quad (A > 0;\ s_0 > 0;\ \nu > 0).$$

Let us remark that, with the notation changes $\nu = 1/(q > 1)$, $1/s_0 = (q-1)/\sigma$ and $A/s_0^{\nu} = f_0$, this expression can be rewritten in the *q*-exponential form $f(s) = f_0\ e_q^{-s/\sigma} \quad (f_0 > 0;\ \sigma > 0;\ q > 1)$. This form and its generalizations enable satisfactory description of one of the basic (quantitative) properties of all languages, namely the frequency of use of words (see Figures 23 e 24).

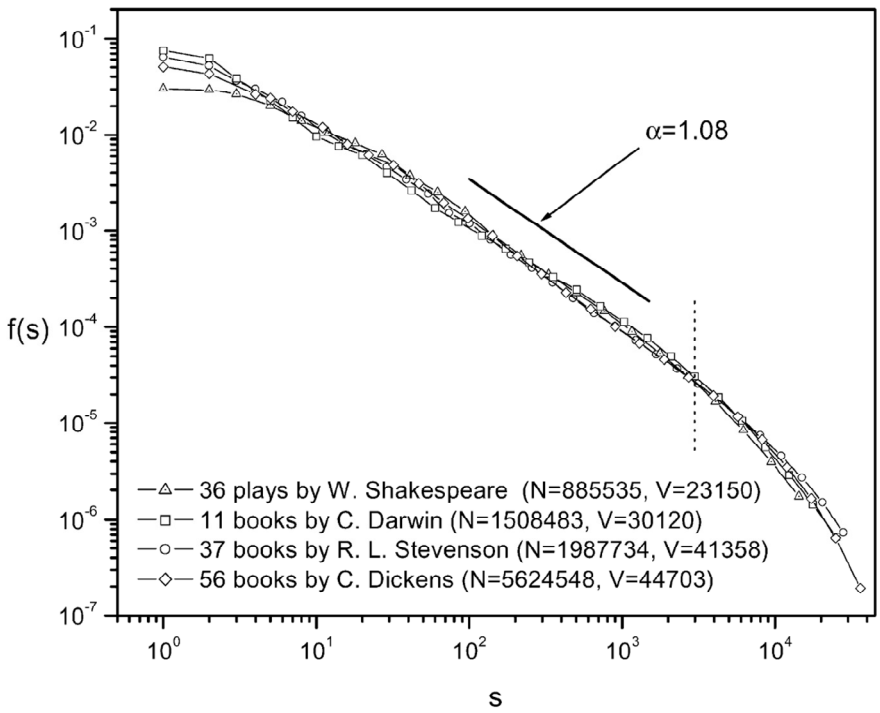


Figure 23 – Rank frequency functions of various authors. It is quite remarkable the fact that the behavior is nearly universal. The same happens with other languages (e.g., Spanish, Italian, Greek): they all appear superimposed on practically the same single curve shown here. From [27], where further details can be seen.

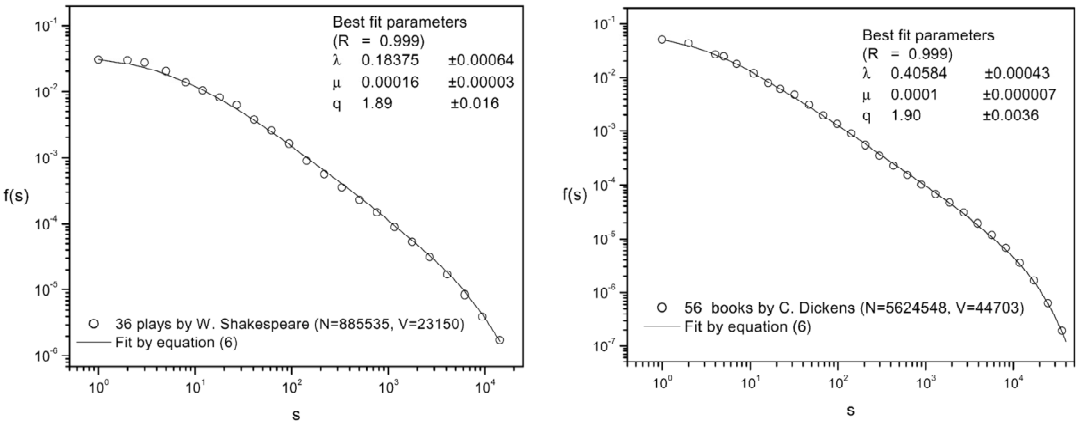


Figure 24 – Rank frequency functions of plays (Shakespeare) and books (Dickens) fitted by the generalizations of the *q*-exponential function used for cosmic rays. From [27], where further details can be seen.

Let us finally address a connection with cognitive psychology. In a learning/memory task, consisting in the memorization of a 5 x 5 matrix with binary symbols (see Figure 25), it was found [28] that computers governed by a nonextensive internal dynamics behave very similarly to humans (see Figure 26). The same fact was verified in the learning of languages [29]. This strongly supports the possibility that humans learn in an essentially global manner (i.e., with $q \neq 1$). This would be the basis of the remarkable capacity of humans to do metaphors – *of all things the greatest*, in Aristotle´s words --, and which led to the characterization of *Homo metaphoricus* [28].

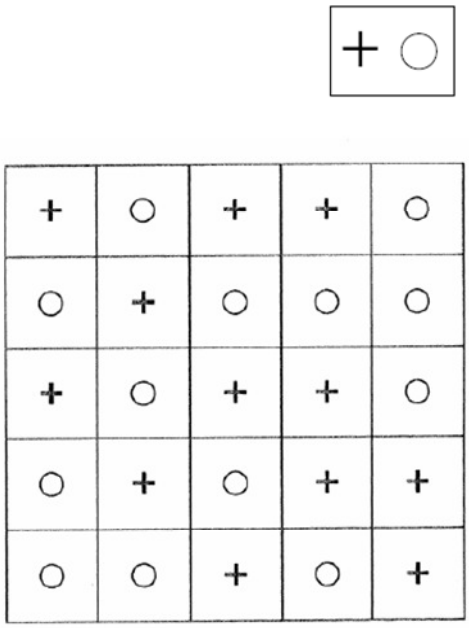


Figure 25 – The 5 x 5 matrix that was learnt through successive exhibitions to the same person. See details in [28].

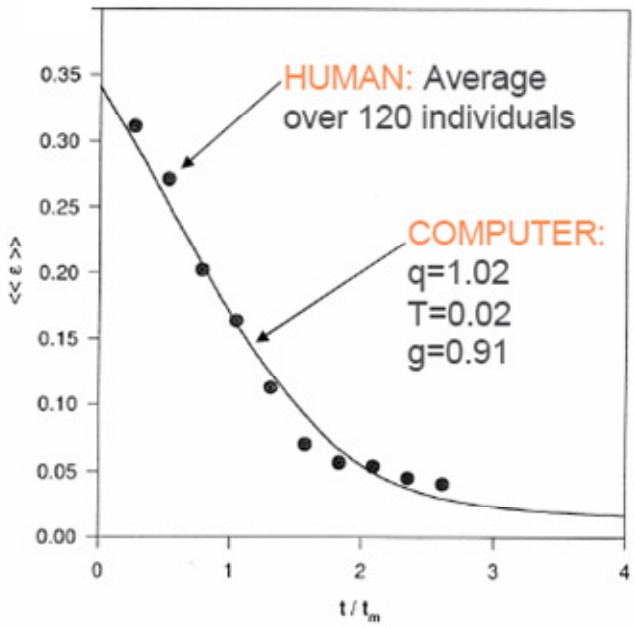


Figure 26 – Average error curve for humans (black dots) and for a nonextensive computational algorithm (continuous curve). The agreement being reasonably satisfactory, we may say that, for this task, humans behave like a computer with global learning dynamics. See details in [28].

4 – Final Remarks

We shall now summarize some of the points that we might have learnt along the various applications of the nonadditive entropy and its associated nonextensive statistical mechanics, as briefly described in the previous Sections.

The focus of the present effort is on *understanding and influencing causality of change of complex socio-technical systems.* A nearly mandatory logical chain must therefore be followed. We must first understand *how* complex natural, artificial and social systems can be identified and characterized, essentially *how* they behave. Then, we must attempt to know *why* they do so. If we succeed in this nontrivial task, we will be at the level of the causes, and we will therefore be in position to understand their basic *causality*. Only then we might have the tools to change it, to determine influence on it. Finally, at the ethical endpoint, we must ask ourselves on whether we wish to do that, why, under what conditions, at what extent, with what purpose, and with what probability of success.

Vast and intricate program! Nevertheless, on quite general grounds, some potentially useful hints do emerge from the analysis that we have undertaken here, along the unifying path offered by the concept of entropy.

We can understand that almost uncorrelated N elements yield exponentially increasing (with N) possible collective configurations. They are many, but possibly uninteresting for high-level purposes. They are typical of thermal equilibrium, where things can blindly just be. In contrast, sensibly correlated elements yield only algebraically increasing (with N) possible collective configurations. They are less in number, but possibly with much larger probability of collective success. They are typical of quasi-stationary states where (slow but efficient) evolution is the protagonist. This is one deep sense that spouses interestingly the title *From Being to Becoming* of Ilya Prigogine´s book.

The role of memory emerges as mostly relevant. Many considerably different possibilities can and ought to be considered between Edith Piaf´s *Non, je ne regrette rien …, Balayé, oublié, je me fous du passé* and Quebec´s *Je me souviens*.

The use of metaphors – that amazing privilege of the *Homo metaphoricus* -- appears not only as possible but also as deeply efficient and fruitful in transposing one complex system into another.

Creativity bridges *fascination* (of discovery) to *knowledge* (of its causes and consequences). Along this line we can give a new sense to Bernard Shaw´s *The reasonable man adapts himself to the world: the unreasonable one persists in trying to adapt the world to himself. Therefore all progress depends on the unreasonable man.* Always however with the necessary touch of freedom: *Si l'action n'a quelque splendeur de liberté, elle n'a point de grâce ni d'honneur,* wrote Montaigne.

Acknowledgments

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