Relativistic Timescale Analysis Suggests Lunar Theory Revision

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Abstract

The International Standard (SI) second of the atomic clock was calibrated to match the Ephemeris Time (ET) second in a mutual four year effort between the National Physical Laboratory (NPL) and the United States Naval Observatory (USNO). The ephemeris time is "clocked" by observing the elapsed time it takes the Moon to cross two positions (usually occultation of stars relative to a position on Earth) and dividing that time span into the predicted seconds according to the lunar equations of motion. The last revision of the equations of motion was the Improved Lunar Ephemeris (ILE), which was based on E. W. Brown's lunar theory. Brown classically derived the lunar equations from a purely Newtonian gravity with no relativistic compensations. However, ET is very theory dependent and is affected by relativity, which was not included in the ILE. To investigate the relativistic effects, a new, noninertial metric for a gravitated, translationally accelerated and rotating reference frame has three sets of contributions, namely (1) Earth's velocity, (2) the static solar gravity field and (3) the centripetal acceleration from Earth's orbit. This last term can be characterized as a pseudogravitational acceleration. This metric predicts a time dilation calculated to be -0.787481 seconds in one year. The effect of this dilation would make the ET timescale run slower than had been originally determined. Interestingly, this value is within 2 percent of the average leap second insertion rate, which is the result of the divergence between International Atomic Time (TAI) and Earth's rotational time called Universal Time (UT or UT1). Because the predictions themselves are significant, regardless of the comparison to TAI and UT, the authors will be rederiving the lunar ephemeris model in the manner of Brown with the relativistic time dilation effects from the new metric to determine a revised, relativistic ephemeris timescale that could be used to determine UT free of leap second adjustments.

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Introduction

Time is measured by counting cycles or fractions of cycles of any physical repeatable phenomenon. The oldest method is based on the rotation of the Earth to define the timescale called Universal Time (UT or UT1 to be more specific). The actual solar day varies by the angles sunlight strikes the Earth as it moves in its inclined elliptical orbit. Through mathematics, the concept of a mean solar day can be established in terms of the sidereal day that Earth takes to rotate 2π radians. As the Earth's rate of rotation was discovered to vary somewhat, a more precise time standard was developed by monitoring the motion of the heavenly bodies and comparing them to the theory of motion for that body. Similar to hands of a clock passing the numbered positions on the clockface, the observed position or ephemeris of a heavenly body against the stellar background determines the timescale, called Ephemeris Time (ET). Unfortunately, ET is very theory dependent. The actual Ephemeris Time of an event was determined well after it occurred due to postprocessing of the observations.

In the mid 1950s, precise atomic frequency standards were developed for ultrastable, long term operation. The atomic vibrations would be monitored so that the number of elapsed cycles could provide the conversion to establish an atomic clock. The primary atomic timescale is currently the International Atomic Time (TAI). The length of the atomic SI second was defined by Markowitz et al. (1958) by an observationally determined value of the ET second obtained from the Improved Lunar Ephemeris (ILE). However, a timing problem surfaced when it was seen that UT ran at a different rate than TAI. Based on conversations with personnel at the US Naval Observatory (USNO) into the derivation of the ILE, it was determined that relativity effects were not incorporated into Brown's lunar theory. Preliminary relativity calculations have yielded a time dilation effect in the lunar ephemeris with a value that is within 2% of the observed divergence between UT and TAI. Work is ongoing to rederive a relativistic lunar ephemeris and obtain a relativistic ET timescale, which will be compared to the TAI and UT timescales.

Development of the Ephemeris and Atomic Timescales

The International Atomic Time (TAI) scale is based on the rate of time defined by the Système International (SI) second. Since 1967, the SI second has been the standard unit of time in all timescales. The calibration study that utilized the ILE to define the SI second averaged the cycles tabulated over 4 years from the cesium standard and compared them to the length of the ET second.[1] So, the SI second matches an ephemeris second very closely and provides continuity between the ET and TAI timescales.[2]

The ILE is a classically derived lunar ephemeris, which is based on E. W. Brown's classical lunar theory as derived from Newtonian gravitation. Brown's original theory as documented in his memoirs[3,4,5,6,7] was finished before general relativity was published in 1916. General relativity theories prior to 1950 using standard spherically symmetric metrics for a single mass produce relativistic corrections well below the level of precision of the empirical corrections applied to the ILE.[8] Therefore, relativistic corrections to the ILE were not considered necessary.

The very first version of ET was defined by Clemence, who used Newcomb's classical theory

for the Tables of the Sun from 1896. Since Einstein published his special and general relativity theories in 1904 and 1916, respectively, it is obvious that ET had no intentional relativistic corrections incorporated in the first ET timescale. From the observational results of Spencer Jones (1939),[9] Clemence derived the fluctuation factor Δ = ET - UT to convert UT to a time measure defined by Newcomb's tables.[10] Because the year was so long, which then took months after an event to determine ET, the Moon's orbit was the best object to study because it had the shortest period. The best lunar theory available was Brown's methodical derivation. But, Brown had to adopt an empirical term from other sources to get better agreement between his lunar theory and the lunar observations used to get the constants of integration for his theory. Clemence determined the correction to Brown's lunar theory so that the independent time variable in the lunar theory would be the same as that in Newcomb's Table of the Sun.[11] Following Clemence's computations published in 1948, the International Astronomical Union agreed to remove Brown's empirical term and to rescale Brown's lunar theory by correcting the mean longitude, L, with the following equation:

$$\Delta L = -8.72'' - 26.74''T - 11.22''T^2 = \Delta L_0 + \Delta nT + \frac{1}{2}\Delta\dot{n}T^2 \tag{1}$$

where T is measured in Julian centuries from 1900 January 0 at Greenwich Mean Noon.

The equation to correct the mean longitude of the Moon can be considered a correction to the mean motion rate of $\dot{n}$ by a value of $\Delta\dot{n} = -22.44''/cy^2$. This modification to the mean longitude agreed with the observations of Spencer Jones (1939). Brown's lunar theory with this correction to the mean longitude and a minor aberration correction term made up the ILE used to compute ET. Recently, Markowitz reported[12] that the SI second and the ILE second were still consistent to a part in 10^{10}, which effectively establishes that the SI and ET seconds are equivalent.

Evidence of Timescale Problem

There has been considerable evidence of timescale inconsistencies between UT and ET. Ephemeris timescales based solely on the orbital periods of the planets appeared to run faster than UT. Data from Spencer Jones showed that the lunar orbital secular acceleration was $5.22''/cy^2 = \Delta\dot{n}_{\text{Moon}}$ and the apparent secular acceleration of the solar orbit was $1.23''/cy^2 = \Delta\dot{n}_{\text{Sun}}$. Spencer Jones attributed the cause to tidal friction slowing down Earth's rotational rate.[13] It also appears that Clemence computed the secular acceleration of Earth's rotation, $\dot{\omega}$, using the secular orbital acceleration of the Moon and Mercury to get $\Delta|dotn = -11.22''/cy^2$. Munk (1963) computed the secular acceleration of Earth's rotation from Spencer Jones' numbers with the following formula for the "weighted discrepancy difference," in which any dependence to a variable Earth rotation was removed.[14] The attempt here was to extract the contribution due to any lunar errors in the timing problem from other sources. So, the weighted discrepancy difference (WDD) is the weighted difference of the secular orbital accelerations between the Moon and Sun that has not been accommodated in the lunar ephemeris used for defining the lunar ET.

$$WDD(t) = \int\int \left[\dot{n}_{\text{Moon}}(t) - \left(\frac{n_{\text{Moon}}}{n_{\text{Sun}}}\right)\dot{n}_{\text{Sun}}(t)\right\} dt\ dt \quad \text{which implies that} \tag{2}$$

$$\frac{d^2}{dt^2}WDD(t) = \left[\dot{n}_{\text{Moon}}(t) - \left(\frac{n_{\text{Moon}}}{n_{\text{Sun}}}\right)\dot{n}_{\text{Sun}}(t)\right] = 5.22''/cy^2 - 13.37(1.23''/cy^2) = -11.22''/cy^2 \tag{3}$$

Based on Clemence's results, WDD(t) could be computed by using Mercury instead of the Sun. Munk assumed that WDD is due to the secular acceleration of Earth's rotation, which will affect values of the independent variable t. He ruled out the alternative option, which is $\dot{n}_{\text{Mercury}} n_{\text{Mercury}} = \dot{n}_{\text{Sun}} n_{\text{Sun}}$, because these secular orbital accelerations are empirical and have no explanation from classical gravitation theory. Lambeck did basically the same thing as Munk using solar, Mercury and Venus data.[9,15,16,17] Using Spencer Jones' work plus three other sources, Lambeck concluded[18]

$$\frac{\dot{n}_{\text{Sun}}}{n_{\text{Sun}}} = \frac{\dot{n}_{\text{Mercury}}}{n_{\text{Mercury}}} = \frac{\dot{n}_{\text{Venus}}}{n_{\text{Venus}}} \tag{4}$$

Again, Lambeck reached the same result as Munk and stated that the empirically derived acceleration has to be caused by a secular deceleration in Earth's rotation as the only plausible mechanism under classical theory.

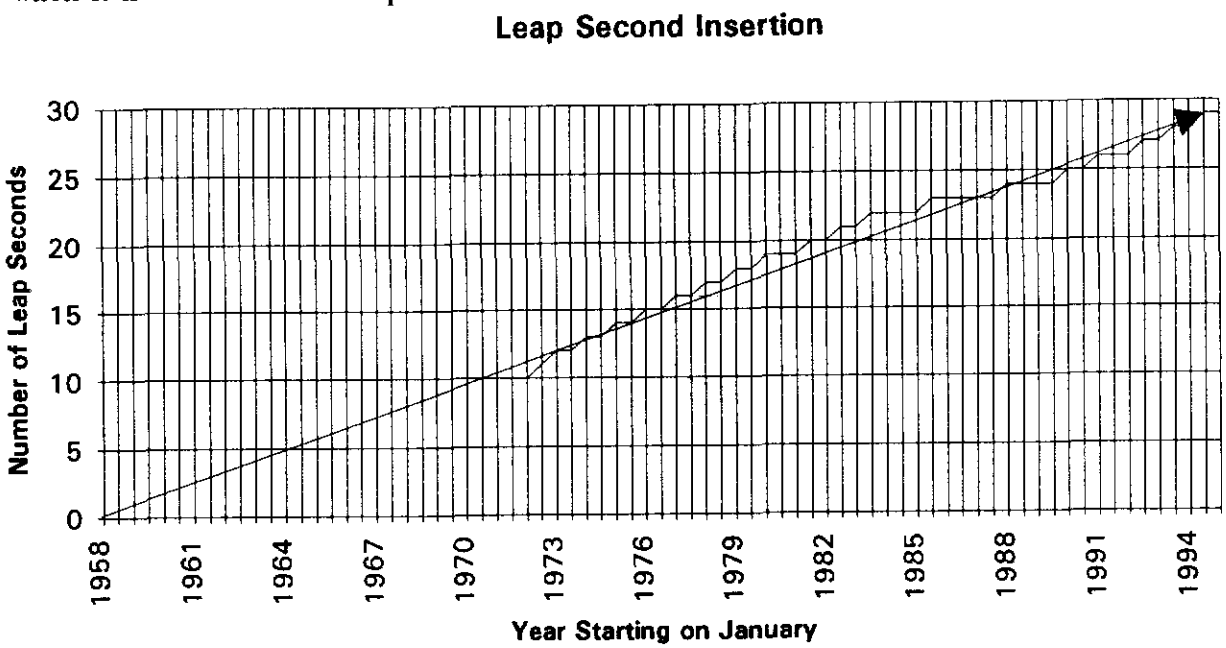
All of these authors would get the same value for what is interpreted as the secular acceleration of Earth's rotation, $-11.22''/cy^2$. Notice this is exactly the value for the quadratic term in the equation used to correct Brown's lunar theory for the ILE. This value corresponds to a corrected secular acceleration in the Moon's mean longitude of $-22.44''/cy^2$. A very recent observation using lunar laser ranging gives $-26.0'' \pm 1.0''/cy^2$ for the Moon's secular acceleration.[19]

When a divergence occurs between two time standards, either the first standard is running slower than the second or the second standard is running faster than the first. All of the authors mentioned in the previous section have identified that there is a timing problem between a timescale based on Earth's rotation and ephemeris time. One option is that ET is running a bit too fast, which could be caused by not including sufficient relativity corrections to lengthen the time unit interval appropriately in the orbital equations of motion. The original ET standard used Earth's orbit to measure one year, which was then divided into ephemeris seconds based on the classically derived theory of the Sun. If the ephemeris second interval were a bit smaller than the proper second interval in a relativistic theory, the ET standard would predict that Earth would complete one entire orbit before Earth actually traveled 2π radians of mean anomaly. Let M represent the observed mean anomaly and T, the orbital period of the Earth. Then, $\Delta M = M - nT$. As T = 2π/n, then $\Delta M = M - 2\pi$. This discrepancy is often interpreted as a secular acceleration, $\Delta M = \frac{1}{2}\dot{n}T^2$. If ΔM is caused by an annual, fixed timing error, ΔT, then one may write $\Delta M = n\Delta T$. The correction between the secular acceleration, and the timing error is given by

$$\frac{\dot{n}}{n} = \frac{2\Delta T}{T^2} = \text{constant} \tag{5}$$

Munk and others have attributed the source of the problem to tidal friction that slows down the Earth's rate of rotation, which then makes the UT timescale run slower, whereas the above ratios suggest that the timing problem is attributed to ET running slightly fast. If the computed ET is running faster than the actual ET, ΔM will be negative. This is confirmed when inserting the negative value of $\dot{n}$.

There has been a general divergence between UT and TAI timescales over the past 30 years. Since the epoch for both UT and TAI is 0 hour of 1958 January 1, UT (as modeled by Universal Coordinate Time UTC as based on the SI second) has trailed behind TAI by 29 seconds.[20] The leap seconds inserted into the UTC timescale, which closely follows UT, are plotted in Figure 1. Leap seconds are inserted at midnight of either December 31 or June 30, depending when it is decided that an update is needed.



Looking at Figure 1, there is a periodic variation in the overall trend as UT and TAI steadily diverge. Fluctuations in the Earth's rotation over timescales of less than a few years are dominated by atmospheric effects,[21,22,23] which affect the atmospheric angular momentum and Earth's moment of inertia and rotation. The average leap second insertion rates for three recent intervals show the effect of the granularity in the data caused by the periodic behavior of the atmosphere and the constraint of inserting leap seconds on the approved dates of June 30 and December 31. The three slopes can also be used to determine the excess length of a mean solar day in terms of SI seconds.

Average Length of Mean Solar Day in SI Seconds	
1992-1958	86400.00214
1993-1958	86400.00216
1994-1958	86400.00218

Subtracting 24 hours of seconds from the average length of day and then inverting gives the

average leap second insertion rate in days as shown in the table below:

Average Leap Second Insertion Rate	
1958-1992	466.6667 day/sec
1958-1993	463.0357 day/sec
1958-1994	459.6551 day/sec

The intent of the cesium clock calibration experiment in 1958 was to calibrate the SI second so that is would be as close as possible to the ET second. It is obvious from the figure that the rates of UT and TAI do not match.

Relativity Effects on Time Standards

Relativity theory has shown that velocity and accelerations affect time, which classical physics does not predict. Relativity requires that a distinction between proper time and coordinate time be made. Proper time is the time kept by an ideal clock attached to the observer, much like a wristwatch tells the observer his time. Coordinate time is equivalent to the instantaneous readout of the master time standard, wherever it may be located, and the output time is communicated instantaneously to the observer at his coordinate position. Any moving, accelerated observer will have a slower proper time than if he was stationary and not gravitated. The Earth is not only rotating, so that an observer on its surface experiences tangential rotational velocity and centripetal acceleration, but it also has orbital dynamics that give Earth, as well as an observer on its surface, additional velocity, centripetal acceleration and gravitational acceleration from the Sun.

For the observer on Earth's geoid (surface where the sum of rotational centripetal acceleration and local gravity from Earth is a constant), a timescale can be defined by Earth's rate of rotation (e.g. UT). This standard does suffer from periodic variations in the atmospheric angular momentum due to expanding and contracting air masses. In general, the rotational time standard is fairly consistent and usable for timekeeping over the long term. Because Earth experiences orbital dynamics and solar gravity, UT slows down (experiences the time dilations that lengthen the second interval compared to operating at a stationary, nongravitated location where no relativity effects exist). Therefore, UT is a proper timescale that has the same time dilations as any fixed place on Earth. So, UT is actually a noninertial time standard, because Earth's reference frame is accelerated.

Ephemeris Time is determined by an Earth observer viewing the position of a heavenly body, like the Moon, and comparing it to a classically predicted orbital position. Postprocessing of the equations of motion will produce a value of the time, a time tag, for the observed position, which is used to define the timescale for ET. With no relativistic perturbations included, the predicted positions are appropriate only for a stationary, gravity-free observer. This is the only location where proper and coordinate times are equivalent which constitutes what we call inertial time. Such a time interval derived by only classical physics is as short as possible.

The equations of motion should be in terms of the observer's own reference frame, which requires that the problem be treated relativistically. Classical equations of motion have no

relativistic time dilations so that the observer's reference frame is interpreted as being stationary and nongravitated. The classical equations of motion establish an inertial time standard. However, the Earth bound observer experiences orbital velocities and associated accelerations that constitute a noninertial reference frame and a noninertial time standard. So, the observers' own proper time rate is slower than classical physics predicts. The time tags given to the observed angular position of a heavenly body is essentially equivalent to Earth's proper time, namely UT. Since ephemeris time was defined with equations of motion that assumed the observer would be stationary and nongravitated, the ET time intervals are a bit short. This would explain why ET would run faster than UT over the long term.

Atomic time standards are defined to operate on Earth's geoid. The atomic clocks are at the same location as the observer on Earth's surface, so that an atomic clock experiences the same relativity effects as a clock in Universal Time.[24] However, atomic clocks were carefully calibrated to match the rate of the ET timescale, which assumed an unaccelerated, stationary frame for the observer. Thus, TAI and ET do not have the same common rate as the UT timescale. Neither TAI, ET nor UT operate in an inertial reference frame. If the complete relativity compensations were included in the lunar ephemeris, then the relationships between these three time rates should be closer.

Noninertial Relativistic Metric and New Time Dilation Effects

Since the Earth and Moon define noninertial systems orbiting each other, then the choice of a relativistic metric must accommodate all relativistic terms for a noninertial dynamical system. Just as measurements taken in noninertial reference frames require that extra classical terms (e.g. centripetal and Coriolis forces) must be taken into account when transforming to inertial frames, then relativistic measurements taken in a noninertial frame must have extra correction terms that would not be found in an inertial frame. Many metrics, such as the Schwarzschild metric, assume the massive object is stationary or nonrotating or inertial. The Nelson metric is an exact, noninertial metric appropriate for a nongravitationally accelerated, rotating reference frame.[25] Deines has extended the exact Nelson metric for nongravitationally accelerated frames to include Newtonian gravity. The inclusion of the Newtonian gravity with the nongravitational accelerations should encompass all significant relativistic terms to second order, since the post–Newtonian approximation from general relativity has the Newtonian gravity as the only second order contribution. The noninertial relativistic contributions are the velocity factor from special relativity, the Newtonian gravitational term from the second order post–Newtonian approximation from general relativity, and a new nongravitational potential contribution that can be treated in general relativity as an effective pseudogravitational factor to account for the centripetal acceleration. The new metric is defined below:

$$g_{ij} = \delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases} \tag{6}$$

$$g_{oj} = \frac{1}{c}(\vec{\omega} \times \vec{R})_j \tag{7}$$

is equivalent to 469.0343 days per leap second. This prediction is within 2% of the average time between leap seconds accumulated between 1994 and 1958. It is also within 0.5% of the observed average time between leap seconds if the average was taken between 1992 and 1958. These preliminary computations indicate that a relativistic lunar ephemeris timescale may well be close to UT.

Also, very preliminary calculations applied to the lunar ephemeris have been made with the time dilation equation. When the total relativistic contributions as calculated to second order are not accommodated in the lunar ephemeris, an apparent secular acceleration in the lunar orbit of $-25.66''/cy^2$ is predicted, which is about 1.3% of the observed value.

Conclusion

As discussed already in this paper, astronomers and geophysicists have, for many years, identified a timescale divergence between Universal Time (UT) and Ephemeris Time (ET). This problem has carried over to the observed divergence between UT and International Atomic Time (TAI), which the latter timescale has a rate defined by the current SI second that was calibrated carefully to the ET second. Previous scientific opinions are that UT is slowing down due to tidal friction. An equally plausible option is that ET had been running slightly faster than UT. The lack of a physical cause has kept this option from serious consideration until now.

An in–depth study of the historical development of our current timescales reveals that the equations of motion that defined the former standard of Ephemeris Time did not include any relativity compensations. Since ET is based on the length of the yearly orbit that was subsequently divided into ET seconds as prescribed by those equations of motion, the ET timescale could be running slightly faster than Earth's proper time standard. Without the relativistic time dilation effects that would "stretch" the ET second slightly, there will be slightly more seconds marked off per year than there should be. In that case, time predictions based on a complete revolution will be ahead compared to when the heavenly body will actually complete an orbit. Studies have shown the planets all lag behind the ET predictions with equal ratios of mean motion rate divided by mean motion. Classical gravitational theory can not explain the existence of these empirical ratios. However, relativity seems to be a possible source of this phenomena.

Because the Earth and Moon are not sufficiently inertial, a relativistic metric that deals with a generalized noninertial reference frame has been developed. Deines has extended the noninertial Nelson metric with Newtonian gravity to satisfy the requirement for modeling a noninertial system in gravity. In noninertial reference frames, three sets of relativistic contributions occur: velocity, gravitational and nongravitational terms. Preliminary research indicates the new relativistic metric will give an updated, theoretical expression for the lunar mean motion and, thereby, a new effect on the lunar timescale to be used for ET. A new time dilation equation has been derived from this new metric and has been used to estimate the time dilation effects of Earth's proper time compared to an inertial coordinate time. Assuming UT typifies Earth's proper time and assuming TAI with the SI second establishes Earth's coordinate time, then the time dilation equation predicts that UT should trail behind TAI by .7787481 seconds per year, which is within 2% of the observed divergence between UT and TAI. Also,

$$g_{oo} = -\left(1 + \frac{\vec{A} \cdot \vec{R} + \Phi}{c^2}\right)^2 + \frac{(\vec{\omega} \times \vec{R})^2}{c^2} \tag{8}$$

where $\vec{A}$ is the time–dependent translational, nongravitated acceleration of the observer's frame relative to a nongravitated inertial frame, Φ is the Newtonian gravitational potential independently existing in the neighborhood of the observer, $\vec{\omega}$ is the time–dependent angular velocity vector of the observer's spatial frame rotating relative to the inertial frame, and $\vec{R}$ is the range vector of the accelerated observer's origin from the inertial frame.

Using the fact that the Nelson metric preserves flat space–time, Deines has rigorously derived a new time dilation equation for a rotating reference frame that is accelerated both nongravitationally and gravitationally.

$$d\tau = \sqrt{\left(1 + \frac{\vec{A} \times \vec{R}}{c^2} + \frac{\Phi^2}{c^2}\right)^2 - \left(\frac{V^2}{c^2}\right)} dt \tag{9}$$

with $\vec{V}$ being the time–dependent velocity of the observer's frame relative to the inertial frame. If proper time τ is associated with UT as Earth's proper time and coordinate time t is considered as TAI with its SI second, then the square root term is the time dilation factor between the UT and TAI seconds.

To estimate the expected time dilation of Earth in its orbit around the Sun, integrate the time dilation equation over one year by the following process. Assume the inertial frame is sufficiently far from the Sun as to experience no gravitational red shift with its ideal master clock (e.g. fixed somewhere on the celestial sphere). Draw the displacement vector $\vec{R}$ from the inertial frame to the barycenter located at the Sun and continue on to the Earth–Moon barycenter. Since the first leg of this vector sum is fixed and assumed sufficiently stationary, the problem now reduces by a transformation to evaluating the time dilation equation from the Sun to Earth. Expand the radical in powers of c2 and retain only the first order terms. Assume Earth's orbit is a perfect ellipse. Substitute the Newtonian potential with the classical representation of the reduced mass divided by the new $\vec{R}$ vector. Derive the expression for the centripetal acceleration due to the elliptical orbit and substitute directly for the dot product term. Give V^2 its value for elliptical orbits. Obtain the differential form of Kepler's equation to express dt as a function of dE where E is the eccentric anomaly.

Collect terms as a function of E and integrate over 2π radians for one anomalistic year (i.e. perigee to perigee or 365.259635 days) to get the effective rate difference between proper and coordinate time as given below:

$$\tau - t = -\frac{\mu}{2ac^2}\sqrt{\frac{a^3}{\mu}} \int_0^{2\pi} (5 + e\cos E) dE = -\frac{\sqrt{\mu a}}{2c^2} 5E \Big|_0^{2\pi} = -0.778748084 \tag{10}$$

seconds per anomalistic year

The result from this integration is that UT will trail TAI by .7787481 seconds in one year, which

very preliminary computations using this time dilation equation indicate that the total relativity effects when ignored can produce an apparent lunar acceleration of $-26.66''/cy^2$, which is within 1.3% of the current observed value of the lunar secular acceleration in mean longitude.

Our future research work will generate a relativistic lunar ephemeris by following Brown's methodical development and using the new noninertial metric. The ongoing project will compare the original ephemeris timescale to a relativistic one. It is expected that the comparison will match the comparison between UT and TAI. One outcome of this effort may be the precise determination of a UT timescale by an appropriate conversion factor applied to an atomic timescale based on the SI second. This could allow an ultraprecise definition of a new UT timescale free of any leap second insertions.

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