Background and target randomization
and
Root Mean Square (RMS) background matching using a new $\Delta T$ metric definition

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ABSTRACT

EO/IR/Laser detection of a target amidst clutter/background is a difficult problem often treated with simplistic models. Unlike noise, clutter is more complex, neither spectrally white nor statistically Gaussian. Therefore, it is insufficient to lump clutter with noise and use standard detection curves. Using current target detection models, it is extremely difficult to perform effectiveness assessments of signature management technologies for survivability of military ground vehicles. Current models do not consider the vehicle on a component-level basis and do not account for artifacts introduced into images from aliasing and varying amounts of clutter. Algorithms must be developed that quantify the effects of random backgrounds on the imaging capability of electro-optical systems to improve false alarm rates. Current trends dictate that EO/IR/Laser imaging systems must consider developments in signature management technologies and countermeasures that are driving clutter magnitudes higher than target signature magnitudes. These trends make the problem of target detection in clutter especially critical. Battelle has produced image randomization software called BATTRAN (Background and Target Randomization) which computes various types of statistical distributions to randomize background and target pixels separately. The types of statistics implemented include exponential, Gaussian, log-normal, and Rice distributions for both the background and target. To generate synthetic images to assess the detection performance of thermal imaging systems and countermeasured platform signatures, a method to characterize the background and target is required so that their signatures can be statistically matched. Current methods use an area-weighted average temperature difference (AWA$\Delta T$), which is regarded as inadequate in representing observer's sensitivity to the inherent detection cues of the target/background/clutter signatures. In an effort to identify a more robust and accurate $\Delta T$ metric definition for background and target matching, Battelle also developed a new $\Delta T$ metric definition and its equation using RMS pixel-based higher order statistics for the background and target signature pixel data in a scene image. This new $\Delta T$ metric provides a better estimate of true signature difference between the background/clutter and target, enabling more accurate matching of the background/clutter and target for use in sensor detection performance assessment.

1. INTRODUCTION

BATTRAN, a software package produced by Battelle, randomizes each background and target pixel in an input scene image. The randomizing distribution of the background and target pixels can be different. The properties of randomizing statistical distributions are provided as a priori distributions. A method to randomize pixels in an image is also provided. Output images from BATTRAN are presented along with inherent input images. The output images are randomized with various pairs of randomizing distributions. These output images are then used for $\Delta T$ analysis.

Currently used $\Delta T$ metric definitions and their equations\textsuperscript{1,2,3} are not proper representations for describing temperature varying features of a target and/or its background. Some $\Delta T$ metrics oversimplify the effects of these features and misrepresent the apparent temperature difference which provides an inherent detection cue of target/background/clutter. Other $\Delta T$ metrics treat the pixels from a target and its background in a sensor image as if they come from two separate images, which ignores a relative temperature apparentness of the target and its background. These facts inadequately represent a human observer's sensitivity to the detection of target from background and clutter. Therefore, these problems associated with the currently used $\Delta T$ metrics necessitate the development of a new $\Delta T$ metric.
**Background and target randomization Root Mean Square (RMS) background matching using a new delta T metric definition**

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**ABSTRACT**
definition and its equation. In this paper, a new $\Delta T$ metric is defined. An equation representing this new definition is then derived. This new $\Delta T$ equation is implemented in BATTRAN along with other $\Delta T$ equations with which the new $\Delta T$ metric is compared.

The remainder of this paper is organized as follows: Section 2 discusses the theory of the background and target randomization algorithm implemented in BATTRAN including the a priori distributions and the pixel randomization method, and presents the randomized output images; Section 3 discusses the new $\Delta T$ definition and its equation derivation, and compares the new $\Delta T$ metric with other $\Delta T$ metrics; Summary and conclusions are given in Section 4; Section 5 contains acknowledgements; and, Section 6 lists the references cited during this research performance period.

2. BACKGROUND/CLUTTER AND TARGET STATISTICS MODELING FOR RANDOMIZATION

This section discusses the theory behind the background and target pixel randomization algorithm and its development and modeling in BATTRAN. The various statistical distributions available for the background and target randomization are provided by their probability density function (PDF) and cumulative distribution function (CDF). The random number generation from a desired statistical distribution using the inverse transform method is presented. To assure that the generated random numbers represent the desired statistics, BATTRAN-simulated PDF and CDF are compared to the calculated (or exact) PDF and CDF. Finally, the BATTRAN-randomized output images are displayed to show effects of the randomization by different statistics on a couple of input inherent images.

2.1 Properties of a priori distributions

The statistical distributions available for image randomization in BATTRAN are exponential, Gaussian, log-normal, and Rice. Because these distributions are not derived from a physical description of clutter, they are referred to as a priori distributions. These are expressed by a PDF and a CDF, denoted by $p(x)$ and $P(x)$, respectively. For a priori distribution equations presented, $\mu$ and $\sigma$ denote the mean and standard deviation of the distribution, respectively. Another parameter provided for the log-normal and Rice distributions is the standard deviation-to-mean ratio, denoted by $R$.

2.1.1 Exponential distribution

The PDF and CDF of the exponential distribution are given in (1.a) and (1.b), respectively. $P(x)$ is the definite integral of $p(x)$, $P(x) = \int_{-\infty}^{x} p(\tau) \, d\tau$.

\[
p(x) = \frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right) \quad (a)
\quad P(x) = 1 - \exp\left(-\frac{x}{\mu}\right) \quad (b)
\]

(1)

2.1.2 Gaussian distribution

The Gaussian PDF is given in (2).

\[
p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}
\]

(2)

Even though there is no analytic closed-form expression of the Gaussian CDF, it can be expressed by using the error function, $Z(\tau) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\tau^2}{2}\right)$. Integrating $Z(\tau)$, the error function’s CDF is obtained as indicated in (3).

\[
P_Z(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} \exp\left\{-\frac{\tau^2}{2}\right\} \, d\tau = \int_{-\infty}^{y} Z(\tau) \, d\tau
\]

(3)

Applying (3) to (2), (4) gives the Gaussian CDF in terms of the error function's CDF, $P_Z(y)$.
\[ P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{(t-x)^2}{2\sigma^2}\right) dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(-\frac{t^2}{2}\right) dt = \Phi\left(\frac{x-\mu}{\sigma}\right) \] (4)

The Gaussian CDF can be then represented as in (5), which is the error function’s CDF approximation. This equation is a polynomial and rational approximation4.

\[ P(x) = P_{\varphi}(x, t) = 1 - Z(v)(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(v) \] (5)

where \( v = \frac{x-\mu}{\sigma} \) and \( t = \frac{1}{1 + \epsilon(x)} \). The error term is bounded such that \( |\epsilon(v)| < 7.5 \times 10^{-8} \). The coefficient values in (5) are \( b_1 = 0.319381530, b_2 = -0.356563782, b_3 = 1.781477937, b_4 = -1.821255978, b_5 = 1.330274429, \) and \( q = 0.2316419 \).

### 2.1.3 Log-normal distribution\(^5\,^6\,^7\)

The PDF of the log-normal distribution is given in (6).

\[ p(x) = \frac{1}{\sqrt{2\pi} \sigma_{ln} x} \exp\left(-\frac{\log(x) - m_{ln}}{2\sigma_{ln}^2}\right) = \frac{1}{\sqrt{2\pi} \sigma_{ln} x} \exp\left(-\frac{1}{2\sigma_{ln}^2} \left[ \log\left(\frac{x}{m}\right) \right]^2\right) \] (6)

Since the log-normal distribution is a Gaussian distribution with the natural logarithm of random variables, (5) is used with \( v = \frac{\log(x) - m_{ln}}{\sigma_{ln}} \) to obtain the log-normal CDF. This is because the log-normal CDF can be written as (7).

\[ P(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(k) - m_{ln}}{\sigma_{ln}}} \exp\left(-\frac{k^2}{2}\right) dk = \Phi\left(\frac{\log_{e} x - m_{ln}}{\sigma_{ln}}\right) \] (7)

Terms in (7) are \( \sigma_{ln} = \log_{e}(\sigma) = \) standard deviation of \( \log\left(\frac{x}{m}\right) \), and \( m_{ln} = \log_{e}(m) \), where \( m \) is the median of \( x \). The mean-to-median ratio \( (M = \mu/m) \) of \( x \) is related to the standard deviation as in (8).

\[ \sigma_{ln} = \left[ 2 \times \log_{e}(M) \right]^{\frac{1}{2}} = \left[ 2 \times \log_{e}\left(\frac{\mu}{m}\right) \right]^{\frac{1}{2}} = \left[ 2 \times \left( \log_{e}(\mu) - m_{ln} \right) \right]^{\frac{1}{2}} \] (8)

The median can then be found by solving (8) when the mean and the standard deviation are given for the log-normal distribution as shown in (9).

\[ m_{ln} = \log_{e}(m) = \log_{e}(\mu) - \frac{\sigma_{ln}^2}{2} \] (9)

The standard deviation-to-mean ratio \( (R) \) is expressed in (10.a), which can be solved for the standard deviation when \( R \) is provided as indicated in (10.b).

\[ R = \sqrt{e^{\sigma_{ln}^2} - 1} \quad (a); \quad \sigma_{ln} = \sqrt{\log_{e}(R^2 + 1)} \] (10)

#### 2.1.4 Rice distribution\(^5\,^6\)

The PDF of the Rice distribution is expressed in (11), where \( I_0(\alpha) \) is the zero-order modified Bessel function of first kind. The standard deviation-to-mean ratio \( (R) \) is expressed in (12.a). When \( R \) is given, the dominant-to-diffuse ratio
\( r \) can be expressed as in (12.b). The dominant-to-diffuse ratio is the ratio of the single dominant scatterer signature to the total signature of the small scatterers.

\[
p(x) = \frac{1 + r}{\mu} \exp \left( -r \left( 1 + r \right) \frac{x}{\mu} \right) I_0 \left( \sqrt{r \left( 1 + r \right) \frac{x}{\mu}} \right)
\]

(11)

\[
R = \frac{\sqrt{1 + 2r}}{1 + r} \quad (a); \quad r = -g + \sqrt{g \times (g - 1)} \quad (b); \quad g = 1 - R^2 \quad (c)
\]

(12)

Numerical integration of the Rice PDF is necessary to obtain its CDF because there is neither an analytic closed-form expression nor a polynomial approximation of the Rice CDF.

2.1.5 Distribution example

Figure 1 shows an example of the four distributions defined above. Shown is each distribution's complement cumulative probability (CCP), \( 1 - \int p(r)dr \). Each distribution has a mean of \( 10^2 \). The second parameter of each distribution is shown in Table 1.

![Figure 1 Example of cumulative distributions](image)

Table 1. The value of the mean and the second parameter.

<table>
<thead>
<tr>
<th>Distribution type</th>
<th>Mean</th>
<th>Second parameter</th>
<th>Input to BATRN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>( \mu = 10^2 )</td>
<td>N/A</td>
<td>( \mu = 10^2 )</td>
</tr>
<tr>
<td>Gaussian</td>
<td>( \mu = 10^2 )</td>
<td>( \sigma = 10^3 )</td>
<td>( \sigma = 10^3 )</td>
</tr>
<tr>
<td>Log-normal</td>
<td>( \mu = 10^2 )</td>
<td>( \sigma = 6.08 ) (or ( \sigma_n = 1.805 ))</td>
<td>( R = 5.0 )</td>
</tr>
<tr>
<td>Rice</td>
<td>( \mu = 10^2 )</td>
<td>( r = 10.9782 )</td>
<td>( R = 0.4 )</td>
</tr>
</tbody>
</table>

2.2 Inverse transform method (ITM) to randomize an input scene image

The inverse transform method is applied to generate random deviates (or random numbers) to randomize the background and target pixels using a desired distribution. This process is modeled in BATRN. Figure 2 is used to illustrate the ITM implementation in BATRN with the following steps.

Step 1: Generate a desired CDF either analytically or numerically, which is denoted by \( P(x) \) and represented by a solid line in Figure 2.

Step 2: Generate uniform random deviates between 0 and 1, which is denoted by \( y \).

Step 3: With \( P(x) \) and \( y \), one-to-one matching of \( y \) and \( x \) can be performed using \( P(x) \) to obtain \( x \), which is a random deviate from the desired distribution.

Applying ITM to an input (inherent) scene image, pixels are randomized with a desired statistical distribution. It is known that a pixel belongs either to the background or to the target prior to the randomization. Therefore, the background pixels can be randomized by one statistical distribution and the target pixels can be randomized by another distribution. The
random numbers, which may represent fluctuations in temperature, intensity, or radiance, are generated and added to the inherent background or target pixel values.

2.4 Comparison of exact distribution and the BATTRAN generated distribution

In this section, random variates generated (or simulated) by BATTRAN are examined to determine whether they closely follow the selected distribution by comparing them to the exact (or calculated) distribution. Figures 3, 4, 5, and 6 show the PDF for exponential, Gaussian, log-normal, and Rice distributions, respectively. Figures 7, 8, 9, and 10 show the CDF for exponential, Gaussian, log-normal, and Rice distributions, respectively. In each figure, the solid and dotted lines represent the calculated and simulated distributions, respectively. The simulated distribution is generated by performing $10^6$ trials. From Figures 3, 4, 5, and 6, it is apparent that the number of trials to generate the simulated distribution is $10^6$ since there are no samples taken beyond the PDF of $10^6$. It is also noted that as the simulated PDF approaches the tail region of the distribution, it deviates from the calculated PDF. This is because not enough samples are taken at each end of the tail regions to replicate the calculated PDF. However, the simulated CDF, as shown in Figures 7, 8, 9, and 10, agrees with the calculated CDF up to the complement cumulative probability (CCP) of $10^5$. Between the CCP of $10^5$ and $10^6$, the simulated CDF deviates from the calculated CDF. Beyond the CCP of $10^6$, no data is available. Figures 3 and 7 show the exponential PDF and CDF, respectively. The mean ($\mu$) of this distribution is 10. Figures 4 and 8 show the Gaussian PDF and CDF, respectively. The mean ($\mu$) is 0 and the standard deviation ($\sigma$) is 20. Figures 5 and 9 show the log-normal PDF and CDF, respectively. The mean ($\mu$) is 5 and the standard deviation-to-mean ratio ($R$) is 1. From the mean and the standard deviation-to-mean ratio, the median ($m$) and the standard deviation ($\sigma$) are 3.5355 and 2.2992, respectively, using (9) and (10). In natural logarithm, the median ($m_{\ln}$) and the standard deviation ($\sigma_{\ln}$) are 1.2629 and 0.8326, respectively. Figures 6 and 10 show the Rice PDF and CDF, respectively. The mean ($\mu$) of the distribution is 35 and the standard deviation-to-mean ratio ($R$) is 0.5. From the standard deviation-to-mean ratio, the dominant-to-diffuse ratio ($r$) is 6.4641 using (12.b) and (12.c). These figures show that the random numbers generated in BATTRAN closely follow the exact PDF and CDF. This not only verifies that the random numbers are generated with the input statistics parameters, but also validates that they are generated by the desired distribution.

2.5 Output images from BATTRAN after background and target pixel randomization

Example output images from BATTRAN after randomization were generated to demonstrate the effects of the randomization with various distributions. A simulated and a measured input inherent images were used for the randomization. The simulated inherent image is shown in Figure 11. The measured inherent image is shown in Figure 12. The simulated image was generated by the Wright Laboratories Tactical Decision Aid (TDA) model at Battelle. The measured image was provided by TARDEC. The size of the simulated and measured images were $378 \times 377$ and $512 \times 512$ pixels, respectively. Background pixel values of the simulated inherent image were set to zero; therefore, no difficulties arose in separating the background pixels from the target pixels. However, the measured image had to be
processed to separate the background pixels from the target pixels. The Geographic Resources Analysis Support System (GRASS) software package was used to cut out the target pixels from the background pixels. The output image file from GRASS also had the background pixels assigned to zero.

Figure 11  Simulated input inherent image.

Figure 12  Measured input inherent image.

2.5.1 Output image examples with the simulated/measured image

For output images shown in Figures 13-16, Table 2 summarizes the types of statistical distributions and associated parameters used to randomize the simulated input inherent image. For output images shown in Figures 17-20, Table 3 summarizes the distributions and parameters used to randomize the measured input inherent image. The randomizing statistics for the target and background pixels are different for each randomized output image. These figures are discussed in detail using various $\Delta T$ metric definitions in Section 3.3.

Table 2  Randomization parameters of BATRAN output image using the simulated input inherent image.

<table>
<thead>
<tr>
<th>Simulated Image</th>
<th>Background Pixel Randomizing Statistics</th>
<th>Target Pixel Randomizing Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure #</td>
<td>Distribution Type</td>
<td>Distribution Parameters</td>
</tr>
<tr>
<td>13</td>
<td>Exponential</td>
<td>$\mu = 10$</td>
</tr>
<tr>
<td>14</td>
<td>Gaussian</td>
<td>$\mu = 24.45$; $\sigma = 5$</td>
</tr>
<tr>
<td>15</td>
<td>Log-normal</td>
<td>$\mu = 6; R = 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$m_{in} = 1.4452$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_{in} = 0.8326$</td>
</tr>
<tr>
<td>16</td>
<td>Rice</td>
<td>$\mu = 15; R = 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3  Randomization parameters of BATRAN output image using the measured input inherent image.

<table>
<thead>
<tr>
<th>Measured Image</th>
<th>Background Pixel Randomizing Statistics</th>
<th>Target Pixel Randomizing Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure #</td>
<td>Distribution Type</td>
<td>Distribution Parameter</td>
</tr>
<tr>
<td>17</td>
<td>Exponential</td>
<td>$\mu = 25$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Gaussian</td>
<td>$\mu = 10$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma = 10$</td>
</tr>
<tr>
<td>19</td>
<td>Log-normal</td>
<td>$\mu = 10; R = 0.9$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$m_{in} = 2.0059$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_{in} = 0.7703$</td>
</tr>
<tr>
<td>20</td>
<td>Rice</td>
<td>$\mu = 34; R = 0.95$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r = 0.45401$</td>
</tr>
</tbody>
</table>
Figure 13  Exponential/Gaussian randomization for background/target using simulated image.

Figure 14  Gaussian/exponential randomization for background/target using simulated image.

Figure 15  Log-normal/Rice randomization for background/target using simulated image.

Figure 16  Rice/log-normal randomization for background/target using simulated image.
Figure 17 Exponential/log-normal randomization for background/target using measured image.

Figure 18 Gaussian/Rice randomization for background/target using measured image.

Figure 19 Log-normal/Gaussian randomization for background/target using measured image.

Figure 20 Rice/exponential randomization for background/target using measured image.
3. A NEW $\Delta T$ DEFINITION

In this section, a new $\Delta T$ metric definition is stated, and the equation based on this definition is derived. This definition is compared with other $\Delta T$ metric definitions such as the conventional $\Delta T$ (= AWA$\Delta T$), the Night Vision Laboratory (NVL) RMS $\Delta T$ in [2], and the $\Delta T$ found in [3]. The $\Delta T$ results from these various definitions are also compared by using Figures 13 - 20. The conventional $\Delta T$, NVL $\Delta T$, and the $\Delta T$ from [3] are denoted by $\Delta T_{\text{conv}}$, $\Delta T_{\text{NVL}}$, and $\Delta T_{[3]}$, respectively. The equations for $\Delta T_{\text{conv}}$, $\Delta T_{\text{NVL}}$, and $\Delta T_{[3]}$ are given in (13.a), (13.b), and (13.c), respectively.

$$\Delta T_{\text{conv}} = \mu_T - \mu_B \quad (a); \quad \Delta T_{\text{NVL}} = \left( \frac{1}{N_T} \sum_{i=1}^{N_T} (P_i^T - \mu_T)^2 \right)^{1/2} = \sigma_T \quad (b); \quad \Delta T_{[3]} = \sqrt{(\mu_T - \mu_B)^2 + \sigma_T^2} \quad (c) \quad (13)$$

where, $\mu_T$: The mean temperature of the target pixels  
$\mu_B$: The mean temperature of the background pixels  
$P_i^T$: Each target pixel’s temperature  
$\sigma_T$: The standard deviation of the target pixel temperature  
$N_T$: The number of pixels on target  
$\sigma_B$: The standard deviation of the background pixels

The $\Delta T_{\text{conv}}$, as shown in (13.a), does not consider temperature varying features on both target pixels and background pixels. This $\Delta T_{\text{conv}}$ definition is an oversimplification of the background and target temperature characteristics. The $\Delta T_{\text{NVL}}$ is simply a standard deviation of the target pixel temperatures. No background terms are involved in (13.b) indicating the background temperature is ignored. The $\Delta T_{[3]}$ considers the target temperature varying feature by having $\sigma_T$; however, the background temperature varying feature is not considered. It is evident in (13.b) and (13.c) that $\Delta T_{\text{NVL}}$ and $\Delta T_{[3]}$ cannot yield a negative $\Delta T$, a case where the target is cooler than the background. These deficiencies indicate the need to define a new $\Delta T$ metric that includes higher order statistics of both background and target pixels’ temperatures.

A human operator does not see a separate target or background, but rather sees both at the same time. Therefore, an overall scene, which includes the target and background, should be involved in defining a new $\Delta T$ metric. This is because when the overall scene is located far enough away such that the entire scene falls onto a pixel, only the mean temperature of the scene is realized. However, this mean temperature is produced not only by the background temperature but also by the target temperature. Therefore, it is adequate to imitate the human perception of the scene by evaluating the standard deviation of the target pixels’ temperatures conditioned on the mean temperature of the scene ($= \sigma_{T[s]}$), and by evaluating the standard deviation of the background pixels’ temperatures conditioned on the mean temperature of the scene ($= \sigma_{B[s]}$). Then, a new $\Delta T$ metric is defined as the difference between the standard deviation of target pixel temperatures conditioned on the mean temperature of scene and the standard deviation of background pixel temperatures conditioned on the mean temperature of scene. This is shown in (14).

$$\Delta T_{\text{new}} = \sigma_{T[s]} - \sigma_{B[s]} \quad (14)$$

3.1 Formulation of the new $\Delta T$ equation

Prior to formulating the new $\Delta T$ metric equation, the pixel temperature statistics of target/background and scene are expressed below. (15.a) and (15.b) show the mean and variance of the target pixel temperatures, respectively. The mean and variance of pixel temperatures on the background are expressed in (16.a) and (16.b), respectively.

$$\mu_T = \frac{1}{N_T} \sum_{i=1}^{N_T} P_i^T \quad (a); \quad \sigma_T^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} (P_i^T - \mu_T)^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} (P_i^T)^2 - \mu_T^2 \quad (b) \quad (15)$$

$$\mu_B = \frac{1}{N_B} \sum_{i=1}^{N_B} P_i^B \quad (a); \quad \sigma_B^2 = \frac{1}{N_B} \sum_{i=1}^{N_B} (P_i^B - \mu_B)^2 = \frac{1}{N_B} \sum_{i=1}^{N_B} (P_i^B)^2 - \mu_B^2 \quad (b) \quad (16)$$

Choe 10
where, $P^T$: Pixel temperature on target  
$P^B$: Pixel temperatures on background  
$\sigma_T^2$: The variance of pixel temperatures on target  
$\sigma_B^2$: The variance of pixel temperatures on background  
$\mu_T$: The mean of pixel temperatures on target  
$\mu_B$: The mean of pixel temperatures on background  
$N_T$: Number of pixels on target  
$N_B$: Number of pixels on background

The mean and variance of the overall scene are given in (17.a) and (17.b), respectively. It is noted that $N_S = N_B + N_T$.

$$
\mu_S = \frac{1}{N_S} \sum_{i=1}^{N_T} P_i^T = \frac{\mu_T N_T + \mu_B N_B}{N_T + N_B} \quad \text{(a)}
$$

$$
\sigma_S^2 = \frac{1}{N_S} \sum_{i=1}^{N_T} (P_i^T - \mu_S)^2 = \frac{1}{N_S} \sum_{i=1}^{N_T} (P_i^T)^2 - \mu_S^2 = \left(\frac{\sigma_T^2 + \mu_T^2}{N_T} + \frac{\sigma_B^2 + \mu_B^2}{N_B}\right) \frac{N_T}{N_T + N_B} - \left(\frac{\sum_{i=1}^{N_T} P_i^T - \sum_{i=1}^{N_B} P_i^B}{N_T + N_B}\right)^2 \quad \text{(b)}
$$

where, $P^S$: Pixel temperatures on scene  
$\mu_S$: The mean of pixel temperatures on scene  
$N_S$: Number of pixels on scene ($= N_B + N_T$)

Then, $\sigma_{TS}^2$ and $\sigma_{BS}^2$ can be expressed in terms of $\mu_T$, $\sigma_T$, $N_T$, $\mu_B$, $\sigma_B$, $N_B$, and $N_S$. The conditional variances are shown in (18.a) and (18.b) for the target and background pixel temperatures, respectively.

$$
\sigma_{TS}^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} (P_i^T - \mu_S)^2 = \sigma_T^2 + \mu_T^2 - 2 \mu_T \mu_T + \mu_T^2 \frac{1}{N_S} \left[N_T^2 \sigma_T^2 + N_B^2 (\mu_T - \mu_B)^2\right] \quad \text{(a)}
$$

$$
\sigma_{BS}^2 = \frac{1}{N_B} \sum_{i=1}^{N_B} (P_i^B - \mu_S)^2 = \sigma_B^2 + \mu_B^2 - 2 \mu_B \mu_B + \mu_B^2 \frac{1}{N_S} \left[N_T^2 \sigma_B^2 + N_B^2 (\mu_B - \mu_T)^2\right] \quad \text{(b)}
$$

Therefore, the new $\Delta T$ metric equation ($= \Delta T_{\text{new}}$) is the difference between the square root of (18.a) and the square root of (18.b). This is written in (19).

$$
\Delta T_{\text{new}} = \sigma_{TS}^2 - \sigma_{BS}^2 = \frac{1}{N_S} \left[N_T \sigma_T^2 + N_B \sigma_B^2 (\mu_T - \mu_B)^2\right] - \sqrt{N_T \sigma_T^2 + N_B \sigma_B^2 (\mu_B - \mu_T)^2} \quad \text{(19)}
$$

In the above equation, it is apparent that the temperature varying feature on the target and background pixels is included in terms of their variances. Also, the number of pixels on target, background, and scene is also included, which shows (19) is a pixel-based temperature difference between the target and background. Of course, the mean temperatures of the target and background are also present in (19).

### 3.2 Comparison of $\Delta T_{\text{new}}$ metric to other $\Delta T$ metric equations

The other $\Delta T$ metric equations introduced earlier ($\Delta T_{\text{conv}}$, $\Delta T_{\text{NL}}$, and $\Delta T_{\text{b}}$) will now be compared to the $\Delta T_{\text{new}}$ metric. The $\Delta T_{\text{conv}}$ metric is evaluated by the mean temperatures of the target and background as shown in (13.a). It ignores temperature variations of the target and background pixels. Thus, the variances of the target and background are zero, $\sigma_T^2 = \sigma_B^2 = 0$. Under these conditions, the $\Delta T_{\text{new}}$ metric reduces to in the $\Delta T_{\text{conv}}$ metric as shown in (20).

$$
\Delta T_{\text{new}}|_{\sigma_T=0, \sigma_B=0} = \frac{1}{N_S} \left[N_B \times (\mu_T - \mu_B) - N_T \times (\mu_B - \mu_T)\right] = \frac{1}{N_S} \times (N_T + N_B) \times (\mu_T - \mu_B) = \mu_T - \mu_B = \Delta T_{\text{conv}} \quad \text{(20)}
$$

The $\Delta T_{\text{NL}}$ metric, as mentioned before, is simply a standard deviation of the target pixel temperatures as shown in (13.b). It does not consider any background characteristics nor the mean target temperature. These conditions can be written such as $\mu_T = \sigma_B = N_B = N_T = 0$. Under these conditions, the $\Delta T_{\text{new}}$ metric reduces to the $\Delta T_{\text{NL}}$ metric as shown in (21).
\[ \Delta T_{\text{new}} \mid \mu_2, \sigma_2 \neq 0, \mu_1, \sigma_1 = 0 = \sigma_T = \Delta T_{\text{NVL}} \]  \hspace{1cm} (21)

The \( \Delta T_{\text{BJ}} \) metric, from the equation shown in (13.c), considers the temperature variance on target pixels but not on background pixels. It also considers the mean temperature of the target and background as the \( \Delta T_{\text{conv}} \) metric. This condition can be written as \( \sigma_b = 0 \), which means a uniform background. For this condition, the \( \Delta T_{\text{new}} \) metric becomes (22).

\[ \Delta T_{\text{new}} \mid \sigma_2 = 0 = \frac{1}{N_2} \left[ \sqrt{N_2^2 \sigma_T^2 + N_2^2 (\mu_T - \mu_B)^2} - N_T (\mu_T - \mu_B) \right] \]  \hspace{1cm} (22)

It seems that there is no direct relationship between (22) and the \( \Delta T_{\text{BJ}} \) metric unless some special conditions are considered. If the mean of the target and background pixel temperatures are the same \( (\mu_T = \mu_B) \), (13.c) and (22) become the standard deviation of the pixel temperatures on the target only, which is the \( \Delta T_{\text{NVL}} \) metric as shown in (23).

\[ \Delta T_{\text{new}} \mid \sigma_2 = 0 = \Delta T_{\text{BJ}} \mid \mu_2 = \mu_B = \Delta T_{\text{NVL}} = \sigma_T \]  \hspace{1cm} (23)

If the variance of the target pixel temperatures \( (\sigma_T^2) \) is zero, i.e., a uniform target temperature, (13.c) and (22) become the difference between the target mean temperature and the background mean temperature. Thus, in this case, \( \Delta T \) is the \( \Delta T_{\text{conv}} \) metric as shown in (24).

\[ \Delta T_{\text{new}} \mid \sigma_2 = 0, \sigma_T = \mu_B = \Delta T_{\text{BJ}} \mid \sigma_T = \mu_T = \mu_B = \Delta T_{\text{conv}} = \mu_T - \mu_B \]  \hspace{1cm} (24)

Therefore, the \( \Delta T_{\text{conv}}, \Delta T_{\text{NVL}}, \) and \( \Delta T_{\text{BJ}} \) metrics describe special cases of the \( \Delta T_{\text{new}} \) metric. The \( \Delta T_{\text{new}} \) metric given in (19) includes various effects due to the target and background characteristics in a temperature scene image by evaluating a number of pixels on the target and the background, the mean of the target and background temperatures, and the variance of the target and background temperatures. The variance represents the temperature varying feature of the target and background. The \( \Delta T_{\text{new}} \) metric is derived from a conditional standard deviation of the target and background pixel temperatures on an overall scene as shown in (18.a), (18.b), and (19). However, the other \( \Delta T \) metrics are derived by evaluating the target and the background separately as if they belong to separate images. This is not a correct interpretation of the image under evaluation, since an operator views the target pixels and background pixels in the evaluated image simultaneously.

3.3 \( \Delta T_{\text{new}} \) metric comparison with other \( \Delta T \) metrics using Figures 13 through 20

Figures 13 - 20 are used to obtain \( \Delta T \) values using the \( \Delta T_{\text{conv}}, \Delta T_{\text{NVL}}, \Delta T_{\text{BJ}}, \) and \( \Delta T_{\text{new}} \) metric equations given by (13.a), (13.b), (13.c), and (19), respectively. Table 4 lists the target and background statistics and \( \Delta T \)s of Figures 13 - 16. Table 5 shows the same for Figures 17 - 20.

Using images in Figures 13 - 16, the various \( \Delta T \) definitions discussed in Sections 3.0, 3.1, and 3.2 were evaluated. All of the target and the background pixels contributed to the \( \Delta T \) results. Observing Figure 13 and its \( \Delta T \) values, it is difficult to tell which \( \Delta T \) represents Figure 13. Nevertheless, the \( \Delta T \) above 10 degrees Kelvin seems too high compared to Figure 13. The \( \Delta T_{\text{new}} \) seems to be an adequate result for Figure 13. In Figure 14, the background mean temperature and the target mean temperature were intentionally matched using BATTRAN to evaluate the performance of each \( \Delta T \) metric. As seen in Figure 14, much of the target features are buried in the background due to the matching. The \( \Delta T_{\text{conv}} \) is zero, which is not a true statement because the target can still be detected and recognized. The \( \Delta T_{\text{NVL}} \) and \( \Delta T_{\text{BJ}} \) may be reasonable only if hot parts of the target are considered. Since all of the target pixels were considered in evaluating \( \Delta T \), the results from these definitions may not properly describe the image. The result from the \( \Delta T_{\text{new}} \) metric provides a more correct representation for the image. In Figure 15, not only were the background mean temperature and target mean temperature closely matched, but also the background variance and the target variance were closely matched using BATTRAN. As seen in Figure 15, the target image is deteriorating. Except for a few hot parts, most of the target parts are relatively cooler than the background. The results from the \( \Delta T_{\text{NVL}} \) and \( \Delta T_{\text{BJ}} \) metrics are only the standard deviation.
of the target pixel temperatures, and the results from these metrics are relatively high compared to the image. The $\Delta T_{\text{new}}$ and $\Delta T_{\text{conv}}$ values are very close to each other. This is because the variance of the background and the target pixels were matched. These results may not reflect target recognition ability. In Figure 15, human eyes can easily detect and recognize the target. The $\Delta T$ results using the image depicted in Figure 16 show that the $\Delta T_{\text{conv}}$, $\Delta T_{\text{NVZ}}$, and $\Delta T_{\text{BJ}}$ values are too high, all being greater than 12 degrees Kelvin. The $\Delta T_{\text{new}}$ value shows a reasonable result for the image.

Table 4 Simulated image output pixel statistics and various $\Delta T$s comparison.

<table>
<thead>
<tr>
<th>Simulated image</th>
<th>Background pixel statistics ($N_b = 130447$)</th>
<th>Target pixel statistics ($N_T = 12059$)</th>
<th>Temperature Difference ($\Delta T$s)</th>
<th>Figure #</th>
<th>$\mu_b$</th>
<th>$\sigma_b$</th>
<th>$\mu_T$</th>
<th>$\sigma_T$</th>
<th>$\Delta T_{\text{conv}}$</th>
<th>$\Delta T_{\text{NVZ}}$</th>
<th>$\Delta T_{\text{BJ}}$</th>
<th>$\Delta T_{\text{new}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>285.01 101.09</td>
<td>294.46 135.76</td>
<td>4.95</td>
<td>13</td>
<td>285.01</td>
<td>101.09</td>
<td>294.46</td>
<td>135.76</td>
<td>4.95</td>
<td>11.65</td>
<td>15.0</td>
<td>4.42</td>
</tr>
<tr>
<td>14</td>
<td>299.46 25.58</td>
<td>300.90 60.20</td>
<td>0.00</td>
<td>14</td>
<td>299.46</td>
<td>25.58</td>
<td>300.90</td>
<td>60.20</td>
<td>0.00</td>
<td>7.76</td>
<td>7.76</td>
<td>2.70</td>
</tr>
<tr>
<td>15</td>
<td>295.51 36.65</td>
<td>306.50 149.72</td>
<td>0.03</td>
<td>15</td>
<td>295.51</td>
<td>36.65</td>
<td>306.50</td>
<td>149.72</td>
<td>0.03</td>
<td>6.10</td>
<td>6.10</td>
<td>0.05</td>
</tr>
<tr>
<td>16</td>
<td>290.55 227.60</td>
<td>306.50 149.72</td>
<td>15.75</td>
<td>16</td>
<td>290.55</td>
<td>227.60</td>
<td>306.50</td>
<td>149.72</td>
<td>15.75</td>
<td>12.24</td>
<td>12.24</td>
<td>3.77</td>
</tr>
</tbody>
</table>

Table 5 Measured image output pixel statistics and various $\Delta T$s comparison.

<table>
<thead>
<tr>
<th>Measured image</th>
<th>Background pixel statistics ($N_b = 239731$)</th>
<th>Target pixel statistics ($N_T = 22413$)</th>
<th>Temperature Difference ($\Delta T$s)</th>
<th>Figure #</th>
<th>$\mu_b$</th>
<th>$\sigma_b$</th>
<th>$\mu_T$</th>
<th>$\sigma_T$</th>
<th>$\Delta T_{\text{conv}}$</th>
<th>$\Delta T_{\text{NVZ}}$</th>
<th>$\Delta T_{\text{BJ}}$</th>
<th>$\Delta T_{\text{new}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>311.33 678.10</td>
<td>311.34 186.47</td>
<td>0.01</td>
<td>17</td>
<td>311.33</td>
<td>678.10</td>
<td>311.34</td>
<td>186.47</td>
<td>0.01</td>
<td>13.66</td>
<td>13.66</td>
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<tr>
<td>18</td>
<td>296.34 154.06</td>
<td>314.37 166.23</td>
<td>18.03</td>
<td>18</td>
<td>296.34</td>
<td>154.06</td>
<td>314.37</td>
<td>166.23</td>
<td>18.03</td>
<td>12.89</td>
<td>22.77</td>
<td>8.42</td>
</tr>
<tr>
<td>19</td>
<td>296.34 134.24</td>
<td>297.30 124.96</td>
<td>0.965</td>
<td>19</td>
<td>296.34</td>
<td>134.24</td>
<td>297.30</td>
<td>124.96</td>
<td>0.965</td>
<td>15.59</td>
<td>15.62</td>
<td>4.03</td>
</tr>
<tr>
<td>20</td>
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<td>333.98 1004.32</td>
<td>13.60</td>
<td>20</td>
<td>320.38</td>
<td>1096.38</td>
<td>333.98</td>
<td>1004.32</td>
<td>13.60</td>
<td>31.69</td>
<td>34.48</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Using images in Figures 17 - 20, the various $\Delta T$ metrics are also discussed. The background mean temperature and the target mean temperature are closely matched using BATRAN in Figure 17. This results in $\Delta T_{\text{conv}}$ close to zero. The $\Delta T_{\text{NVZ}}$ and $\Delta T_{\text{BJ}}$ values are the standard deviation of the target pixel temperatures, and their results are too high compared to Figure 17. The overall target temperature is cooler than the overall background. This is shown in $\Delta T_{\text{new}}$ with a negative value. As previously noted in Section 3.0, the $\Delta T_{\text{NVZ}}$ and $\Delta T_{\text{BJ}}$ metrics cannot discern whether the background is cooler than the target, or vice versa. Figure 18 closely represents the image taken during 30 mm/hr rain fall.* The BATRAN randomization can be used to generate images which may represent the target and background under various weather conditions. The $\Delta T$ results from Figure 19 show that the $\Delta T_{\text{NVZ}}$ and $\Delta T_{\text{BJ}}$ values are too high compared to the image. Either $\Delta T_{\text{conv}}$ or $\Delta T_{\text{new}}$ value represents a reasonable temperature difference between the background and the target. The $\Delta T$ results calculated from Figure 20 clearly tell the tolerance of each $\Delta T$ metric. The image may represent the background and the target under a severe weather condition. The target is barely distinguishable in the image. However, the $\Delta T_{\text{conv}}$, $\Delta T_{\text{NVZ}}$, and $\Delta T_{\text{BJ}}$ metrics yield the temperature differences of 13.6, 31.69, 34.48 degrees Kelvin, respectively. These unreasonable results were found because the definition of these metrics considers the target and the background separately as if they belong to separate images. Also, these definitions do not consider the background temperature varying features in their equations. However, the $\Delta T_{\text{new}}$ metric interpreted the image under a severe weather condition, as shown in Figure 20, quite well.

4. SUMMARY AND CONCLUSIONS

Background, clutter, and target field measurements from EO/IR, ladar (laser radar), visual, and other sensor systems show their statistics can be represented by the distributions implemented in the BATRAN (Background and Target Randomization) software, namely exponential, Gaussian, log-normal, and Rice distributions. The background and target pixel randomization algorithm using these distributions was first developed analytically prior to implementation in the BATRAN software, which performs the randomization on an input scene image. The background can be randomized by a statistical distribution which differs from the target randomizing statistical distribution. Also, background and target randomizing statistical distributions may be the same. The BATRAN software allows for insertion of randomness, which may represent artifacts due to aliasing, background characteristics, and varying amounts of clutter, into sensor images. The randomness is produced according to the given statistical distributions so that the randomized image can be used to

* Figure 18 was compared to a real image taken by TARDEC during 30 mm/hr rain fall.
analyze various detection and ATR algorithms. BATTRAN is also able to incorporate weather effects on sensor image if the weather statistical characteristics are known. With these capabilities, BATTRAN is a tool for enhancing the effectiveness assessment of signature management technologies for military ground vehicles, thus increasing their survivability.

The background RMS matching with a target was performed with a newly developed $\Delta T$ metric definition. This new $\Delta T$ metric, denoted by $\Delta T_{\text{new}}$ in this paper, performed better than the other definitions reviewed in this paper when applied to various scene images, both simulated and measured. Once the background/clutter and target statistics are known, and can be described any one of the BATTRAN implemented statistics, the BATTRAN software can match the target signature to the background/clutter well by adjusting the given target and/or background statistical parameters to provide a desired temperature difference ($\Delta T$). Various combinations of background/clutter and target statistics generated the output images (Figures 13 - 20) from the input scene images (Figures 11 and 12). These output images indicate each combination's ability to match a target to the background/clutter to a human observer. Using the $\Delta T_{\text{new}}$ metric, the target and background can be matched not only in their mean temperatures but also in their temperature varying features, i.e., variance or standard deviation. This corrects the deficiencies of the existing $\Delta T$ metrics mentioned in Section 3.0. It was also concluded that the $\Delta T_{\text{new}}$ metric gave the best apparent match of the suppressed vehicle with the background. Therefore, the $\Delta T_{\text{new}}$ metric allows for a more robust and accurate assessment of the detection performance of thermal imaging systems than the other $\Delta T$ metrics.

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6. REFERENCES