# GM-CPHD and ML-PDA Applied to the Metron Multi-Static Sonar Dataset *

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**Abstract –** *The Gaussian Mixture Cardinalized Probability Hypothesis Density (GM-CPHD) Tracker and the Maximum Likelihood-Probabilistic Data Association (ML-PDA) Tracker were applied to the Metron simulated multi-static sonar dataset created for the MSTWG (Multistatic Tracking Working Group). The large number of measurements at each scan was a problem for the GM-CPHD. Winnowing by a detection test on SNR and Doppler followed by predetection fusion (contact sifting) had to be performed before tracking with the GM-CPHD to obtain good performance. Due to the nature of its likelihood formulation, ML-PDA was able to process the measurements directly and reasonable results for all five scenarios were achieved. Plots of the tracks obtained on the five scenarios the Metron dataset with both trackers are provided.*

**Keywords:** GM-CPHD, ML-PDA, Metron, predetection fusion.

## 1 Introduction

The Metron dataset [1] was generated by Kirill Orlov of Metron, Inc. to continue the evaluation of the tracking algorithms of the MSTWG participants. It was intended to be the follow-up to the SEABAR and TNO-Blind datasets in terms of providing a challenging/realistic scenario. A couple inaccuracies have been discovered in the documentation describing the dataset and are corrected here.

GM-CPHD and ML-PDA represent different paradigms and we are interested to see how they compare when faced with a difficult dataset such as Metron. Our work with the GM-CPHD and the ML-PDA applied to the previous MSTWG datasets can be found in [2] and [3].

## 2 GM-CPHD

The Cardinalized Probability Hypothesis filter is a recursive filter that propagates both the posterior likelihood of (an unlabeled) target state and the posterior cardinality density, i.e. the probability mass function of the number of targets [4].

Under linear Gaussian dynamics and the assumption of state independence for the probability of detection and the probability of survival, closed form filter equations were given in [5]. In that work, the posterior PHD surface was approximated by a Gaussian Mixture and is shown to remain a Gaussian Mixture after the update step. Hence, the propagation of the whole surface could be replaced by the propagation of the weight, mean and covariance of each mode in the mixture. Mode means and covariances are propagated by an Extended Kalman Filter while mode weights are calculated using the PHD equations. In common with other similar trackers such as the Multiple Hypothesis Tracker (MHT), the number of Gaussian modes could increase exponentially with the number of scans, and as such track-management (pruning, merging, etc.) is necessary to make the approach practical.

In our analysis [6], we divide the state space into infinitesimal bins and for each bin, we ask the question, "Is there a target at this location?" The filter contains equations for the prediction and the Bayesian update of the probability of each bin containing a target. Integration of the PHD surface over a volume gives the expected number of targets therein. In the limiting case in which bins' volumes go to zero, the filter's equations converge to the PHD/CPHD filter of Mahler[4] and to the Intensity Filter of Streit [7].

We employ the GM-CPHD filter with a linear motion model and a nonlinear measurement model in which range, bearing and range rate (when available) form the measurement. Our implementation is thus capable of processing both Doppler sensitive (i.e., a constant frequency pulse - CW) and Doppler insensitive waveforms (i.e., a linear frequency modulated pulse - LFM). For LFM waveforms, the range rate measurement ($\dot{r}$) is not significant and hence ignored. In its original form the GM-CPHD filter is not able to provide scoreable tracks, so a track management scheme was devised [8] [9]. This is a set of policies dealing with events such as track initiation, update, merging, spawning and deletion.

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14. ABSTRACT
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# 3 ML-PDA

The Maximum Likelihood Probabilistic Data Association algorithm is a batch tracker that assumes some parameterized, determinisic motion for a target. It makes some simple assumptions about a target and the environment, and then maximizes the likelihood ratio function that results from these assumptions. In a sense, it is simply a batch detector. It was first developed in a passive narrowband application in [10]. Subsequently, in [11] it was expanded to a bistatic active application, which is how we currently employ it. The assumptions that go into ML-PDA and the development of the likelihood function are well documented in [12].

For this work, we implemented ML-PDA as a sliding batch/window tracker. The ML-PDA window length was set to 1800 seconds (11 pings), and all frames that fell within this window were used for the tracker cycle (each frame consists of the measurements from a unique source-receiver combination at a certain time). At every tracker update, the window was shifted forward 360 seconds. Within the window, target motion was parameterized as a straight line. This window size is long enough to pick out deterministic target motion from random clutter, while short enough to approximate the motion of a moderately maneuvering target with a series of line segments.

The ML-PDA, as formulated in [12], is a single target tracker. Work has been done [13], [14] to extend ML-PDA to a multi-target framework, but in this implementation, we took a different approach for dealing with multiple targets. For each window, the (single-target) likelihood ratio function was maximized. If a target was present, as determined by the peak of the likelihood ratio exceeding some threshold, a track was declared. Next, the most probable measurement associated with the discovered track was removed from each frame of data. (This is possible because one of the assumptions of ML-PDA is that at most one target detection is present in each frame.) The optimization over the window was then redone, and this process was repeated until no more targets were found in a window.

For the Metron dataset, ML-PDA was optimal in the sense that it used all available data in the measurements. In addition to using the positional measurement to find targets, it used amplitude data as well as Doppler information when available. Specifically, in the case of Doppler, our implementation took advantage of both the Doppler from the target, which increased the value of the likelihood ratio around an actual target, as well as the distribution of the Doppler from clutter points, which helped to suppress false alarms.

# 4 Metron Dataset

The surveillance area is $72000 \times 72000\ m^2$. The 25 stationary sensors are laid out as two concentric square grids as seen in Figure 1. All the sensors are receivers with the exception of the 4 marked sensors which are colocated source/receiver units.

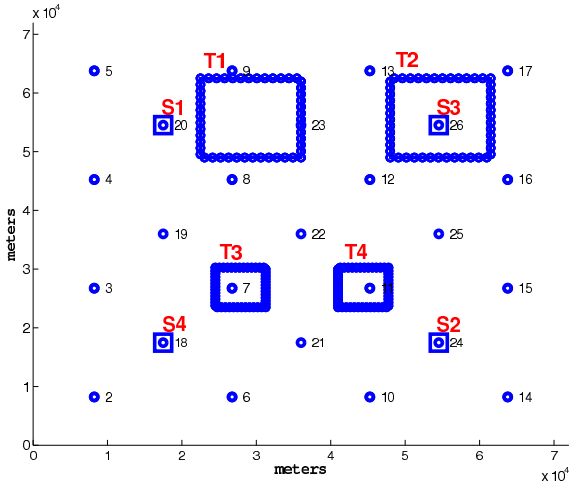


Figure 1: Ground truth in scenario 1.

Two waveform types are simulated: CW and FM. CW yields position and Doppler information for contacts, whereas FM only yields position information. The ping schedule is the following: S1 CW, S2 FM, S3 CW, S4 FM, S1 FM, S2 CW, S3 FM, S4 CW with a ping occuring every 180 seconds for a total of 200 pings.

There are 4 targets. Each target completes four cycles of motion about the perimeter of a square region. Target motion specifications can be found in Table 1. The contacts generated by targets 2 and 3 have been labeled [1].

Measurements were generated with the following errors:

- Sound speed is 1500 m/sec
- Bearing error is normally distributed with mean 0.0° and standard deviation 8.0°
- Time difference of arrival (TDOA) error is normally distributed with mean 0.0 sec and standard deviation 0.4 sec
- Bistatic Doppler error is normally distributed with mean 0.0 m/sec and standard deviation 0.5 m/sec (CW only)

# 5 Winnowing

The Metron dataset contains too many contacts at each scan for the GM-CPHD to handle successfully and therefore, a winnowing step was devised. For a CW contact, we winnow by a simple detection test on SNR and Doppler (see Eq. 1). For an FM contact, we winnow by SNR only. Selecting a suitable threshold requires tuning. The contacts that survive the winnowing are fed to the predetection fusion stage.

$$\frac{f_1}{f_0}(snr) \times \frac{f_1}{f_0}(Doppler) \gtrless threshold \qquad (1)$$

Table 1: Target Parameters.

| | Target1 | Target2 |
|---|---|---|
| Speed | 6 | 6 |
| Start | (36000,49000) | (48000,49000) |
| Box | $13500 \times 13500$ | $13500 \times 13500$ |
| Motion | Clockwise | CounterClockwise |
| Label | 0 | 1 |

| | Target3 | Target4 |
|---|---|---|
| Speed | 3 | 3 |
| Start | (24250,30250) | (47750,30250) |
| Box | $6750 \times 6750$ | $6750 \times 6750$ |
| Motion | CounterClockwise | Clockwise |
| Label | 1 | 0 |

## 5.1 Doppler

In the Metron dataset, the Doppler for clutter contacts is drawn from a normal distribution with a mean of 0 knots and standard deviation of 2.5 knots. Therefore we can calculate:

$$f_0(x) = \frac{1}{\sqrt{2\pi} * 2.5knots} * exp\left(-\frac{x^2}{2*(2.5knots)^2}\right) \quad (2)$$

The predicted Doppler for a true contact is:

$$\hat{x} = \frac{\dot{x}_t(x_t - x_s) + \dot{y}_t(y_t - y_s)}{2ST} + \frac{\dot{x}_t(x_t - x_r) + \dot{y}_t(y_t - y_r)}{2TR} \quad (3)$$

where $\dot{x}_t$, $\dot{y}_t$ are the target speed components, $(x_s, y_s)$ is the location of the source, $(x_r, y_r)$ is the location of the receiver and $ST$ and $TR$ are the distances between source and target, target and receiver.

Given the Doppler measurement error described in Section 3, we can write:

$$f_1(x) = \frac{1}{\sqrt{2\pi} * 0.5m/sec} * exp\left(-\frac{(x-\hat{x})^2}{2*(0.5m/sec)^2}\right) \quad (4)$$

## 5.2 SNR

We have observed that the clutter SNR is exponential in dB (see Figure 2). This is consistent with the documentation, but such distributions are usually encountered before conversion to decibels. We can write:

$$f_0(y) = \frac{1}{\theta} * exp\left(-\frac{(y-\tau)}{\theta}\right) * u(y-\tau) \quad (5)$$

where $u$ is the step function.

We have applied a fit to the plot in Figure 2 to determine the parameter $\theta$. Metron contains only contacts with SNR above $\tau = -1.5$dB for FM pings and above $\tau = -5.5$dB for CW pings; we obtained $\theta = 2.95$ for CW and $\theta = 2.11$ for FM.

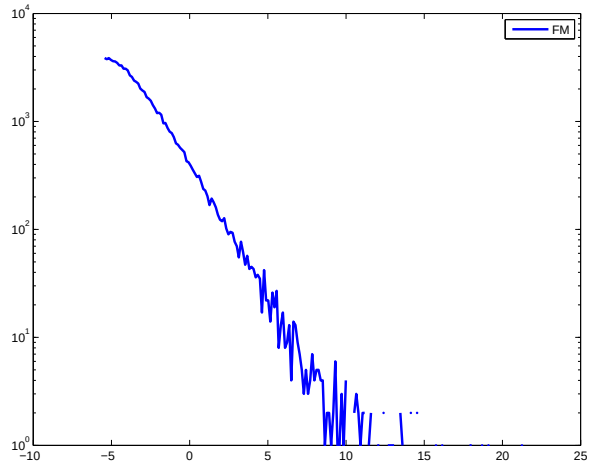


Figure 2: Clutter SNR (semilog plot).

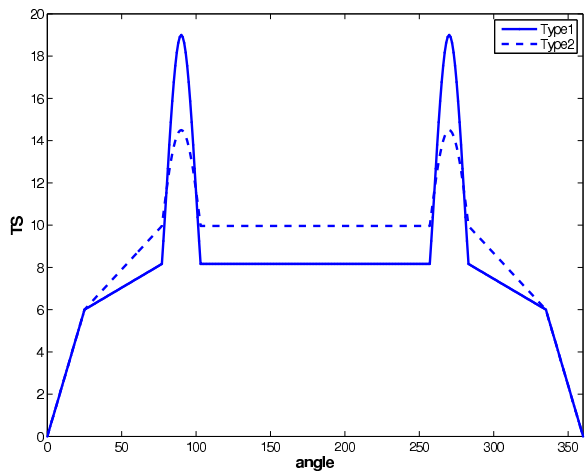


Figure 3: Target strength.

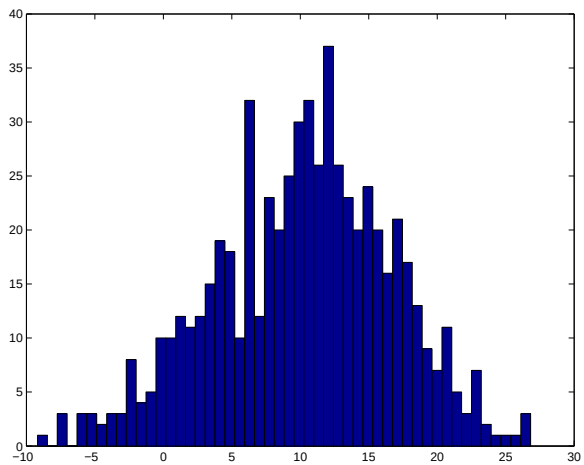

Figure 4: Histogram of tagged observed-predicted SNR.

The predicted contact SNR for target originated contacts, $\hat{y}$, is modeled using the following equations:

$$\hat{y}_{CW} = 65 - TL_{ST} - TL_{TR} + TS \tag{6}$$

$$\hat{y}_{FM} = 69 - TL_{ST} - TL_{TR} + TS \tag{7}$$

where $TL_{ST}$ is the transmission loss between the source and target, $TL_{TR}$ is the transmission loss between the target and the receiver, and $TS$ is the target strength. Given entities A and B that are R meters apart, $TL_{AB}$ is computed as $10log_{10}(R)$. $TS$ varies by target type, and is a function of the bistatic aspect angle. Our approximation of the target strength shown in the documentation can be seen in Figure 3. All the 4 targets in scenario 1 of the Metron dataset are of type 1.

The documentation also specifies that a simulated contact SNR is drawn from a normal distribution with the mean computed in this way and a standard deviation of 8 dB.

By plotting the difference between the observed contact SNRs and the predicted contact SNRs (Figure 4), some inaccuracies in the documentation were revealed: a standard deviation of 6dB should be used instead of 8dB as described and the predicted SNR in Eq. 6 and Eq. 7 should be 10dB higher.

With these in mind, we can write:

$$f_1(y) = \frac{1}{\sqrt{2\pi} * 6dB} * exp\left( - \frac{(y - \hat{y})^2}{2 * (6dB)^2} \right) \tag{8}$$

# 6 Predetection Fusion

We believe that the multi-sensor PHD needs further investigation. There are good but contending ideas from Dr. Roy Streit and Dr. Ron Mahler. The fusion concern stems from the PHD assumption that the bins are independent after every update, and if the updates are simultaneous this is a problem. Hence we chose to do predetection fusion/contact sifting [15].

Usual predetection fusion is simply a likelihood ratio test. But in this (practical) case the alternative hypothesis is composite: the location of the target is not known so a more sophisticated fusion scheme must be used. We have observed that the measurement errors of the Metron dataset result in the measurements' Cartesian covariance ellipses being very eccentric. Some uncertainties are as much as 10-20km (major axis of ellipse). Therefore, for each contact, we generate 100 samples via Monte Carlo according to the contact's measurement error covariance matrix. Without this step a large covariance measurement would still only be "seen" in the grid cell at the measurement's nominal value.

We then "sift" these according to a grid (we use $25 \times 25$): if a contact yields a sample that is quantized to a grid cell, then that contact is added to the cell's list. We test each grid cell's number of hits against a threshold calculated according to binomial distribution and desired fused probability of false alarms, $P_{fa}$. For cells that pass the test, a Probabilistic Multi Hypothesis Tracker (PMHT) measurement model [16] and the Expectation-Maximization (EM) algorithm are used on that cell's listed contacts to refine the estimated measurement location and to estimate the posterior covariance. Afterwards, we merge detections that gate with each other, since often neighboring cells have used the same detections from the initial Monte Carlo step.

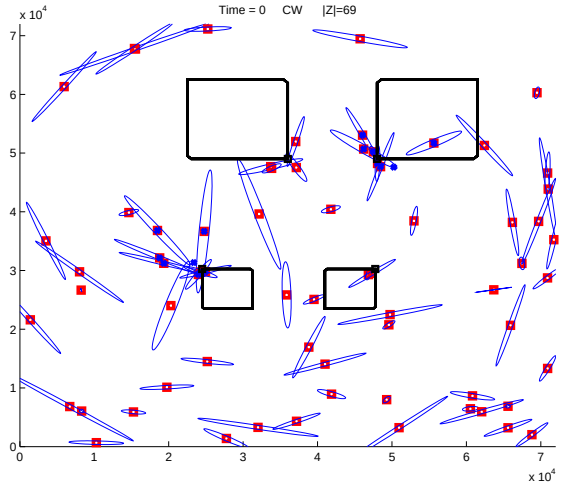


Figure 5: Winnowing on the first scan of data.

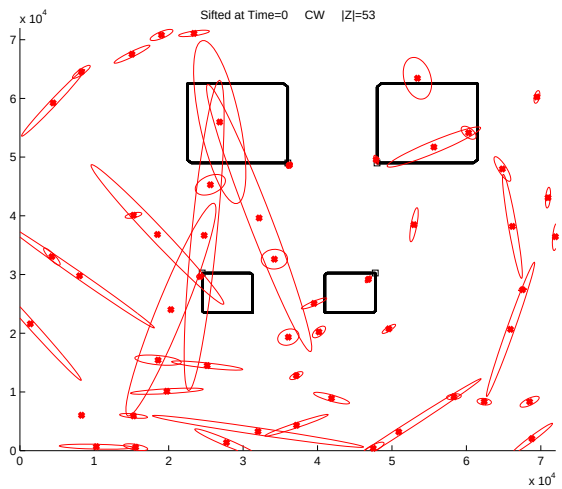


Figure 6: Predetection fusion on the first scan of data.

Figure 5 shows the initial scan of data (25 pings, one from each receiver) from the Metron dataset after contact winnowing based on amplitude and Doppler. Note that for target 3, without the Monte Carlo step the tagged measurements (the measurements with a blue dot inside the red square) would not be associated together. Figure 6 shows the same scan of data after the predetection fusion step. Although there are still many contacts, note the single low-covariance ones at the southwest corner of target 1, southeast corner of target 2, northeast corner of target 3 and northwest corner of target 4 (the starting positions of the four targets).

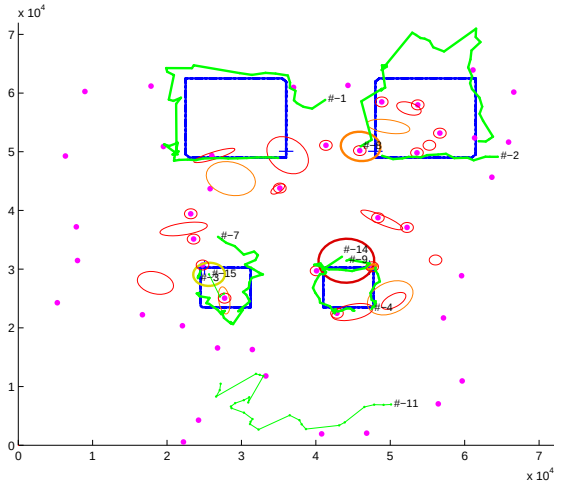

Figure 7: Scenario 1 GM-CPHD tracks (scans 1-50).

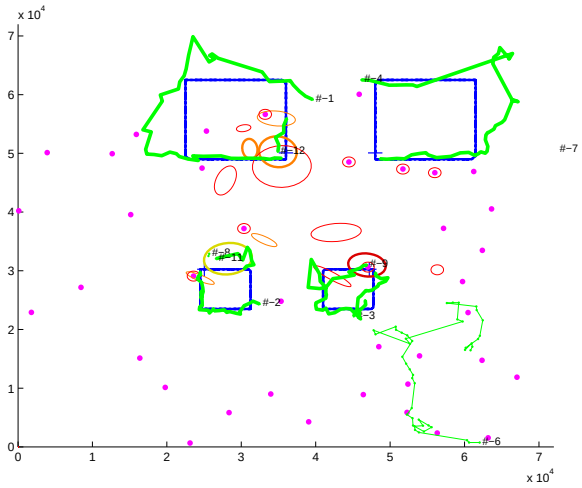

Figure 8: Scenario 1 GM-CPHD tracks (scans 50-100).

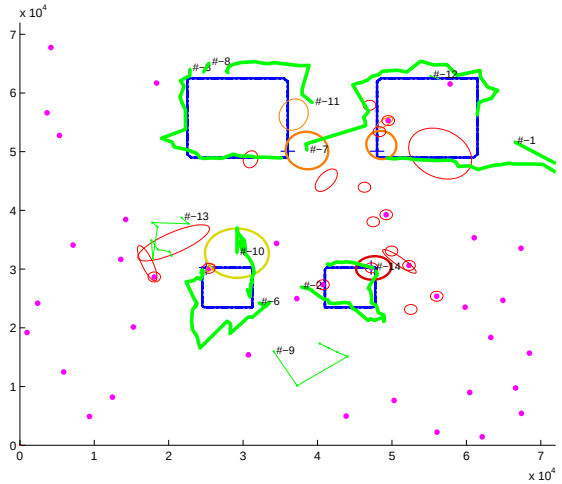

Figure 9: Scenario 1 GM-CPHD tracks (scans 100-150).

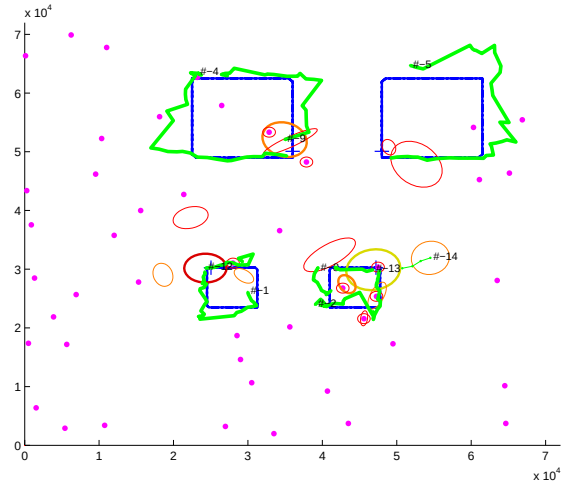

Figure 10: Scenario 1 GM-CPHD tracks (scans 150-200).

# 7 GM-CPHD Results

It should be mentioned that running the GM-CPHD without winnowing and without contact sifting resulted in unsatisfactory performance. Our GM-CPHD results on the Metron dataset were made possible by the addition of winnowing and predetection fusion steps before the tracker is run. Plots of the generated tracks are displayed in Figures 7-14. The ellipses reveal the location and covariance of the Gaussian modes, the magenta dots represent the measurements at the current (here, last) scan.

The results on scenario 1, for which the ground truth is provided, were obtained with perfect initialization, i.e. a mode of weight 1 had been placed at the initial scan at the exact location of each target. This was not the case for the other scenarios. GM-CPHD successfully followed all targets as they move four times around their square trajectories.

In scenario 2, GM-CPHD discovered 3 targets, each following a rectangular trajectory. Their being in close proximity while the measurements were very noisy made for a challenging situation but the tracker was able to keep up and output a nice figure eight. In scenario 3, we believe that there are 2 targets present: a target following a bow tie path which intersects with a target following a straight line. In scenario 4, GM-CPHD has revealed 3 targets. Two of them have parallel straight paths and the third crosses these at almost a right angle. Scenario 5 has proven to be the most difficult scenario. We estimate there are two targets present - one with a sinusoidal trajectory and one moving in a straight line.

For all scenarios, the tracking performance was good as little fragmentation was present, the track detection probability was high, the number of false tracks was very low and therefore the false alarm rate was very low, while the RMS error was acceptable.

Our tracker does not include an IMM at the current time and our work on these scenarios has underlined the need

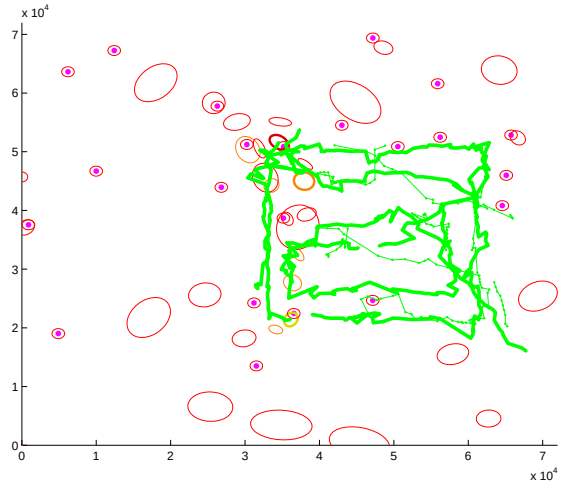


Figure 11: Scenario 2 GM-CPHD tracks.

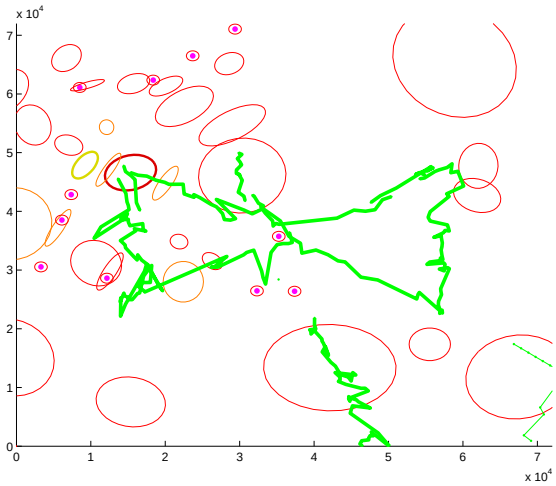


Figure 12: Scenario 3 GM-CPHD tracks.

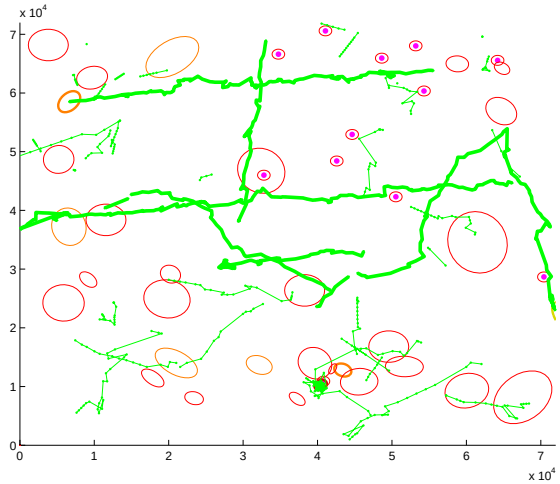


Figure 13: Scenario 4 GM-CPHD tracks.

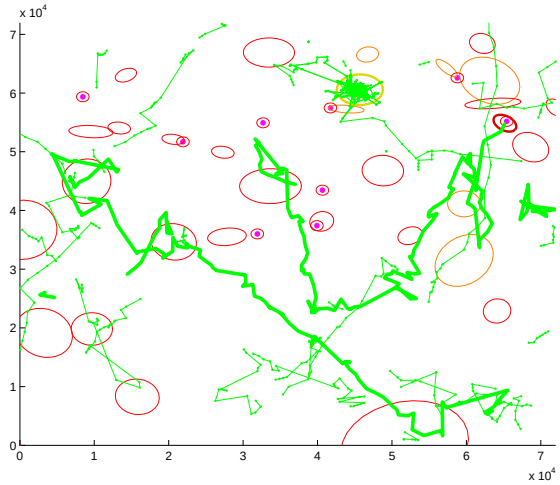


Figure 14: Scenario 5 GM-CPHD tracks.

for it, since an Interacting Multiple Model (IMM) estimator would increase the performance on datasets, such as Metron, in which there are sharp turns along the target trajectory.

# 8 ML-PDA Results

The ML-PDA algorithm perfomed rather well on the Metron dataset. It did not require any preprocessing of the data (i.e. it did not require winnowing or contact fusing). The ML-PDA is in a sense a batch detector, and its likelihood function formulation essentially has the winnowing function described above built into it. Furthermore, the ML-PDA formulation does not depend on the number of sensors that are providing data to it. As long as one of the basic assumptions of ML-PDA is not violated – at most one return from the target is present in any given frame – the only limitation to ML-PDA in terms of the number of sensors feeding information to it is computer processing time. Thus, the ML-PDA algorithm does not require the predetection fusion algorithm described above.

Three track initialization methods were tried for ML-PDA: a two-point initialization, a one-point initialization and a grid space initialization. In general, the grid space initialization worked the best, although it required the longest computation time. In some cases, though, the two-point initialization scheme worked very well, which is desireable since it ran the fastest, and it required no assumptions about the target motion. The grid initialization and the one-point initialization both required assumptions about possible initial courses and speeds.

For Metron scenarios 1-3, the ML-PDA algorithm produced good results. For scenario 1, it picked up all four targets doing multiple rectangular revolutions. On scenario 2, the algorithm detected targets that appeared to trace out an “eight.” For scenario 3, a target appeared to follow a “bowtie” pattern, and there was a second target that crossed the middle of the bowtie. Scenarios 4 and 5 appeared some-

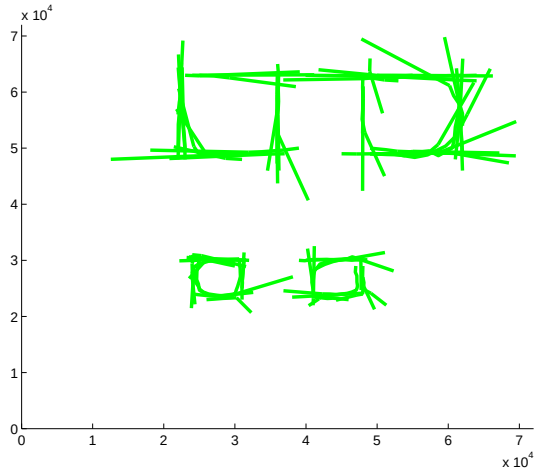


Figure 15: ML-PDA for Metron scenario 1.

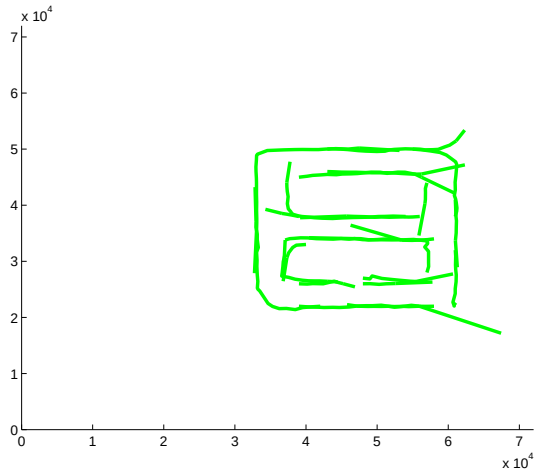


Figure 16: ML-PDA for Metron scenario 2.

what more cluttered, although in each one, a stationary target was definitely picked out. In scenario 4, this appeared at $(x, y) = (40000, 10000)$ meters, while it scenario 5, the stationary target was at $(x, y) = (45000, 60000)$ meters. These stationary targets were also found with the GM-CPHD (see Figures 13 and 14).

# 9 Conclusions

The GM-CPHD and ML-PDA trackers were applied to the Metron simulated multi-static sonar dataset created for the MSTWG. Metron proved to be an extremely challenging dataset in which the data is extremely noisy and obeys strong statistical models (Doppler, clutter, signal-excess, aspect-dependent SNR), more than would practically be found.

Feeding the tracker all the measurements in a scan, i.e. the combined measurements from 25 receivers obtained at the same time stamp, resulted in considerably long run time and poor performance of the GM-CPHD as such a large number of measurements at each scan combined with an ex-

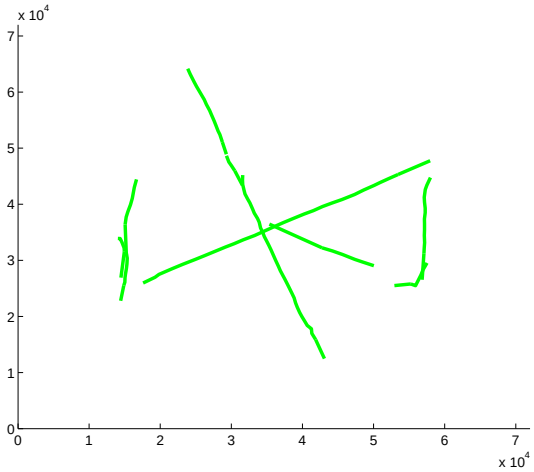


Figure 17: ML-PDA for Metron scenario 3.

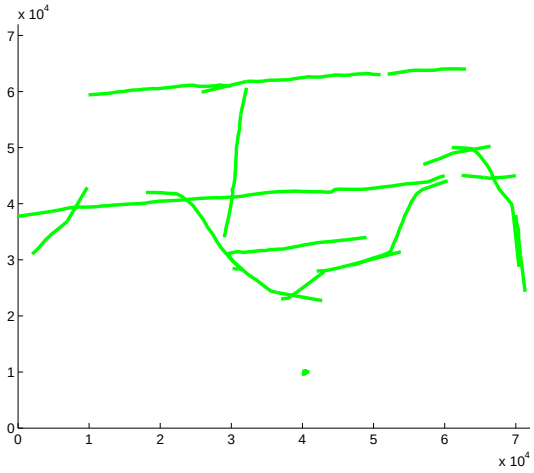


Figure 18: ML-PDA for Metron scenario 4.

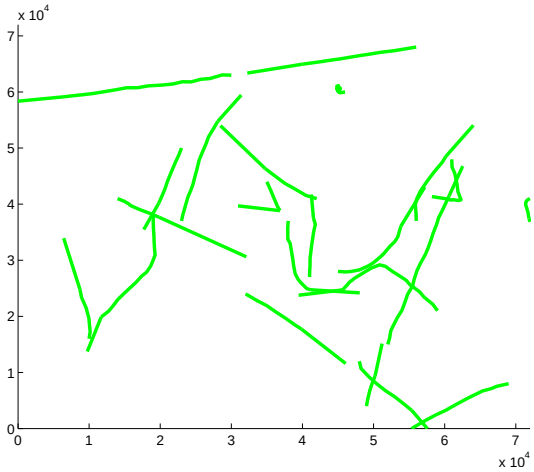


Figure 19: ML-PDA for Metron scenario 5.

tremely low probability of detection was a problem for the track management. The ML-PDA was not affected by this problem.

The Metron multisensor data required winnowing by amplitude and Doppler and a predetection fusion scheme to be applied before the GM-CPHD tracker. This approach worked well: on scenario 1 the GM-CPHD tracks are in the vicinity of the ground truth and on scenarios 2-5 the GM-CPHD results are in agreement with the ML-PDA results.

The ML-PDA algorithm was also applied to the Metron dataset. Due to how its likelihood function is formulated, it was able to process the Metron data directly and obtain results without the need for winnowing or predetection fusion. It was able to get good results for scenarios 1, 2 and 3, with slightly noisier results for scenarios 4 and 5. The ML-PDA did sometimes have trouble with contacts that abruptly maneuvered. Future work should add the capability of maneuver models into the ML-PDA framework.

GM-CPHD and ML-PDA discovered the same targets in all scenarios and were very similar in performance, with the ML-PDA generating slightly more clear tracks with no predetection fusion needed. This reinforced the claim that ML-PDA is effective in tracking very low observable targets where target signal-to-noise ratios require very low detection processing thresholds to reliably give target detections [14]; the Metron dataset provided such a situation as the probability of detection, $P_D$, per sensor per scan was around 0.12 and the target SNR was generally not much higher than the clutter SNR. However, ML-PDA uses low-thresholded measurement data over a batch of measurement frames and computes track estimates using a sliding window. On the other hand, GM-CPHD is not a batch algorithm yet it shows comparable performance to the ML-PDA.

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