Effect of Earth’s Rotation and Range Foldover on Space Based Radar Performance

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ABSTRACT

A space based radar (SBR) by virtue of its motion generates a Doppler frequency component to the clutter return from any point on the earth as a function of the SBR – earth geometry. The effect of earth’s rotation around its own axis also adds an additional component to this Doppler frequency. The overall effect of the earth’s rotation on the Doppler turns out to be two correction factors in terms of a crab angle affecting the azimuth angle, and a crab magnitude scaling the Doppler magnitude of the clutter patch. Interestingly both factors depend only on the SBR orbit inclination and its latitude and not on the specific location of the clutter patch of interest. Further, it is shown that the crab angle has maximum effect for an SBR on a polar orbit that is above the equator. The crab magnitude on the other hand peaks for an SBR on an equatorial orbit. It is also shown that earth’s rotation together with range foldover phenomenon significantly degrade the clutter suppression performance of adaptive processing algorithms. Detailed derivations of these results are presented in this paper.

1. INTRODUCTION

Space based radar (SBR) because of its height can cover a very large area on earth for intelligence, surveillance and monitoring of ground moving targets. Once launched into orbit, the SBR moves around the earth while the earth continues to rotate on its own axis. By adjusting the SBR speed and orbit parameters it is thus possible to scan various parts of the earth periodically and collect data. Such an SBR based surveillance system can be remotely controlled and may require very little human intervention. As a result, targets of interest can be identified and tracked in greater detail and/or images can be made with a very high resolution.

In SBR systems, the range foldover phenomenon – clutter returns that correspond to previous/later radar pulses – contributes to the SBR clutter [1, 2, 3, 4]. Another important phenomenon that affects the clutter data is the effect of earth’s motion around its own axis [1, 5]. At various points on earth this contributes differently to Doppler, and the modification to Doppler due to earth’s rotation is derived in this paper. Detailed performance analysis together with methods to minimize these effects is also presented here.

2. MODELING EARTH’S ROTATION

Consider an SBR located at height \( H \) above the earth’s surface, and for any point of interest \( D \) on earth at range \( R \), define the elevation angle \( \theta_{el} \) and azimuth angle \( \theta_{az} \) (measured between the SBR velocity vector and the range vector \( BD \)) as shown in Fig. 1 and Fig. 2. In this case, the conventional Doppler shift relative to the SBR equals [2, 5]

\[
\omega_d = \frac{2V_T}{\lambda/2} \sin \theta_{el} \cos \theta_{az},
\]

where \( T_r \) represents the pulse repetition rate, \( \lambda \) the operating wavelength and \( V_T = \sqrt{GM_e/(R_e + H)} \) the SBR speed. Here \( G \) is the universal gravitational constant and \( M_e \) is the mass of earth.

As the SBR moves around the earth, the earth itself is rotating around its own axis on a 23.9345 hour basis. This contributes an eastward motion with equatorial velocity of

\[
V_e = \frac{2\pi R_e}{23.9345 \times 3600} = 0.4651 \text{ km/sec}.
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Let ($\alpha_1$, $\beta_1$) refer to the latitude and longitude of the SBR nadir point $B$ and ($\alpha_2$, $\beta_2$) those of the point of interest $D$ as shown in Fig. 2 and Fig. 3. As a result, the point of interest $D$ on the earth at latitude $\alpha_2$ rotates eastward with velocity $V_e \cos \alpha_2$ and this will contribute to the Doppler in (1) as well.

Fig. 2 Doppler contributions from SBR velocity and earth’s rotation.

To compute this new component in Doppler shift, note from Fig. 3 that the azimuth angle $BDY$ between the ground range vector $R$ and the earth velocity vector at $D$ equals $(\pi/2 + \beta)$ so that $V_e \cos \alpha_2 \cos(\pi/2 + \beta) = -V_e \cos \alpha_2 \sin \beta = V_e \sin \gamma$, represents the earth’s relative velocity at $D$ along the ground range towards $B$.

Since the grazing angle $\psi$ represents the slant range angle with respect to the ground range at $D$ (see Fig. 2), $V_e \cos \psi = -V_e \cos \alpha_2 \sin \beta \cos \psi$ represents the relative velocity contribution between the SBR and the point of interest $D$ due to the earth’s rotation. Together with (1), this gives the modified Doppler frequency that also accounts for the earth’s rotation to be

$$\omega_d = \frac{2T_e}{A/2} \left( V_e \sin \theta_{EL} \cos \theta_{AZ} - V_e \cos \alpha_2 \sin \beta \cos \psi \right). \quad (2)$$

To simplify (2), from triangle $ACD$ in Fig. 1 and Fig. 2, we have

$$\sin \left(\frac{\pi}{2} + \psi \right) = \sin \theta_{EL} \quad \text{and} \quad \cos \psi = \left(1 + \frac{H}{R_y}\right) \sin \theta_{EL}, \quad (3)$$

and hence (2) becomes

$$\omega_d = \frac{2V_e T_e}{A/2} \sin \theta_{EL} \left[ \cos \theta_{AZ} - \frac{V_e}{V_p} \left(1 + \frac{H}{R_y}\right) \cos \alpha_2 \sin \gamma \right], \quad (4)$$

where we have used the identity

$$\cos \alpha_2 \sin \beta = \cos \alpha_1 \sin \gamma \quad (5)$$

that follows from considering the spherical triangle $ZBD$ formed by the intersection of three great circles (see [1] for details). To simplify (4), from Fig. 3 and defining the following triangles $ZBD = \gamma$, $ZBB_o = \delta$ and $B_BB_D = \theta_{AZ}$, we have $\gamma = \pi - \delta + \theta_{AZ}$, so that

$$\sin \gamma = \sin \delta \cos \theta_{AZ} - \cos \delta \sin \theta_{AZ} \quad (6)$$

with $\eta_i$ representing the orbit inclination at the equator for an SBR (see Fig. 3) and considering the spherical triangle $ZBB_o$, it can be shown that

$$\sin \delta = \frac{\cos \eta_i}{\cos \alpha_i} \quad \text{and} \quad \cos \delta = -\sqrt{\cos^2 \alpha_i - \cos^2 \eta_i} \cos \alpha_i. \quad (7)$$

Substituting (7) into (4) and simplifying we get the modified Doppler frequency to be [6]

$$\omega_d = \frac{2V_e T_e}{A/2} \rho_i \sin \theta_{EL} \cos(\theta_{AZ} + \phi_i) \quad (8)$$

where

$$\phi_i = \tan^{-1} \left( \frac{\Delta \sqrt{\cos^2 \alpha_i - \cos^2 \eta_i}}{1 - \Delta \cos \eta_i} \right), \quad (9)$$

$$\rho_i = \sqrt{1 + \Delta^2 \cos^2 \alpha_i - 2 \Delta \cos \eta_i}, \quad (10)$$

and

$$\Delta = \frac{V_e}{V_p} \left(1 + \frac{H}{R_y}\right). \quad (11)$$

In (8) - (10), $\phi_i$ represents the crab angle and $\rho_i$ represents the crab magnitude. In summary, the effect of earth’s rotation

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on the Doppler frequency is to introduce a crab angle and crab magnitude into the SBR azimuth angle and modify it accordingly. Interestingly both these quantities depend only on the SBR orbit inclination and its latitude and not on the latitude/longitude of the clutter patch of interest. Eqs. (8) - (10) correspond to the case where the region of interest $D$ is to the east of the SBR path as shown in Fig. 3. If the region of interest is to the west of the SBR path, the crab angle can be shown to be (derivation omitted)

$$\phi_c = -\tan^{-1}\left(\frac{\Delta\cos^2 \alpha_c - \cos^2 \eta}{1 - \Delta \cos \eta}\right).$$

and the modified Doppler and crab magnitude are as given in (8) and (10), respectively.

The effect of crab angle on Doppler as a function of range for different azimuth angles is shown in Fig. 4.

The clutter corresponding to these range bins will be associated with these modified Doppler frequencies. To illustrate the limitations imposed on processing gains due to earth’s rotational phenomenon and range foldover, Fig. 7 and Fig. 8 show the matched filter (MF) output in terms of ideal $\text{SINR}$ loss without and with range foldover and earth’s rotation. Recall that the $\text{SINR}$ loss is defined as \[3, 5, 7\]

$$\text{SINR} = s^H(c, \omega_d) R^{-1} s(c, \omega_d) \triangleq s^H R^{-1} s.$$ \hspace{1cm} (13)

(The superscript $H$ denotes the complex conjugate transpose operation). Here $s(c, \omega_d)$ represents the normalized space time steering vector for the desired point of interest located at $\theta = (\theta_{izl}, \theta_{Az})$ that corresponds to the cone-angle

$$c = \sin \theta_{izl} \cos \theta_{Az},$$

and Doppler frequency $\omega_d$ for the SBR configuration under consideration. For a uniformly spaced array consisting of $N$ elements, the space-time steering vector is given by

$$s(c, \omega_d) = b(\omega_d) \otimes a(c),$$ \hspace{1cm} (15)

where

$$a(c) = \frac{1}{\sqrt{N}} [1, \nu, \nu^2, \ldots, \nu^{N-1}]^T,$$ \hspace{1cm} (16)

$$\nu = e^{-j \pi d \sin \theta_{izl} \cos \theta_{Az}} = e^{-j \pi d c},$$ \hspace{1cm} (17)

and $d$ represents the inter-element distance normalized to half wavelength. Further, for a data vector containing $M$ pulses

$$b(\omega_d) = \frac{1}{\sqrt{M}} [1, \lambda, \lambda^2, \ldots, \lambda^{M-1}]^T$$ \hspace{1cm} (18)

where $\lambda = e^{-j \pi \omega_d}$. The ideal clutter covariance matrix $R = E\{xx^H\}$ corresponding to an $MN \times 1$ clutter data vector $x$ at a specific range $R_o$ has the form
\[ R = \sum_{k} \sum_{m=0}^{N_i} P_{lm} G(\theta_{km}) s_{lm}^{H} \sigma^2 I \]  

(19)

where the inner summation is over the \( N_a \) range foldovers at \( R_1, R_2, \ldots \), and the outer summation is over all azimuth angles \( \theta_{AZ} = \theta_{AZ} + k \Delta \theta \), \( k = \pm 1, \pm 2, \ldots \), of interest. Here, \( G(\theta_{km}) \) and \( P_{lm} \) correspond to the array gain and clutter power from the \((k, m)^{th}\) ground patch at \( \theta_{lm} = (\theta_{AZ}, \theta_{EL}) \), respectively. The associated steering vector is given by \( s_{lm} = s(\omega_{lm}, \omega_{AZ}) \), where the cone angle for the \((k, m)^{th}\) patch equals \( c_{lm} = \sin \theta_{AZ} \cos \theta_{EL} \) and the corresponding Doppler frequency is given by (from (1) or (8))

\[ \omega_{AZ} = \begin{cases} \beta c_{lm}, & \text{w/ earth's rotation,} \\ \beta \rho \sin \theta_{AZ} \cos(\theta_{EL} + \phi), & \text{w/o earth's rotation,} \end{cases} \]

(20)

depending on whether earth’s rotation is absent or present in Eq. (19). Fig. 6 shows Doppler as a function of the cone angle with and without earth’s rotation. Note that in the absence of earth’s rotation, there is a one-to-one correspondence between Doppler and cone angle. As a result, points that project the same cone angle in the range-azimuth domain generate identical Doppler. Thus, all points on the iso-cone plot (see Fig. 9 (a)) project the same Doppler frequency. However, in the presence of earth’s rotation, this is no longer true, and for a given cone angle, different range foldovers generate different Doppler frequencies. This results in Doppler spread as shown in Fig. 6, which displays the entire band of Doppler curves corresponding to all range points of interest and their foldovers as a function of azimuth angle. It is seen that in the limit, they generate a continuous band of Doppler frequencies. In summary, in the presence of earth’s rotation, every point on earth projects a different Doppler that is confined within a band of frequencies.

Fig. 6 Doppler spilling due to earth’s rotation.

Fig. 7 shows the ideal SINR performance as a function of range and Doppler with and without range foldover and earth’s rotation. Note from Fig. 7 (c) that the resulting known bias can be adjusted, if necessary, to reflect the normalized performance as shown in Fig. 7 (a). However, this is not the case in Fig. 7 (d), which shows a severe performance degradation when both range foldover and earth’s rotation are jointly present.

This effect can also be seen in Fig. 8, which corresponds to a range of 2,400 km. It is seen that performance in terms of clutter nulling is significantly inferior when both these phenomena are simultaneously present. This is evidently the case in practice, which presents a significant challenge for target detection [5, 6].

Fig. 6 Doppler spilling due to earth’s rotation.

Fig. 7 Matched filter output with and without range foldover and earth’s rotation, SBR height \( H = 506 \) km, \( PRF = 500 \) Hz, and \( \theta_{AZ} = 90^\circ \).

Fig. 8 Matched filter output with and without (ideal ) range foldover and earth’s rotation, range = 2400 km and \( \theta_{AZ} = 90^\circ \).

To understand why the clutter notch widens in the presence of earth’s rotation as shown in Fig. 8, it is necessary to review (13) - (20) simultaneously. Fig. 8 is plotted for a fixed \( (R_n, \theta_{AZ}) \), which fixes the cone angle \( c \) in (14) while varying \( \omega_{AZ} \) in (18). As a result, the cone angle

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\[ c_o = \sin \theta_{EL} \cos \theta_{AZ} \]  
(21)

remains fixed. However, referring to (19) and (20), there are several other points \((R_x, \theta_{AZ})\) or equivalently locations at \((\theta_{EL}, \theta_{AZ})\) that satisfy the identity

\[ c_o = \sin \theta_{EL} \cos \theta_{AZ} \]  
(22)

Hence, secondary nulls occur at \(\tilde{\omega}_d, \tilde{\omega}_d, \ldots\), and when taken together they result in a wider clutter null (see Fig. 8).

The problem is thus to rectify the situation in Fig. 7 (d), so that it becomes closer to the symmetric curves of Fig. 7 (a), (b), and (c). Having both range foldover and earth’s rotation at the same time results in unacceptable performance degradation as shown in Fig. 7 (d) - Fig. 8; whereas when either one is present separately, the effect can be rectified.

4. ORTHOGONAL PULSING SCHEME

We propose to use waveform diversity on the sequence of transmitted radar pulses to realize the above goal by suppressing the range foldover returns. Recall that in ordinary practice, a set of identical pulses are transmitted. To suppress returns due to range foldover, for example, individual pulses \(f_i(t), f_j(t), \ldots\), can be made orthogonal to each other so that

\[ \int_0^T f_i(t) f_j(t) dt = \delta_{ij}, \quad i, j = 1, 2, \ldots, N_u, \]  
(23)

where \(N_u\) is the maximum number of distinct range foldovers present in the data and \(\delta_{ij}\) is the Kronecker delta product.

Then, with appropriate matched filtering [8], the range ambiguous returns can be minimized from the main return corresponding to the range of interest. In this case, performance will be closer to that shown in Fig. 7 (c). Note that for range foldover elimination, waveform diversity needs to be implemented only over \(N_u\) pulses. For an SBR located at a height of 506 km and an operating PRF = 500 Hz, \(N_u = 7\). This is the case since matched filtering will eliminate the superimposed range foldover returns since they correspond to waveforms that are orthogonal to the one in the mainbeam.

Fig. 10 shows the SINR improvement using eight up/down chirp waveforms all with equal bandwidth [8].
These waveforms are either up/down ramps or triangular in shape in the frequency domain. Quadrature phase shifting of these waveforms will generate an additional set of four waveforms, resulting in a pool of eight waveforms. Notice that these eight waveforms are only approximately orthogonal. However, they all maintain the desirable pulse compression property. In this case, although the performance has improved over the conventional pulsing scheme considerably, the remaining degradation can be attributed to the approximate orthogonal nature of these waveforms.

Fig. 11 (b) shows the improvement in SINR as a function of range and Doppler obtained by using these chirp waveforms. For comparison purposes, Fig. 11 (a) shows the performance using conventional pulsing in the presence of both range foldover and earth’s rotation. Note that using waveform diversity on transmit, the performance shown in Fig. 11 (b) is restored to that shown in Fig. 7 (c), where only earth’s rotation present.

In summary, using waveform diversity on transmit, it is possible to eliminate the effect of range foldover with performance results as shown in Fig. 11 (b). This situation on the other hand corresponds to that shown in Fig. 7 (c), where a known crab angle generates the range dependency on the Doppler. As a result, using an appropriate inverse compensation factor, it is possible to correct this Doppler dependency. The resulting performance will be approximately the same as the performance shown in Fig. 7 (a), indicating that using waveform diversity on transmit, it is possible to achieve performance close to the ideal case even in the presence of both range foldover and earth’s rotation. The results presented here correspond to the case where the ensemble averaged clutter covariance matrix is given. The case where the covariance matrix is estimated from secondary data is under investigation.

5. CONCLUSIONS

This paper gives a detailed derivation for modeling earth’s rotation on SBR Doppler. The overall effect of earth’s rotation on Doppler can be modeled as two correction factors in terms of a crab angle affecting the azimuth and a crab magnitude affecting the Doppler magnitude of the clutter path. The earth’s rotation together with range foldover significantly degrade the clutter suppression performance of adaptive processing algorithms, thus making for target detection more difficult. Waveform diversity on transmit is shown to minimize these effects and restores performance close to that of the ideal case.

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BIOGRAPHIES

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