REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. **PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

1. REPORT DATE *(DD-MM-YYYY)*
30-09-2009

2. REPORT TYPE
Final technical

3. DATES COVERED *(From - To)*
01-Aug-2006 to 31-Jul-2009

4. TITLE AND SUBTITLE
Networked and Distributed Convex Optimization for Design, Estimation, and Verification

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-06-1-0514

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Boyd, Stephen

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The Leland Stanford Jr. University
Stanford Sponsored Projects
340 Panama Street
Stanford, CA 94305-6203

8. PERFORMING ORGANIZATION REPORT NUMBER
SPO 35730

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AF Office of Scientific Research
875 N. Randolph St
Suite 325, Room 3112
Arlington, VA 22203
c/o Dr. Donald Hearn, Program Mgr
Computational Mathematics

10. SPONSOR/MONITOR'S ACRONYM(S)
AFOSR/PKR1

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Unlimited/A

13. SUPPLEMENTARY NOTES

14. ABSTRACT
This research explored a variety of applications of distributed optimization, in which local decisions are coordinated to produce a good overall solution. These problems arise in many areas, such as monitoring the health of a complex system, coordinating the design of a complex system, or fusing together information from disparate sources. The researchers exploited mathematical properties of the underlying problems to be solved (specifically, convexity) to develop effective methods.

15. SUBJECT TERMS
convex optimization, distributed optimization, power optimization, network optimization, fault estimation, distributed design

16. SECURITY CLASSIFICATION OF:
Nothing classified

a. REPORT	b. ABSTRACT	c. THIS PAGE

17. LIMITATION OF ABSTRACT
UU

18. NUMBER OF PAGES
8

19a. NAME OF RESPONSIBLE PERSON
Stephen Boyd

19b. TELEPHONE NUMBER *(include area code)*
650-723-0002

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

Networked and Distributed Convex Optimization for Design, Estimation, and Verification

Principal Investigator:
Stephen Boyd
Stanford University

October 1, 2009

Final Report
AFOSR FA9550-06-1-0514
August 1, 2006, through July 31, 2009
Dr. Donald Hearn, Program Manager

1 Abstract

This research explored a variety of applications of distributed optimization, in which local decisions are coordinated to produce a good overall solution. These problems arise in many areas, such as monitoring the health of a complex system, coordinating the design of a complex system, or fusing together information from disparate sources. The researchers exploited mathematical properties of the underlying problems to be solved (specifically, convexity) to develop effective methods.

Keywords: Convex optimization, distributed optimization, power optimization, network optimization, fault estimation, distributed design.

2 Core research

The core research conducting over the period of this grant focussed on distributed and networked optimization in a variety of practical settings, including design (such as, for circuits) [5,13,39], parameter estimation and fault identification (for integrated vehicle health management) [3,10,12,22], network flow control [28], control of wireless and sensor networks [1,19,26,32,42,44,48,53], methods for general purpose control [2,6,15,27], and power optimization in processors [4,35,45].

Some of these advances were described in previous annual reports; here we will describe some others that were not described in those reports. In [17], Yang and Boyd developed a new computational method for finding bounds on performance for causal state feedback stochastic control with linear dynamics, arbitrary noise distribution, and arbitrary input constraint set. This can be very useful as a comparison to the performance of suboptimal control policies, which we can evaluate using Monte Carlo simulation. Our method involves solving a semidefinite program (a linear optimization problem with linear matrix inequality constraints), a convex optimization problem which can be efficiently solved. Numerical experiments show that the lower bound obtained by our method is often close to the performance achieved by several widely-used suboptimal control policies, which shows that both are nearly optimal. As a by-product, our performance bound yields approximate value functions that can be used as control Lyapunov functions for suboptimal control policies.

In [27], Yang and Boyd developed a new method for fast computation of model predictive control (MPC) control policies. It has been used (with great success) for 20 years in applications with slow dynamics, where the sample time is measured in seconds or minutes. A well known technique for implementing fast MPC is to compute the entire control law offline, in which case the online controller can be implemented as a lookup table. This method works well for systems with small state and input dimensions (say, no more than 5), and short time horizons. In this paper we describe a collection of methods for improving the speed of MPC, using online optimization. These custom methods, which exploit the particular structure of the MPC problem, can compute the control action on the order of 100 times faster than a method that uses a generic optimizer. As an example, our method computes the control actions for a problem with 12 states, 3 controls, and horizon of 30 time steps (which entails solving a quadratic program with 450 variables and 1284 constraints) in around 5msec, allowing MPC to be carried out at 200Hz.

In [28], Trichakis et al consider a multi-period variation of the network utility maximization problem that includes delivery constraints. We allow the flow utilities, link capacities and routing matrices to vary over time, and we introduce the concept of delivery contracts, which couple the flow rates across time. We describe a distributed algorithm, based on dual decomposition, that solves this problem when all data is known ahead of time. We briefly describe a heuristic, based on model predictive control, for approximately solving a variation on the problem, in which the data are not known ahead of time.

3 Archival Publications

1. R. Madan, S. Boyd, and S. Lall, "Fast Algorithms for Resource Allocation in Cellular Networks," to appear in *IEEE Transactions on Networking*, 2010.

2. J. Skaf and S. Boyd, "Design of Affine Controllers via Convex Optimization," to appear, *IEEE Transactions on Automatic Control.*

3. A. Zymnis, S. Boyd, and D. Gorinevsky, "Mixed Linear System Estimation and Identification," to appear in *Proceedings of the IEEE Conference on Decision and Control,* December 2009.

4. A. Mutapcic, S. Boyd, S. Murali, D. Atienza, G. De Micheli, and R. Gupta, "Processor Speed Control with Thermal Constraints," *IEEE Transactions on Circuits and Systems I*, 56(9):1994-2008, September 2009.

5. S. Joshi and S. Boyd, "An Efficient Method for Large-Scale Slack Allocation," to appear in *Engineering Optimization*, 2009.

6. J. Skaf and S. Boyd, "Nonlinear Q-Design for Convex Stochastic Control," to appear in *IEEE Transactions on Automatic Control*, October 2009.

7. S. Joshi and S. Boyd, "Subspaces that Minimize the Condition Number of a Matrix," *Rejecta Mathematica*, 1(1):4-9, July 2009.

8. S. Boyd, P. Diaconis, P. Parrilo, and L. Xiao, "Fastest Mixing Markov Chain on Graphs with Symmetris," *SIAM Journal on Optimization*, 20(2):792-819, June 2009.

9. A. Mutapcic and S. Boyd, "Cutting-Set Methods for Robust Convex Optimization with Pessimizing Oracles," *Optimization Methods and Software,* 24(3):381-406, June 2009.

10. A. Zymnis, S. Boyd and D. Gorinevsky, "Relaxed Maximum a Posteriori Fault Identification," *Signal Processing*, 89(6):989-999, June 2009.

11. A. Julius, M. Zavlanos, S. Boyd, and G. Pappas, "Genetic network Identification using Convex Programming," *IET Systems Biology*, 3(3):155-166, May 2009.

12. S.-J. Kim, K. Koh, S. Boyd, and D. Gorinevsky, "l_1 Trend Filtering," *SIAM Review,* 51(2):339-360, May 2009.

13. Y. Xu, K.-L. Hsiung, X. Li, I. Nausieda, L. Pileggi, and S. Boyd, "Regular Analog/RF Integrated Circuits Design Using Optimization with Recourse Including Ellipsoidal Uncertainty," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems,* 28(5):623-637, May 2009.

14. A. Mutapcic, S. Boyd, A. Farjadpour, S.G. Johnson, and Y. Avniel, "Robust Design of Slow-Light Tapers in Periodic Waveguides," *Engineering Optimization*, 41(4):365-384, April 2009.

15. J. Skaf and S. Boyd, "Analysis and Synthesis of State-Feedback Controllers with Timing Jitter," *IEEE Transactions on Automatic Control*, 54(3):652-657, March 2009.

16. A. Magnani and S. Boyd, "Convex Piecewise-Linear Fitting," *Optimization and Engineering*, 10(1):1-17, March 2009.

17. Y. Wang and S. Boyd, "Performance Bounds for Linear Stochastic Control," *Systems and Control Letters*, 58(3):178-182, March 2009.

18. D. Gorinevsky, S.-J. Kim, S. Beard, S. Boyd and G. Gordon, "Optimal Estimation of Deterioration from Diagnostic Image Sequence," *IEEE Transactions on Signal Processing*, 57(3):1030-1043, March 2009.

19. S. Joshi and S. Boyd, "Sensor Selection via Convex Optimization," *IEEE Transactions on Signal Processing*, 57(2):451-462, February 2009.

20. J. Mattingley and S. Boyd, "Automatic Code Generation for Real-Time Convex Optimization," to appear as a chapter in *Convex Optimization in Signal Processing and Communications,* Y. Eldr and D. Palomar, eds., Cambridge University Press, 2009.

21. E. Candes, M. Wakin, and S. Boyd, "Enhancing Sparsity by Reweighted l_1 Minimization," *Journal of Fourier Analysis and Applications*, 14(5):877-905, December 2008.

22. A. Zymnis, S. Boyd, and D. Gorinevsky, "Mixed State Estimation for a Linear Gaussian Markov Model," *Proceedings of the IEEE Conference on Decision and Control,* pp.3219-3226, December 2008.

23. S. Joshi and S. Boyd, "An Efficient Method for Large-Scale Gate Sizing," *IEEE Transactions on Circuits and Systems I*, 55(9):2760-2773, November 2008.

24. S.-J. Kim and S. Boyd, "A Minimax Theorem with Applications to Machine Learning, Signal Processing, and Finance," *SIAM Journal on Optimization*, 19(3):1344-1367, November 2008.

25. J. Skaf and S. Boyd, "Filter Design with Low Complexity Coefficients," *IEEE Transactions on Signal Processing,* 56(7):3162-3169, July 2008.

26. Z. Wang, S. Zheng, Y. Ye, and S. Boyd, "Further Relaxations of the Semidefinite Programming Approach to Sensor Network Localization," *SIAM Journal on Optimization,* 19(2):655-673, July 2008.

27. Y. Wang and S. Boyd, "Fast Model Predictive Control Using Online Optimization," *Proceedings of the IFAC World Congress,* pp.6974-6997, July 2008.

28. N. Trichakis, A. Zymnis, and S. Boyd, "Dynamic Network Utility Maximization with Delivery Contracts," *Proceedings of the IFAC World Congress*, pp.2907-2912, July 2008.

29. K.-L. Hsiung, S.-J. Kim, and S. Boyd, "Tractable Approximate Robust Geometric Programming," *Optimization and Engineering*, 9(2):95-118, June 2008.

30. M. Zavlanos, A. Julius, S. Boyd and G. Pappas, "Identification of Stable Genetic Networks Using Convex Programming," *Proceedings of the American Control Conference*, pp.2755-2760, June 2008.

31. R. Panicker, J. Kahn, and S. Boyd, "Compensation of Multimode Fiber Dispersion using Adaptive Optics via Convex Optimization," *IEEE Journal of Lightwave Technology*, 26(10):1295-1303, May 2008.

32. S. O'Neill, A. Goldsmith, and S. Boyd, "Optimizing Adaptive Modulation in Wireless Networks via Utility Maximization," *Proceedings of the IEEE International Conference on Communications*, pp.3372-3377, May 2008. Best paper award.

33. S.-J. Kim, A. Zymnis, A. Magnani, K. Koh, and S. Boyd, "Learning the Kernel via Convex Optimization," *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP),* pp.1997-2000, April 2008.

34. S.-J. Kim, A. Magnani, A. Mutapcic, S. Boyd and Z.-Q. Luo, "Robust Beamforming via Worst-Case SINR Maximization," *IEEE Transactions on Signal Processing*, 56(4):1539-1547, April 2008.

35. S. Murali, A. Mutapcic, D. Atienza, R. Gupta, S. Boyd, L. Benini, and G. De Micheli, "Temperature Control of High-Performance Multi-Core Platforms Using Convex Optimization," *Proceedings of IEEE Design, Automation and Test in Europe (DATE)*, pp.110-115, March 2008.

36. A. Ghosh, S. Boyd, and A. Saberi, "Minimizing Effective Resistance of a Graph," *SIAM Review,* 50(1):37-66, February 2008.

37. D. Gorinevsky, S. Boyd, and G. Stein, "Design of Low-Bandwidth Spatially Distributed Feedback," *IEEE Transactions on Automatic Control*, 53(2):257-272, February 2008. M. Grant and S. Boyd, "Graph Implementations for Nonsmooth Convex Programs," in *Recent Advances in Learning and Control*, V. Blondel, S. Boyd and H. Kimura, eds., pp.95-110, Springer, 2008.

38. S. Boyd and B. Wegbreit, "Fast Computation of Optimal Contact Forces," *IEEE Transactions on Robotics*, 23(6):1117-1132, December 2007.

39. S.-J. Kim, S. Boyd, S. Yun, D. Patil, and M. Horowitz, "A Heuristic for Optimizing Stochastic Activity Networks with Applications to Statistical Digital Circuit Sizing," *Optimization and Engineering*, 8(4):397-430, December 2007.

40. S.-J. Kim, K. Koh, M. Lustig, S. Boyd, and D. Gorinevsky, "An Interior-Point Method for Large-Scale l_1-Regularized Least Squares," *IEEE Journal on Selected Topics in Signal Processing*, 1(4):606-617, December 2007.

41. A. Zymnis, S.-J. Kim, J. Skaf, M. Parente, and S. Boyd, "Hyperspectral Image Unmixing via Alternating Projected Subgradients," *Proceedings of the Asilomar Conference on Signals, Systems and Computers*, pp.1164-1168, November 2007.

42. A. Mutapcic, S.-J. Kim, and S. Boyd, "A Tractable Method for Robust Downlink Beamforming in Wireless Communications," *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, pp.1224-1228, November 2007.

43. A. Mutapcic, S.-J. Kim, and S. Boyd, "Robust Chebyshev FIR Equalization," *Proceedings of the 50th IEEE Global Communications Conference (GLOBECOM'07),* pp.3074-3079, November 2007.

44. A. Zymnis, N. Trichakis, S. Boyd, and D. O'Neill, "An Interior-Point Method for Large Scale Network Utility Maximization," *Proceedings of the Allerton Conference on Communication, Control, and Computing,* September 2007.

45. S. Murali, A. Mutapcic, D. Atienza, R. Gupta, S. Boyd, and G. De Micheli, "Temperature-Aware Processor Frequency Assignment for MPSoCs Using Convex Optimization," *Proceedings of the 5th International Conference on Hardware/Software Codesign and Systems Synthesis (CODES+ISSS)*, pp.111-116, October 2007.

46. S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, "A Tutorial on Geometric Programming," *Optimization and Engineering*, 8(1):67-127, 2007.

47. K. Koh, S.-J. Kim, and S. Boyd, "An Interior-Point Method for Large-Scale l_1-Regularized Logistic Regression," *Journal of Machine Learning Research,* 8:1519-1555, July 2007.

48. S. Samar, S. Boyd, and D. Gorinevsky, "Distributed Estimation via Dual Decomposition," *Proceedings of the European Control Conference*, pp.1511-1516, July 2007.

49. J. Skaf and S. Boyd, "Controller Coefficient Truncation Using Lyapunov Performance Certificate," *Proceedings of the European Control Conference*, pp.4699-4706, July 2007.

50. M. Lobo, M. Fazel, and S. Boyd, "Portfolio Optimization with Linear and Fixed Transaction Costs," *Annals of Operations Research*, 152(1):341-365, July 2007.

51. A. Mutapcic, S.-J. Kim, and S. Boyd, "Beamforming with Uncertain Weights," *IEEE Signal Processing Letters*, 14(5):348-351, May 2007.

52. L. Vandenberghe, S. Boyd, and K. Comanor, "Generalized Chebyshev Bounds via Semidefinite Programming," *SIAM Review*, 49(1):52-64, March 2007.

53. L. Xiao, S. Boyd, and S.-J. Kim, "Distributed Average Consensus with Least-Mean-Square Deviation," *Journal of Parallel and Distributed Computing*, 67(1):33-46, 2007.

54. K. Koh, S.-J. Kim, and S. Boyd, "An Efficient Method for Large-Scale l_1-Regularlized Convex Loss Minimization," *Proceedings of the IEEE Information Theory and Applications Workshop,* pp.223-230, January 2007.

55. A. Ghosh and S. Boyd, "Growing Well-Connected Graphs," *Proceedings of the IEEE Conference on Decision and Control*, pp.6605-6611, December 2006.

56. A. Mutapcic, S.-J. Kim, and S. Boyd, "Array Signal Processing with Robust Rejection Constraints via Second-Order Cone Programming," *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, pp.2267-2270, October-November 2006.

4 Student doctoral dissertations

1. Alessandro Magnani, "Primal-Dual Cutting-Plane Method for Distributed Design," December 2006.

2. Kan-Lin Hsiung, "Geometric Programming Under Uncertainty with Engineering Applications," June 2008.

3. Siddharth Joshi, "Large-Scale Geometric Programming for Devices and Circuits," June 2008.

4. Almir Mutapcic, "Robust Optimization: Methods and Applications," June 2008.

5. Joelle Skaf, "Convex Optimization Formulation of Controller Design Problems," November 2008.

6. Kwangmoo Koh, "Methods for Large-Scale Convex Optimization problems with l_1 Regularization," December 2008.

7. Argyrios Zymnis, "Convex Relaxation Methods for Fault Estimation," June 2009.

8. Jun Sun, "The Map Stitching Problem and Its Applications in Locally Isometric Embedding," September 2009.

5 Software

All code for papers with computational experiments posted on-line. In addition, we developed several now packages in wide use.

1. CVX (general purpose parser/solver for convex optimization)

2. `l1_logreg` (ℓ_1 regularized logistic regresssion)

3. `l1_ls`, for large-scale ℓ_1-regularized least-squares
4. `l1_tf`, for ℓ_1 trend filtering

6 Honors

2008 *Best Paper Award*, IEEE International Conference on Communications, Wireless Communications Symposium, for "Optimizing Adaptive Modulation in Wireless Newtorks via Utility Maximization," by D. O'Neill, A. Goldsmith, and S. Boyd.

2006 *Honorary Doctorate*, Royal Institute of Technology (KTH), Stockholm.

2006 *Best Student Paper Award*, IEEE International Conference on Control Applications, for "Embedded Estimation of Fault Parameters in an Unmanned Aerial Vehicle," by (student) S. Samar, D. Gorinevsky, and S. Boyd.

7 Plenary/Keynote Talks

2009 Plenary lecture, *International Symposium on Advanced Control of Chemical Processes*, Koc, Turkey.

2009 Plenary lecture, *International Symposium on Mathematical Programming*, Chicago.

2009 Plenary lecture, *International Conference on Optimization and Control with Applications*, Harbin.

2008 Keynote lecture, *Conference in Control, Communications and Signal Processing*, University of Illinois.

2007 *Simon Stevin Lecture*, Katholieke Universiteit Leuven.

2007 *DYSCO (Dynamic systems, control, and optimization) lecture*, Louvain la Neuve.

2006 *Linneaus Center Distinguished Speaker Series*, KTH, Stockholm.