Spatial Evolution of the Frequency Distribution of Dissipation and Implications on Frequency Domain Modeling

James M. Kaihatu, Jayaram Veeramony, Kacey L. Edwards

Naval Research Laboratory
Oceanography Division
Stennis Space Center, MS 39529-5004

Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217-5660

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The evolution of the frequency dependence of dissipation coefficient $a_n$ of shoaling and breaking waves is investigated. Prior studies have established the observation of, and physical reasoning behind, $a_n \sim f^2$, or that the dissipation should be weighted as the square of the wave frequency in the spectrum. A recent study, however, showed that this weighting evolves over the shoaling and breaking zone, with $a_n \sim f$ acting as an inner surf zone asymptote. Parameterization of the evolution of the weighting as a function of depth brings forward several questions, the most important being whether the sum of individual breaking events is equivalent to the total dissipation as described by lumped parameterizations. Overall generality of the parameterizations will require more data to establish.

Parameterization, wave spectra, wave propagation
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SPATIAL EVOLUTION OF THE FREQUENCY DISTRIBUTION OF DISSIPATION AND IMPLICATIONS ON FREQUENCY DOMAIN MODELING

James M. Kauhatu1, Jayaram Veeramony2 and Kacey L. Edwards3

The evolution of the frequency dependence of dissipation coefficient αω of shoaling and breaking waves is investigated. Prior studies have established the observation of, and physical reasoning behind, αω ∝ ω^2, or that the dissipation should be weighted as the square of the wave frequency in the spectrum. A recent study, however, showed that this weighting evolves over the shoaling and breaking zone, with αω ∝ ω acting as an inner surf zone asymptote. Parameterization of the evolution of the weighting as a function of depth brings forward several questions, the most important being whether the sum of individual breaking events is equivalent to the total dissipation as described by lumped parameterizations. Overall generality of the parameterizations will require more data to establish.

INTRODUCTION

Wave Breaking and Wave Spectra
Waves in the nearshore are significantly modified by nonlinear interactions, transitioning from quasi-sinusoidal forms to cnoidal-type forms, with markedly flatter troughs and peaked crests. These processes are caused by nonlinear wave-wave interactions among triads of frequencies. These interactions occur at length scales of O(m), in contrast to the O(km) length scales of quartet interactions of deep water waves. As these waves approach the shoreline, they reach a limiting height and break; the type of breaking ranges from gentle spilling breakers to violent plunging breakers. The breaking leads to a release of momentum into the water column, and is responsible for the generation of nearshore currents, the transport of sediments, and bathymetric change.

The transformations described above are reflected in wave spectra measurements (Freilich and Guza 1984). These processes are manifested in the changes of the spectral shape. Harmonics of the spectral peak are amplified due to superharmonic nonlinear interaction during shoaling. Low frequencies are also amplified as long wave motions are generated by nonlinear interactions. In the surf zone, the high frequency energy begins to decrease as dissipation continues. Smith and Vincent (2003), using wavenumber spectra derived from recorded frequency spectra in the laboratory and field, determined that the shape of the spectra in the surf zone had characteristic shapes explicable by the theories of

1 Zachary Department of Civil Engineering, Texas A&M University, 3136 TAMU, College Station, TX, 77843-3138, USA
2 Oceanography Division (Code 7322), Naval Research Laboratory, Stennis Space Center, MS, 39529-5004, USA
Toba (1973) and Zakharov (1999). In contrast, Kairahtu et al. (2007), using the data of Bowen and Kirby (1994) and Mase and Kirby (1992), showed how the wave spectra reached an asymptotic shape in the inner surf zone, where the high frequency tail appeared to have a slope proportional to $f^{-2}$, in accord with the observation of a sawtooth wave (Kirby and Kairahtu 1996). Interestingly, however, this value was not constant throughout the surf zone, but varied through the surf zone as the depth changed.

Kirby and Kairahtu (1996) developed a method of evaluating the frequency-dependent dissipation from data, assuming that the dissipation was primarily that due to eddy viscosity, in the manner of Zelt (1991). The resulting mechanism demonstrated an inverse relationship between the frequency dependence of dissipation and the shape of the frequency spectrum. Along with the correspondence between sawtooth waves and the associated $f^{-2}$ dependence on the spectral variance, Kirby and Kairahtu (1996) demonstrated that the optimal frequency distribution for the dissipation is $f^{-2}$. Chen et al. (1997) tested this assumption distribution with data by using various distributions in a nonlinear wave transformation model (Chen and Liu 1995) and comparing them to data. They determined that the $f^{-2}$ worked best.

Interestingly, though, Kairahtu et al. (2007) showed that, while the inverse relationship between spectral slope and dissipation frequency dependence was evident, the frequency dependence of the dissipation evolved through the surf zone. The $f^{-2}$ weighted dependence for dissipation was the asymptote in the very nearshore.

**Frequency Domain Models and Dissipation**

Numerical models of nonlinear wave propagation are generally divided into time-domain and frequency domain formulations. Time domain models evolve the free surface as a function of time and space; extended Boussinesq models such as Madsen et al. (1991), Nwogu (1993), Wei et al. (1995) and Lynett and Liu (2004) fall into this category. These models have the advantage of allowing non-periodic waves and currents to be simulated, but can also be numerically complex. Frequency-domain models, in contrast, assume time-periodic motion from the outset. Triad resonance is assumed in order to completely factor out time dependence from the model, allowing explicit formulation of the three-wave interaction terms.

Frequency domain models require a spectral dissipation mechanism (e.g., Battjes and Janssen 1978; Thornton and Guza 1983). Generally these mechanisms are only functions of integrated parameters of the spectrum. No frequency dependence is specified, so it is generally assumed that the dissipation is either applied as a constant over frequency, or is weighted as frequency squared. This latter weighting was found by Chen et al. (1997) to provide accurate results for not only frequency spectra, but also with wave shape statistics such as skewness and asymmetry. However, since Kairahtu et al. (2007) showed evolution of the dissipation's frequency dependence through the surf zone, it would be interesting to see how this evolution affects the models.

In this study we investigate the trends in the slope of the spectral tail for shoaling and breaking waves, with an emphasis on the implications for numerical modeling of nonlinear shoaling and breaking waves.

**LABORATORY EXPERIMENTS**

Bowen and Kirby (1994)

Bowen and Kirby (1994) conducted several laboratory experiments, in which single peaked wave spectra were generated in a laboratory flume and allowed to propagate over a sloping bottom. The layout of the experiment is shown in Figure 1. Three different incident wave conditions (referred to as Case A, B and C) were generated at the wave maker; the measurement procedures for each case were identical. One feature of this experiment is the dense coverage of data in the surf zone. Free surface elevations were measured at 47 locations in the tank by utilizing a carriage situated on two rails on the top edge of the tank, and moving it in increments.

![Figure 1. Experimental layout of Bowen and Kirby (1994).](image)

Data were taken at 25 Hz for approximately 17 minutes, with the first 925 points disregarded to allow the domain to fill up with waves. The resulting records were each divided into 12 realizations of 2,048 points apiece. Frequency spectra were calculated for each realization, averaged over the realizations, and then averaged over eight adjacent bands, leading to 192 degrees of freedom. Spectra from the data are shown in Figure 2 (Case B).

**ANALYSIS**

**Frequency Dependent Dissipation Coefficient from Data**

Many frequency domain shoaling wave models (e.g. Agnon et al. 1993; Kairahtu and Kirby 1995) have the following form:
\[
\frac{\partial A_n}{\partial x} + \frac{1}{2C_m} \frac{\partial C_m}{\partial x} + \alpha_n \Delta A_n = n \text{terms}
\]

where \( n \) is the integer multiple of the frequency resolution \( \Delta f \). Note in Equation (3) that the dissipation coefficient \( \alpha_n \) and the spectral density \( S(n\Delta f) \) are inversely related. Kaihatu et al. (2007) showed that the inverse nature of these relationships was strongest in the inner surf zone, where \( \alpha_n \sim f^{-2} \) and \( S(f) \sim f^{-2} \).

**Evolution of Frequency Dependence of Dissipation with Depth**

One immediate method for incorporating this evolution of the frequency dependence into modeling is to fit curves through the measured evolution of the frequency dependence of the dissipation coefficient in the model. In the model of Kaihatu and Kirby (1995), this is achieved via the following:

\[
\alpha_n = \frac{1}{\rho g \sqrt{gh}} \frac{1}{\sqrt{2\Delta f}} \frac{S_n(n\Delta f)}{S(n\Delta f)}
\]

in which \( m \) is the exponent of frequency dependence. For the case of frequency-dependent dissipation coefficient studied by Kirby and Kaihatu (1996) and Chen et al. (1997), \( m = 2 \). The term \( \beta(x) \) is the total dissipation in the wavefield due to breaking at any location \( x \); this can be described by such lumped parameter descriptions as Battjes and Janssen (1978) and Thornton and Guza (1983), but requires modification to fit within the context of frequency domain models (Mase and Kirby 1992; Kaihatu and Kirby 1995; Eldeberky and Battjes 1996).

As done in Kaihatu et al. (2007), the frequency dependence of \( \alpha_n \) was found by performing a power fit for the frequency range from zero up to one-half of the Nyquist frequency. The resulting power fits were then themselves curve-fit as a function of water depth. Two curve fits of these frequency dependencies were performed for each of the three cases of the Bowen and Kirby (1994) experiments. The result for Case B is shown in Figure 3. It is apparent that the frequency dependence of dissipation undergoes substantial evolution over the shoaling and breaking region. However, use of this information in frequency domain models via (4) assumes that equivalence exists between the integral of instantaneous dissipation events (Equation 2) and the total lumped parameter dissipation mechanism of Battjes and Janssen (1978) or Thornton and Guza (1983). This is not entirely clear. Most models using lumped energy parameterizations for breaking assume that averaging of the individual breaking events leads to the parameterization. This is, in general, not guaranteed. This is particularly true when nonlinearity is taken into account; any preferential frequency distribution of the dissipation will affect inter-frequency wave-wave
interaction. Kirby and Kaimhatu (1996) note that individual breaking events can impart a discrete "kick" into the water column; if these discrete events are widely spaced in time, they can themselves generate low frequency motions. These events would likely not be represented in an integrated bulk parameterization.

In practical modeling of these issues, it is not likely that frequencies up to one-half the Nyquist frequency would be included, as this would generally require \( O(500) \) frequency components to be retained. Bredmose (2002) noted that the number of computations required for nonlinear frequency domain models were on the order of \( O(N^2) \), where \( N \) is the total number of frequency components. Typically, around 200 frequencies would be retained for the nonlinear frequency domain models, which, in the case of Bowen and Kirby (1994) would result in a maximum frequency of 2.4 Hz, about three times the peak frequency. However, this would also imply that the frequency range over which the dissipation \( \alpha \) would be calculated would also be concomitantly smaller. To investigate this, the frequency range for the calculation of the exponent \( m \) was limited to between 0 and 2.4 Hz. The values of \( m \) were then recalculated, and then plotted as a function of the water depth. This plot, along with the trial curve fits, is shown in Figure 4. It is clear that this is quite different than the previous result in Figure 4. The impact on modeling is thus not clear, especially since the accurate modeling of high frequency evolution is important for reliable wave shape statistics (Kaihatu and Kirby 1996). It is likely that the large coefficients of the fourth-order polynomial will cause some difficulty with the numerical solution. In any event, more data will be needed to provide a clearer picture of whether or not a parameterization in this manner is sensible.

CONCLUSIONS AND FUTURE WORK

The issue of the frequency dependence of dissipation coefficient \( \alpha \), was addressed in this study, particularly as regards its impact on frequency domain numerical wave modeling. Kirby and Kaihatu (1996) discussed the choice of \( \alpha_f \), in concert with the inverse relationship between the spectral shape and the frequency dependence of \( \alpha \). This was confirmed with numerical simulations by Chen et al. (1997) and with additional data analysis by Kaihatu et al. (2007). The latter study also demonstrated the evolution of this frequency dependence of \( \alpha \), implying that improvement can be made by replicating this evolution in the frequency domain models via Equation (4). Analysis of the data in this regard reveals that a) an equivalence between the integral over individual instantaneous breaking events, and a lumped parameterization describing overall dissipation, needs to be established; and b) the number of retained frequency components in modeling has a significant effect on the form of the parameterized frequency distribution. The adequacy of these descriptions remains to be seen, and will require more data to establish.

It can also be argued that a well-suited alternative to this approach would be to allow the time-dependent breaking mechanism to establish its own statistics.
For frequency domain models, this would entail using a hybrid time/frequency
domain modeling approach, similar to that of Bredmose (2003). This would
allow breaking parameters to be calculated based on local front face slopes (Zelt
1991) and not on presumed frequency distributions of lumped dissipation
parameters. This approach is presently being pursued.

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