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Fusion of imperfect information in the unified framework of random sets theory

Application to target identification

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Defence R&D Canada – Valcartier
Technical Report
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Canada

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Abstract

This is a study of the applicability of random sets theory to target identification problems as a technique for fusion of imperfect information. For target identification, several sources of information (radar, ESM - Electronic Support Measures, SAR - Synthetic Aperture Radar, IR images) are available. Since the information provided is always imperfect and several kinds of imperfection may be encountered (imprecision, uncertainty, incompleteness, vagueness, etc.), several theories were developed to assist probability theory (long the only tool available to deal with uncertainty) in data fusion problems. In recent decades fuzzy sets theory was developed to deal with vague information, possibility theory was developed to deal with incomplete information, evidence theory was developed to deal with imprecise and uncertain information, and rough sets theory was developed to deal with vague and uncertain information. These theories have several points in common; here we study random sets theory, which is a unifying framework for all the aforementioned theories. In two simple test scenarios, we demonstrate the effectiveness of this unifying framework for representing and fusing imperfect information in the target identification application.

Résumé

Ce travail présente une étude de l'applicabilité de la théorie des ensembles aléatoires aux problèmes d'identification de cibles en tant que technique de fusion des informations imparfaites. Pour identifier une cible, plusieurs sources d'information (radar, ESM - Electronic Support Measures, SAR - Synthetic Aperture Radar, images IR) sont disponibles. Parce que ces informations sont toujours imparfaites et que différents types d'imperfections peuvent être répertoriés (incertaines, imprécises, incomplètes, vagues), plusieurs théories ont été développées pour assister la théorie des probabilités (longtemps le seul outil pour traiter l'incertitude), dans sa tâche de combinaison d'informations imparfaites. Depuis plusieurs décennies ont vu le jour la théorie des sous-ensembles flous pour traiter les informations vagues, la théorie des possibilités pour les informations incomplètes, la théorie de l'évidence pour les informations incertaines, la théorie des ensembles approchés pour les informations vagues et incertaines. Il apparaît que ces différentes théories présentent plusieurs points communs et nous étudions dans ce travail la théorie des ensembles aléatoires qui s'avère un cadre unificateur pour la plupart d'entre elles. Sur deux scénarios tests simples, on démontre l'utilité d'un cadre unificateur pour représenter et fusionner les informations imparfaites, et ce, dans une application particulière qu'est l'identification de cibles.

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Executive summary

Fusion of imperfect information in the unified framework of random sets theory: Application to target identification

Mihai Cristian Florea, Anne-Laure Jousselme, Éloi Bossé; DRDC Valcartier TR 2003-319; Defence R&D Canada – Valcartier; November 2007.

Background: For target identification, several sources of information (radar, ESM - Electronic Support Measures, SAR - Synthetic Aperture Radar, IR images) are available. Since the information provided is always imperfect and several kinds of imperfection may be encountered (imprecision, uncertainty, incompleteness, vagueness, etc.), several theories were developed to assist probability theory (long the only tool available to deal with uncertainty) in data fusion problems. In recent decades, fuzzy sets theory was developed to deal with vague information, possibility theory was developed to deal with incomplete information, evidence theory was developed to deal with imprecise and uncertain information, and rough sets theory to deal with vague and uncertain information. These theories have several points in common; here we study random sets theory, which is a unifying framework for all the aforementioned theories.

Principal results: This is a study of the applicability of random sets theory to target identification problems as a technique for fusing imperfect information. In two simple test scenarios, we demonstrate the effectiveness of this unifying framework for representing and fusing imperfect information in the target identification application.

Future work: More complex test scenarios should be analyzed to further investigate the effectiveness of a unifying framework for data fusion. Performance measurements could also be introduced to gage the advantages of random sets theory over the classical theories.

Sommaire

Fusion of imperfect information in the unified framework of random sets theory: Application to target identification

Mihai Cristian Florea, Anne-Laure Jousselme, Éloi Bossé ; DRDC Valcartier TR 2003-319 ; R & D pour la défense Canada – Valcartier ; novembre 2007.

Contexte : Pour identifier une cible, plusieurs sources d'information (radar, ESM - Electronic Support Measures, SAR - Synthetic Aperture Radar, images IR) sont disponibles. Parce que ces informations sont toujours imparfaites et que différents types d'imperfections peuvent être répertoriés (incertaines, imprécises, incomplètes, vagues), plusieurs théories ont été développées pour remplacer la théorie des probabilités (longtemps le seul outil pour le traitement de l'incertitude), dans sa tâche de représentation et de combinaison d'informations imparfaites. Depuis plusieurs décennies, ont vu le jour la théorie des sous-ensembles flous pour les informations vagues, la théorie des possibilités pour les informations incomplètes, la théorie de l'évidence pour les informations incertaines, la théorie des ensembles approchés pour les informations vagues et incertaines. Il apparaît que ces différentes théories présentent plusieurs points communs et nous étudions dans ce travail la théorie des ensembles aléatoires qui s'avère un cadre unificateur pour la plupart d'entre elles.

Principaux résultats : Ce mémoire est une étude de l'applicabilité de la théorie des ensembles aléatoires aux problèmes d'identification de cibles en tant que technique de représentation et de fusion des informations imparfaites. Sur deux scénarios tests, on démontre l'utilité d'un cadre unificateur pour représenter et fusionner les informations imparfaites, et ce pour l'application particulière qu'est l'identification de cibles.

Travaux futurs : Des scénarios tests plus complexes peuvent être analysés pour mieux étudier la pertinence d'un cadre unificateur pour la fusion de données. De plus, des mesures de performance pourraient être introduites pour étudier l'amélioration apportée par la théorie des ensembles aléatoires par rapport aux théories classiques.

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1 Introduction

Animate life forms (both animal and human) have always used their senses (sight, hearing, touch, taste and smell) to recognize objects and other life forms and make decisions. Usually a combination of different senses is needed for correct identification, since only one sense may lack information.

Hence, the data fusion concept is not new. The result of fusion (or combination) is information that, on the one hand, can be combined with a new piece of information, and on the other, is usable for decision-making.

Military applications like target tracking and target identification, and non-military applications like medicine and robotics, have been developed using the mathematics of data fusion.

The target identification question is particularly complex and difficult. Although the diversity of sensors can be a great help, it also creates the greatest difficulties. Information sources may be human (such as opinions "I believe that..." or descriptions "the object is long"), electronic (radar or electronic support measures (ESM), etc.), or even statistical.

The difficulties of target identification indeed result from the variety of sensors, but another difficulty stems from imperfections in the information collected: imprecision, uncertainty, fuzziness, incompleteness, inconsistency or randomness.

Many topologies or classifications of imperfect information have been proposed (e.g., Klir [1], Krause and Clark [2]). In this study we rely on the classification proposed by Smets [3]. This classification supposes that imperfections may be characterized in terms of uncertainty or imprecision. A piece of imperfect information is therefore uncertain, imprecise or both at the same time.

An information theory that enables the user to reason under uncertainty is required if problems related to imperfect information are to be resolved. Owing to the diversity of imperfections, many theories have been developed in recent decades. Each of them offers alternatives to the use of probability theory, which was the first and only theory for dealing with uncertain information for many years.

Probability theory is one way to process uncertain information. It is also useful for processing other kinds of information (imprecise information, for example). Yet it lacks certain capabilities, which makes it too restrictive. New theories have therefore been developed to deal with these aspects.

Vague pieces of information, characterized by fuzzy attributes, are modelled using fuzzy sets theory (Zadeh [4]). For example, the piece of information "John is tall" gives information on John's size, but the classifier *tall* is ill defined, even unknown. This kind of information is called fuzzy information, and is well modelled by fuzzy sets theory.

Dempster-Shafer evidence theory (Dempster [5] followed by Shafer [6]) is well suited to studying information that is imprecise and uncertain at the same time. It is a generalization of probability theory.

Possibility theory, the basis of which was proposed by Zadeh [7], is derived from fuzzy sets theory (developed by the same author) and deals with incomplete and uncertain information.

All these theories in one way or another characterize uncertainty?the confidence we have for a particular event. One difficult aspect of modelling information in these theories is quantifying the confidence levels that someone has for the events considered. Pawlak [8] developed rough sets theory to deal with imprecise information of which there is total certainty (be it known or unknown).

Since many different types of imperfection coexist in a data fusion process, different formalisms may be used at different levels of the process. The main problem of data fusion is to use these different pieces of information, modelled by different mathematical formalisms, to arrive at a global decision. This problem may be solved in two ways:

1. Choose one theory already used to represent all pieces of information, and transform the pieces of information modeled in other formalisms into this theory. Many authors have worked on formulas for transforming from one theory to another. For instance, Smets [9] and Voorbraak [10] developed a transformation between evidence and probability theories. Klir and Parviz [11] give different formulas for transforming between probability and possibility theories. Finally, Skowron [12] developed a link between rough sets and evidence theories.

2. Use a unifying formalism that is able to represent all theories. Random sets theory seems to be well suited for this application. Indeed, for several years now, many authors have shown its capacity to represent the different theories of uncertainty reasoning. For example, the link with evidence theory was mostly developed by Nguyen [13] and the link with fuzzy sets theory was studied by several authors (Orlov [14], Goodman [15]).

The main purpose of this study is to demonstrate the effectiveness of random sets theory in the problem of target identification through the fusion of pieces of information of different types with different kinds of imperfection. This will be done by means of two test scenarios using different sensors observing one target. The target must be identified from a database of 143 objects.

The second section is an introduction to the data fusion concept, in particular the use of data fusion in target identification. We will briefly describe the database and the two test scenarios used. The first test scenario includes 13 pieces of information, all of which are consistent with the observed target. In the second test scenario we introduce a countermeasure to test the fusion system's ability to eliminate this data.

The third section describes the classification and modelization of imperfect information according to Smets' model. Many examples are given to show clearly the differences between the various types of imperfect information.

The fourth section presents the common theories of uncertainty reasoning (probability, evidence, fuzzy sets, possibility, and rough sets theory. We will show through examples the mathematical tools used by each theory) representation of information, combination rules, ignorance representation and decision-making rules. Each of these theories is then applied to both test scenarios.

In the fifth section, we enumerate several links between the different theories previously described.

The sixth and final section introduces random sets theory and its link with the classic theories advanced by various authors. We conclude this section by applying the theory to both scenarios.

We demonstrate that random sets theory offers an effective unifying framework for the theories of uncertainty reasoning. We also show how it can be applied to the target identification problem to great effect.

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2 Data fusion applied to target identification

2.1 Data fusion

The term **data fusion** was introduced to denote the combination of pieces of information coming from different sensors, databases, expert opinions, etc. The variety of existing information sources led to the creation of different mathematical models to represent the information collected. This field has shown considerable development since the advent of computers able to store large amounts of information and to execute calculus fast enough. Data fusion is an important field of study for the development of expert systems.

In the beginning, data fusion applications were mostly developed for military purposes, e.g., target tracking and identification or threat identification (friend/neutral/foe). More recently it has been used in non-military applications such as robotics or medicine. The purpose of this section is to describe the target identification problem in greater detail.

2.2 Target identification

A target identification problem is illustrated in figure 1. It shows a platform with its various sensors spotting different moving targets, which are to be identified out of a database of known targets.

A multiple target identification problem is generally broken down into several subproblems as follows:

1. **detection** of raw information (signals);
2. **tracking**;
3. **association** of detected information with targets;
4. **modelling of raw data** coming from targets;
5. **data fusion** ;
6. **decision-making**.

Contacts (positions of possible targets) are detected in the first step. Tracking provides a prediction of the position of the targets known at the time of acquisition. Multiple target tracking is realized with the help of a Kalman filter. Then a decision is made to associate a track with each of the contacts among the previously predicted tracks. These first three steps are represented in figure 2. In the current study, these steps are considered to be achieved. Also, we consider the association between

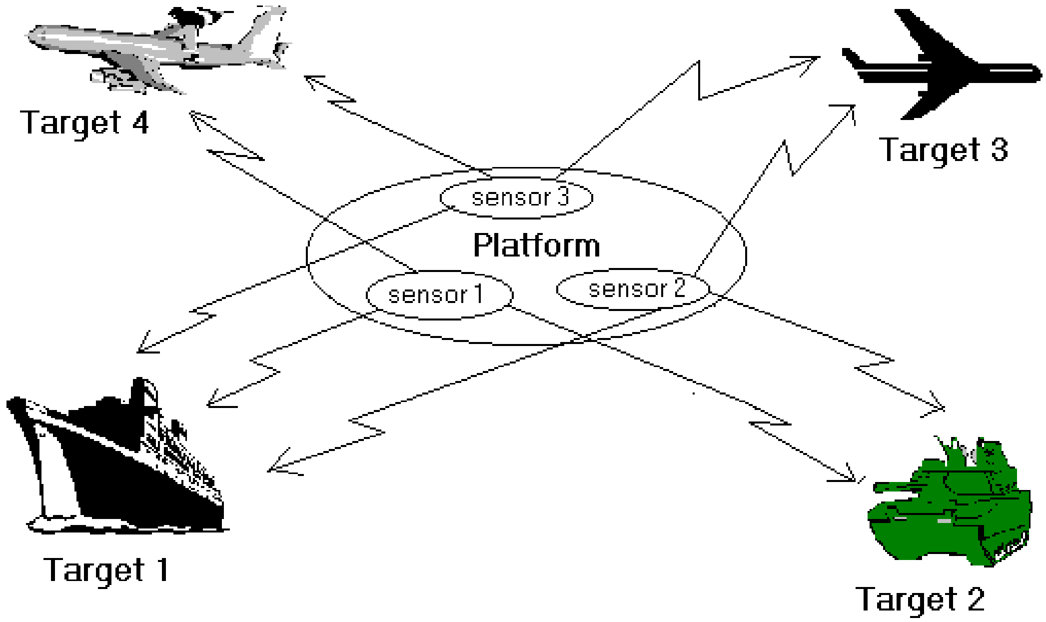

Figure 1: *Target identification problem*

contacts and tracks to be perfect, and therefore that each target can be analyzed separately, as if we were dealing with one target at a time.

According to Hall and Llinas [16], the three major problems that are still to be solved in today's data fusion systems are:

1. the quantification of uncertainty in measurements or observations;
2. the combination of distinct pieces of information;
3. the enormous number of possible alternatives to deal with.

The fusion process reduces to the version represented in figure 3, considering that detection, tracking and association are already effected.

A target is observed by different sensors, such as radars, electronic support measures (ESM), imaging infrared (IRR), identification, friend or foe (IFF), etc. Data provided by these sensors are not necessarily ideal, but rather imperfect (imprecise, uncertain, vague, incomplete, inconsistent, etc.). The data can be modelled using the different mathematical theories to be analyzed. For this study, we consider the different sources to be independent.

With the aid of mathematical formalisms, we fuse imperfect information, to obtain a final piece of information that, on the one hand, can be combined with a new piece of information, and on the other, is usable for decision-making.

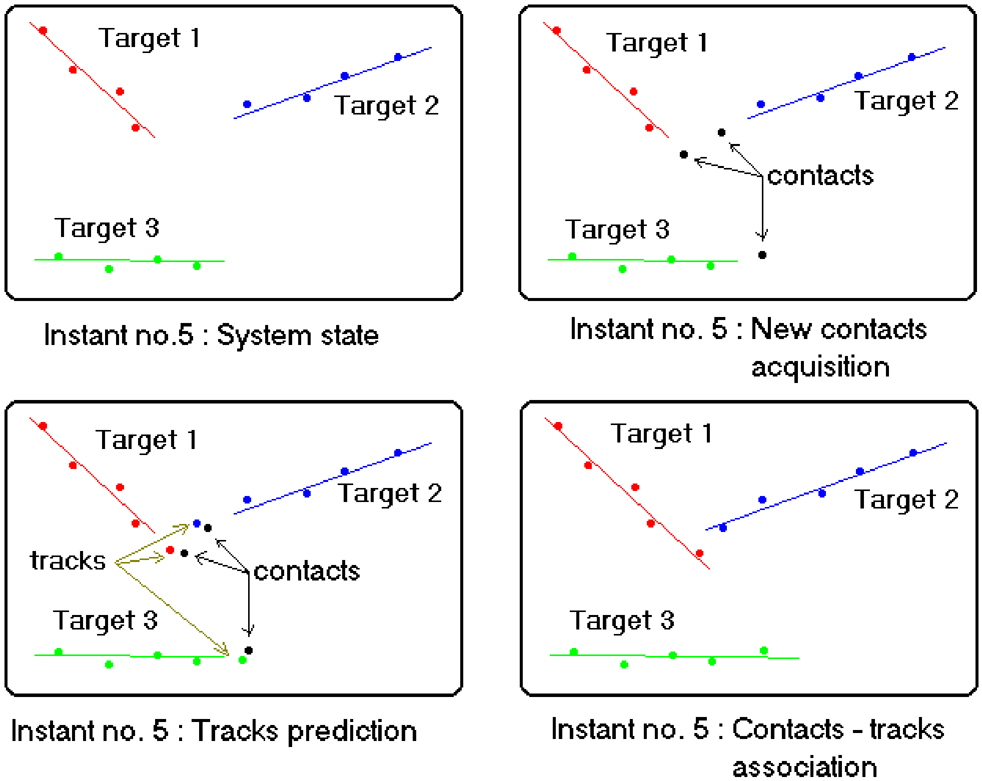

Figure 2: *Contact acquisition, track prediction and association between contacts and tracks*

The decision as to the identity of a target is taken on the final information with reference to the database of known targets. Using algorithmic and mathematical methods, we look for the most probable target.

2.3 Uncertainty and imperfect information

Each of the six steps described above is well defined and very important. However, each one may introduce imperfections into the system (unsuitable or non-optimal measurements, associations, models, fusions or decision methods).

By **uncertainty reasoning** we mean working with imperfect information, such as may be provided by sensors, experts or other sources. Klir [1], Krause and Clark [2] use the term **uncertainty** to denote the imperfection of information. Thus, we refer to one or more of the characteristics imprecise, uncertain, incomplete, inconsistent and vague when using this term.

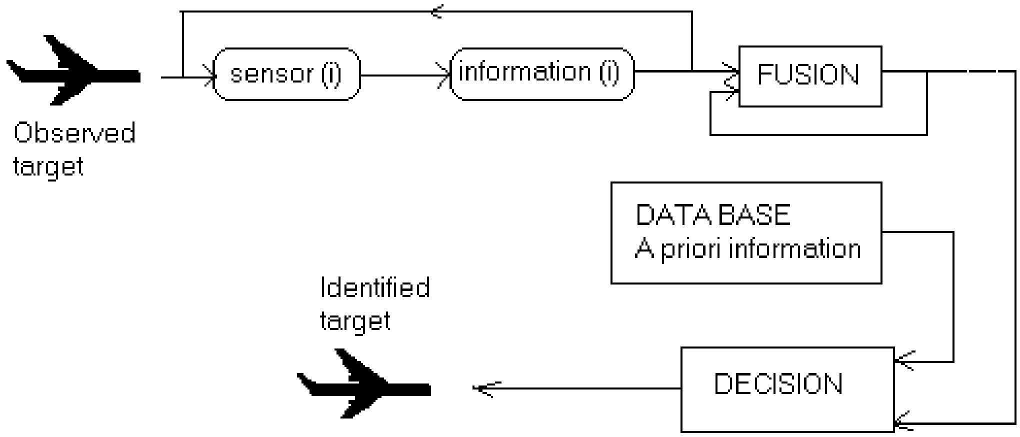

Figure 3: *Target identification process (simplified version)*

2.4 The Database

Data on the known potential targets are classified in a database accessible to the fusion system. The data will help the system identify the observed target. For each target in the database we have the following parameters:

1. identification number (database index)
2. acronym (name)
3. type
4. sub-type
5. offensiveness classification
6. country
7. minimum and maximum cruise speed
8. maximum acceleration
9. maximum altitude
10. dimensions (length, height, width)
11. radar cross-section (RCS) (side, front and top)
12. number of engines
13. engine type(s)

14. number of cylinders in the engines
15. infrared signature
16. blade list
17. number of emitters
18. list of emitters.

An ideal database would contain information on all the parameters of all possible targets. But in reality the database is never complete, and that makes the identification process harder. In the identification process, the parameters listed above represent the information we look for. Knowledge of all the parameters allows accurate target identification. A second database containing geopolitical information is associated with the first. With each country are associated an allegiance (friend, neutral or foe) and the number and names of the languages spoken. A third database, linked to the first, lists the emitters and their characteristics. The databases used in this study are described in Appendix D.

2.5 Example of a scenario

An example of a scenario for the situation described in figure 1 can be composed with the pieces of raw information contained in table 1.

Instant	Characteristic	Target 1	Target 2	Target 3	Target 4
1	Type of target	ship	terrestrial target	plane	plane
2	Emitter 44 on board	yes	no	no	yes
3	Length	small	small	medium	medium
4	RCS_{side}	medium	small	small	medium
5	Emitter 77 on board	yes	yes	no	no
6	Height	small	small	small	small
7	Emitter 47 on board	yes	no	yes	no
8	Emitter 55 on board	yes	no	yes	yes
9	Width	medium	small	medium	medium
10	Emitter 56 on board	yes	no	yes	yes
11	Emitter 103 on board	yes	yes	no	yes
12	Emitter 109 on board	yes	no	no	no
13	RCS_{top}	small	medium	small	medium
14	RCS_{front}	very small	small	medium	medium

Table 1: *Example scenario*

In the following, we use target #1 to test the different theories of uncertainty reasoning. Two scenarios are proposed:

1. **Test Scenario 1** is composed of the pieces of information 1 to 4 and 6 to 14, which are measurements consistent with object #19 in the database;

2. **Test Scenario 2** is composed of the pieces of information 1 to 14; the fifth piece of information is a countermeasure inserted in the middle of the fusion process to observe how the system reacts.

2.6 Data fusion tools

Many theories have been developed to deal with uncertainty. The following are described in this study:

1. probability theory
2. evidence theory
3. fuzzy sets theory
4. possibility theory
5. rough sets theory
6. random sets theory.

Each one proposes a mathematical model for imperfect information and one or more methods of combination and decision-making.

3 Uncertainty and imperfect information

3.1 Introduction

Uncertainty is the general term used by several authors (Klir [1], Krause and Clark [2]) to denote imperfect information. However, this name can create confusion in Smets' classification [3], where uncertainty is only one class of information imperfection. Next, we present the concept of information related to the target identification problem, we examine the various kinds of imperfect information, and we analyze some easy examples to facilitate understanding of the new theories. In this study we use only the classification of imperfect information proposed by Smets [3].

3.2 The Information

In the target identification problem, information can be described by at least the following three parameters:

1. an object or class of objects that refers to the target of interest;
2. a characteristic (attribute) describing the object or class of objects;
3. a degree of confidence associated with the pair {object(s), attribute(s)}.

Here are some examples of information:

- "The observed target **could be** the object θ_i with a **degree of confidence of 0.8**";
- "The observed target is **large**";
- "The observed target is **one of the objects** $\{\theta_i, \theta_j, \theta_k\}$ with a **belief of 0.9**".

According to Smets [3], perfect information must be precise and certain. According to Solaiman [17], perfect information must also be exhaustive. Thus, perfect information according to the definition and constraints presented earlier must have the following properties:

1. the referring class of objects must be a singleton (only one object) - **precise information**;
2. the confidence degree we associate with a proposition or set of propositions is equal to 1 (total confidence) - **certain information**;
3. the proposed object is part of a set of known targets recorded in a database (closed word hypothesis) - **exhaustive information**.

But in the real world, perfect information is almost never obtained. Sensors always provide imperfect information. We devote the following section to the study and classification of imperfect information.

3.3 Imperfect information

An initial first classification of information may be made according to its objectivity:

1. We say an **information** is **objective** if it is independent of human opinion;

2. We say an **information** is **subjective** if it is dependent on human opinion.

Example 1 Saying that when a die is rolled number 4 can come up with a probability of 1/6 represents objective information because it is an interpretation in equiprobable terms of appearances (traditional interpretation based on the symmetry of the die). Human opinion is not a factor in this case. On the other hand, the information "I believe that number 4 can come up with a probability of 0.2" represents subjective information. The opinion of a person is a factor in the judgment.

In section 3.2 we saw the criteria for describing information as perfect. If one condition is not met, then the information is imperfect.

In the following sections we will consider the closed word hypothesis to be true (exhaustive information). According to Smets' classification, the only remaining causes of imperfection are imprecision and uncertainty (figure 4). So imperfect information may be imprecise, uncertain, or both imprecise and uncertain.

3.3.1 Uncertain information

Uncertain information is information with an associated confidence degree smaller than 1 (with which total confidence is not associated). This imperfection is mainly due to an expert's lack of confidence in the information provided by a sensor.

Here are some examples of uncertain information:

1. "I **think** that the observed target is object θ_i in the database";

2. "**It is probable** that the observed target is object θ_i in the database and the confidence degree associated with it is 0.7".

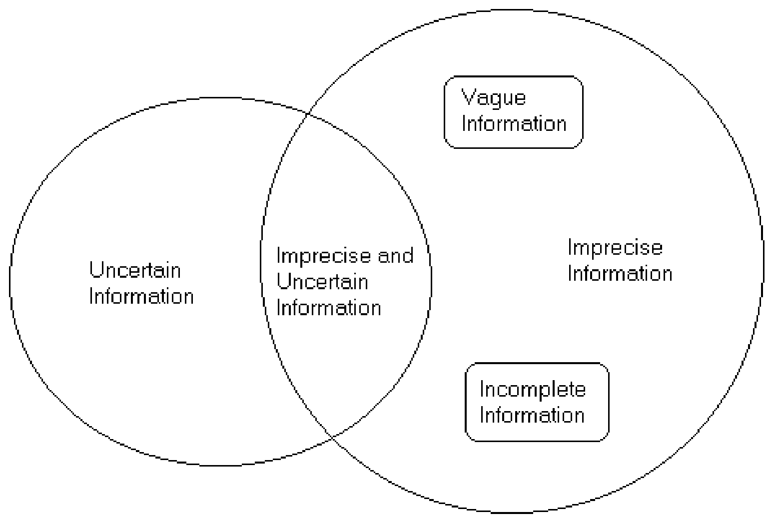

Figure 4: *The classification of imperfect information proposed by Philippe Smets*

3.3.2 Imprecise information

Imprecise information is information that refers to not just one object but several objects in the database. The following are examples of imprecise information:

1. "The observed target is **one of the objects** $\{\theta_i, \theta_j, \theta_k\}$" ;
2. "The length of the target **is between** 28 and 34 metres".

Incomplete information

Incomplete information is information that has an unknown degree of confidence but for which we know the upper limit of confidence. The term "incomplete information" comes from the knowledge of this upper limit instead of the confidence degree (imprecision of the confidence degree). Some examples of incomplete information are shown below:

1. "**It is possible that** the target is object 19";
2. "**It is possible to** detect the target type".

Vague information

Vague information is information characterized by ill-defined attributes, by some classes having ill-defined limits. Instead of defining the membership of a known object in an ill-known class by a strict relation (1 - belongs to, 0 - does not belong to), we consider that the membership can be fuzzy. So we define a membership measure of an object in some class (which has ill-defined limits). Vague information is imprecise information. Here are some examples of vague information:

1. "The observed target is **large**";

2. "The observed target has a **very large** radar cross section";

3.3.3 Uncertain and imprecise information

Previously we saw some examples of imprecise information and uncertain information. These two situations are extreme, since it is supposed that information has only one imperfection at once. But real situations often involve information exhibiting both imperfections. For example :

1. "A **confidence degree of 0.7** is associated with the fact that the observed target **is one of** $\{\theta_1, \theta_2, \theta_3\}$";

2. "A **confidence degree of 0.9** is associated with the fact that the observed target has a **length between** 28 and 34 metres".

3.4 Modelling information

Sensors usually provide information that refers to the known characteristics of the objects in the database (see section 2.4 for a complete list of the characteristics). Figure 5 illustrates the process of information modelling.

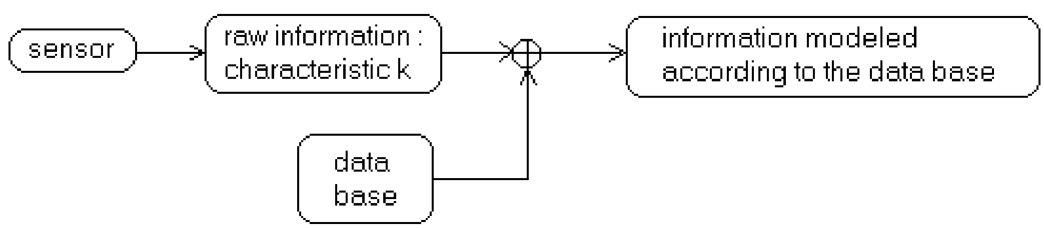

Figure 5: *The process of information modelling*

The raw data provided for one characteristic in the database is information like "we estimate with 80% confidence that the observed object has a length of 15 metres". Analyzing the database, we can extract the objects having a length of 15 metres and associate a degree of confidence of 0.8 with them. We call this set of objects a **proposition**.

The raw data may also be vague information. Attributes like "large" or "very big" introduce sets with ill-defined limits. So attributes must be defined according to their usage in the database. A target in the database with a length of 180 metres may be considered a "large target". But an object measuring 250 metres is considered to have a higher degree of membership in the "large target" class than the 180 m target.

Depending on the problem, sometimes we have to deal with imprecise or uncertain information. The uncertainty of information may be replaced by the imprecision, and inversely, according to our needs. A relation exists between these two kinds of imperfection characterizing the same situation:

> **In a given situation, when uncertainty increases, imprecision may decrease.**

The next example will help clarify the relation between uncertainty and imprecision.

Example 2 Let a sensor provide information on a detected target for the purpose of identifying it. The sensor measures the length of the target, but the measurement may not be certain. Now consider these 5 objects from the database:

Θ_5	object #1	object #2	object #3	object #4	object #5
length (metres)	29	31	30	24	34

Table 2: *Example of database*

Since the measurement is not perfect, the information provided by the sensor can be modelled as "A degree of confidence of 0.7 is associated with the fact that the detected target is 30 metres long". Then we check the database for known targets with a length of 30 metres. The information now becomes: "Researched target = object 3 with a degree of confidence of 0.7". Knowing the external factors that can affect measurement, we can seek to model the information provided by the sensor differently: "Total confidence is associated with the fact that the detected target has a length between 28 and 32 metres". We observe that the number of targets in the database having a length between 28 and 32 metres is greater than the number of targets exactly 30 metres long (object 1, object 2 and object 3). Thus, the information expressed by the second proposition is imprecise but certain. This information now becomes "Researched target is one of objects 1, 2 or 3". We can see how the same situation may be modelled by two different pieces of information: one precise but uncertain, the other imprecise but certain.

3.5 Conclusion

This section provided an introduction to the concept of imperfect information. After a short description of different kinds of imperfection, we presented some examples to facilitate understanding. To summarize, the principal sources of imperfection are uncertainty and imprecision. Imperfect information is classified as either uncertain information (objective or subjective) or imprecise information (objective), with two subcategories: vague information and incomplete information. We may also encounter information that exhibits both types of imperfection. Owing to this diversity of states, both general and particular, processing imperfect data is fraught with difficulties. The following section presents several theories that have been developed to deal with imperfect information.

4 Reasoning under uncertainty

4.1 Introduction

In the last section, we discussed several kinds of imperfect information and showed some examples to illustrate the differences between them. Several theories have been developed to deal with this variety of imperfect information. Each theory aims to model real situations more accurately. In this section, we present the best known and most frequently used theories and we apply them to a target identification problem. For each theory we also present the most commonly used combination rules.

Let us first introduce some notations that will be used in this section:

- θ_i is the i-th object in the database;
- N represents the number of objects in the database;
- $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$ represents the set of all objects in the database;
- 2^Θ represents the power set of Θ (the set of all subsets of Θ). Therefore $A \subseteq \Theta$ is equivalent to $A \in 2^\Theta$;
- Capital letters denote the subsets of Θ ($A_i \subseteq \Theta$);
- A **singleton** is a subset of Θ which contains only one element;
- An **event** is a subset A of Θ.

4.2 Probability theory

4.2.1 Introduction

Probability theory is the best known and oldest theory of uncertainty. The objective view of probability theory was developed in the 17th century to study random events such as occur in gambling (dice games and card games). Three versions of the objective view of probability theory can be distinguished:

1. Laplace considered the probability of an event to be equal to the ratio between the number of realizations producing a given outcome and the total number of possible realizations. This view requires that the possible realizations be equiprobable.

   Example 3 The probability that tossing a die will produce an even value is equal to the number of possible even values ($\{2, 4, 6\}$) divided by the total number of possible outcomes ($\{1, 2, 3, 4, 5, 6\}$). In this case the probability is 0.5.

2. The frequentist Richard von Mises considered the probability to be equal to the relative frequency of the realization of this event after repeating the experiment an infinite number of times. The major drawback of this view is the impossibility of conducting the same experiment an infinite number of times, and it is possible that it may not be realized more than once.

   Example 4 The probability that tossing a die will produce an even value is obtained by conducting the experiment (tossing a die) an infinite number of times and counting the occurrences of *even values*.

3. Bernoulli's *big numbers law* supposed that if an outcome is obtained k times from n identical and independent experiments, then the objective probability is very close to k/n when n is very large. This view of objective probability is akin to the frequentist view of von Mises.

Another interpretation of probability theory is the subjective one, which was formulated during the 20th century. It supposes that there are no random experiments, only ill-known experiments which cause uncertain results. Here the probability of an event is interpreted as the degree of belief that a person associates with the result of the experiment. This is the Bayesian view of probability theory.

Example 5 Tossing a die is considered an ill-known experiment instead of a random experiment. The outcome of the die tossing experiment can be estimated if we have enough information, like the speed and height of the toss, the friction coefficient between the die and the table, the trajectory of the toss, the die position before the toss, etc. Lack of information and the impossibility for a human to equate and compute the parameters of the toss make the result hard to predict. But that does not mean the experiment is a random experiment.

In the next section we will study probability theory without considering how to interpret objective or subjective probability.

4.2.2 Theory description

Axioms and definitions

A **probability measure** is a function $P : 2^{\Theta} \to [0, 1]$ which satisfies the following three axioms:

1. the probability of the empty set is null and the probability of the frame of discernment is equal to unity:

$$P[\varnothing] = 0 \tag{1}$$
$$P[\Theta] = 1 \tag{2}$$

2. the probability of an event is always positive and equal to or less than unity:

$$0 \leq P[A] \leq 1 \qquad \forall A \subseteq \Theta \tag{3}$$

3. the probability measure is additive:

$$P[A \cup B] = P[A] + P[B] - P[A \cap B] \qquad \forall\, A, B \subseteq \Theta \tag{4}$$

   If the intersection of two events is empty $A \cap B = \varnothing$, the events are said to be independent and (4) reduces to(5):

$$P[A \cup B] = P[A] + P[B] \qquad \forall\, A, B \subseteq \Theta \tag{5}$$

Consequence 1 The probability not assigned to A is implicitly assigned to $\overline{A}$, the complement of A:

$$P[\overline{A}] = 1 - P[A] \tag{6}$$

Consequence 2 The probability of a subset $A \subseteq \Theta$ can be expressed by the probabilities of the singletons which form A:

$$P[A] = \sum_{\theta \in A} P[\theta] \qquad \forall\, A \subseteq \Theta \tag{7}$$

Example 6 In a target identification problem, an uncertain piece of information such as "*The observed target can be the object #1 with a confidence degree of 70%*" can be modelled in probability theory by:

$$P[\theta_1] = 0.7$$

The remaining confidence is automatically assigned to the other objects in the database:

$$P[\theta_2 \cup \theta_3 \cup \ldots \cup \theta_N] = P[\theta_2] + P[\theta_3] + \ldots + P[\theta_N] = 0.3$$

Ignorance

A situation of **partial ignorance** arises when no additional knowledge exists about an element of Θ belonging to a subset $A \subseteq \Theta$ with known probability $P[A]$. The only information available is that θ belongs to A with a probability of $P[A]$. But the ambiguity remains if A contains many elements. One way to remedy this situation is to consider all the singletons of A to be equiprobable. In this manner, both imprecision and uncertainty can be considered in probability theory. An imprecise information (defined by (7)) is thereby transformed into a precise information (modelled only by its singletons):

$$P[\theta_i] = \frac{P[A]}{\mathrm{card}(A)} \qquad \forall\, \theta_i \in A \tag{8}$$

A situation of **total ignorance** arises when there exists no additional knowledge on the entire frame of discernment and we need to suppose that all the singletons are equiprobable:

$$P[\theta_i] = \frac{1}{N} = \frac{1}{\mathrm{card}(\Theta)} \qquad \forall\, \theta_i \in \Theta \tag{9}$$

Example 7 Suppose that a sensor provides the length of a target with great imprecision but with strong certainty. The piece of information is then "*The length of the target is between 15 and 40 metres with a confidence degree of 90%*". Of the $N = 143$ objects in the database, the first 133 match this description and the last 10 have a length that falls outside the specified range. Considering the partial ignorance, we can calculate the probability of each proposition:

$$P[\theta_1] = P[\theta_2] = \ldots = P[\theta_{133}] = \frac{0.9}{133} = 0.0068$$

$$P[\theta_{134}] = P[\theta_{134}] = \ldots = P[\theta_{143}] = \frac{0.1}{10} = 0.0100$$

Here we face a paradox: the propositions derived from the most certain piece of information have been assigned a lower probability than the propositions derived from the less certain piece of information. This example shows that the ignorance definition must be used with caution because it can lead to some erroneous models of information.

Information combination

Definition 1 The **conditional probability** of an event A, knowing the event B has occurred, is given by:

$$P[A|B] = \frac{P[A \cap B]}{P[B]} \tag{10}$$

1. **Bayes' rule:**

$$P[A|B] = \frac{P[B|A]P[A]}{P[B]} \tag{11}$$

where the total probability is $P[B] = \sum_{C \in \mathcal{P}} P[B|C]P[C]$, $\mathcal{P}$ being a partition of Θ.

2. **Consensus rule:** Let $\{S_k\}_{k=1}^{k=K}$ be a partition of Θ_0 (set of the information sensors) and A an event of Θ. Then the total probability of A is given by:

$$P[A] = \sum_{k=1}^{K} P[A|S_k]P[S_k] \tag{12}$$

This is a disjunctive combination rule for two pieces of information in probability theory.

Example 8 Let Θ_0 be a set of three sources of information. The confidence associated with the sources and the conditional probabilities provided by the sources are presented in Table 3. Applying the consensus rule we obtain:

	Sensor S_1	Sensor S_2	Sensor S_3
Confidence associated to the sensors $P[S_k] \quad k = 1, 2, 3$	$P[S_1] = 0.25$	$P[S_2] = 0.25$	$P[S_3] = 0.5$
$P[\theta_1\|S_k] \quad k = 1, 2, 3$	0.4	0	0.6
$P[\theta_2\|S_k] \quad k = 1, 2, 3$	0.3	0.5	0
$P[\theta_3\|S_k] \quad k = 1, 2, 3$	0.2	0.3	0.2
$P[\theta_4\|S_k] \quad k = 1, 2, 3$	0.1	0.2	0.2

Table 3: *Example of pieces of information for the consensus rule of combination.*

$$\begin{aligned}
P[\theta_1] =& P[\theta_1|S_1]P[S_1] + P[\theta_1|S_2]P[S_2] + P[\theta_1|S_3]P[S_3] = \\
=& 0.4 \times 0.25 + 0 \times 0.25 + 0.6 \times 0.5 = 0.4 \\
P[\theta_2] =& P[\theta_2|S_1]P[S_1] + P[\theta_2|S_2]P[S_2] + P[\theta_2|S_3]P[S_3] = \\
=& 0.3 \times 0.25 + 0.5 \times 0.25 + 0 \times 0.5 = 0.2 \\
P[\theta_3] =& P[\theta_3|S_1]P[S_1] + P[\theta_3|S_2]P[S_2] + P[\theta_3|S_3]P[S_3] = \\
=& 0.2 \times 0.25 + 0.3 \times 0.25 + 0.2 \times 0.5 = 0.225 \\
P[\theta_4] =& P[\theta_4|S_1]P[S_1] + P[\theta_4|S_2]P[S_2] + P[\theta_4|S_3] \times P[S_3] = \\
=& 0.1 \times 0.25 + 0.2 \times 0.25 + 0.2 \times 0.5 = 0.175
\end{aligned}$$

The final piece of information allows us to conclude that object θ_1 is the most probable target.

3. Two events A and B are independent if and only if:

$$P[A \cap B] = P[A]P[B] \tag{13}$$

Equation (13) can be considered as a third combination rule in probability theory, specifically, a conjunctive combination rule. Dempster's rule [5] reduces to (13) when the belief functions are defined only for the singletons (see Section 4.3).

Consequence 3 The two events A and B being independent implies (from equation (13)) that:

$$P[B|A] = P[B] \quad \text{and} \quad P[A|B] = P[A] \tag{14}$$

This means that the knowledge of the occurrence of event A does not influence the probability of the event B.

Example 9 Suppose two sensors provide information on the length and speed of an observed object: These two pieces of information being independent, we can combine

Length (m)	58	59	60	61	62
Confidence	0.1	0.2	0.4	0.2	0.1

Speed (m/s)	490	500	510
Confidence	0.2	0.6	0.2

Table 4: *Example of information on length and speed.*

them using Equation (13). The objects having the characteristics from Table 4 are presented in Table 5.

Propositions	θ_3	θ_7	θ_1	θ_2	θ_9
Probabilities	0.1	0.2	0.4	0.2	0.1

Propositions	θ_1	θ_4	θ_9
Probabilities	0.2	0.6	0.2

Table 5: *Example of propositions derived the information on length and speed.*

Applying the combination rule for independent pieces of information yields only objects θ_1 and θ_9, with final probabilities of 0.08 and 0.02, respectively. When these values are normalized, the final probabilities of the propositions are as given in Table 6.

Final propositions	θ_1	θ_9
Probabilities	0.8	0.2

Table 6: *Final propositions.*

Decision

In probability theory, the singleton with the highest probability is the chosen singleton. If several singletons have the same final probability, they are equiprobable and no decision can be made.

$$\theta_{\text{observed}} = \text{Arg}\{\max_{\theta \in \Theta}\{P[\theta]\}\} \tag{15}$$

Instant	Information	Modelled information in probability theory	Associated propositions
1	Target type = ship	$P[\text{ship}] = 0.8$ $P[\overline{\text{ship}}] = 0.2$	ship = $\{ \theta_1, \theta_2, \ldots, \theta_{70}, \theta_{71}, \theta_{73} \}$
2	Emitter 44 on board	$P[\text{emitter 44}] = 0.8$ $P[\overline{\text{emitter 44}}] = 0.2$	emitter 44 = $\{ \theta_{11}, \theta_{19}, \theta_{63} \}$
3	Small length length $\in [0, 100]$ m	$P[\text{small length}] = 0.8$ $P[\overline{\text{small length}}] = 0.2$	small length = $\{ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{62}, \theta_{72}, \theta_{73}, \theta_{75}, \ldots, \theta_{127}, \theta_{131}, \theta_{135}, \theta_{137}, \ldots, \theta_{142} \}$
4	RCS_{side} medium $\text{RCS}_{\text{side}} \in [2, 15] \times 10^4$ dm^2	$P[\text{RCS}_{\text{side}} \text{ medium}] = 0.8$ $P[\overline{\text{RCS}_{\text{side}} \text{ medium}}] = 0.2$	RCS_{side} medium = $\{ \theta_2, \theta_3, \theta_5, \theta_6, \theta_8, \theta_9, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \ldots, \theta_{31}, \theta_{34}, \theta_{36}, \theta_{39}, \theta_{45}, \ldots, \theta_{49}, \theta_{60}, \theta_{62}, \ldots, \theta_{71}, \theta_{73}, \theta_{95}, \theta_{96}, \theta_{125}, \theta_{131}\}$
5	Emitter 77 on board	$P[\text{emitter 77}] = 0.8$ $P[\overline{\text{emitter 77}}] = 0.2$	emitter 77 = $\{ \theta_{29}, \theta_{33}, \theta_{35}, \theta_{36} \}$
6	Small height height $\in [0, 5]$ m	$P[\text{small height}] = 0.8$ $P[\overline{\text{small height}}] = 0.2$	small height = $\{ \theta_2, \theta_3, \theta_8, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{73}, \theta_{77}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{98}, \theta_{101}, \theta_{115}, \theta_{117}, \theta_{118}, \theta_{120}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{137}, \theta_{138}, \theta_{140} \}$
7	Emitter 47 on board	$P[\text{emitter 47}] = 0.8$ $P[\overline{\text{emitter 47}}] = 0.2$	emitter 47 = $\{ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{46}, \theta_{47}, \theta_{63} \}$
8	Emitter 55 on board	$P[\text{emitter 55}] = 0.8$ $P[\overline{\text{emitter 55}}] = 0.2$	emitter 55 = $\{ \theta_{18}, \theta_{19} \}$
9	Small width width $\in [0, 15]$ m	$P[\text{small width}] = 0.8$ $P[\overline{\text{small width}}] = 0.2$	small width = $\{ \theta_3, \theta_5, \theta_6, \theta_9, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{70}, \ldots, \theta_{73}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \theta_{98}, \theta_{101}, \theta_{113}, \ldots, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141} \}$
10	Emitter 56 on board	$P[\text{emitter 56}] = 0.8$ $P[\overline{\text{emitter 56}}] = 0.2$	emitter 56 = $\{ \theta_{18}, \theta_{19}, \theta_{34} \}$
11	Emitter 103 on board	$P[\text{emitter 103}] = 0.8$ $P[\overline{\text{emitter 103}}] = 0.2$	emitter 103 = $\{ \theta_{11}, \theta_{18}, \ldots, \theta_{24}, \theta_{30}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{36}, \theta_{45}, \ldots, \theta_{49}, \theta_{63}, \theta_{67}, \theta_{69} \}$
12	Emitter 109 on board	$P[\text{emitter 109}] = 0.8$ $P[\overline{\text{emitter 109}}] = 0.2$	emitter 109 = $\{ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{63} \}$
13	RCS_{top} small $\text{RCS}_{\text{top}} \in [1, 10] \times 10^4$ dm^2	$P[\text{RCS}_{\text{top}} \text{ small}] = 0.8$ $P[\overline{\text{RCS}_{\text{top}} \text{ small}}] = 0.2$	SER_{top} small = $\{ \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{73}, \theta_{75}, \theta_{91}, \theta_{92}, \theta_{97}, \theta_{99}, \theta_{102}, \ldots, \theta_{112}, \theta_{116}, \theta_{131}, \theta_{139}, \theta_{142} \}$
14	$\text{RCS}_{\text{front}}$ very small $\text{RCS}_{\text{front}} \in [0, 3000]$ dm^2	$P[\text{RCS}_{\text{front}} \text{very small}] = 0.8$ $P[\overline{\text{RCS}_{\text{front}} \text{very small}}] = 0.2$	$\text{RCS}_{\text{front}}$ very small = $\{ \theta_7, \theta_{18}, \theta_{19}, \theta_{72}, \theta_{73}, \theta_{74}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{97}, \theta_{98}, \theta_{100}, \theta_{101}, \theta_{104}, \theta_{105}, \theta_{106}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141} \}$

Table 7: *Information used in the test scenarios modelled in probability theory.*

4.2.3 Scenarios study

We study two scenarios focussing on target #1 from the example 1. The first one is a test scenario formed from imperfect but coherent information. In the second test scenario we simulate a countermeasure. This countermeasure is inserted in the data fusion process at instant #5, to study how (if) the fusion system is able to overcome it. We emphasize that a piece of information cannot be inserted just anywhere in the data fusion system, because not all combination rules satisfy the associativity propriety.

In Table 7, we present, for each piece of information, a possible model in probability theory and the set of propositions. The results obtained for scenario 1, applying the two combination rules-the consensus rule (equation (12)) and Dempster's rule for singletons (equation (13))-are presented in Figures 6 and 7.

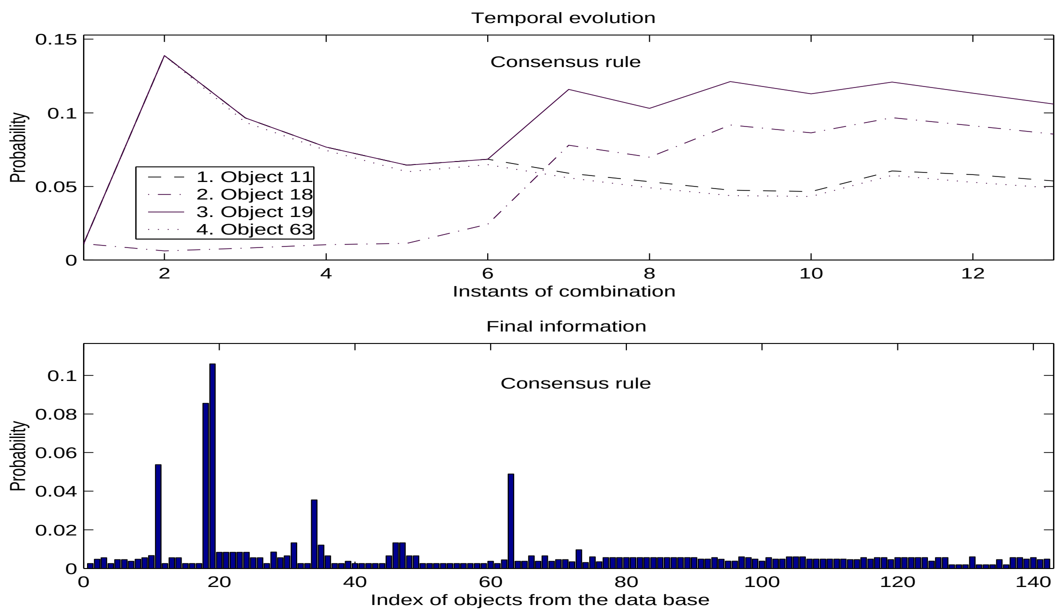

Figure 6: *Probability theory - Test scenario 1 - Consensus rule*

The instants of combination $i \geq 5$ in the first test scenario correspond to the fusion of the information $i + 1$, since the information #5 is not considered in the fusion process. This remains valid throughout this study.

The result obtained in the second scenario, applying Dempster's rule of combination for singletons, is presented in Figure 8.

The results obtained for the first and second scenarios show that object #19 in the database has the highest final probability. It is the object actually observed. The identification is made without ambiguity.

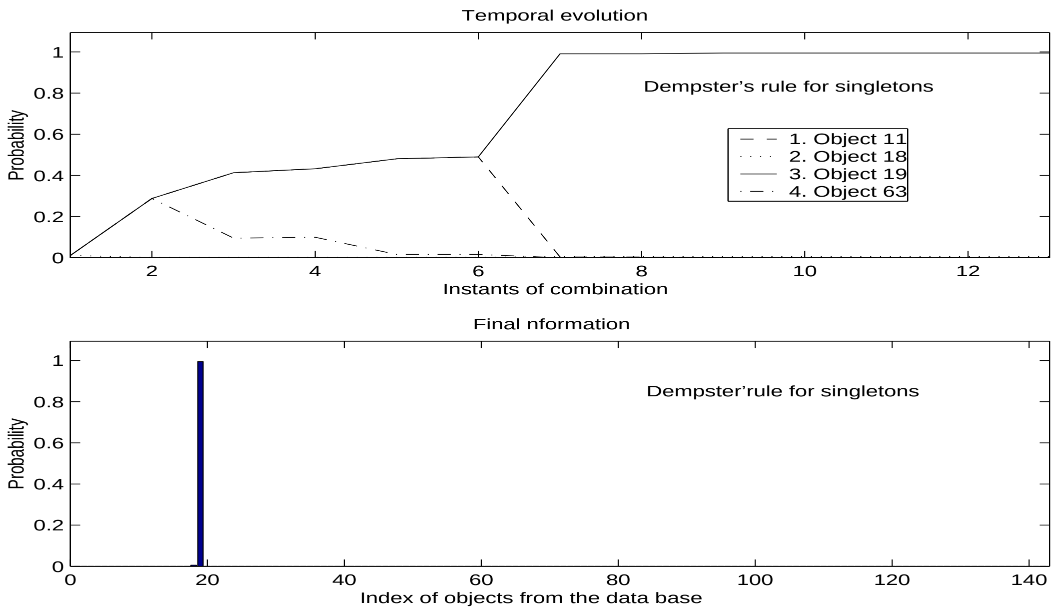

Figure 7: *Probability theory - Test scenario 1 - Dempster's rule for singletons*

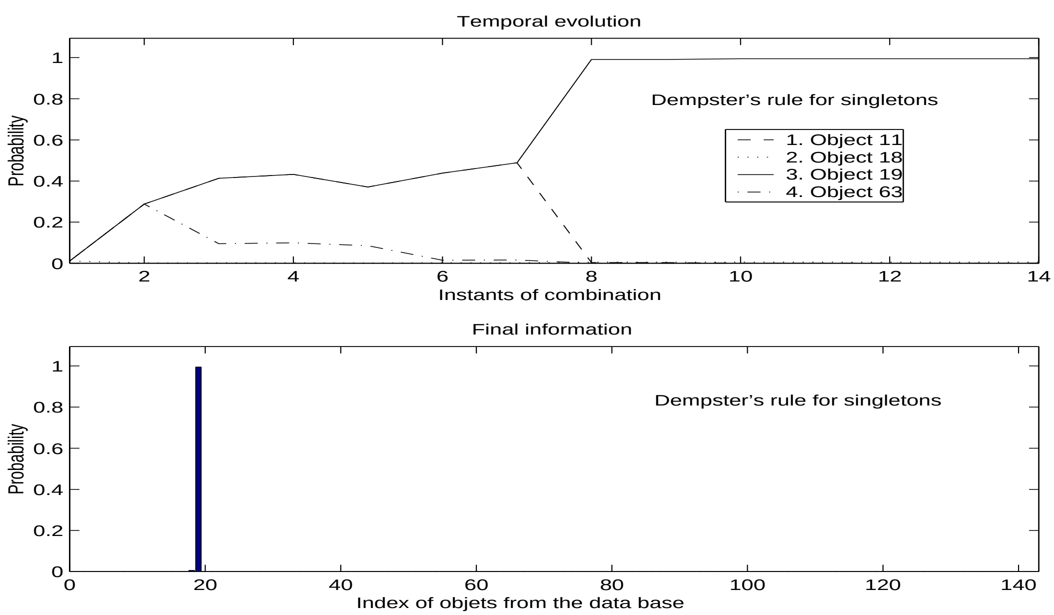

Figure 8: *Probability theory - Test scenario 2 - Dempster's rule for singletons*

When Dempster's rule for singletons is used with the first or second scenario, the final results are almost the same. In Figure 8 we observe only a small decrease in the probability of the representative singletons at the instant of combination #5, which correspond to the instant of fusion of the countermeasure. But after the next two pieces of information are fused, the countermeasure is completely eliminated.

Since we consider the different pieces of information to be independent, Bayes's rule of combination (conditional probability) cannot be used in this study. To apply Bayes's rule of combination, the different pieces of information introduced in the fusion process must be dependent and we must know their cross-correlation matrix.

4.3 Evidence theory

4.3.1 Introduction

Evidence theory, also known as Dempster-Shafer theory (Dempster [5] and Shafer [6]), is often presented as a generalization of probability theory, although this interpretation was not originated by the authors of the theory. It is often referred to as the theory of belief function, since it provides a better representation of subjective belief than probability does. As such, unlike probability theory, it is able to represent both imprecision and uncertainty.

4.3.2 Theory description

Axioms and definitions

In probability theory, a probability measure quantifies the confidence associated with a piece of information. An imprecise information must satisfy condition 3 (equation (5)), which imposes a dependency between the probability of the set A and the probabilities of the singletons $\theta_i \in A$. This constraint does not create the best conditions for modelling imprecise information in probability theory. Evidence theory provides an alternative solution.

Definition 2 The **basic probability assignment** (BPA) of a subset A of Θ is the degree of confidence assigned specifically to that subset. The basic probability function $m : 2^{\Theta} \rightarrow [0, 1]$ must satisfy three conditions:

1. the BPA of the empty set is zero (hypothesis of a closed world):

$$m[\varnothing] = 0 \tag{16}$$

2. the BPA of any event is always positive and equal to or less than unity:

$$0 \leq m[A] \leq 1 \qquad \forall A \subseteq \Theta \tag{17}$$

3. the sum of the BPAs of all events is unity:

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{18}$$

The probability distribution function and the basic probability function are defined on the set 2^Θ. However, the probability distribution function characterizes entirely only the set Θ (because of the additivity axiom, Equation (5)), while the basic probability function characterizes entirely the set 2^Θ.

Definition 3 The **belief** of an event A is the total confidence associated with it. This belief function satisfies the following three axioms:

1. the belief of the empty set is null and the belief of the frame of discernment is unity:

$$\mathrm{Bel}(\varnothing) = 0 \tag{19}$$

$$\mathrm{Bel}(\Theta) = 1 \tag{20}$$

2. the belief of any event is always positive and equal to or less than unity:

$$0 \leq \mathrm{Bel}(A) \leq 1 \qquad \forall A \in 2^\Theta \tag{21}$$

3. the belief of the union of two events satisfies:

$$\mathrm{Bel}(A \cup B) \geq \mathrm{Bel}(A) + \mathrm{Bel}(B) - \mathrm{Bel}(A \cap B) \qquad \forall A, B \in 2^\Theta \tag{22}$$

The belief function, $\mathrm{Bel} : 2^\Theta \to [0, 1]$, can also be defined from the basic probability function by:

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B) \quad \forall A \subseteq \Theta \tag{23}$$

When equality is achieved in (22), the belief function is called **Bayesian belief function**. Then, evidence theory reduces to probability theory.

Definition 4 The **plausibility** function $\mathrm{Pl} : 2^\Theta \to [0, 1]$ is defined as the dual of the belief function:

$$\mathrm{Bel}(A) = 1 - \mathrm{Pl}(\overline{A}) \tag{24}$$

Plausibility represents the maximum confidence one is willing to assign to A.

Plausibility can also be defined from the BPA:

$$\mathrm{Pl}(A) = \sum_{B \cap A \neq \varnothing} m(B) \quad \forall A \subseteq \Theta \tag{25}$$

Ignorance

$m(\Theta)$ represents the confidence assigned to the idea that the search object belongs to the set Θ, and thus $m(\Theta)$ quantifies one's ignorance. A situation of total ignorance arises when $m(\Theta) = 1$, i.e. the only information is that the object belongs to the database.

Example 10 Let us consider the speed of an observed target that is given in Table 8.

Speed (m/s)	10	11	12	other values
Confidence	0.1	0.7	0.1	0.1

Table 8: *Example of speed*

We then extract from the database all objects with a cruise speed equal to the values listed in table above, and the pieces of information obtained are presented in Table 9.

Propositions	$\{\theta_1, \theta_7, \theta_9\}$	$\{\theta_{12}, \theta_{19}, \theta_2, \theta_{25}, \theta_6\}$	$\{\theta_4, \theta_{10}\}$	Θ
BPA	0.1	0.7	0.1	0.1

Table 9: *Propositions for speed.*

This example illustrates one way to model information using ignorance.

Information combination

Definition 5 Combination rule Let m_1 and m_2 be two BPAs defined on Θ. The **conjunctive combination** of these functions is given by:

$$(m_1 \wedge m_2)(C) = \sum_{A \cap B = C} m_1(A) m_2(B) \quad \forall C \subseteq \Theta \tag{26}$$

The **disjunctive combination** of these functions is defined by:

$$(m_1 \vee m_2)(C) = \sum_{A \cup B = C} m_1(A) m_2(B) \quad \forall C \subseteq \Theta \tag{27}$$

The conjunctive combination may require normalization, since the final piece of information must also satisfy the closed world hypothesis ($m(\varnothing) = 0$) because the BPA of the empty set after combination is not necessarily zero. Therefore, the **normalized conjunctive combination**, also known as **Dempster's rule of combination**, is:

$$(m_1 \oplus m_2)(C) = \frac{\displaystyle\sum_{A \cap B = C} m_1(A) m_2(B)}{1 - \displaystyle\sum_{A \cap B = \varnothing} m_1(A) m_2(B)} \quad \forall C \subseteq \Theta \tag{28}$$

Example 11 Let a sensor provide information about the presence of emitters 44 and 55 on board the observed target (Table 10).

	Emitter 44	Emitter 55
Detected presence	present	present
Sensor reliability	80%	80%

Table 10: *Presence of emitters 44 and 55 on board the observed target.*

We then extract the propositions from the database corresponding to the information from Table 10, and we model this information in evidence theory (Table 11).

Propositions	$\{\theta_{11}, \theta_{19}, \theta_{63}\}$	Θ
m_{44}	0.8	0.2

Propositions	$\{\theta_{18}, \theta_{19}\}$	Θ
m_{55}	0.8	0.2

Table 11: *Propositions for the presence of emitters 44 and 55.*

Based on the pieces of information shown in Table 11, the conjunctive and disjunctive combinations are performed in Tables 12 and 13, respectively.

m_{55} \ m_{44}	$\{\theta_{11}, \theta_{19}, \theta_{63}\}$ - 0.8	Θ - 0.2
$\{\theta_{18}, \theta_{19}\}$ - 0.8	$\{\theta_{19}\}$ - 0.64	$\{\theta_{18}, \theta_{19}\}$ - 0.16
Θ - 0.2	$\{\theta_{11}, \theta_{19}, \theta_{63}\}$ - 0.16	Θ - 0.04

Plausibility				
θ_{11}	θ_{18}	θ_{19}	θ_{63}	Others
0.2	0.2	1	0.2	0.04

Table 12: *Result of the conjunctive combination.*

m_{55} \ m_{44}	$\{\theta_{11}, \theta_{19}, \theta_{63}\}$ - 0.8	Θ - 0.2
$\{\theta_{18}, \theta_{19}\}$ - 0.8	$\{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{63}\}$ - 0.64	Θ - 0.16
Θ - 0.2	Θ - 0.16	Θ - 0.04

Plausibility				
θ_{11}	θ_{18}	θ_{19}	θ_{63}	Others
1	1	1	1	0.36

Table 13: *Results of the disjunctive combination.*

We note that the conjunctive combination identifies object #19 as the object most likely (with a maximum of plausibility) to be the observed target.

The result of the disjunctive combination is too imprecise and too difficult to interpret, since several objects in the database have a maximum plausibility, making them all potential candidates for a decision.

Note also that the more the two pieces of information are in conflict, the higher the BPA of the empty set.

Definition 6 The **degree of conflict** between two BPAs m_1 and m_2, denoted by $\mathrm{Con}(m_1, m_2)$, is:

$$\mathrm{Con}(m_1, m_2) = -\log\left(1 - \sum_{A \cap B = \varnothing} m_1(A) m_2(B)\right) \tag{29}$$

The conjunctive rules of combination can be used if the degree of conflict between the two BPAs is not too high.

Decision

Several decision rules apply to evidence theory:

1. The **maximum of plausibility**:

$$\text{Max Pl: } \theta_{\text{observed}} = \mathrm{Arg}\{\max_{\theta \in \Theta}[\mathrm{Pl}(\theta)]\} \tag{30}$$

2. The **maximum of BetP**:

$$\text{Max BetP: } \theta_{\text{observed}} = \mathrm{Arg}\{\max_{\theta \in \Theta}[\mathrm{BetP}(\theta)]\} \tag{31}$$

   where the function $\mathrm{BetP}(\theta)$ is the pignistic transformation of a belief function proposed by Smets [9]:

$$\mathrm{BetP}(\theta) = \sum_{\theta \in A,\ A \subseteq \Theta} \frac{m(A)}{\mathrm{card}(A)} \quad \forall \theta \in \Theta \tag{32}$$

3. The **maximum of the mean utility interval** is another decision rule proposed by Cheaito [18].

Several other decision rules are studied in the literature, but they are not the focus of this study.

Instant	Information	Modelled information in evidence theory	Associated propositions
1	Target type = ship	$m(\text{ship}) = 0.8$ $m(\Theta) = 0.2$	ship = $\{\ \theta_1,\ \theta_2,\ \ldots,\ \theta_{70},\ \theta_{71},\ \theta_{73}\ \}$
2	Emitter 44 on board	$m(\text{emitter 44}) = 0.8$ $m(\Theta) = 0.2$	emitter 44 = $\{\ \theta_{11},\ \theta_{19},\ \theta_{63}\ \}$
3	Small length length $\in [0, 100]$ m	$m(\text{small length}) = 0.8$ $m(\Theta) = 0.2$	small length = $\{\ \theta_{11},\ \theta_{18},\ \theta_{19},\ \theta_{28},\ \theta_{62},\ \theta_{72},\ \theta_{73},\ \theta_{75},\ \ldots,\ \theta_{127},\ \theta_{131},\ \theta_{135},\ \theta_{137},\ \ldots,\ \theta_{142}\ \}$
4	RCS_{side} medium $\text{RCS}_{\text{side}} \in [2, 15] \times 10^4\ \text{dm}^2$	$m(\text{RCS}_{\text{side}}\ \text{medium}) = 0.8$ $m(\Theta) = 0.2$	RCS_{side} medium = $\{\ \theta_2,\ \theta_3,\ \theta_5,\ \theta_6,\ \theta_8,\ \theta_9,\ \theta_{11},\ \theta_{13},\ \theta_{14},\ \theta_{18},\ \ldots,\ \theta_{26},\ \theta_{28},\ \ldots,\ \theta_{31},\ \theta_{34},\ \theta_{36},\ \theta_{39},\ \theta_{45},\ \ldots,\ \theta_{49},\ \theta_{60},\ \theta_{62},\ \ldots,\ \theta_{71},\ \theta_{73},\ \theta_{95},\ \theta_{96},\ \theta_{125},\ \theta_{131}\}$
5	Emitter 77 on board	$m(\text{emitter 77}) = 0.8$ $m(\Theta) = 0.2$	emitter 77 = $\{\ \theta_{29},\ \theta_{33},\ \theta_{35},\ \theta_{36}\ \}$
6	Small height height $\in [0, 5]$ m	$m(\text{small height}) = 0.8$ $m(\Theta) = 0.2$	small height = $\{\ \theta_2,\ \theta_3,\ \theta_8,\ \ldots,\ \theta_{11},\ \theta_{13},\ \theta_{14},\ \theta_{18},\ \ldots,\ \theta_{26},\ \theta_{28},\ \theta_{29},\ \theta_{34},\ \theta_{73},\ \theta_{77},\ldots,\ \theta_{90},\ \theta_{93},\ \theta_{94},\ \theta_{98},\ \theta_{101},\ \theta_{115},\ \theta_{117},\ \theta_{118},\ \theta_{120},\ldots,\ \theta_{124},\ \theta_{126},\ \theta_{127},\ \theta_{137},\ \theta_{138},\ \theta_{140}\ \}$
7	Emitter 47 on board	$m(\text{emitter 47}) = 0.8$ $m(\Theta) = 0.2$	emitter 47 = $\{\ \theta_{11},\ \theta_{18},\ \theta_{19},\ \theta_{31},\ \theta_{34},\ \theta_{35},\ \theta_{46},\ \theta_{47},\ \theta_{63}\ \}$
8	Emitter 55 on board	$m(\text{emitter 55}) = 0.8$ $m(\Theta) = 0.2$	emitter 55 = $\{\ \theta_{18},\ \theta_{19}\ \}$
9	Small width width $\in [0, 15]$ m	$m(\text{small width}) = 0.8$ $m(\Theta) = 0.2$	small width = $\{\ \theta_3,\ \theta_5,\ \theta_6,\ \theta_9,\ \ldots,\ \theta_{11},\ \theta_{13},\ \theta_{14},\ \theta_{18},\ \ldots,\ \theta_{26},\ \theta_{28},\ \theta_{29},\ \theta_{34},\ \theta_{70},\ \ldots,\ \theta_{73},\ \theta_{76},\ \ldots,\ \theta_{90},\ \theta_{93},\ \theta_{98},\ \theta_{101},\ \theta_{113},\ \ldots,\ \theta_{115},\ \theta_{117},\ \ldots,\ \theta_{124},\ \theta_{126},\ \theta_{127},\ \theta_{135},\ \theta_{137},\ \theta_{138},\ \theta_{140},\ \theta_{141}\ \}$
10	Emitter 56 on board	$m(\text{emitter 56}) = 0.8$ $m(\Theta) = 0.2$	emitter 56 = $\{\ \theta_{18},\ \theta_{19},\ \theta_{34}\ \}$
11	Emitter 103 on board	$m(\text{emitter 103}) = 0.8$ $m(\Theta) = 0.2$	emitter 103 = $\{\ \theta_{11},\ \theta_{18},\ \ldots,\ \theta_{24},\ \theta_{30},\ \theta_{31},\ \theta_{34},\ \theta_{35},\ \theta_{36},\ \theta_{45},\ \ldots,\ \theta_{49},\ \theta_{63},\ \theta_{67},\ \theta_{69}\ \}$
12	Emitter 109 on board	$m(\text{emitter 109}) = 0.8$ $m(\Theta) = 0.2$	emitter 109 = $\{\ \theta_{11},\ \theta_{18},\ \theta_{19},\ \theta_{63}\ \}$
13	RCS_{top} small $\text{RCS}_{\text{top}} \in [1, 10] \times 10^4\ \text{dm}^2$	$m(\text{RCS}_{\text{top}}\ \text{small}) = 0.8$ $m(\Theta) = 0.2$	RCS_{top} small = $\{\ \theta_{10},\ \theta_{11},\ \theta_{18},\ \theta_{19},\ \theta_{28},\ \theta_{73},\ \theta_{75},\ \theta_{91},\ \theta_{92},\ \theta_{97},\ \theta_{99},\ \theta_{102},\ \ldots,\ \theta_{112},\ \theta_{116},\ \theta_{131},\ \theta_{139},\ \theta_{142}\ \}$
14	$\text{RCS}_{\text{front}}$ very small $\text{RCS}_{\text{front}} \in [0, 3000]\ \text{dm}^2$	$m(\text{RCS}_{\text{front}}\text{very small}) = 0.8$ $m(\Theta) = 0.2$	$\text{RCS}_{\text{front}}$ very small = $\{\ \theta_7,\ \theta_{18},\ \theta_{19},\ \theta_{72},\ \theta_{73},\ \theta_{74},\ \theta_{76},\ \ldots,\ \theta_{90},\ \theta_{93},\ \theta_{94},\ \theta_{97},\ \theta_{98},\ \theta_{100},\ \theta_{101},\ \theta_{104},\ \theta_{105},\ \theta_{106},\ \theta_{113},\ \theta_{114},\ \theta_{115},\ \theta_{117},\ \ldots,\ \theta_{124},\ \theta_{126},\ \theta_{127},\ \theta_{135},\ \theta_{137},\ \theta_{138},\ \theta_{140},\ \theta_{141}\ \}$

Table 14: *Information used in the test scenarios, modelled in evidence theory.*

4.3.3 Scenarios study

Table 14 presents the information modelled in evidence theory for the two test scenarios.

The main difference between probability theory and evidence theory is that the latter does not require the use of the additivity axiom found in probability theory, which imposes that $P[\overline{A}] = 1 - P[A]$. Indeed, in evidence theory the corresponding weight of $P[\overline{A}]$ can be assigned to $m(\Theta)$.

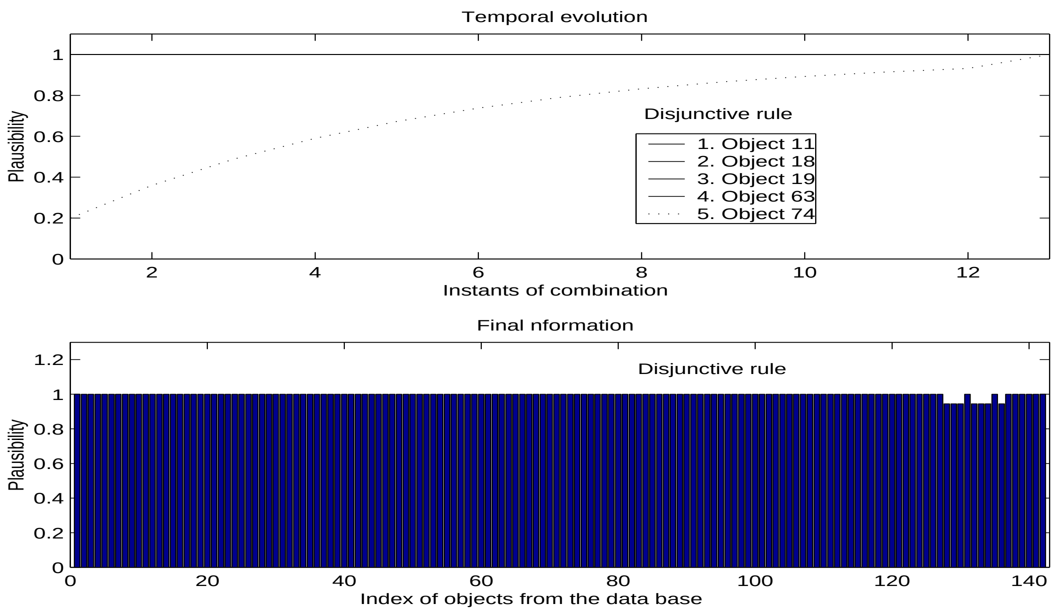

Figure 9: *Evidence theory - Test scenario 1 - Disjunctive rule.*

The results of the first scenario using a disjunctive rule of combination and Dempster's rule of combination are shown in figures 9 and 10, respectively. We note that with the disjunctive rule of combination, the BPA associated with ignorance increases after the fusion of each new piece of information, which makes it impossible for any decision process to converge toward a singleton. Therefore the disjunctive rule of combination is not recommended for this type of problem.

The conjunctive rule (Figure 10) is effective in this situation of low conflict. We note as well that whenever an object does not appear in a proposition to be combined, its plausibility decreases.

Introducing a countermeasure in the second scenario (Figure 11) shows that the fusion process provides the same results as in the first scenario, i.e. without the countermeasure. The plausibilities of the most representative singletons (those with highest plausibility) decrease after the combination of the countermeasure (instant of combi-

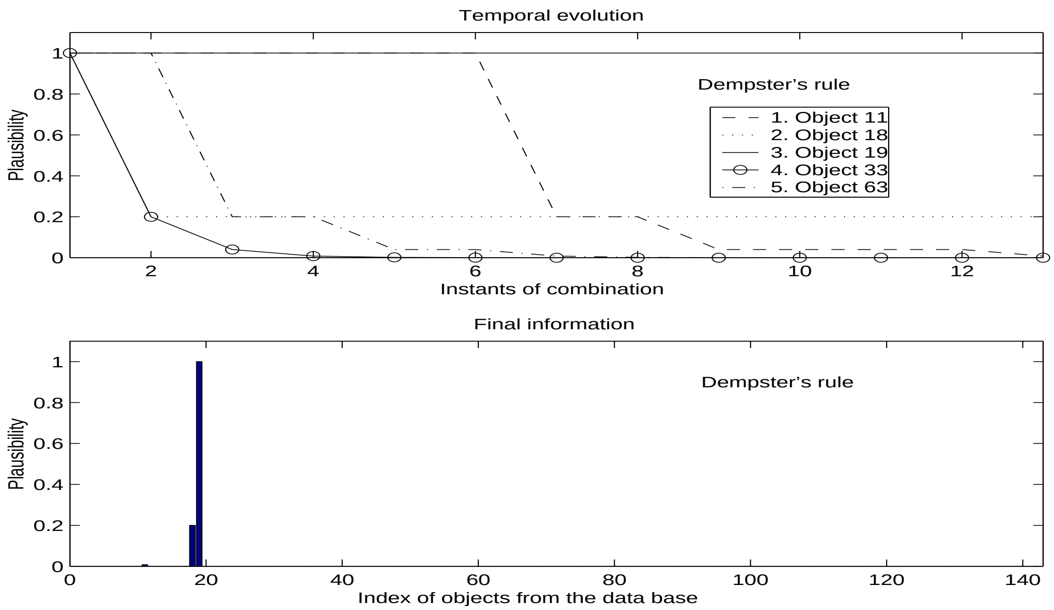

Figure 10: *Evidence theory - Test scenario 1 - Dempster's rule.*

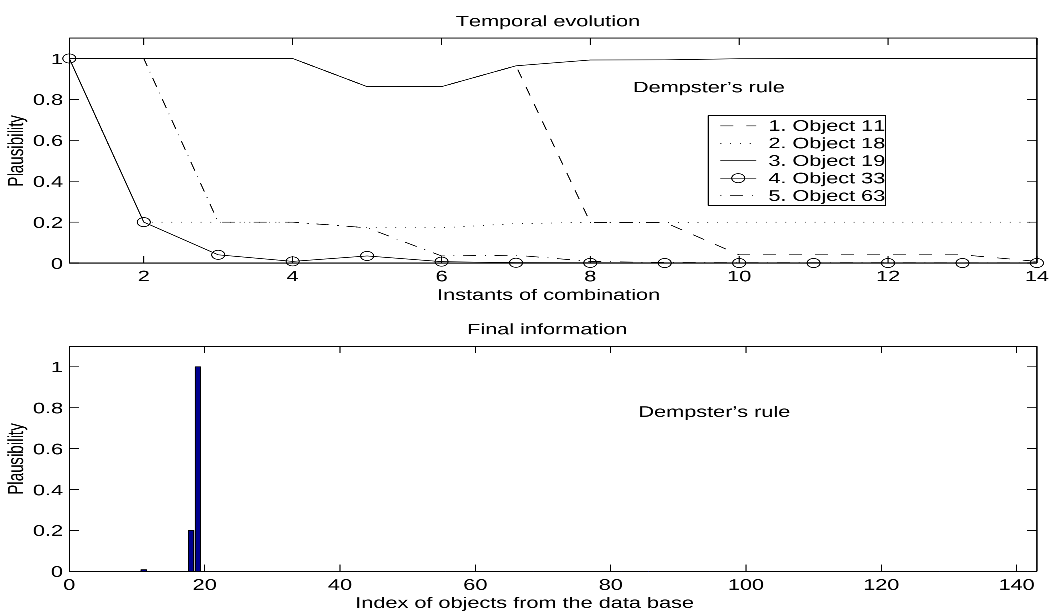

Figure 11: *Evidence theory - Test scenario 2 - Dempster's rule.*

nation #5). On the other hand, the plausibilities of the singletons characterizing the countermeasure increase.

4.4 Fuzzy sets theory

4.4.1 Introduction

Fuzzy sets theory was proposed in 1965 by Zadeh [4] to model vague information. While probability and evidence theories are well suited to modelling uncertain information, i.e., the uncertainty of membership of a target in a well-defined class of objects, fuzzy sets theory is well suited to vague information, i.e., the fuzzy membership of a target in an ill-defined class.

4.4.2 Theory description

Axioms and definitions

In the classical set theory, a subset $A \subseteq \Theta$ can be represented by a binary membership function:

$$\mu_A(\theta) = \begin{cases} 1 & \text{if} \quad \theta \in A \\ 0 & \text{otherwise} \end{cases} \qquad \forall \theta \in \Theta \tag{33}$$

An element θ of Θ belongs or does not belong to a crisp set (or classical set). Probability theory and evidence theory rely on this assumption of a two-value logic: A is either true or false.

Fuzzy sets theory models imprecision and especially vagueness or fuzziness. It relies on a multi-value logic, where events are allowed to be more true or less true.

Definition 7 A **fuzzy set** $\underline{A}$ of Θ is defined by the gradual membership function taking its values in the interval $[0, 1]$:

$$\mu_{\underline{A}}(\theta) \in [0, 1] \quad \forall \theta \in \Theta \tag{34}$$

For each θ of Θ, $\mu_{\underline{A}}(\theta)$ denotes the membership degree of θ to the subset A of Θ. The higher the degree, the more θ belongs to A.

Example 12 Figure 12 illustrates the difference between the two concepts of crisp and fuzzy membership of a subset to a subset A ($A \subseteq \Theta$ vs. $\underline{A} \subseteq \Theta$).

In classical set theory, a length of 50 metres belongs to the subset **large length**. But in fuzzy sets theory, the same length of 50 m belongs to the same class **large length** with a degree of membership of 0.85. Note that fuzzy sets theory allows us to classify the same length into several classes (with different membership degrees). For example, the length 50 m can belong to **large length** with a degree of 0.85, belong to **small length** with a degree of 0.2, etc.

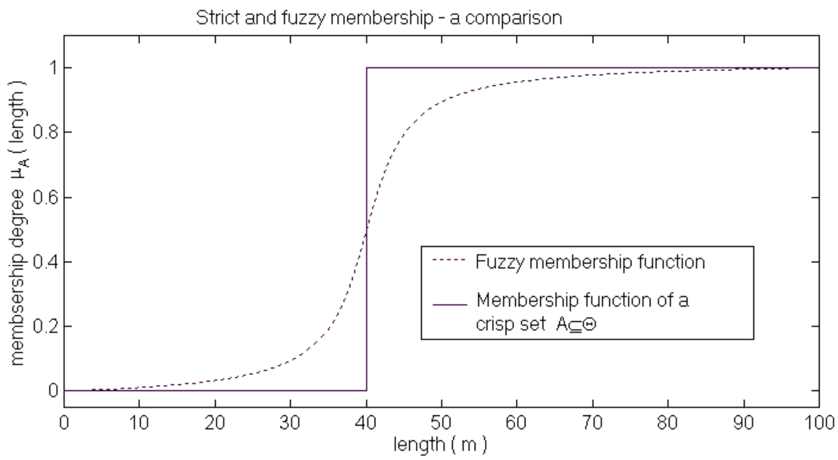

***Figure 12:* Large length** - *Difference between the concepts of crisp and fuzzy membership.*

Definition 8 An α-cut of a fuzzy subset $\underline{A} \subseteq \Theta$ is a crisp set of Θ defined by:

$$A_\alpha = \{\theta | \mu_{\underline{A}}(\theta) \geq \alpha\} \tag{35}$$

A fuzzy set $\underline{A} \subseteq \Theta$ can also be defined as the set of its α-cuts.

Information combination

In fuzzy sets theory, several combination rules are defined as being conjunctive or disjunctive.

1. Conjunctive combinations

Definition 9 The **standard intersection** of the two fuzzy sets $\underline{A}$ and $\underline{B}$ is a new fuzzy set defined by:

$$\mu_1^\cap(\theta) = \min\{\mu_{\underline{A}}(\theta), \mu_{\underline{B}}(\theta)\} \quad \forall\theta \in \Theta \tag{36}$$

Definition 10 The **product** of the two fuzzy sets $\underline{A}$ and $\underline{B}$ is a new fuzzy set defined by:

$$\mu_2^\cap(\theta) = \mu_{\underline{A}}(\theta)\mu_{\underline{B}}(\theta) \quad \forall\theta \in \Theta \tag{37}$$

Several other conjunctive rules are presented in the following equations:

$$\mu_3^\cap(\theta) = \max\{0, \mu_{\underline{A}} + \mu_{\underline{B}} - 1\} \quad \forall\theta \in \Theta \tag{38}$$

$$\mu_4^\cap(\theta) = \frac{\min\{\mu_{\underline{A}}(\theta), \mu_{\underline{B}}(\theta)\}}{h(\mu_{\underline{A}}, \mu_{\underline{B}})} \quad \forall\theta \in \Theta \tag{39}$$

$$\mu_5^\cap(\theta) = \min\{\mu_{\underline{A}}(\theta), \mu_{\underline{B}}(\theta)\} + 1 - h(\mu_{\underline{A}}, \mu_{\underline{B}}) \quad \forall\theta \in \Theta \tag{40}$$

where $h(\mu_{\underline{A}}, \mu_{\underline{B}}) = \max\Big(\min_{\theta\in\Theta}\{\mu_{\underline{A}}(\theta), \mu_{\underline{B}}(\theta)\}\Big)$ quantifies the degree of conflict between the two membership functions.

2. Disjunctive combinations

Definition 11 The **standard union** of the two fuzzy sets $\underline{A}$ and $\underline{B}$ is a new fuzzy set defined by:

$$\mu_1^{\cup}(\theta) = \max\{\mu_{\underline{A}}(\theta), \mu_{\underline{B}}(\theta)\} \quad \forall\theta \in \Theta \tag{41}$$

Definition 12 The **algebraic sum** of the two fuzzy sets $\underline{A}$ and $\underline{B}$ is a new fuzzy set defined by:

$$\mu_2^{\cup}(\theta) = \mu_{\underline{A}}(\theta) + \mu_{\underline{B}}(\theta) - \mu_{\underline{A}}(\theta) \times \mu_{\underline{B}}(\theta) \quad \forall\theta \in \Theta \tag{42}$$

Another disjunctive combination rule is defined by:

$$\mu_3^{\cup}(\theta) = \min\{\mu_{\underline{A}}(\theta) + \mu_{\underline{B}}(\theta), 1\} \quad \forall\theta \in \Theta \tag{43}$$

3. Adaptive combination

An adaptive combination rule was proposed by Dubois and Prade [19] as a cross between a conjunctive rule and a disjunctive rule. This rule uses a conjunctive combination rule when the sources are both reliable, and uses a disjunctive combination rule when one of the sources is unreliable (but we don't know which one):

$$\mu_{AD}(\theta) = \max\left\{\frac{\mu_i^{\cap}(\theta)}{h(\mu_{\underline{A}}(\theta), \mu_{\underline{B}}(\theta))}, \min\{1 - h(\mu_{\underline{A}}(\theta), \mu_{\underline{B}}(\theta)), \mu_j^{\cup}(\theta)\}\right\} \quad \forall\theta \in \Theta \tag{44}$$

The combination $\mu_i^{\cap}(\theta)$ and $\mu_j^{\cup}(\theta)$ can be chosen from any of the above conjunctive and disjunctive combination rules.

Decision

The most likely object is the one with the highest degree of membership in the final fuzzy set after a sequence of combination steps:

$$\theta_{\text{observed}} = \text{Arg}\big\{\max_{\theta\in\Theta}[\mu_{\underline{A}}(\theta)]\big\} \tag{45}$$

Because of the normalization step required in many of the above combination rules, the associativity property is not satisfied, and thus the order of combination is important. Indeed, if the decision is taken after the last combination step, the result will depend on the order in which the pieces of information have been fused.

Example 13 Let us consider the two pieces of information presented in figure 13, characterized by their fuzzy membership functions $\mu_{\text{small length}}(\theta)$ and $\mu_{\text{medium height}}(\theta)$, respectively.

Figure 14 shows the results of three different combination rules: one disjunctive rule (the maximum) yields several objects as solution (objects 3, 5 or 7), and two conjunctive combinations (the minimum and the normalized minimum), which are more selective, yield just one solution (object 3).

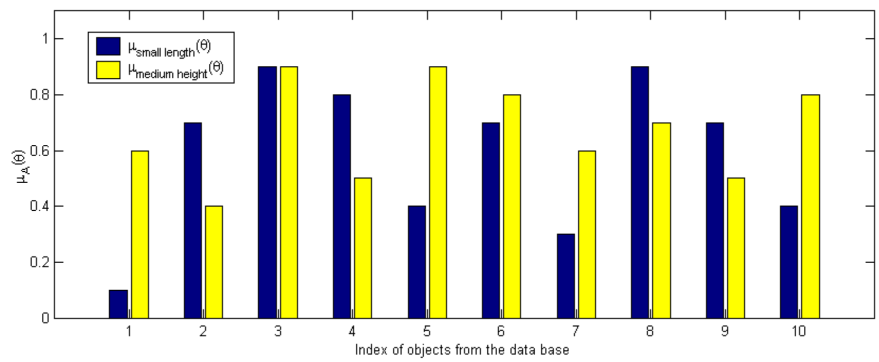

Figure 13: *Example of fuzzy membership functions* ***small length*** *and* ***medium height****.*

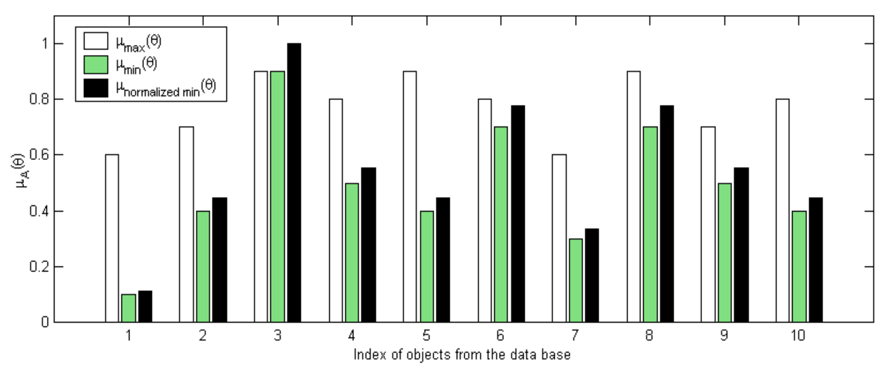

Figure 14: *Example of three combination rules in fuzzy sets theory.*

Note that in this example, the normalization of the third combination rule (normalized minimum) is not necessary because it does not influence the decision-making in this simple case of one step of combination. However, the normalization step can be useful when more pieces of information are fused. Note finally that θ_3 has the maximum membership degree using the three combination rules, and thus will be the selected object by the decision rule.

4.4.3 Scenarios study

Some pieces of information in the test scenario, namely the target type and list of the emitters, are modelled better by crisp sets, which are special cases of fuzzy sets. Other pieces of information characterizing the physical dimensions of the target are modelled better by fuzzy sets. Tables 15 and 16 show crisp classifications of attributes, i.e., how it would have been modelled in classical set theory. The extension to fuzzy sets theory is illustrated in Figure 15 below.

Attribute \ Class	small	medium	large
length (m)	0 - 100	100 - 200	200 - 500
width (m)	0 - 15	15 - 40	40 - 80
height (m)	0 - 5	5 - 15	15 - 35

Table 15: *Classifications in classical sets theory of length, width and height.*

SER (dm^2) \ Class	very small	small	medium	large	very large
RCS_{side}	0 - 4000	0.4 - 2 $\times 10^4$	2 - 15 $\times 10^4$	1.5 - 5 $\times 10^5$	$\geq 5 \times 10^5$
RCS_{top}	0 - 10000	1 - 10 $\times 10^4$	1 - 5 $\times 10^5$	5 - 10 $\times 10^5$	$\geq 10^6$
RCS_{front}	0 - 3000	3 - 7 $\times 10^3$	7 - 15 $\times 10^3$	1.5 - 3 $\times 10^4$	$\geq 3 \times 10^4$

Table 16: *Classifications in classical sets theory of the side, top and front radar cross sections.*

These classifications are based on those defined in [20], but without any consideration of the target type (air, naval, ground targets, etc.).

While a crisp classification is at the basis of probability and in evidence theories, it cannot be used to model vague information. This previous classification must then be fuzzified so that different fuzzy classes can be defined. The upper graph in Figure 15 shows three fuzzy classes for the attribute *length* (small, medium, large), while the other three graphs show the fuzzy membership degrees of all the objects in the database in these three fuzzy classes. Between the upper graph and the three others there is a change of variable: the first domain is a continuous and ordered one, characterizing the attribute over real values (i.e., length between 0 and 500 m), and the second domain is the index of objects in the database, which is a discrete and non-ordered domain. More details on the domain change and also the complete models of the fuzzy classes of the pieces of information for the scenario are presented in Annexes B.1 and B.2.

The other pieces of information not modelled by fuzzy sets are modelled using crisp sets, as summarized in Table 17.

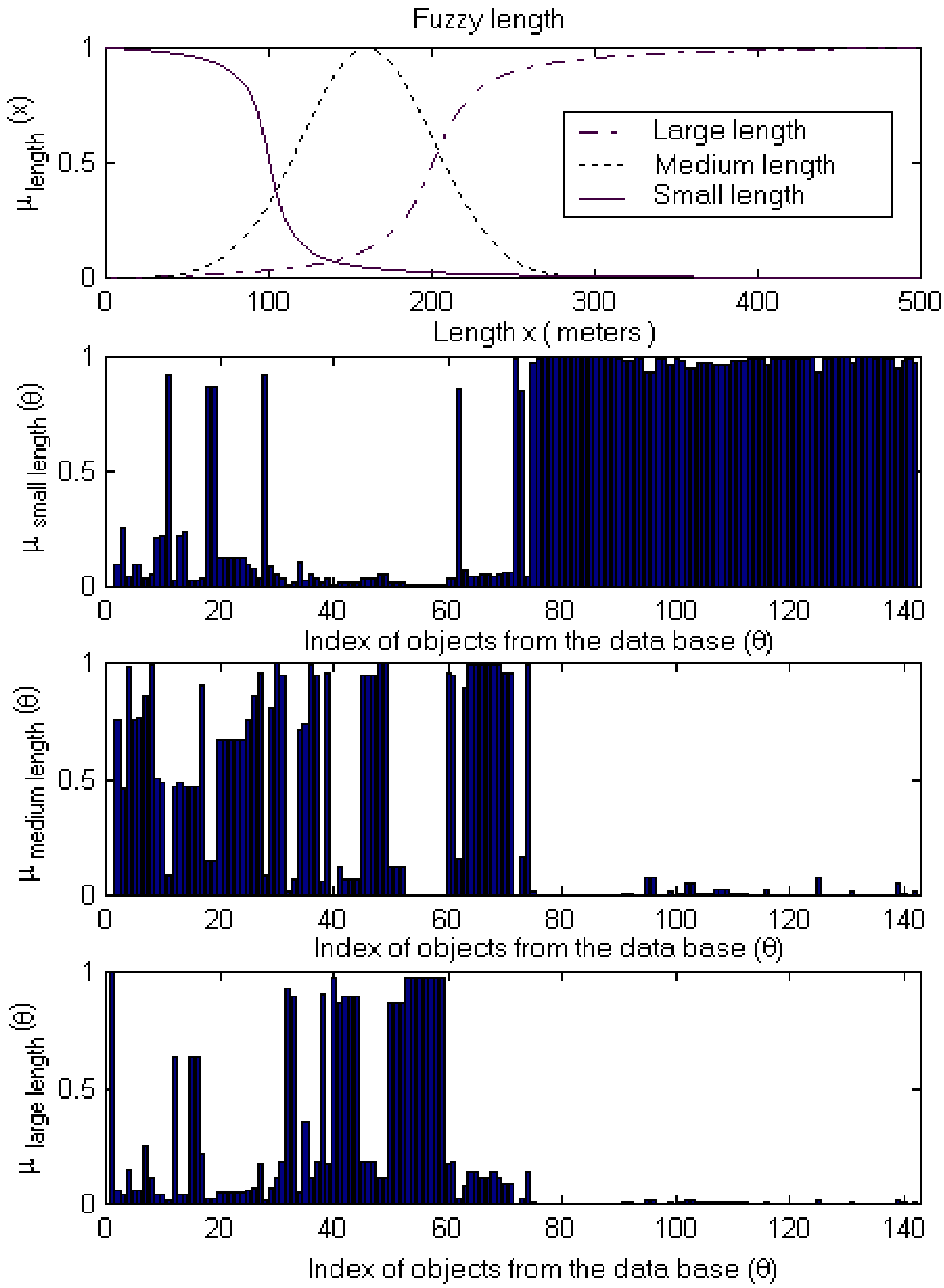

Figure 15: *Fuzzy length - class small / medium / large. Characterization of the database by the classes small length, medium length and large length*

Instant	Information	Modelled information in fuzzy sets theory	Associated propositions
1	Target type = ship	$\mu_{\text{ship}}(\theta) = 1\ \forall\theta \in \text{ship}$ $\mu_{\text{ship}}(\theta) = 0\ \forall\theta \in \overline{\text{ship}}$	ship = $\{\ \theta_1, \theta_2, \ldots, \theta_{70}, \theta_{71}, \theta_{73}\ \}$
2	Emitter 44 on board	$\mu_{\text{em. 44}}(\theta) = 1\ \forall\theta \in \text{emitter 44}$ $\mu_{\text{em. 44}}(\theta) = 0\ \forall\theta \in \overline{\text{emitter 44}}$	emitter 44 = $\{\ \theta_{11}, \theta_{19}, \theta_{63}\ \}$
3	Length small	Fig.B.1	Θ
4	RCS_{side} medium	Fig.B.4	Θ
5	Emitter 77 on board	$\mu_{\text{em. 77}}(\theta) = 1\ \forall\theta \in \text{emitter 77}$ $\mu_{\text{em. 77}}(\theta) = 0\ \forall\theta \in \overline{\text{emitter 77}}$	emitter 44 = $\{\ \theta_{29}, \theta_{33}, \theta_{35}, \theta_{36}\ \}$
6	Small height	Fig.B.3	Θ
7	Emitter 47 on board	$\mu_{\text{em. 47}}(\theta) = 1\ \forall\theta \in \text{emitter 47}$ $\mu_{\text{em. 47}}(\theta) = 0\ \forall\theta \in \overline{\text{emitter 47}}$	emitter 47 = $\{\ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{46}, \theta_{47}, \theta_{63}\ \}$
8	Emitter 55 on board	$\mu_{\text{em. 55}}(\theta) = 1\ \forall\theta \in \text{emitter 55}$ $\mu_{\text{em. 55}}(\theta) = 0\ \forall\theta \in \overline{\text{emitter 55}}$	emitter 55 = $\{\ \theta_{18}, \theta_{19}\ \}$
9	Small width	Fig.B.2	Θ
10	Emitter 56 on board	$\mu_{\text{em. 56}}(\theta) = 1\ \forall\theta \in \text{emitter 56}$ $\mu_{\text{em. 56}}(\theta) = 0\ \forall\theta \in \overline{\text{emitter 56}}$	emitter 56 = $\{\ \theta_{18}, \theta_{19}, \theta_{34}\ \}$
11	Emitter 103 on board	$\mu_{\text{em. 103}}(\theta) = 1\ \forall\theta \in \text{emitter 103}$ $\mu_{\text{em. 103}}(\theta) = 0\ \forall\theta \in \overline{\text{emitter 103}}$	emitter 103 = $\{\ \theta_{11}, \theta_{18}, \ldots, \theta_{24}, \theta_{30}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{36}, \theta_{45}, \ldots, \theta_{49}, \theta_{63}, \theta_{67}, \theta_{69}\ \}$
12	Emitter 109 on board	$\mu_{\text{em. 109}}(\theta) = 1\ \forall\theta \in \text{emitter 109}$ $\mu_{\text{em. 109}}(\theta) = 0\ \forall\theta \in \overline{\text{emitter 109}}$	emitter 109 = $\{\ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{63}\ \}$
13	RCS_{top} small	Fig.B.5	Θ
14	$\text{RCS}_{\text{front}}$ very small	Fig.B.6	Θ

Table 17: *Information used in the test scenarios, modelled in fuzzy sets theory.*

The result of the conjunctive normalized minimum rule (Figure 16) shows that after the final step of combination, only one element has a membership degree equal to 1, and the decision rule then easily selects it to be the identity of the observed target, namely object #18. The other elements in the database disappeared from the propositions list as soon as a new piece of information to be combined does not fit with them. In the database we considered, all the features of objects #18 and #19 are the same except one. That is why both objects remain possible candidates for the observed target.

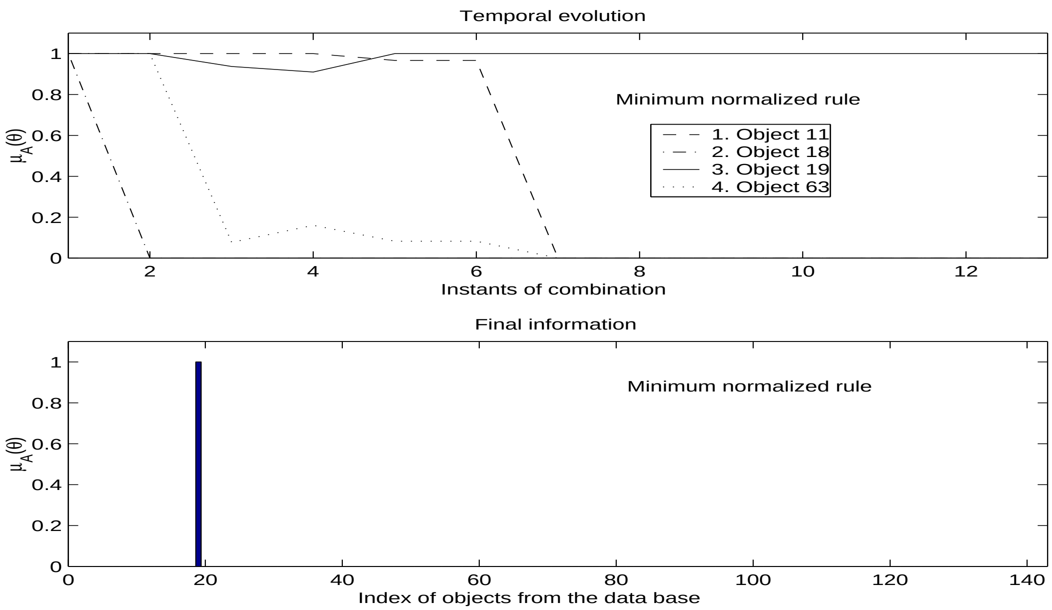

Figure 16: *Fuzzy sets theory - Test scenario 1 - Normalized minimum rule*

In the first test scenario, all pieces of information to be combined are coherent in the sense that they do not create a total conflict between the sources. However, we are now interested in situations where a countermeasure is included in the fusion process, so that the conflict must be managed. This is illustrated in test scenario 2. One the one hand, it appears that the conjunctive combination rules like the minimum or the product will eliminate all the propositions, the likely ones included. On the other hand, the disjunctive combination rules like the maximum or the algebraic sum will consider all the propositions with indiscernible weights. Hence, neither a disjunctive rule nor a conjunctive rule yields an effective solution for this kind of problem. Being a compromise between the conjunctive and disjunctive rules, an adaptive combination rule like the one proposed by Dubois and Prade (equation (44)) seems to provide better results for target identification problems using data fusion techniques. Figure 17 shows the results after using the adaptive rule in test scenario 1. Although two possible matches for the observed target appear at the end of the combination process

(Objects #18 and #19), Object #19 will be selected by the decision rule because it has a higher degree of membership than Object #18. Note that the order of

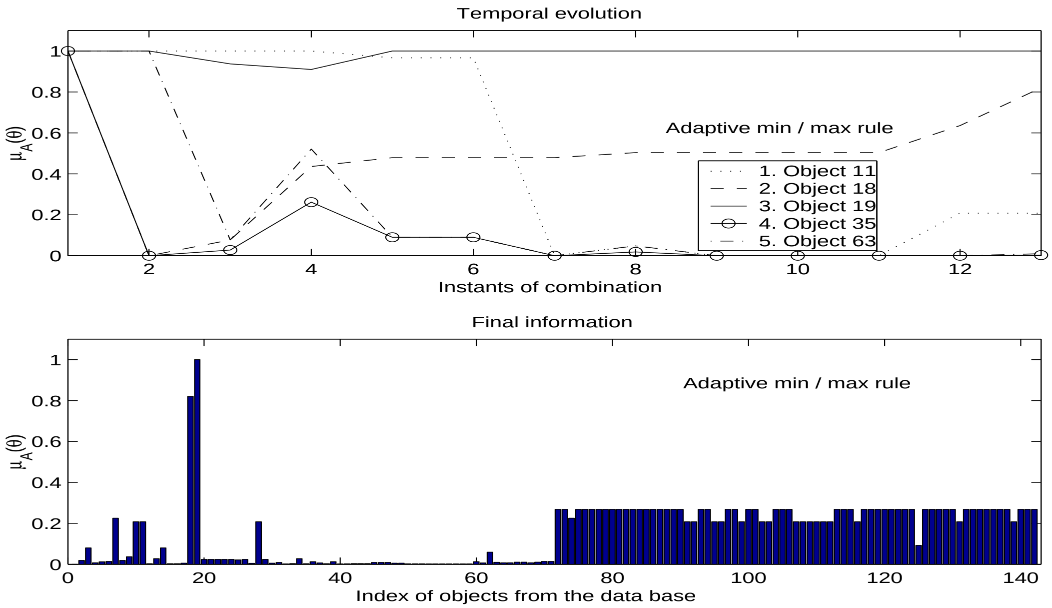

Figure 17: *Fuzzy sets theory - Test scenario 1 - Adaptive min/max rule.*

combination of the pieces of information is important, since the normalization used in the adaptive combination rule makes it non-associative.

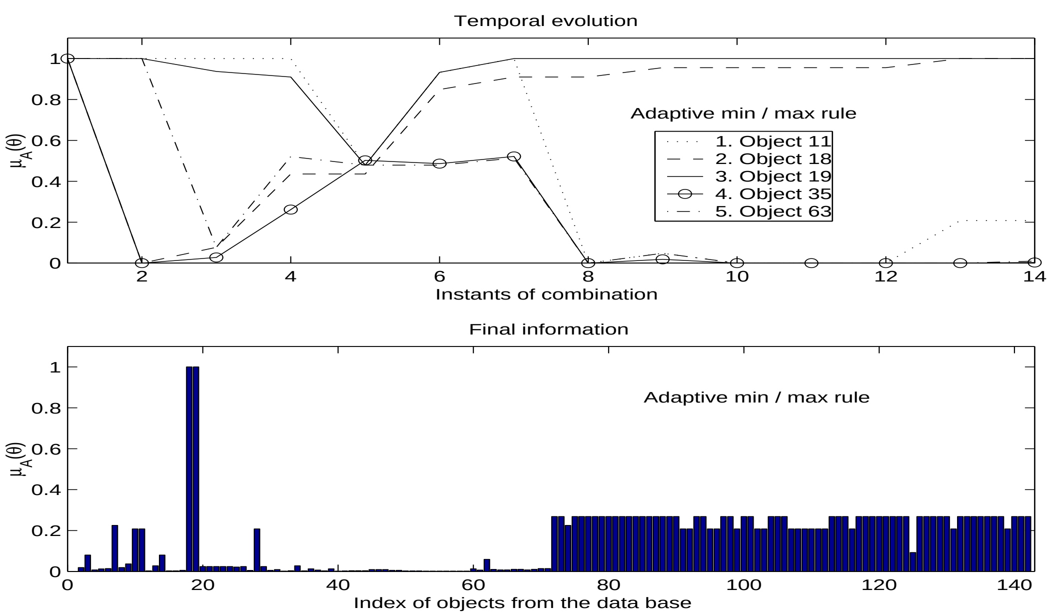

Figure 18: *Fuzzy sets theory - Test scenario 2 - Adaptive min/max rule.*

Figure 18 shows the result of the adaptive combination rule applied to the second scenario where a countermeasure has been introduced at step #5 of combination. We note that the objects belonging to the proposition representing piece of information #5 are eliminated during the fusion process. However, Object #18 has the same final fuzzy membership degree as Object #19, so no decision can be made about a single object. So if the adaptive combination rule eliminates the countermeasure, it also eliminates the differences between the two most representative elements, and the identification remains imprecise in this case.

Conjunctive combination rules used with the second test scenario do not produce any relevant result, since after the countermeasure is combined, all the objects in the database are assigned a nil fuzzy membership degree.

4.5 Possibility theory

4.5.1 Introduction

Possibility theory was developed also by Zadeh [7] and proposed another alternative in the representation of uncertainty, i.e., for representing incomplete information. Based on fuzzy sets theory, possibility theory is however different since it does not address vague information. Indeed, fuzzy sets theory models fuzzy information (membership of an object in a ill-defined class) without any consideration for randomness. But possibility theory models the uncertainty of a well-defined class, in an equivalent spirit as probability and evidence theories, although uncertainty is quantified differently.

4.5.2 Theory description

Axioms and definitions

According to Dubois and Prade [21], the concept of possibility can be regarded from several points of view. We can refer to possibility as *feasibility*, as in the example "*It is possible* ***to*** *solve this problem*". And we can use the possibility concept to express the idea of *plausibility*, as in "*It is possible* ***that*** *the target is a plane*". Possibility can also be viewed as *consistency* to express the compatibility or incompatibility of two pieces of information, as in the example "*It is impossible for the observed target to be at 3000 metres of altitude,* ***since*** *it is a submarine*". Information expressed in possibility theory can be either objective or subjective.

Zadeh [7] defined the concept of possibility using the concept of fuzzy restriction. Let $\underline{A}$ be a fuzzy set of Θ, where $\mu_{\underline{A}}(\theta)$ represents the membership degree of an element θ in the fuzzy set $\underline{A}$. Considering the fuzzy set $\underline{A}$, Zadeh defines the **possibility degree** of the event $X = \theta$, noted $\pi_X(\theta)$, as the membership degree of θ in the fuzzy set $\underline{A}$:

$$\pi_X(\theta) = \mu_{\underline{A}}(\theta) \tag{46}$$

The possibility distribution function $\pi_X(\theta) \in [0,1]\ \forall\theta \in \Theta$, is the model of uncertain information in possibility theory. The only restriction of this distribution function is its normalization, requiring at least one element $\theta_0 \in \Theta$ to have a possibility degree equal to 1.

Equation (46) may create confusion by implying that fuzzy sets theory and possibility theory are the same. To avoid this ambiguity, Dubois and Prade [22] proposed a new notation:

- the possibility degree characterizing the uncertain membership of an element θ in a known class is denoted by $\pi(\theta|\underline{A})$;
- the membership degree characterizing a fuzzy set, which is ill-defined, is denoted by $\mu(\underline{A}|\theta)$

Example 14 Let us consider the piece of information "*The observed target has a large length*". The fuzzy set **large length** is imperfect information. In fuzzy sets theory we can quantify the membership degree of any object in the database in the fuzzy class **large length**. For example, we can consider that an object with a length of 100 m can be classified into the *large length* class with a membership degree of 0.7 ($\mu_{\text{large length}}(100) = 0.7$). Zadeh [7] makes the link between the concepts of fuzzy set and possibility (equation (46)). He considers that the possibility that the length of the observed object is 100 m is 0.7 ($\pi_X(100) = 0.7$).

From the possibility distribution function $\pi_X(\theta)$ expressing the possibility of $X = \theta$, where $\theta \in \Theta$, two other measures are defined: the possibility measure Π, where $\Pi(A)$ characterizes the possibility of $X \in A$, and the necessity measure N, where $N(A)$ characterizes the necessity of $X \in A$:

$$\Pi(A) = \max_{\theta \in A}\{\pi_X(\theta)\} \quad \forall A \subseteq \Theta \tag{47}$$

$$N(A) = \min_{\theta \notin A}\{1 - \pi_X(\theta)\} \quad \forall A \subseteq \Theta \tag{48}$$

Between these two measures there is a duality relation (equivalent to that existing in evidence theory between plausibility and belief - equation (24)):

$$N(A) = 1 - \Pi(\overline{A}) \quad \forall A \subseteq \Theta \tag{49}$$

$\Pi(A) = 1$ corresponds to the situation where at least one singleton $\theta \in A$ has a possibility degree equal to 1 and it is likely to be the element sought. This means that the researched element **can** belong to subset A, without being necessary. $N(A) = 1$ corresponds to the situation where $\Pi(\overline{A}) = 0$, meaning that the researched element certainly does not belong to subset $\overline{A}$, so it certainly belongs to subset $A \subseteq \Theta$.

In possibility theory, knowledge of $\Pi(A)$ does not involve knowledge of $\Pi(\overline{A})$, but only knowledge of $N(\overline{A})$.

Moreover, possibility and necessity degrees satisfy the following axioms:

$$\Pi(A \cup B) = \max\{\Pi(A), \Pi(B)\} \quad \forall A, B \subseteq \Theta \tag{50}$$

$$N(A \cap B) = \min\{N(A), N(B)\} \quad \forall A, B \subseteq \Theta \tag{51}$$

Information combination

As fuzzy sets, possibility distributions can be combined using conjunctive or disjunctive combination rules, these rules being the same as those introduced for fuzzy sets. The only difference is that all the combination rules must be normalized so that the final information is normalized.

Decision

The decision rule in possibility theory is based on the greatest possibility, making the element with the highest possibility measure the best candidate for the identity of the observed target:

$$\theta_{\text{observed}} = \text{Arg}\{\max_{\theta \in \Theta}[\pi_X(\theta)]\} \tag{52}$$

4.5.3 Scenarios study

Table 18 lists the pieces of information used in the test scenarios modelled in possibility theory. The result of the combination process using a normalized product combination rule is shown in Figure 19. Object #19 appears as the sole candidate, having the highest final possibility.

4.6 Rough sets theory

4.6.1 Introduction

In 1982, Pawlak introduced a new concept to deal with imperfect information: rough sets. The basic idea of rough sets theory is to replace an imprecise and uncertain concept by a pair of precise concepts obtained from lower and upper approximations. Then, classical sets theory is used to deal with the approximated precise concepts. Rough sets theory deals only with the imprecision of information, ignoring the uncertainty. Usually, the knowledge coming from the new piece of information is modelled

Instant	Information	Modelled information in possibility theory	Associated propositions
1	Target type = ship	$\pi(\theta) = 1 \quad \forall\theta \in \text{ship}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{ship}}$	ship = $\{ \theta_1, \theta_2, \ldots, \theta_{70}, \theta_{71}, \theta_{73} \}$
2	Emitter 44 on board	$\pi(\theta) = 1 \quad \forall\theta \in \text{emitter 44}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{emitter 44}}$	emitter 44 = $\{ \theta_{11}, \theta_{19}, \theta_{63} \}$
3	Small length length $\in [0, 100]$ m	$\pi(\theta) = 1 \quad \forall\theta \in \text{small length}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{small length}}$	small length = $\{ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{62}, \theta_{72}, \theta_{73}, \theta_{75}, \ldots, \theta_{127}, \theta_{131}, \theta_{135}, \theta_{137}, \ldots, \theta_{142} \}$
4	RCS_{side} medium $\text{RCS}_{\text{side}} \in [2, 15] \times 10^4\ \text{dm}^2$	$\pi(\theta) = 1 \quad \forall\theta \in \text{RCS}_{\text{side}}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{RCS}_{\text{side}}}$	RCS_{side} medium = $\{ \theta_2, \theta_3, \theta_5, \theta_6, \theta_8, \theta_9, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \ldots, \theta_{31}, \theta_{34}, \theta_{36}, \theta_{39}, \theta_{45}, \ldots, \theta_{49}, \theta_{60}, \theta_{62}, \ldots, \theta_{71}, \theta_{73}, \theta_{95}, \theta_{96}, \theta_{125}, \theta_{131}\}$
5	Emitter 77 on board	$\pi(\theta) = 1 \quad \forall\theta \in \text{emitter 77}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{emitter 77}}$	emitter 77 = $\{ \theta_{29}, \theta_{33}, \theta_{35}, \theta_{36} \}$
6	Small height height $\in [0, 5]$ m	$\pi(\theta) = 1 \quad \forall\theta \in \text{small height}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{small height}}$	small height = $\{ \theta_2, \theta_3, \theta_8, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{73}, \theta_{77}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{98}, \theta_{101}, \theta_{115}, \theta_{117}, \theta_{118}, \theta_{120}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{137}, \theta_{138}, \theta_{140} \}$
7	Emitter 47 on board	$\pi(\theta) = 1 \quad \forall\theta \in \text{emitter 47}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{emitter 47}}$	emitter 47 = $\{ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{46}, \theta_{47}, \theta_{63} \}$
8	Emitter 55 on board	$\pi(\theta) = 1 \quad \forall\theta \in \text{emitter 55}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{emitter 55}}$	emitter 55 = $\{ \theta_{18}, \theta_{19} \}$
9	Small width width $\in [0, 15]$ m	$\pi(\theta) = 1 \quad \forall\theta \in \text{small width}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{small width}}$	small width = $\{ \theta_3, \theta_5, \theta_6, \theta_9, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{70}, \ldots, \theta_{73}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \theta_{98}, \theta_{101}, \theta_{113}, \ldots, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141} \}$
10	Emitter 56 on board	$\pi(\theta) = 1 \quad \forall\theta \in \text{emitter 56}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{emitter 56}}$	emitter 56 = $\{ \theta_{18}, \theta_{19}, \theta_{34} \}$
11	Emitter 103 on board	$\pi(\theta) = 1 \quad \forall\theta \in \text{emitter 103}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{emitter 103}}$	emitter 103 = $\{ \theta_{11}, \theta_{18}, \ldots, \theta_{24}, \theta_{30}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{36}, \theta_{45}, \ldots, \theta_{49}, \theta_{63}, \theta_{67}, \theta_{69} \}$
12	Emitter 109 on board	$\pi(\theta) = 1 \quad \forall\theta \in \text{emitter 109}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{emitter 109}}$	emitter 109 = $\{ \theta_{11}, \theta_{18}, \theta_{19}, \theta_{63} \}$
13	RCS_{top} small $\text{RCS}_{\text{top}} \in [1, 10] \times 10^4\ \text{dm}^2$	$\pi(\theta) = 1 \quad \forall\theta \in \text{RCS}_{\text{top}}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{RCS}_{\text{top}}}$	RCS_{top} small = $\{ \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{73}, \theta_{75}, \theta_{91}, \theta_{92}, \theta_{97}, \theta_{99}, \theta_{102}, \ldots, \theta_{112}, \theta_{116}, \theta_{131}, \theta_{139}, \theta_{142} \}$
14	$\text{RCS}_{\text{front}}$ very small $\text{RCS}_{\text{front}} \in [0, 3000]\ \text{dm}^2$	$\pi(\theta) = 1 \quad \forall\theta \in \text{RCS}_{\text{front}}$ $\pi(\theta) = 0.2 \quad \forall\theta \in \overline{\text{RCS}_{\text{front}}}$	$\text{RCS}_{\text{front}}$ very small = $\{ \theta_7, \theta_{18}, \theta_{19}, \theta_{72}, \theta_{73}, \theta_{74}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{97}, \theta_{98}, \theta_{100}, \theta_{101}, \theta_{104}, \theta_{105}, \theta_{106}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141} \}$

Table 18: *Information used in the test scenarios, modelled in possibility theory.*

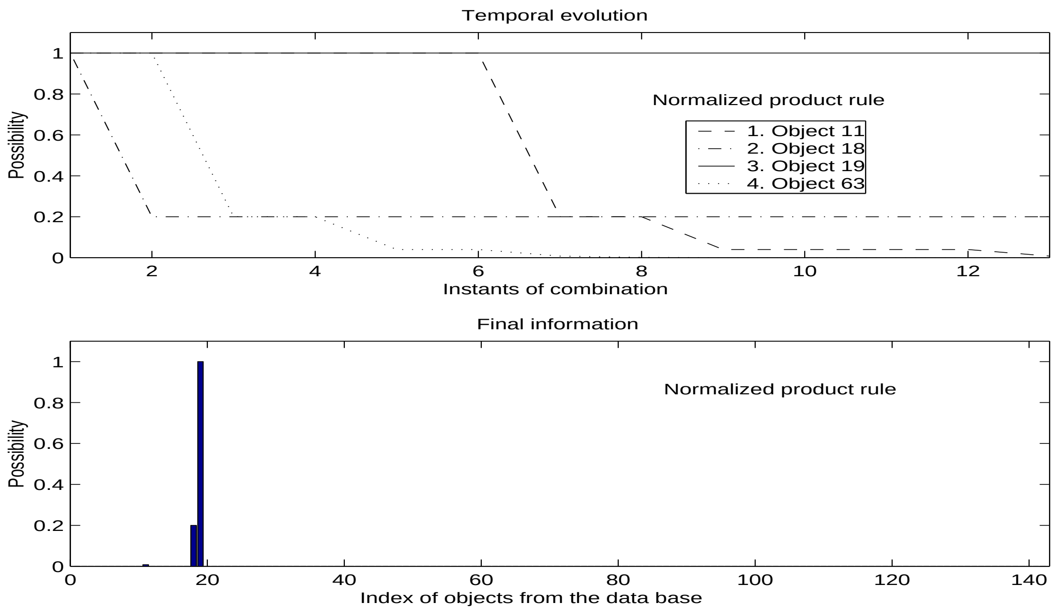

Figure 19: *Possibility theory - Test scenario 1 - Normalized product rule.*

as an equivalence relation on the objects in the data base, i.e., building classes in which some objects are indiscernible.

4.6.2 Theory description

Axioms and definitions

Definition 13 A **concept** of Θ a subset $A \subseteq \Theta$.

Definition 14 A set of subsets of Θ generates **knowledge** of Θ. We denote items of knowledge by R ($R \subseteq 2^{\Theta}$).

Knowledge of Θ is often defined as a partition of the frame of discernment, according to a given attribute. This is a restriction of the general case of rough sets theory, since a partition is a special subset of the power set 2^{Θ}. This knowledge is *a priori* knowledge of the studied system, and it does not represent known information already integrated into the fusion process.

In an ideal situation, an item of knowledge R is formed by a set of singleton concepts of Θ, such knowledge being called **fine** knowledge.

Let $A \subseteq \Theta$ be a concept of Θ and let R be an item of knowledge of Θ. If $A \in R$, the concept A is certain according to knowledge R. Otherwise it is uncertain. If the concept A is uncertain, the question is how to approximate this concept according to knowledge R.

Definition 15 The **lower approximation** of A according to R, noted $\underline{R}A$, is the set of elements $[\theta]_R \in R$ included in A.

$$\underline{R}A = \{[\theta]_R | [\theta]_R \subseteq A\} \tag{53}$$

The lower approximation is obtained from all the concepts of Θ which are certain according to knowledge R.

Definition 16 The **upper approximation** of A according to R, noted $\overline{R}A$, is the set of elements $[\theta]_R$ of R whose intersection with A is not empty.

$$\overline{R}A = \{[\theta]_R | [\theta]_R \cap A \neq \varnothing\} \tag{54}$$

The upper approximation is obtained by subtracting from all the concepts of Θ those concepts which are certainly not included in the knowledge R.

The terms lower and upper approximations were proposed by Pawlak [8]. Other designations are **core** and **envelop**, respectively (Parsons [23]).

Example 15 Let us consider a frame of discernment Θ_{10} with 10 objects:

$$\Theta_{10} = \{\theta_0, \theta_1, \ldots, \theta_9\}$$

and a knowledge R of Θ_{10} given by:

$$R = \Big\{\{\theta_0, \theta_4\}, \{\theta_1\}, \{\theta_2\}, \{\theta_3\}, \{\theta_5, \theta_6, \theta_9\}, \{\theta_7\}, \{\theta_8\}\Big\}$$

This knowledge means that we are not able to distinguish between objects θ_0, θ_4 nor between θ_5, θ_6, θ_9.

An information can be represented by any subset of Θ_{10}:

$$A = \{\theta_0, \theta_1, \theta_4, \theta_5\}$$

This rough set model of the information takes into account only the imprecision of the information; its certainty degree is not considered.

We can determine lower and upper approximations from knowledge R and information A, respectively:

$$\underline{R}A = \Big\{\{\theta_0, \theta_4\}, \{\theta_1\}\Big\} \tag{55}$$

$$\overline{R}A = \Big\{\{\theta_0, \theta_4\}, \{\theta_1\}, \{\theta_5, \theta_6, \theta_9\}\Big\} \tag{56}$$

These approximations describe on the one hand the set of objects to which we grant our complete confidence ($\underline{R}A$), and on the other hand the set of all possible objects ($\overline{R}A$).

Information combination

Definition 17 (Concepts combination) Let Θ be the frame of discernment and let R be a knowledge of Θ. Let A and B be two concepts of Θ approximated according to the knowledge R by the couple of sets ($\underline{R}A$, $\overline{R}A$) and ($\underline{R}B$, $\overline{R}B$), respectively. Imperfect pieces of information $A \cap B$ and $A \cup B$ are characterized by their approximations ($\underline{R}(A \cap B)$, $\overline{R}(A \cap B)$) and ($\underline{R}(A \cup B)$, $\overline{R}(A \cup B)$), respectively. These combined approximations are defined as follows:

1. conjunctive combination:

$$\underline{R}(A \cap B) = \underline{R}A \cap \underline{R}B \tag{57}$$

$$\overline{R}(A \cap B) \subseteq \overline{R}A \cap \overline{R}B \tag{58}$$

2. disjunctive combination:

$$\underline{R}(A \cup B) \supseteq \underline{R}A \cup \underline{R}B \tag{59}$$

$$\overline{R}(A \cup B) = \overline{R}A \cup \overline{R}B \tag{60}$$

Definition 18 (Knowledge combination) Let $\mathbf{R}$ be a knowledge family of Θ. An equivalent knowledge of this family is given by the intersection of all the pieces of knowledge of this family:

$$\mathrm{IND}(\mathbf{R}) = \bigcap_{R_i \in \mathbf{R}} R_i \tag{61}$$

$\mathrm{IND}(\mathbf{R})$ is the set of all classes of objects which are indiscernible.

Ignorance

Definition 19 Let Θ be a frame of discernment and let R be a knowledge of Θ. A complete lack of knowledge on the concept A is modelled by lower and upper approximations such that $\underline{R}A = \varnothing$ and $\overline{R}A = \Theta$.

Independence

Definition 20 Let $\mathbf{F} = \{A_1, A_2, ..., A_n\}$ be a family of concepts of Θ. We say that the concept A_i is **dispensable** for $\mathbf{F}$ if

$$\bigcap_{j=1}^{j=n}\,_{j\neq i} A_j = \bigcap_{j=1}^{j=n} A_j \tag{62}$$

If not, we say that the concept A_i is **indispensable** for $\mathbf{F}$.

Definition 21 A family of concepts $\mathbf{F}$ is said to be **independent** if all its concepts are indispensable for $\mathbf{F}$; if not the family is called **dependent**.

When a family of concepts or pieces of knowledge is dependent, there are several ways to transform it into an equivalent independent family (see Pawlak [8] fro details).

Decision

After the combination of rough pieces of information, using the equations (57) to (60), the following conclusion can be drawn: the researched object is probably in $\underline{R}A_{final}$ but it also could be in $\overline{R}A_{final}$.

1. If the knowledge R is not fine enough, the lower approximation of any information can be empty; this is a situation we must avoid as much as possible.
2. If the knowledge R is totally rough, rough sets theory reduces in this special case to classical sets theory.

So, to successfully apply rough sets theory, *a priori* knowledge of the database must be neither too fine nor too rough. In the previous extreme conditions, rough sets theory should not be used to combine pieces of information, since the results could be irrelevant.

4.6.3 Scenarios study

The database can be modelled by a very fine knowledge

$$R_0 = \Big\{\{\theta_1\}, \{\theta_2\}, \ldots, \{\theta_{143}\}\Big\}$$

This model expresses the fact that all objects are distinct, none of them having the same features as another. We will, however, consider that our *a priori* knowledge of the database is not complete and we know only a partition of the database according to, for example:

1. type and sub-type;

2. offensive classification.

In practice, these partitions are defined from the information provided by the sensors in use. Pieces of knowledge R_1 and R_2, defining the partitions of the database by object type and sub-type and by offensive classification, respectively, are presented in Annex C.1. The equivalent knowledge R is given by:

R= $\Big\{$ $\{\theta_1\}$, $\{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}$, $\{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}$, $\{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{37}, \theta_{45}, \theta_{46}, \theta_{47}\}$, $\{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}$, $\{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}$, $\{\theta_{12}, \theta_{15}, \theta_{16}\}$, $\{\theta_{13}, \theta_{14}\}$, $\{\theta_{25}\}$, $\{\theta_{27}\}$, $\{\theta_{28}\}$, $\{\theta_{30}, \theta_{38}, \theta_{48}, \theta_{49}\}$, $\{\theta_{32}\}$, $\{\theta_{33}, \theta_{42}, \theta_{43}, \theta_{44}, \theta_{61}\}$, $\{\theta_{34}, \theta_{65}, \theta_{68}\}$, $\{\theta_{35}\}$, $\{\theta_{36}\}$, $\{\theta_{40}, \theta_{53}, \ldots, \theta_{59}\}$, $\{\theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\}$, $\{\theta_{62}\}$, $\{\theta_{74}\}$, $\{\theta_{75}, \theta_{99}, \theta_{116}\}$, $\{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}$, $\{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}$, $\{\theta_{83}, \theta_{87}\}$, $\{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}$, $\{\theta_{91}\}$, $\{\theta_{92}, \theta_{101}, \theta_{140}\}$, $\{\theta_{95}, \theta_{96}, \theta_{102}, \theta_{103}, \theta_{125}, \theta_{139}\}$, $\{\theta_{97}, \theta_{110}, \theta_{131}\}$, $\{\theta_{98}\}$, $\{\theta_{100}, \theta_{111}\}$, $\{\theta_{104}, \ldots, \theta_{109}, \theta_{112}, \theta_{142}\}$, $\{\theta_{118}\}$, $\{\theta_{120}, \theta_{124}\}$, $\{\theta_{119}, \theta_{123}, \theta_{127}\}$, $\{\theta_{126}\}$, $\{\theta_{135}\}$, $\{\theta_{128}, \theta_{129}, \theta_{130}, \theta_{132}, \theta_{133}, \theta_{134}, \theta_{136}, \theta_{143}\}$ $\Big\}$

Then, each piece of information considered in the test scenarios is modelled by a concept (a subset of Θ) using rough sets theory:

$A_1 = \{\theta_1, \theta_2, \ldots, \theta_{70}, \theta_{71}, \theta_{73}\}$
$A_2 = \{\theta_{11}, \theta_{19}, \theta_{63}\}$ $A_3 = \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{62}, \theta_{72}, \theta_{73}, \theta_{75}, \ldots, \theta_{127}, \theta_{131}, \theta_{135}, \theta_{137}, \ldots, \theta_{142}\}$
$A_4 = \{\theta_2, \theta_3, \theta_5, \theta_6, \theta_8, \theta_9, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \ldots, \theta_{31}, \theta_{34}, \theta_{36}, \theta_{39}, \theta_{45}, \ldots, \theta_{49}, \theta_{60}, \theta_{62}, \ldots, \theta_{71}, \theta_{73}, \theta_{95}, \theta_{96}, \theta_{125}, \theta_{131}\}$
$A_5 = \{\theta_{29}, \theta_{33}, \theta_{35}, \theta_{36}\}$
$A_6 = \{\theta_2, \theta_3, \theta_8, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{73}, \theta_{77}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{98}, \theta_{101}, \theta_{115}, \theta_{117}, \theta_{118}, \theta_{120}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{137}, \theta_{138}, \theta_{140}\}$
$A_7 = \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{46}, \theta_{47}, \theta_{63}\}$
$A_8 = \{\theta_{18}, \theta_{19}\}$
$A_9 = \{\theta_3, \theta_5, \theta_6, \theta_9, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{70}, \ldots, \theta_{73}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \theta_{98}, \theta_{101}, \theta_{113}, \ldots, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141}\}$
$A_{10} = \{\theta_{18}, \theta_{19}, \theta_{34}\}$
$A_{11} = \{\theta_{11}, \theta_{18}, \ldots, \theta_{24}, \theta_{30}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{36}, \theta_{45}, \ldots, \theta_{49}, \theta_{63}, \theta_{67}, \theta_{69}\}$
$A_{12} = \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{63}\}$
$A_{13} = \{\theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{73}, \theta_{75}, \theta_{91}, \theta_{92}, \theta_{97}, \theta_{99}, \theta_{102}, \ldots, \theta_{112}, \theta_{116}, \theta_{131}, \theta_{139}, \theta_{142}\}$
$A_{14} = \{\theta_7, \theta_{18}, \theta_{19}, \theta_{72}, \theta_{73}, \theta_{74}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{97}, \theta_{98}, \theta_{100}, \theta_{101}, \theta_{104}, \theta_{105}, \theta_{106}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141}\}$

The complete models for the test scenarios are presented in Annex C.3. Some examples for the lower and upper approximations of pieces of information A_2 and A_6 are given below:

$\underline{R}A_2 = \Big\{ \varnothing \Big\}$
$\overline{R}A_2 = \Big\{ \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\} \Big\}$
$\underline{R}A_6 = \Big\{ \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_{25}\}, \{\theta_{28}\}, \{\theta_{98}\},$
$\{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{118}\}, \{\theta_{126}\} \Big\}$
$\overline{R}A_6 = \Big\{ \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_{13},$
$\theta_{14}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{25}\}, \{\theta_{28}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}$
$, \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88},$
$\theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{98}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{119}, \theta_{123}, \theta_{127}\}, \{\theta_{118}\},$
$\{\theta_{126}\} \Big\}$

Let us denote by $A^{\cap}$ and $A^{\cup}$ the final information obtained using the intersection and the union of information in the first scenario:

$$A^{\cap} = A_1 \cap \ldots A_4 \cap A_6 \cap \cdots \cap A_{14} \tag{63}$$
$$A^{\cup} = A_1 \cup \ldots A_4 \cup A_6 \cup \cdots \cup A_{14} \tag{64}$$

The results of this first scenario are given by the lower and upper approximations:

$\underline{R}A^{\cap} = \Big\{ \varnothing \Big\}$
$\overline{R}A^{\cap} \subseteq \Big\{ \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\} \Big\}$

and

$\underline{R}A^{\cup} \supseteq \Big\{ \{\theta_1\}, \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{37},$
$\theta_{45}, \theta_{46}, \theta_{47}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{12},$
$\theta_{15}, \theta_{16}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_{25}\}, \{\theta_{27}\}, \{\theta_{28}\}, \{\theta_{30}, \theta_{38}, \theta_{48}, \theta_{49}\}, \{\theta_{32}\}, \{\theta_{33}, \theta_{42},$
$\theta_{43}, \theta_{44}, \theta_{61}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{35}\}, \{\theta_{36}\}, \{\theta_{40}, \theta_{53}, \ldots, \theta_{59}\}, \{\theta_{41}, \theta_{50}, \theta_{51},$
$\theta_{52}, \theta_{63}\}, \{\theta_{62}\}, \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \{\theta_{74}\}, \{\theta_{75},$
$\theta_{99}, \theta_{116}\} \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\},$
$\{\theta_{91}\}, \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{95}, \theta_{96}, \theta_{102}, \theta_{103}, \theta_{125}, \theta_{139}\}, \{\theta_{97}, \theta_{110}, \theta_{131}\}, \{\theta_{98}\},$
$\{\theta_{100}, \theta_{111}\}, \{\theta_{104}, \ldots, \theta_{109}, \theta_{112}, \theta_{142}\}, \{\theta_{118}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{119}, \theta_{123}, \theta_{127}\},$
$\{\theta_{126}\}, \{\theta_{135}\} \Big\}$
$\overline{R}A^{\cup} = \Big\{ \{\theta_1\}, \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{37},$
$\theta_{45}, \theta_{46}, \theta_{47}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{12},$
$\theta_{15}, \theta_{16}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_{25}\}, \{\theta_{27}\}, \{\theta_{28}\}, \{\theta_{30}, \theta_{38}, \theta_{48}, \theta_{49}\}, \{\theta_{32}\}, \{\theta_{33}, \theta_{42},$
$\theta_{43}, \theta_{44}, \theta_{61}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{35}\}, \{\theta_{36}\}, \{\theta_{40}, \theta_{53}, \ldots, \theta_{59}\}, \{\theta_{41}, \theta_{50}, \theta_{51},$
$\theta_{52}, \theta_{63}\}, \{\theta_{62}\}, \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \{\theta_{74}\}, \{\theta_{75},$
$\theta_{99}, \theta_{116}\} \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\},$
$\{\theta_{91}\}, \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{95}, \theta_{96}, \theta_{102}, \theta_{103}, \theta_{125}, \theta_{139}\}, \{\theta_{97}, \theta_{110}, \theta_{131}\}, \{\theta_{98}\},$

$\{\theta_{100},\, \theta_{111}\}$, $\{\theta_{104},\, \ldots,\, \theta_{109},\, \theta_{112},\, \theta_{142}\}$, $\{\theta_{118}\}$, $\{\theta_{120},\, \theta_{124}\}$, $\{\theta_{119},\, \theta_{123},\, \theta_{127}\}$, $\{\theta_{126}\}$, $\{\theta_{135}\}$ $\Big\}$

Since $\forall A \subseteq \Theta$, $\underline{R}A \subseteq \overline{R}A$ and considering the results obtained previously, we obtain $\underline{R}A^{\cup} = \overline{R}A^{\cup}$.

The result of the first scenario can be summarized as follows:

1. Using a conjunctive combination rule (the intersection), the identity of the observed object is probably one of the objects of the class $\{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}$ which belongs to $\overline{R}A^{\cap}$. No object can be identified since the subset $\underline{R}A^{\cap}$ is empty.

2. Using a disjunctive combination rule (the union), the identity of the observed object is certainly one of the objects of the class $\underline{R}A^{\cup} = \overline{R}A^{\cup}$. Like all other disjunctive combination rules, this one provides an imprecise result, which makes identification impossible.

The particular feature of the second scenario where a countermeasure is introduced at step #5 is that $\underline{R}A^{\cap} = \overline{R}A^{\cap} = \{\varnothing\}$. This allows us to conclude that the conjunctive combination rule is unable to eliminate the countermeasure from the fusion process, and invalidates any decision. So, rough sets theory does not seem to be useful in this special example of target identification, since it is unable to eliminate the countermeasure.

4.7 Conclusion

Probability theory is able to deal effectively with uncertainty problems like gambling situations. However, in problems where imprecision is involved, the modelization provided by the probability theory is inadequate. A consensus combination rule considers known *a priori* reliability degrees that one can assign to different sensors. Where reliability degrees are unavailable, the better method of modelling for combining pieces of information is to consider them as equally reliable.

Evidence theory models uncertain and imprecise information. The conjunctive combination rule used to combine different pieces of information produces more realistic results, compared to probability theory, as it offers a way to manage the conflict between two sources of information. The disjunctive combination rule avoids excessively drastic conclusions where sources are unreliable, but on the other hand it cannot converge towards a singleton, which means it cannot be used alone on target identification problems.

Vague information cannot be modelled in probability theory or evidence theory. Fuzzy sets theory is the appropriate framework for this kind of information. Fuzzy sets theory is particularly effective for modelling information supplied by human sources in natural language (*small length, high speed*, etc). As probability requires prior probability distributions, fuzzy sets theory requires prior membership functions for the different fuzzy classes.

Possibility theory was developed to deal with incompleteness, which is a particular case of uncertainty not addressed by probability theory. Incomplete pieces of information are fused in possibility theory using the same combination rules as in fuzzy sets theory. The combination rules need to be normalized, making them non-associative, which requires that the order of combination of the pieces of information must be considered.

Imprecise information having an unknown or unquantifiable uncertainty degree can be modelled in rough sets theory. In this theory, knowledge is represented by classes of indiscernible objects. A piece of information is then projeted on this partition, leading to the definition of lower and upper bounds, in the case where it does not fit perfectly into a cell of the partition. The test scenario with a countermeasure provides an irrelevant result.

5 Links between the different theories

5.1 Introduction

In the previous sections we considered raw information, and we were able to model it into the appropriate theory considering the information content itself or the situation we were dealing with. However, when different modules of a data fusion process need to be connected, the information is already modelled in a given theory such as those presented in Section 4. In this case, different theories may come into play, since the different modules do not necessarily have to deal with the same kind of information. In this section we present the most significant transformations between the theories under study so that users can build bridges between the different mathematical models. Another approach consists in defining a framework that is general enough to allow the different theories to be described within it. This will be discussed in Section 6.

Figure 20 presents the links between the different theories for representing and combining imperfect information. These links are examined beginning with probability theory and following the arrows in the order indicated.

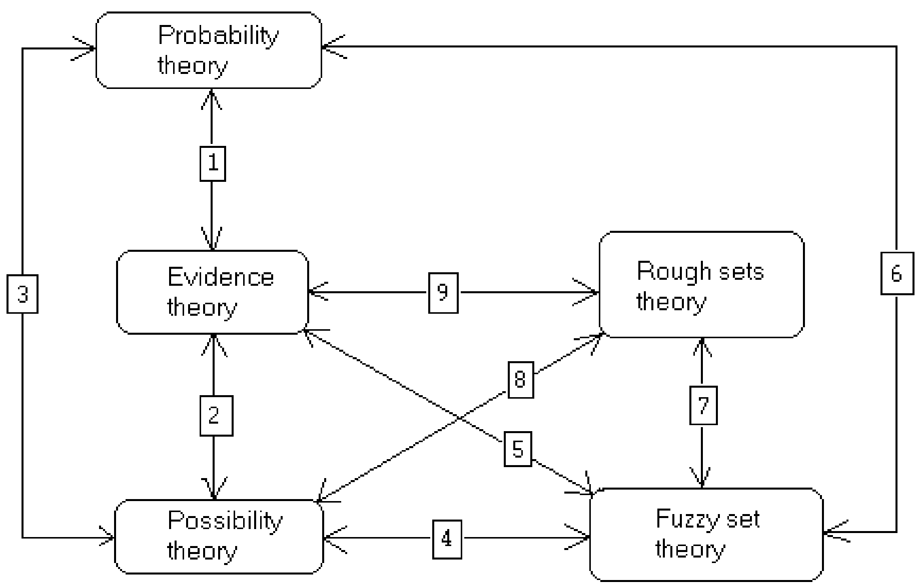

Figure 20: *Links between the different theories*

5.2 Probability theory/Evidence theory

Evidence theory can be seen as an extension of probability theory, since the latter does not consider the imprecision of imperfect information. While a probability distribution function characterizes only the set Θ (because of the additivity axiom), the BPA in evidence theory characterizes the entire power set 2^{Θ}. Because probability theory is a special case of evidence theory, no one-to-one correspondence exists between them and many transformations can therefore be defined. A transformation from evidence theory to probability theory necessarily involves a loss of information and, consequently, a loss of uncertainty.

1. **Pignistic transformation** (Smets [9]) is the most commonly used transformation from evidence theory to probability theory. This transformation distributes the BPA*bpa* of the imprecise subsets equally among the singletons composing the specified subset:

$$\mathrm{BetP}[\theta] = \sum_{\theta \in A,\, A \subseteq \Theta} \frac{m(A)}{\mathrm{card}(A)} \quad \forall \theta \in \Theta \tag{65}$$

Example 16 Let a piece of information be modelled in evidence theory by the BPA defined on the frame of discernment $\Theta_{10} = \{\theta_0, \theta_1, \theta_2, \ldots, \theta_9\}$:

$$\begin{cases} m(\{\theta_0, \theta_1, \theta_2\}) = 0.6 \\ m(\Theta_{10}) = 0.4 \end{cases} \tag{66}$$

Applying the pignistic transformation (equation (65)) leads to:

$$\begin{aligned} \mathrm{BetP}[\theta_0] = \mathrm{BetP}[\theta_1] = \mathrm{BetP}[\theta_2] &= \frac{0.6}{3} + \frac{0.4}{10} = 0.24 \\ \mathrm{BetP}[\theta_3] = \mathrm{BetP}[\theta_4] = \ldots = \mathrm{BetP}[\theta_9] &= \frac{0.4}{10} = 0.04 \end{aligned} \tag{67}$$

2. **Voorbraak transformation** (Voorbraak [10]) proposes to normalize the plausibility of singletons:

$$P_v[\theta] = \frac{\displaystyle\sum_{\theta \in A} m(A)}{\displaystyle\sum_{B \subseteq \Theta} m(B)\mathrm{card}(B)} \qquad \forall \theta \in \Theta \tag{68}$$

Example 17 Let us consider the basic probability assignment (equation (66)) from the previous example. Applying Voorbraak's transformation leads to:

$$\begin{aligned} P_v[\theta_0] = P_v[\theta_1] = P_v[\theta_2] &= \frac{1}{3 \times 1 + 7 \times 0.4} = 0.172 \\ P_v[\theta_3] = P_v[\theta_4] = \ldots = P_v[\theta_9] &= \frac{0.4}{3 \times 1 + 7 \times 0.4} = 0.069 \end{aligned} \tag{69}$$

Transformations from evidence theory to probability theory make several BPAs corresponding to one probability distribution function. If we perform the reverse transformation from probability theory to evidence theory, we get an infinite number of possible solutions. A Bayesian belief function $\mathrm{Bel}(\theta_i) = m(\theta_i) = P[\theta_i]$ can be used as a transformation from probability theory to evidence theory.

Sudano [24] proposes a way to construct a **consonant** BPA m (with the focal elements being nested sets) from a probability distribution function P defined on the set Θ. Let σ be the permutation of $\{1, 2, \ldots, N\}$ ($N = \mathrm{card}(\Theta)$) defined by:

$$P[\theta_{\sigma(1)}] \geq P[\theta_{\sigma(2)}] \geq \cdots \geq P[\theta_{\sigma(N)}]$$

The BPA m_s proposed by Sudano is given by:

$$\begin{aligned}
m_s(\{\theta_{\sigma(1)}\}) &= 1 \times \big(P[\theta_{\sigma(1)}] - P[\theta_{\sigma(2)}]\big) \\
m_s(\{\theta_{\sigma(1)}, \theta_{\sigma(2)}\}) &= 2 \times \big(P[\theta_{\sigma(2)}] - P[\theta_{\sigma(3)}]\big) \\
&\cdots\cdots\cdots \\
m_s(\{\theta_{\sigma(1)}, \theta_{\sigma(2)}, \ldots, \theta_{\sigma(N-1)}\}) &= (N-1) \times \big(P[\theta_{\sigma(N-1)}] - P[\theta_{\sigma(N)}]\big) \\
m_s(\{\theta_{\sigma(1)}, \theta_{\sigma(2)}, \ldots, \theta_{\sigma(N-1)}, \theta_{\sigma(N)}\}) &= N \times P[\theta_{\sigma(N)}]
\end{aligned}$$

5.3 Evidence theory/Possibility theory

Let m be a BPA and let π be a possibility distribution defined on the frame of discernment Θ. We consider $\{A_1, A_2, \ldots, A_N\}$ to be a subset of the power set 2^Θ, such as $A_1 \subseteq A_2 \subseteq \cdots \subseteq A_N$:

$$\begin{aligned}
&A_1 = \{\theta_{\sigma(1)}\} \\
&A_2 = \{\theta_{\sigma(1)}, \theta_{\sigma(2)}\} \\
&\ldots\ldots\ldots\ldots \\
&A_N = \{\theta_{\sigma(1)}, \theta_{\sigma(2)}, \ldots, \theta_{\sigma(N)}\}
\end{aligned}$$

where $\theta_1, \theta_2, \ldots, \theta_N$ are the singletons of Θ and σ is a permutation of $\{1, 2, \ldots, N\}$.

If the BPA is consonant, the plausibility of singletons is then:

$$\begin{aligned}
&\mathrm{Pl}(\{\theta_{\sigma(1)}\}) = m(A_1) + m(A_2) + m(A_3)... + m(A_N) = 1 \\
&\mathrm{Pl}(\{\theta_{\sigma(2)}\}) = m(A_2) + m(A_3)... + m(A_N) \\
&\ldots\ldots\ldots\ldots \\
&\mathrm{Pl}(\{\theta_{\sigma(N)}\}) = m(A_N)
\end{aligned}$$

So, we can find an order relation between these values:

$$1 = \mathrm{Pl}(\theta_{\sigma(1)}) \geq \mathrm{Pl}(\theta_{\sigma(2)}) \geq ... \geq \mathrm{Pl}(\theta_{\sigma(N)})$$

When the BPA is consonant, the plausibility becomes:

$$\mathrm{Pl}(A) = \max_{\theta_i \in A}\{\mathrm{Pl}(\theta_i)\}$$

Let $\pi(\theta)$ be a possibility distribution taking values in the interval [0,1], let $\theta_1, \theta_2, \ldots, \theta_N$ be the singletons of Θ, and let σ be a permutation of $\{1, 2, \ldots, N\}$ such that:

$$1 = \pi(\theta_{\sigma(1)}) \geq \pi(\theta_{\sigma(2)}) \geq \ldots \geq \pi(\theta_{\sigma(N)})$$

Comparing the couples of equations defining the two theories, Dubois and Prade [25, 21] conclude that possibility theory can be included in evidence theory, since the possibility and necessity degrees are the same as the plausibility and belief, defined for nested subsets. This is consistent with the frequentist interpretation of possibility theory [21] when the possibility measure is defined as an upper bound of the frequency of the probability.

Considering equality $\pi(\theta_i) = \mathrm{Pl}(\theta_i)$, it seems that the two couples of measures are equivalent. However, the two couples define the same characteristics for two different kinds of imperfection and they are not equivalent. The two theories deal with different aspects of imperfection, and the two mathematical models of plausibility and possibility do not deal with the same kinds of information, as argued by Smets [26] and Sudkamp [27].

However, even if the two theories are not equivalent, several transformations from one theory to the other are possible, as indicated below.

1. From evidence theory to possibility theory:

$$\pi(\theta_i) = \mathrm{Pl}(\theta_i) \tag{70}$$

   This transformation is valid only if the BPA is consonant. If not, the constraint imposed on the possibility distribution is not respected ($\exists \theta \in \Theta$ such that $\pi(\theta) = 1$). If the BPA is not consonant, we can use this transformation, which requires a normalization to obtain the final possibility distribution.

2. From possibility theory to evidence theory:

$$m(A_{\sigma(i)}) = \begin{cases} \pi(\theta_{\sigma(N)}) & \text{if} \quad i = N \\ \pi(\theta_{\sigma(i+1)}) - \pi(\theta_{\sigma(i)}) & \text{if} \quad i \neq N \end{cases} \tag{71}$$

Example 18 Let us consider a piece of information modelled in evidence theory by the BPA:

$$\begin{cases} m(\{\theta_1, \theta_2, \theta_3\}) = 0.4 \\ m(\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\}) = 0.2 \\ m(\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\}) = 0.3 \\ m(\Theta) = 0.1 \end{cases} \tag{72}$$

We calculate the plausibility value for each singleton of Θ using the transformation (70), and we obtain the following possibility distribution:

$$\begin{cases} \pi(\theta_1) = \pi(\theta_2) = \pi(\theta_3) = 1 \\ \pi(\theta_4) = \pi(\theta_5) = \pi(\theta_6) = 0.6 \\ \pi(\theta_7) = \pi(\theta_8) = 0.4 \\ \pi(\theta_9) = \pi(\theta_{10}) = 0.1 \end{cases} \tag{73}$$

Conversely, if we want to model the information from possibility theory into evidence theory, we can use the transformation (71), and we obtain the following BPA:

$$\begin{cases} m(\{\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7,\theta_8,\theta_9,\theta_{10}\}) & = \pi(\theta_{10}) = 0.1 \\ m(\{\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7,\theta_8,\theta_9\}) & = \pi(\theta_9) - \pi(\theta_{10}) = 0 \\ m(\{\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7,\theta_8\}) & = \pi(\theta_8) - \pi(\theta_9) = 0.3 \\ m(\{\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7\}) & = \pi(\theta_7) - \pi(\theta_8) = 0 \\ m(\{\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6\}) & = \pi(\theta_6) - \pi(\theta_7) = 0.2 \\ m(\{\theta_1,\theta_2,\theta_3,\theta_4,\theta_5\}) & = \pi(\theta_5) - \pi(\theta_6) = 0 \\ m(\{\theta_1,\theta_2,\theta_3,\theta_4\}) & = \pi(\theta_4) - \pi(\theta_5) = 0 \\ m(\{\theta_1,\theta_2,\theta_3\}) & = \pi(\theta_3) - \pi(\theta_4) = 0.4 \\ m(\{\theta_1,\theta_2\}) & = \pi(\theta_2) - \pi(\theta_3) = 0 \\ m(\{\theta_1\}) & = \pi(\theta_1) - \pi(\theta_2) = 0 \end{cases}$$

If the frame of discernment has a very large number of objects, the transformation of a possibility distribution can produce a BPA with a large number of focal elements, which makes it very difficult to manipulate. There is a way to limit the number of nested sets according to our needs. Let $\alpha \in [0,1]$ be a constant. We can construct the nested sets according to the constant α. Consider:

$$M = \left\lceil \frac{1}{\alpha} \right\rceil \tag{74}$$

where $\lceil x \rceil$ is the approximation of x by the nearest upper integer.

The number of nested sets is then lower than or equal to the maximum M. Then the BPA is given by:

$$\begin{cases} m(A_1) = \alpha & A_1 = \{\theta | \pi(\theta) \geq 1 - \alpha\} \\ m(A_2) = \alpha & A_2 = \{\theta | \pi(\theta) \geq 1 - 2\alpha\} \\ \dots & \dots \\ m(A_{M-1}) = \alpha & A_{M-1} = \{\theta | \pi(\theta) \geq 1 - (M-1)\alpha\} \\ m(A_M) = 1 - (M-1)\alpha & A_M = \Theta \end{cases} \tag{75}$$

In this manner, the complexity of the mathematical model has been reduced with a controlled loss of information.

Example 19 Let Θ be a frame of discernment with card(Θ) = 1000, and let π be a possibility distribution on Θ. Instead of using the classical transformation to calculate the equivalent information in evidence theory (equation (71)), we use the aforementioned approximation calculus using $\alpha = 0.15$. The final piece of information has only 7 nested sets:

$$\begin{cases} m(A_1) = 0.15 & A_1 = \{\theta|\pi(\theta) \geq 0.85\} \\ m(A_2) = 0.15 & A_2 = \{\theta|\pi(\theta) \geq 0.7\} \\ m(A_3) = 0.15 & A_3 = \{\theta|\pi(\theta) \geq 0.55\} \\ \dots & \dots \\ m(A_6) = 0.15 & A_6 = \{\theta|\pi(\theta) \geq 0.1\} \\ m(A_7) = 0.1 & A_7 = \Theta \end{cases} \tag{76}$$

5.4 Possibility theory/Probability theory

Evidence theory can be seen as a generalization of both probability theory and possibility theory. However, because these theories represent different kinds of information, it is sometimes useful to combine pieces of information that have been modelled in these two theories. Before defining transformations, we must find the permutation σ of $\{1, 2, \dots, N\}$, such that:

1. to pass from possibility theory to probability theory:

$$\pi(\theta_{\sigma(i)}) \geq \pi(\theta_{\sigma(i+1)}) \quad \forall i\ ,\ 1 \leq i \leq N-1 \tag{77}$$

2. to pass from probability theory to possibility theory:

$$P(\theta_{\sigma(i)}) \geq P(\theta_{\sigma(i+1)}) \quad \forall i\ ,\ 1 \leq i \leq N-1 \tag{78}$$

Several transformations have been proposed to achieve the link between these two theories:

1. The transformations put forward by Dubois and Prade are the most common in the literature:

$$\pi(\theta_{\sigma(i)}) = \sum_{j=1}^{N} \min\{P(\theta_{\sigma(i)}), P(\theta_{\sigma(j)})\} \tag{79}$$

$$P(\theta_{\sigma(i)}) = \sum_{j=i}^{N} \frac{\pi(\theta_{\sigma(j)}) - \pi(\theta_{\sigma(j+1)})}{j} \tag{80}$$

2. We can also recall the transformation of Klir and Parviz [11], which preserves the imperfection measure. For a probability distribution function, the entropy is considered to be the measure of imperfection:

$$H(P) = -\sum_{\theta \in \Theta} P(\theta) \log_2(P(\theta)) \tag{81}$$

For a possibility distribution, several measures have been proposed (Klir [1]) to quantify the imperfection. The non-specificity measure is defined as:

$$NS(\pi) = \sum_{i=2}^{N} \Big(\pi(\theta_{\sigma(i)}) - \pi(\theta_{\sigma(i+1)})\Big) \log_2 \frac{i^2}{\displaystyle\sum_{j=1}^{i} \pi(\theta_{\sigma(j)})} \tag{82}$$

Considering this, Klir and Parviz proposed the following transformation:

$$\pi(\theta_{\sigma(i)}) = \frac{P(\theta_{\sigma(i)})^{H(P)}}{P(\theta_{\sigma(1)})^{H(P)}} \tag{83}$$

$$P(\theta_{\sigma(i)}) = \frac{\pi(\theta_{\sigma(i)})^{1/NS(\pi)}}{\displaystyle\sum_{k=1}^{N} \pi(\theta_{\sigma(k)})^{1/NS(\pi)}} \tag{84}$$

3. The most common transformations, based on proportionality, are special cases of the equations (83) and (84):

$$\pi(\theta_{\sigma(i)}) = \frac{P(\theta_{\sigma(i)})}{P(\theta_{\sigma(1)})} \tag{85}$$

$$P(\theta_{\sigma(i)}) = \frac{\pi(\theta_{\sigma(i)})}{\pi(\theta_{\sigma(1)}) + \pi(\theta_{\sigma(2)}) + ... + \pi(\theta_{\sigma(N)})} \tag{86}$$

Besides all these transformations, Klir and Parviz [11] conclude that one theory is more robust (more refined) than the other. This seems to be a contradiction, and the authors conclude that while both theories deal with uncertainty, they analyze different aspects of uncertainty. The same idea holds in Zadeh's paper [7], where he defined possibility theory from fuzzy sets theory.

Example 20 Let us consider a frame of discernment of 10 objects $\Theta_{10} = \{\theta_1, \theta_2, \ldots, \theta_{10}\}$, and the probability and possibility distributions shown in Figure 21. We notice that the two distributions are not at all correlated, hence no ideal transformation can be performed between them. Therefore, if one of the distributions is unavailable, we cannot recover it from the other distribution using one of the transformations discussed

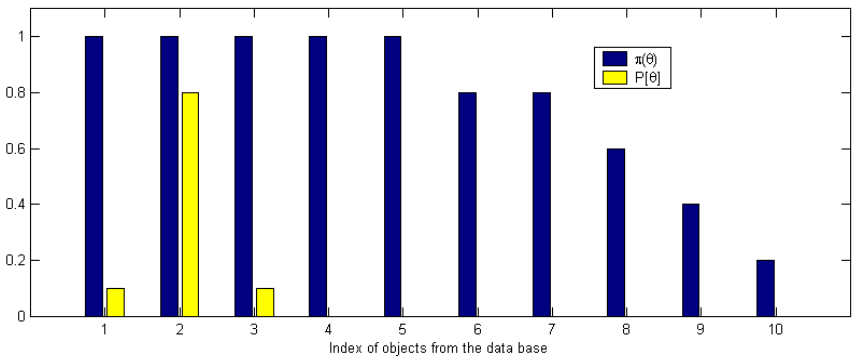

Figure 21: *Example of complementary probability and possibility distributions.*

above. Considering only one of the two distributions will result in a significant loss of information. Thus, we must consider the two pieces of information simultaneously if they are available, since they are complementary and non-redundant. Because the transformation between the two theories involves a loss of information, it must be considered only as a last resort.

5.5 Possibility theory/Fuzzy sets theory

Fuzzy sets theory and possibility theory are strongly related, since possibility theory was developed by Zadeh [7] on the basis of fuzzy sets theory, as indicated above in Section 4. If $\underline{A}$ is a fuzzy subset of Θ, and π is a possibility distribution over Θ, then we have:

$$\pi(\theta) = \mu_{\underline{A}}(\theta) \tag{87}$$

So if a piece of information is modelled by a fuzzy set $\underline{A}$, the same piece of information can also be modelled by a possibility distribution $\pi(\theta) = \mu_{\underline{A}}(\theta)$. Conversely, if a piece of information is modelled by a distribution possibility, $\pi(\theta)$, it can also be modelled by a fuzzy set $\underline{A}$, such that $\mu_{\underline{A}}(\theta) = \pi(\theta)$.

5.6 Fuzzy sets theory/Evidence theory

1. To find a link between fuzzy sets theory and evidence theory, we can design a composed transformation: first, we perform a transformation from fuzzy sets theory to possibility theory, and second, from possibility theory to evidence theory. So we have:

$$\mu(\theta) = \pi(\theta) \quad \forall \theta \in \Theta \tag{88}$$

$$\pi(\theta) = \mathrm{Pl}(\theta) \quad \forall \theta \in \Theta \tag{89}$$

implying that

$$\mu(\theta) = \mathrm{Pl}(\theta) \quad \forall \theta \in \Theta \tag{90}$$

2. A more complex transformation was proposed by Kampé de Fériet [28] in the early 1980s. He considered that the membership function can be seen either as a belief function or as a plausibility function, and he proposed a method for distinguishing between the two. He defined the restricted framework Θ_+ as the set of θ such that $\mu(\theta) > 0$. If Θ_+ is not countable, the membership function $\mu(\theta)$ is interpreted as a plausibility function. If Θ_+ is countable, the membership function is interpreted according to the quantity $\sum_{\theta \in \Theta_+} \mu(\theta)$. If this quantity is greater than 1, the membership function $\mu(\theta)$ is considered as a plausibility function. If not, it is considered as a belief function.

3. Another link between the two theories is made from the α-cuts representation of the membership function of a fuzzy set. Let $\{\alpha_i\}_{i=1}^{i=M}$ be a set of ordered values from the interval [0,1], $1 = \alpha_0 > \alpha_1 > \ldots > \alpha_M = 0$, and let $A_{\alpha_i} \subseteq \Theta$ be subsets of Θ such that $A_{\alpha_i} = \{\theta \,|\, \theta \in \Theta\,,\, \mu(\theta) \geq \alpha_i\}$. We can define a BPA from the α_i values, given by:

$$m(A_{\alpha_i}) = \alpha_{i-1} - \alpha_i \quad \forall i\ ,\ 1 \leq i \leq N \tag{91}$$

5.7 Fuzzy sets theory/Probability theory

Fuzzy sets theory and probability theory do not deal with the same kind of imperfection. Probability theory deals with the membership of an uncertain element in a certain subset, while fuzzy sets theory deals with the membership of a certain element in an uncertain subset (ill-defined) subset. A transformation between the two theories can be performed using the following equation:

$$\mu_{\underline{A}}(\theta) = P[\underline{A}|\theta] \tag{92}$$

where $P[\underline{A}|\theta]$ is the conditional probability of $\underline{A}$ knowing θ (Dubois and Prade [22]). The same authors considered that probability theory and fuzzy sets theory have several common traits but also some distinguishing traits, possibility theory being placed between them (a view shared by Klir and Parviz [11]).

5.8 Fuzzy sets theory/Rough sets theory

Pawlak [29] compared rough sets and fuzzy sets. He showed that the standard union and the intersection defined for the membership function in Zadeh's fuzzy sets theory [4], have no equivalent operations in rough sets theory. He concluded that rough sets is a more general concept than fuzzy sets. According to Pawlak, if the couple of equations (58) and (59):

$$\overline{R}(X \cap Y) \subseteq \overline{R}X \cap \overline{R}Y$$
$$\underline{R}(X \cup Y) \supseteq \underline{R}X \cup \underline{R}Y$$

yield equality, rough sets theory is reduced to fuzzy sets theory.

5.9 Rough sets theory/Possibility theory

Dubois and Prade [30, 31] consider that Pawlak's view [29] is incorrect and that rough sets and fuzzy sets must not be regarded as rival theories, since they deal with different kinds of imperfection. Rough sets theory deals with the indiscernible character of information, while fuzzy sets theory deals with vague information. But it is possible to encounter both imperfections in the same situation. A hybrid of the two theories could deal with more complex imperfect information situations. So the concepts of *fuzzy rough set* and *rough fuzzy set* were introduced.

Definition 22 Let Θ be a frame of discernment and let R be a knowledge of Θ. Instead of a crisp concept of Θ, we consider a fuzzy concept $\underline{A}$. A **fuzzy rough set** is the couple $(\underline{RA}, \overline{R}\underline{A})$ given by:

$$\mu_{\underline{RA}}(\theta_i) = \inf\{\mu(\theta) | [\theta]_R\} \tag{93}$$

$$\mu_{\overline{R}\underline{A}}(\theta_i) = \sup\{\mu(\theta) | [\theta]_R\} \tag{94}$$

It has been shown [30, 31] that equations (93) and (94) are the same equations describing the **degree of possibility** and the **degree of necessity**, respectively, of a fuzzy event. However, these equations are the basis for the C-calculi where the concept of C-set is the same as that of a fuzzy rough set. It appears, therefore, that the concept of rough sets was introduced before Pawlak but under different forms.

5.10 Rough sets theory/Evidence theory

An important link between rough sets theory and evidence theory was described by Skowron [12] and Skowron and Grzymala-Busse [32]. They demonstrated that any problem modelled in rough sets theory can also be modelled in evidence theory, and they defined a transformation between the two theories.

Definition 23 Consider a frame of discernment Θ, a knowledge R and a concept A, with its lower ($\underline{R}A$) and upper bounds ($\overline{R}A$). The belief and the plausibility functions are defined by:

$$\mathrm{Bel}(A) = \frac{\mathrm{card}(\underline{R}A)}{\mathrm{card}(\Theta)} = \underline{k}(A) \tag{95}$$

$$\mathrm{Pl}(A) = \frac{\mathrm{card}(\overline{R}A)}{\mathrm{card}(\Theta)} = \overline{k}(A) \tag{96}$$

Definition 24 In rough sets theory, the functions $\underline{k}(A)$ and $\overline{k}(A)$ are called the **the lower quality function** and **the upper quality function**, respectively.

Note that this transformation is not reversible.

5.11 Conclusion

The classical theories for representing uncertainty deal with different aspects of the imperfection of information. Rather than being rivals, they should be considered as complementary and used to model information better. In this section we presented some transformations between these theories, although almost all of them resulted in a loss of information. Consequently, while these transformations are sometimes necessary, they should be performed with care to minimize information loss.

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6 Random sets theory: a unifying framework for fusing imperfect information

6.1 Introduction

Each of the theories presented in section 4 are well suited to one particular kind of imperfect information. In practice, complex situations involve the processing of several pieces of information, each exhibiting one or more types of imperfection. Therefore, no single theory can deal alone with these complex situations. Rather than performing transformations between these theories, an alternative solution is to employ a unifying framework in which each theory is a special case. In this section, we study random sets theory and its capabilities as a unifying framework.

The bases of random sets theory were set down by Mathéron [33] and Kendall [34] in the mid-1970s to study integral geometry. Since then, several authors have tried to demonstrate the unifying capability of random sets theory in data fusion problems (Kreinovich [35], Mori [36], Goodman et al. [37], Quinio etand Matsuyama [38]). Most of the classical theories for reasoning under uncertainty can be considered as particular cases of random sets theory, which is able to model all types of imperfect information. The statistical framework of random sets theory can model and has efficient tools for combining different pieces of information, and can even be used to solve other steps in data fusion problems (Goodman et al. [37]):

1. target detection;
2. target identification;
3. target tracking and locating;
4. modelling of *a priori* information for detection, classification and tracking tasks;
5. modelling of perfect and imperfect information;
6. definition of combination rules;
7. sensor management.

In this section, random sets theory is briefly presented. The links between random sets theory and the theories discussed in Section 4 are also shown, and an example of an application to the target identification problem is studied.

6.2 Theory description

The concept of random set can be seen as a generalization of the concepts of random variable and random vector A probability distribution is defined on the power set

2^Θ, instead of the set Θ as in probability theory. The following study is limited to a discrete and finite frame of discernment.

Definition 25 Let Θ be a discrete and finite frame of discernment. Let also $(\Omega,\mathbf{A}, \mathcal{P})$ be a probabilistic space and let $(\boldsymbol{B},\sigma(\boldsymbol{B}))$ be a measurable space, where $\boldsymbol{B}\subseteq 2^\Theta$ and $\sigma(\boldsymbol{B})$ is a σ-algebra over $\boldsymbol{B}$. A mapping $\mathcal{X} : \Omega \to \boldsymbol{B}$, being$\mathcal{A}$-$\sigma(\boldsymbol{B})$ measurable, is called a **random set**.

A random set on Θ is therefore characterized by a probability distribution defined on $(\boldsymbol{B}, \sigma(\boldsymbol{B}))$.

For example, for a discrete and finite frame of discernment Θ, if $\boldsymbol{B} = 2^\Theta$ and if $\sigma(\boldsymbol{B})$ is the power set of 2^Θ (2^{2^Θ}), any probability measure defined by a probability distribution $f : 2^\Theta \to [0, 1]$ with $f(A) = P[\mathcal{X} = A]$ $\forall A \subseteq \Theta$ defines a random set $\mathcal{X}$ on Θ.

This probability distribution entirely characterizes the random set. Moreover, it has the same properties as any probability distribution defined for a random variable:

$$P[\mathcal{X} = A] \in [0, 1] \tag{97}$$

$$P[\varnothing] = 0 \tag{98}$$

$$\sum_{A\subseteq\Theta} P[\mathcal{X} = A] = 1 \tag{99}$$

Three other functions, namely **hitting capacity** $T_\mathcal{X}(A)$, **implying functional** $R_\mathcal{X}(A)$, and **inclusion capacity** $P_\mathcal{X}(A)$, also entirely characterize a random set $\mathcal{X}$:

$$T_\mathcal{X}(A) = P[\mathcal{X} \cap A \neq \varnothing] \tag{100}$$

$$R_\mathcal{X}(A) = P[A \subseteq \mathcal{X}] \tag{101}$$

$$P_\mathcal{X}(A) = P[\mathcal{X} \subseteq A] \tag{102}$$

In a simplified version (a Dempster-Shafer version), a random set $\mathcal{X}$ is represented as a set of couples:

$$\mathcal{X} = \{A_i, m_i\} \quad \forall i\ ,\ 1 \leq i \leq 2^N \tag{103}$$

such as such that $A_i \subseteq \Theta$ and $\displaystyle\sum_{i=1}^{2^N} m_i = \sum_{i=1}^{2^N} P[\mathcal{X} = A_i] = 1$, where $N = \mathrm{card}(\Theta)$.

6.3 Representation of classical theories in the random sets formalism

In the following paragraphs, we show how information represented in the classical theories discussed in Section 4 can be expressed in random sets theory.

6.3.1 Evidence theory

Quinio and Matsuyama [38] showed that the plausibility function Pl defined on a finite frame of discernment is exactly the hitting capacity $T_{\mathcal{X}}$. Moreover, the belief function Bel corresponds to the inclusion capacity $P_{\mathcal{X}}$ and the commonality function Q corresponds to the implying functional $R_{\mathcal{X}}$.

Let us consider two random sets $\mathcal{X}_1$ and $\mathcal{X}_2$, and assume that the event $\big\{\mathcal{X}_1 \cap \mathcal{X}_2 = \varnothing\big\}$ has a nil probability (closed-world hypothesis). We consider that the event $\big\{\mathcal{X}_1 \cap \mathcal{X}_2 = A\big\}$ is replaced by $\big\{\mathcal{X}_1 \cap \mathcal{X}_2 = A \,|\, \mathcal{X}_1 \cap \mathcal{X}_2 \neq \varnothing\big\}$, to normalize the final probability distribution. So the intersection of the hitting capacity of the two random sets is given by:

$$\begin{aligned} T_{\mathcal{X}_1 \cap \mathcal{X}_2 | \mathcal{X}_1 \cap \mathcal{X}_2 \neq \varnothing}(D) &= P[(\mathcal{X}_1 \cap \mathcal{X}_2) \cap D \neq \varnothing | \mathcal{X}_1 \cap \mathcal{X}_2 \neq \varnothing] \\ &= \frac{P[(\mathcal{X}_1 \cap \mathcal{X}_2) \cap D \neq \varnothing]}{P[\mathcal{X}_1 \cap \mathcal{X}_2 \neq \varnothing]} \\ &= \frac{\displaystyle\sum_{A \cap D \neq \varnothing} P[\mathcal{X}_1 \cap \mathcal{X}_2 = A]}{P[\mathcal{X}_1 \cap \mathcal{X}_2 \neq \varnothing]} \\ &= \frac{\displaystyle\sum_{A \cap D \neq \varnothing} \Big(\sum_{B \cap C = A} P[\mathcal{X}_1 = B \,,\, \mathcal{X}_2 = C] \Big)}{1 - \displaystyle\sum_{B \cap C = \varnothing} P[\mathcal{X}_1 = B \,,\, \mathcal{X}_2 = C]} \end{aligned} \tag{104}$$

Dempster's rule of combination is a particular case of the intersection in random sets theory when the two random sets $\mathcal{X}_1$ and $\mathcal{X}_2$ are statistically independent. Equation (104) in this case becomes:

$$T_{\mathcal{X}_1 \oplus \mathcal{X}_2}(K) = \frac{\displaystyle\sum_{A \cap K \neq \varnothing} \Big(\sum_{B \cap C = A} P[\mathcal{X}_1 = B] P[\mathcal{X}_2 = C] \Big)}{1 - \displaystyle\sum_{B \cap C = \varnothing} P[\mathcal{X}_1 = B] P[\mathcal{X}_2 = C]} \tag{105}$$

which is Dempster's rule of combination defined in evidence theory.

Evidence theory has an identical information representation as the simplified random sets representation (Nguyen [13], Nguyen and Wang [39]) given by the equation (103). So, the transformation between the two theories is immediate:

$$m_i = m(A_i) = P[\mathcal{X} = A_i] \qquad \forall A_i \subseteq \Theta$$

6.3.2 Probability theory

If random sets are a generalization of random variables and random vectors, then in probability theory random sets are reduced to singletons (sets with only one element).

A piece of information modelled in probability theory by a probability distribution function $P[\theta_i]$, $\forall \theta_i \in \Theta$, is modelled in random sets theory by the set of couples $\{(A_i, m_i)\}$, given by:

$$A_i = \theta_i, \quad \text{card}(A_i) = 1 \tag{106}$$

$$m_i = P[\theta_i] \tag{107}$$

6.3.3 Possibility theory

Let us consider a piece of information modelled in possibility theory by a discrete possibility distribution $\pi(\theta_i) \in [0,1]$, $\forall \theta_i \in \Theta$, and let $0 < \alpha_1 < \alpha_2 < < \alpha_3 < \ldots < \alpha_M = 1$ be the values of that possibility distribution in the interval [0, 1] ($M \leq N = \text{card}(\Theta)$). We recall that a possibility distribution is defined such that at least one element of the frame of discernment has a unity possibility degree ($\exists \theta_k \in \Theta | \pi(\theta_k) = 1$). The number of realizations of the random set $\mathcal{X}$ is also equal to M. We can now construct the equations defining the optimal transformation (i.e., without any approximation) of a piece of information modelled in possibility theory to an information modelled in random sets theory (Dubois and Prade [40]):

$$A_i = \{\theta_j | \pi(\theta_j) \geq \alpha_i\} \quad \forall i\ ,\ 1 \leq i \leq M \tag{108}$$

$$m_i = \begin{cases} \alpha_i - \alpha_{i-1} & \forall i\ ,\ 2 \leq i \leq M \\ \alpha_1 & i = 1 \end{cases} \tag{109}$$

If the number M of nested sets is too large, we can reduce it according to some α_i values. With the new α_i values, Equation (108) is still valid for computing the A_i sets; however, the values m_i given by Equation (109) must be replaced by:

$$m_i = \begin{cases} a_i - a_{i-1} & \forall i\ ,\ 2 \leq i \leq M \\ a_1 & i = 1 \end{cases} \tag{110}$$

where

$$\begin{cases} a_M = 1 \\ a_i = \max\{\mu(\theta_j) | \theta_j \in \Theta \setminus A_i\} \quad \forall i, 1 \leq i \leq M-1 \end{cases} \tag{111}$$

When no approximation is required, we let $a_i = \alpha_i \quad \forall i$.

When the chosen values of α_i are such that there are k levels having no $\theta_i \in \Theta$ with $\pi(\theta_i) \in [\alpha_{k-1}, \alpha_k]$, we eliminate the corresponding α_k value from the $\{\alpha_i\}$ set and we apply Equations (108) and (110) to the reduced set of $\{\alpha_i\}$.

6.3.4 Fuzzy sets theory

Let $0 < \alpha_1 < \alpha_2 < \alpha_3 < \ldots < \alpha_M \leq 1$ be the different values of the membership function $\mu_{\underline{A}(\theta)}$ in the interval $[0, 1]$ ($M \leq \text{card}(\Theta)$). The transformation of a piece of information modelled in fuzzy sets theory into a piece of information modelled in the random sets formalism (Goodman [15], Orlov [14]) is equivalent to the transformation of a possibility distribution into the random sets formalism. The only difference is that the set of values $\{m_i\}$ must be normalized (according to the α_M value). So we have:

$$A_i = \{\theta_j | \mu(\theta_j) \geq \alpha_i\} \quad \forall i\,,\, 1 \leq i \leq M \tag{112}$$

$$m_i = \begin{cases} \dfrac{\alpha_i - \alpha_{i-1}}{\alpha_M} & \forall i\,,\, 2 \leq i \leq M \\ \dfrac{\alpha_1}{\alpha_M} & i = 1 \end{cases} \tag{113}$$

As in the previous section, if we choose a set of ordered values $\{\alpha_i\}$ from the interval $[0, 1]$, we can still use Equation (112) to find the nested sets A_i. However, the values of m_i defined by Equation (113) must be replaced by:

$$m_i = \begin{cases} \dfrac{a_i - a_{i-1}}{a_M} & \forall i\,,\, 2 \leq i \leq M \\ \dfrac{a_1}{a_M} & i = 1 \end{cases} \tag{114}$$

where

$$\begin{cases} a_M = \max\{\mu(\theta_j) | \theta_j \in \Theta\} \\ a_i = \max\{\mu(\theta_j) | \theta_j \in \Theta \setminus A_i\} \quad \forall i, 1 \leq i \leq M-1 \end{cases} \tag{115}$$

When no approximation is required, we let $a_i = \alpha_i \quad \forall i$.

6.3.5 Rough sets theory

To the best of our knowledge, the only link to date between random sets theory and rough sets theory was made by means of evidence theory. Other avenues remain an open field of research.

Table 19 recapitulates the transformations from classical theories of uncertainty to random sets theory.

6.4 Scenarios study

Table 20 shows the random sets modelization of the pieces of information used with the two test scenarios. The third column, headed “Theory”, indicates the classical theory that is best suited to the imperfect information involved.

Information modelled from fuzzy sets theory and from possibility theory are not presented in this document, since their mathematical representation is highly complex. Those pieces of information are obtained using an optimal transformation from their initial models.

Classical theory	Information model	Transformation to random sets theory
Probability theory	$P[\theta_i] \in [0,1] \quad \forall \theta_i \in \Theta$ $\sum_{\theta_i \in \Theta} P[\theta_i] = 1$	$A_i = \theta_i \quad \mathrm{card}(A_i) = 1$ $m_i = P[\theta_i]$
Evidence theory	$m(A_i) \in [0,1] \quad \forall A_i \subseteq \Theta$ $\sum_{A_i \subseteq \Theta} m(A_i) = 1$	$A_i = A_i$ $m_i = m(A_i)$
Possibility theory	$\pi(\theta_i) \in [0,1] \quad \forall \theta_i \in \Theta$ $\exists \theta_k \in \Theta$ such that $\pi(\theta_k) = 1$	$A_i = \{\theta_j \vert \pi(\theta_j) \geq \alpha_i\} \quad \forall i\ ,\ 1 \leq i \leq M$ Ideal transformation: $m_i = \begin{cases} \alpha_i - \alpha_{i-1} & \forall i\ ,\ 2 \leq i \leq M \\ \alpha_1 & i = 1 \end{cases}$ Approximated transformation using a fixed $\{\alpha_i\}$ set: $m_i = \begin{cases} a_i - a_{i-1} & \forall i, 2 \leq i \leq M \\ a_1 & i = 1 \end{cases}$ with $\begin{cases} a_M = 1 \\ a_i = \max\{\mu(\theta_j) \vert \theta_j \in \Theta \setminus A_i\} \end{cases}$
Fuzzy sets theory	$\mu(\theta_i) \in [0,1] \quad \forall \theta_i \in \Theta$	$A_i = \{\theta_j \vert \mu(\theta_j) \geq \alpha_i\} \quad \forall i\ ,\ 1 \leq i \leq M$ Ideal transformation: $m_i = \begin{cases} \frac{\alpha_i - \alpha_{i-1}}{\alpha_M} & \forall i\ ,\ 2 \leq i \leq M \\ \frac{\alpha_1}{\alpha_M} & i = 1 \end{cases}$ Approximated transformation using a fixed $\{\alpha_i\}$ set: $m_i = \begin{cases} \frac{a_i - a_{i-1}}{a_M} & \forall i, 2 \leq i \leq M \\ \frac{a_1}{a_M} & i = 1 \end{cases}$ with $\begin{cases} a_M = \max\{\mu(\theta_j) \vert \theta_j \in \Theta\} \\ a_i = \max\{\mu(\theta_j) \vert \theta_j \in \Theta \setminus A_i\} \end{cases}$

Table 19: *Random sets model of information transformed from the classical theories.*

Instant	Information	Theory	Random sets model
1	Target type = ship	probability	$m_i = 0.011111 \quad A_i = \theta_i \quad \forall i \in \{1, 2, 3, \ldots, 70, 71, 73\}$ $m_i = 0.002857 \quad A_i = \theta_i \quad \forall i \in \{72, 74, 75, \ldots, 141, 142\}$
2	Emitter 44 on board	evidence	$m_1 = 0.800000 \quad A_1 = \{\theta_{11}, \theta_{19}, \theta_{63}\}$ $m_2 = 0.200000 \quad A_2 = \Theta$
3	Small length	fuzzy sets	ideal transformation of $\mu_{\text{small length}}$ (figure B.1)
4	RCS_{side} medium	fuzzy sets	Ideal transformation of $\mu_{\text{RCS-side medium}}$ (figure B.5)
5	Emitter 77 on board	evidence	$m_1 = 0.800000 \quad A_1 = \{\theta_{29}, \theta_{33}, \theta_{35}, \theta_{36}\}$ $m_2 = 0.200000 \quad A_2 = \Theta$
6	Small height	fuzzy sets	Ideal transformation of $\mu_{\text{small height}}$ (figure B.3)
7	Emitter 47 on board	evidence	$m_1 = 0.800000 \quad A_1 = \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{46}, \theta_{47}, \theta_{63}\}$ $m_2 = 0.200000 \quad A_2 = \Theta$
8	Emitter 55 on board	evidence	$m_1 = 0.800000 \quad A_1 = \{\theta_{18}, \theta_{19}\}$ $m_2 = 0.200000 \quad A_2 = \Theta$
9	Small width	fuzzy sets	Ideal transformation of $\mu_{\text{small width}}$ (figure B.2)
10	Emitter 56 on board	evidence	$m_1 = 0.800000 \quad A_1 = \{\theta_{18}, \theta_{19}, \theta_{34}\}$ $m_2 = 0.200000 \quad A_2 = \Theta$
11	Emitter 103 on board	evidence	$m_1 = 0.800000 \quad A_1 = \{\theta_{11}, \theta_{18}, \ldots, \theta_{24}, \theta_{30}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{36}, \theta_{45}, \ldots, \theta_{49}, \theta_{63}, \theta_{67}, \theta_{69}\}$ $m_2 = 0.200000 \quad A_2 = \Theta$
12	Emitter 109 on board	evidence	$m_1 = 0.800000 \quad A_1 = \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{63}\}$ $m_2 = 0.200000 \quad A_2 = \Theta$
13	RCS_{top} small	fuzzy sets	Ideal transformation of $\mu_{\text{RCS-top small}}$ (figure B.6)
14	$\text{RCS}_{\text{front}}$ very small	fuzzy sets	Ideal transformation of $\mu_{\text{RCS-front very small}}$ (figure B.8)

Table 20: *Information used in the test scenarios modelled in random sets theory.*

Figures 22, 23 and 24 show the results of the fusion process, first using an optimal transformation of the information coming from fuzzy sets theory, and second using two different approximations (with 10 and 5 α-cuts, respectively):

1.	$\alpha_1 = 0.1$	$\alpha_2 = 0.2$	$\alpha_3 = 0.3$	$\alpha_4 = 0.4$	$\alpha_5 = 0.5$
	$\alpha_6 = 0.6$	$\alpha_7 = 0.7$	$\alpha_8 = 0.8$	$\alpha_9 = 0.9$	$\alpha_{10} = 1$
2.	$\alpha_1 = 0.1$	$\alpha_2 = 0.4$	$\alpha_3 = 0.6$	$\alpha_4 = 0.8$	$\alpha_5 = 1$

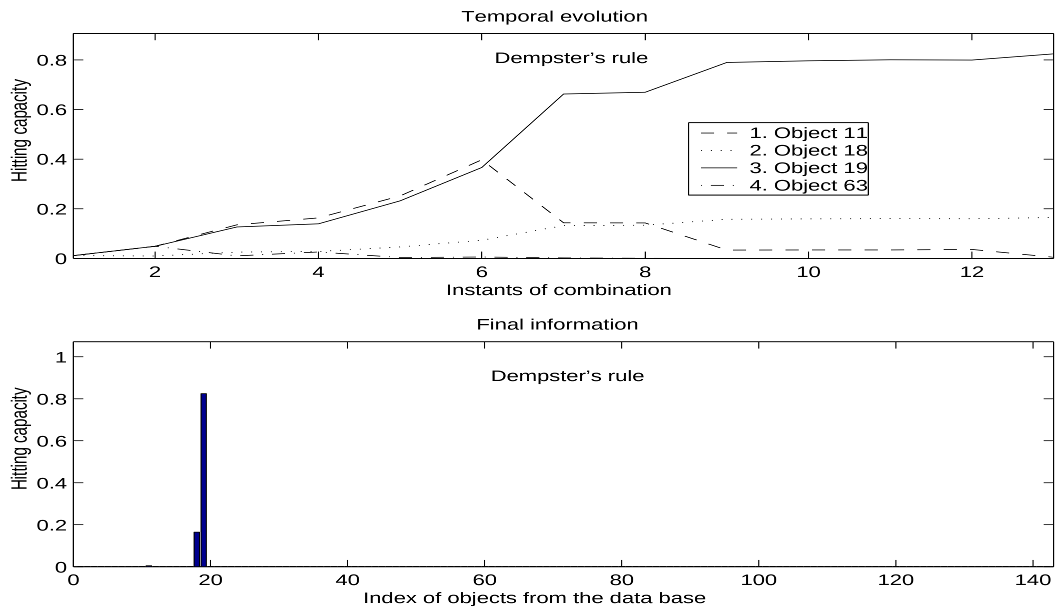

Figure 22: *Random sets theory - Test scenario 1 - Transformation without approximation.*

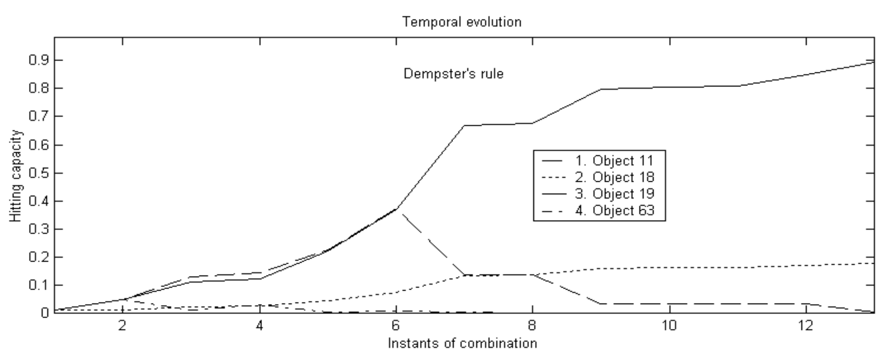

Figure 23: *Random sets theory - Test scenario 1 - Transformation with approximation: 10 α-cuts.*

These three figures show similar results (Figure 25), but the test scenario used was probably not complex enough to challenge the comparison task. We note, however,

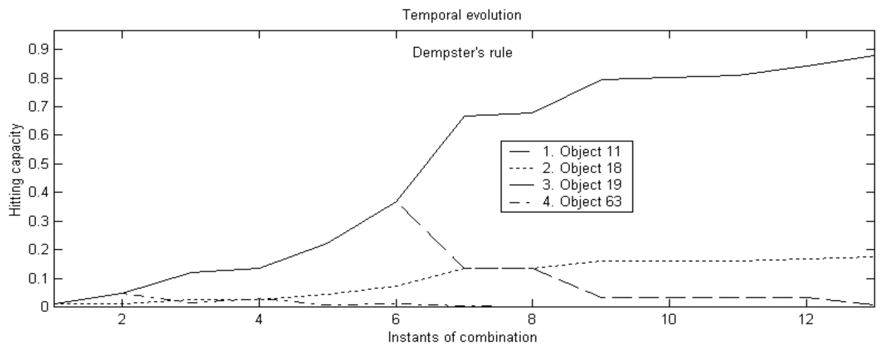

Figure 24: *Random sets theory - Test scenario 1 - Transformation with approximation: 5 α-cuts.*

that using a transformation without approximation leads to more precise results. As expected, using an approximate transformation with 10 α-cuts yields closer results than those obtained using a transformation without approximation. Still, with an optimal transformation, the algorithm is very costly in computing time, and therefore a balance must be struck between quality of approximation and computational burden.

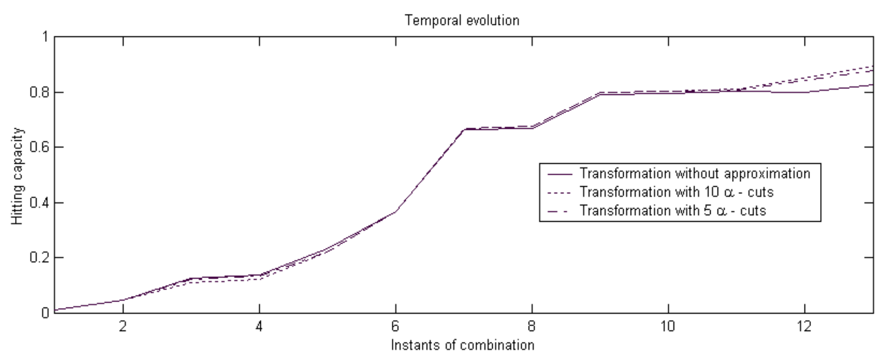

Figure 25: *Random sets theory - Test scenario 1 - Comparison of results for database object #19*

Table 21 compares computing times for the data fusion process using the first test scenario, between a transformation without approximation and two transformations with approximation (one 10 α-cuts, the other 5 α-cuts).

Transformation	Flops	Reduction to (in %)
without approximation	575 678 261	100 %
with approximation : 10 α-cuts	15 776 204	2.75 %
with approximation : 5 α-cuts	5 824 933	1 %

Table 21: *Comparison of computing times (in flops)*

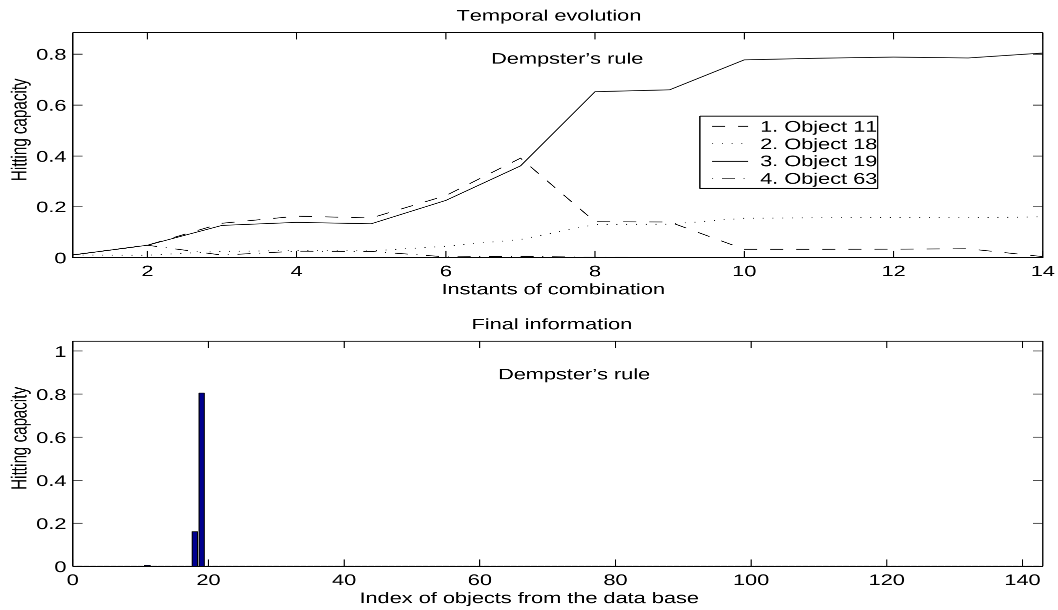

Figure 26: *Random sets theory - Test scenario 2 - Transformation without approximation*

The second scenario (includuing a countermeasure in the fusion process) produces results similar to those of the first scenario (Figure 26). The confidence level assigned to Object #19 is only barely reduced and a precise decision can be made.

6.5 Conclusion

Random sets theory allows several kinds of imperfect information to be modelled in a unified framework. Random sets theory is very similar to evidence theory, and in problems of modelling and combining information it can be reduced to evidence theory. Moreover, random sets theory can be used to tackle several problems in the data fusion process (like target detection, target locating, target tracking, sensor management, etc.) for multi-sensor, multi-target applications. This makes it a good candidate to unify not only the theories for representing and combining imperfect information, but also the theories of finite set statistics, which deal with the problem of target identification, including tracking and association (Goodman et al. [37]).

7 Conclusions

In this study we examined different modelling methods for imperfect information. Each of the classical theories, namely probability, evidence, fuzzy sets, possibility and rough sets theory, can model different aspects of information imperfection - uncertainty, imprecision, incompleteness, fuzziness. In a general manner, an information fusion system receives pieces of information from different types of sources such as electronic devices (radars, ESM, IFF, etc.), as well as expert opinions expressed in natural language, resulting in a wide variety of imperfections in the pieces of information to be fused (uncertainty, vagueness, incompleteness). Each classical theory is well suited to particular types of imperfection, but none of them is able to model all types of uncertainty. Probability theory can model uncertainty well but cannot model imprecision. Evidence theory can model imprecise and uncertain information. Fuzziness is only represented correctly by fuzzy sets theory; neither probability nor evidence theory can do this. Possibility theory provides a good means of modelling incompleteness. Vagueness and indistinguishability are modelled by rough sets theory. Each of these theories offers a mathematical formalism to model ignorance, as well as several combination rules and decision rules.

Combination rules can be segregated into two categories: conjunctive and disjunctive rules. Conjunctive rules produce good results if the pieces of information to be fused are not in conflict. Moreover, an associative conjunctive rule like Dempster's rule has the ability to eliminate some countermeasures. In the case of fuzzy sets, possibility theory and rough sets theory, it was not possible to eliminate the countermeasure. Disjunctive rules are usually used when the pieces of information to be fused are contradictory (because some sensors are not reliable). The disjunctive rules used in the test scenarios yielded several alternatives for best candidate for the observed target, hence allowing a decision to be made on a single object. In this sense, it is not recommended that disjunctive rules be used alone in target identification systems. However, disjunctive rules display an interesting behaviour in adaptive rules. They act as a conjunctive rule when the pieces of information are not in conflict (assuming that both are reliable) and as a disjunctive rule when they are in conflict (assuming at least one of them is unreliable). The second test scenario showed that this combination method can eliminate the countermeasure, but unfortunately it also eliminates the difference between degrees of confidence (membership or possibility) assigned to the most representative objects, thereby precluding a decision.

In some data fusion applications, when different modules of processing need to be connected, some transformations between theories are required. If each module is designed separately to handle a specific type of imperfect information better, then to build a whole system we need to transform some pieces of information from one theory to another. Most of these transformations are not one-to-one correspondences, and they often result in a loss of information. In this study, no effort was made to quantify the loss induced by each transformation, but that could be undertaken in future work.

An alternative to transformations between theories is a unifying framework in which each kind of imperfect information can be modelled. The framework would then be used to design the different modules of a fusion system, and no transformation is required to make connections between the modules. In this study we assessed the ability of random sets theory to perform this task in the target identification application. Imperfect information, be it imprecise, uncertain, fuzzy, incomplete or inconsistent, can be represented in random sets theory.

Moreover, random sets theory can also be used to build models in multi-target, multi-sensors applications, including sensor management, detection, tracking, etc. All these considerations make random sets theory a promising candidate for data fusion problems, particularly multiple target identification problems.

The material presented in this document is mainly academic, since it shows through examples how the different theories of uncertain reasoning can be applied to target identification problems. However, the examples and the test scenarios used were too simple to permit any conclusions as to the performance of modelization in one theory compared to another. In this sense, it was mainly a theoretical study of the capability of random sets theory in information fusion applications.

Future study should be concerned with the quantification of information, in particular the loss of information resulting from the different transformations.

References

[1] Klir, G. J. and Wierman, M. J. (1999). Uncertainty-Based Information, New York: Physica-Verlag.

[2] Krause, Paul and Clark, Dominic (1993). Representing Uncertain Knowledge: An Artificial Intelligence Approach, Kluwer Academic Publishers.

[3] Smets, P. (1997). Imperfect Information : Imprecision - Uncertainty. In Motro, A. and Smets, Ph., (Eds.), *Uncertainty Management in Information Systems. From Needs to Solutions*, pp. 225–254. Kluwer Academic Publishers.

[4] Zadeh, L. A. (1965). Fuzzy Sets. *Information and control*, **8**, 338–353.

[5] Dempster, A. (1967). Upper and Lower Probablilities Induced by Multivalued Mapping. *Ann. Math. Statist.*, **38**, 325–339.

[6] Shafer, G. (1976). A Mathematical Theory of Evidence, Princeton University Press.

[7] Zadeh, L.A. (1978). Fuzzy Sets as a Basis for a Theory of Possibility. *Fuzzy Sets and Systems*, **1**, 3–28.

[8] Pawlak, Z. (1991). Rough Sets - Theorical Aspects of Reasoning about Data, Kluwer Academic Publisher.

[9] Smets, P. (1990). Constructing the pignistic probability function in a context of uncertainty. *Uncertainty in Artificial Intelligence*, **5**, 29–39.

[10] Voorbraak, F. (1989). A computationally efficient approximation of Dempster-Shafer theory. *Int. J. Man-Machine Studies*, **30**, 525–536.

[11] Klir, G. J. and Parviz, B. (1992). Probability-Possibility Transformations : A Comparison. *International Journal of General Systems*, **21**, 291–310.

[12] Skowron, A. (1989). The Relationship Between the Rough Sets and the Evidence Theory. *Bulletin of the Polish Academy of Science*, **37**(1-2), 87–90.

[13] Nguyen, H. T. (1978). On Random Sets and Belief Functions. *Journal of Mathematical Analysis and Applications*, **65**, 531–542.

[14] Orlov, A. I. (1978). Fuzzy and Random Sets. *Prikladnoi Mnogomerni Statisticheskii Analys*, pp. 262–280.

[15] Goodman, I.R. (1982). Fuzzy Sets as Equivalence Classes of Random Sets. In Yager, R. R., (Ed.), *Fuzzy Sets and Possibility Theory*, pp. 327–342. Pergamon Press, Oxford.

[16] Hall, David and Llinas, T. James (2001). Handbook of Multisensor Data Fusion, CRC Press.

[17] Solaiman, B. (2001). Information Fusion Concepts. From Information Elements Definition to the Application of Fusion Approaches. *Proceedings of SPIE*, **4385**, 205–212.

[18] Cheaito, A., Lecours, M., and Bossé, É. (1998). A non-*ad hoc* Decision Rule for Dempster-Shafer Method of Evidential Reasoning. In SPIE, (Ed.), *Sensor Fusion: Architecture, Algorithm, and Application II*, Vol. 3376, pp. 44–57. Orlando, FL.

[19] Dubois, D. and Prade, H. (1994). La fusion d'informations imprécises. *Traitement du Signal*, **11**(6), 447–458.

[20] Lookheed-Martin Canada (1998). MSDF Implementation and Test Document for "Real-Time Issues and Demonstartion of Data Fusion Concepts for Airborne Surveillance". DM No. 990001008, Contract No. W7701-6-4081.

[21] Dubois, D. and Prade, H. (2000). Fundamentals of Fuzzy Sets, Kluwer Academic Publishers.

[22] Dubois, D. and Prade, H. (1993). Fuzzy Sets and Probability: Misunderstandings, Bridges and Gaps. In *Proceedings of the Second IEEE International Conference on Fuzzy Systems*, pp. 1059–1068.

[23] Parsons, S., Kubat, M., and Dohnal, M. (1995). A Rough Sets Approach to Reasoning under Uncertainty. *Journal of Experimental and Theoretical Artificial Intelligence*, **7**, 175–193.

[24] Sudano, John (2002). Inverse Pignistic Probability Transforms. In *ISIF*, pp. 763–768.

[25] Dubois, D. and Prade, H. (1982). On Several Representations of an Uncertain Body of Evidence. In *Fuzzy Information and Decision Processes*, pp. 167–181. North Holland.

[26] Smets, P. (1990). The Transferable Belief Model and Possibility Theory. *Proc. NAFIPS*, pp. 215–218.

[27] Sudkamp, T. (1992). The Semantics of Plausibility and Possibility. *International Journal of General Systems*, **21**, 273–289.

[28] Kampé de Fériet, J. (1982). Interpretation of Membership Functions of Fuzzy Sets in Terms of Plausibility and Belief. In *Fuzzy Information and Decision Processes*, pp. 93–98. North Holland.

[29] Pawlak, Z. (1985). Rough Sets and Fuzzy Sets. *Fuzzy Sets and Systems*, **17**, 99–102.

[30] Dubois, D. and Prade, H. (1990). Rough Fuzzy Sets and Fuzzy Rough Sets. *International Journal of General Systems*, **17**, 191–209.

[31] Dubois, D. and Prade, H. (1992). Putting Rough Sets and Fuzzy Sets Together. In *Intelligent Decision Support: A Handbook of Applications and Advances in Rough Sets*, pp. 203–232. Kluwer Academic Publishers.

[32] Skowron, A. and Grzymala-Busse, J. (1994). From Rough Sets Theory to Evidence Theory. In *Advances in the Dempster-Shafer Theory of Evidence*, pp. 193–235. John Wiley and Sons, NY.

[33] Matheron, G. (1975). Random Sets and Integral Geometry, J. Wiley.

[34] Kendall, D. G. (1974). Foundations of a Theory of Random Sets. In Hardings, E. F. and Kendall, D.G., (Eds.), *Stochastic Geometry*, pp. 322–376. J. Wiley - London.

[35] Kreinovich, V. (1997). Random Sets Unify, Explain, and Aid Known Uncertainty Methods in Expert Systems. In *Random Sets : Theory and Applications*, pp. 321–345. Springer-Verlag.

[36] Mori, S. (1997). Random Sets in Data Fusion. Multi-Objet State-Estimation as a Foundation of Data Fusion Theory. In *Random Sets : Theory and Applications*, pp. 185–207. Springer-Verlag.

[37] Goodman, I. R., Mahler, R. P., and Nguyen, H. T. (1997). Mathematics of Data Fusion, Kluwer Academic Publishers.

[38] Quinio, P. and Matsuyama, T. (1991). Random Closed Sets : A Unified Approach to the Representation of Imprecision and Uncertainty. In Kruse, R. and Siegel, P., (Eds.), *Symbolic and Quantitative Approaches to Uncertainty*, pp. 282–286. Springer-Verlag.

[39] Nguyen, H.T. and Wang, T. (1997). Belief Functions and Random Sets. In *Random Sets : Theory and Applications*, pp. 243–255. Springer-Verlag.

[40] Dubois, D. and Prade, H. (2000). Possibility Theory in Information Fusion. In *Fusion 2000*, p. Plenary Speech. Paris.

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Annex A: Notations used

Symbol	Meaning
Θ	The **frame of discernment** (the set of all known targets from the data base). Also known as the **universe** or **sample space**.
N	The number of objects in the data base.
2^Θ	The **power set** of Θ (the set of all sub-sets of Θ).
$\theta_i \quad \forall i \; , \; 1 \leq i \leq N$	The i-th object of the data base.
$A_i \quad \forall i \; , \; 1 \leq i \leq 2^N$	A sub-set of the set Θ.
Θ_0	The set of the information sensors.
$S_k \quad \forall k \; , \; 1 \leq k \leq K$	The k-th information sensor.
X	Random variable
$\mathcal{X}$	Random set
$X = \theta_i$ or $X \in A$	The event θ_i or A respectively (θ_i represent the realization of a random variable).

Table A.1: *Notations used in target identification problems*

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Annex B: Modeling vague information using fuzzy sets theory

B.1 Changing the definition domain

Let Θ be the set of all objects from the data base :

$$\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$$

Each object θ_i from the data base is characterized by a set of parameters :

$$\theta_i = [x_1^i, x_2^i, \ldots, x_P^i]$$

where x_j^i is the parameter $\#j$ of the object $\#i$ (see the list of all parameters in section 2.4).

The parameter x_j has a domain of definition $\mathcal{D}_j$ which may be discrete or continuous, ordered or not ($x_j \in \mathcal{D}_j$).

We define a fuzzy set $\underline{A}_k \subseteq \mathcal{D}_j$ by the membership function $\mu_{\underline{A}_k}(x)$. $\underline{A}_k$ may correspond for example to the class *small length* and the parameter x may correspond then to the values of the domain $\mathcal{D}_j$ where the length of an object may be defined (0 - 500 meters).

We are now looking for a fuzzy set $\underline{B}_k \subseteq \Theta$ defined by a membership function $\mu_{\underline{B}_k}(\theta_i)$ which must be equivalent to the fuzzy set $\underline{A}_k$. Obviously the new fuzzy set will be defined on a discretized domain. And its membership function $\mu_{\underline{B}_k}(\theta_i)$ is defined in function of $\mu_{\underline{A}_k}(x)$ by :

$$\mu_{\underline{B}_k}(\theta_i) = \mu_{\underline{A}_k}(x_j^i) \tag{B.1}$$

B.2 Modeling vague information using fuzzy sets theory - modeling information in classes

The following figures present the partition in classes of the parameters defining the objects from the data base :

1. length (figure B.1);
2. width (figure B.2);
3. height (figure B.3);
4. side radar cross section (figures B.4 and B.5);

5. top radar cross section (figures B.6 and B.7);
6. front radar cross section (figures B.8 and B.9).

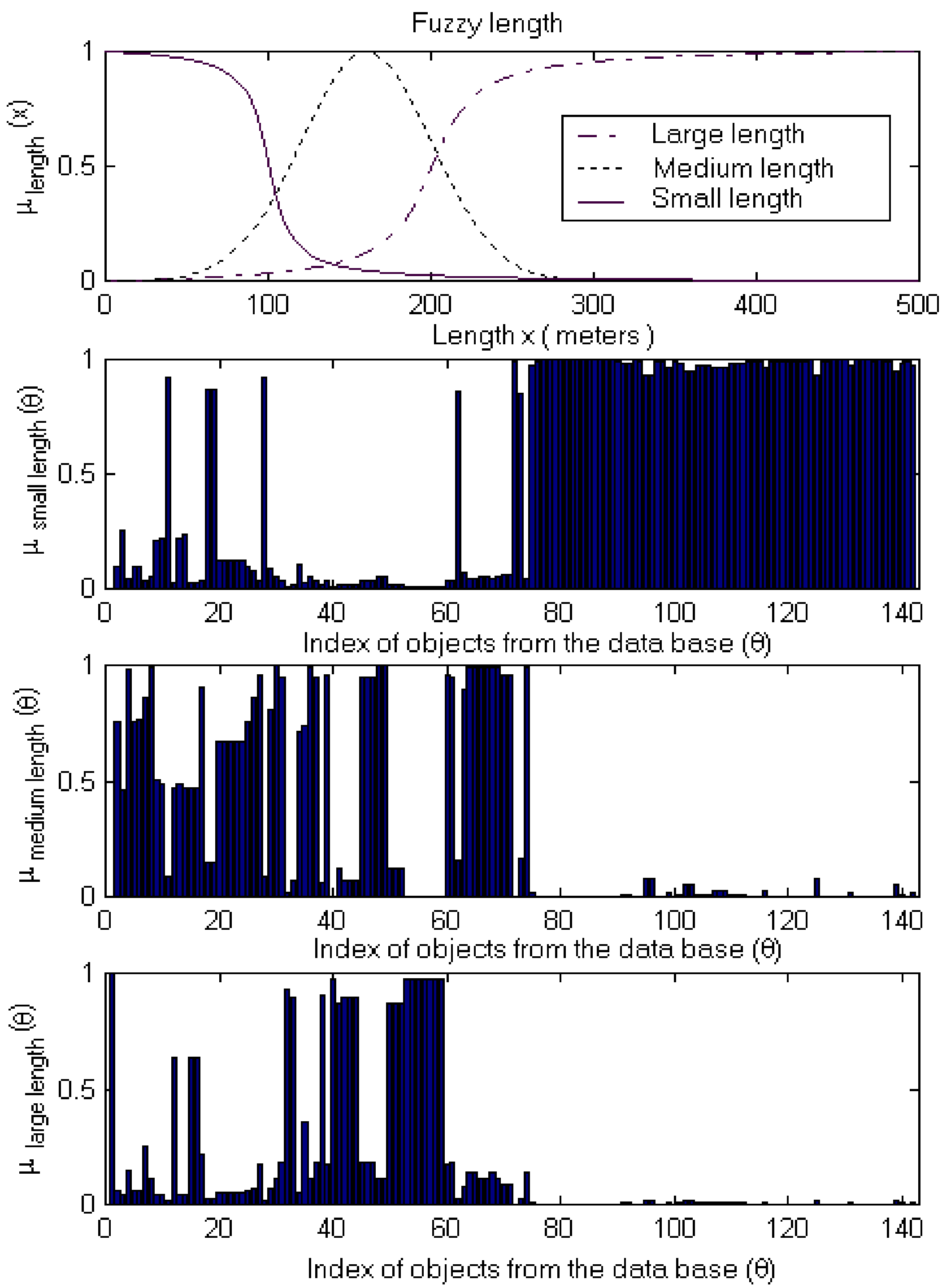

Figure B.1: *Fuzzy length - class small / medium / large. Characterization of the data base by the classes small length, medium length and large length*

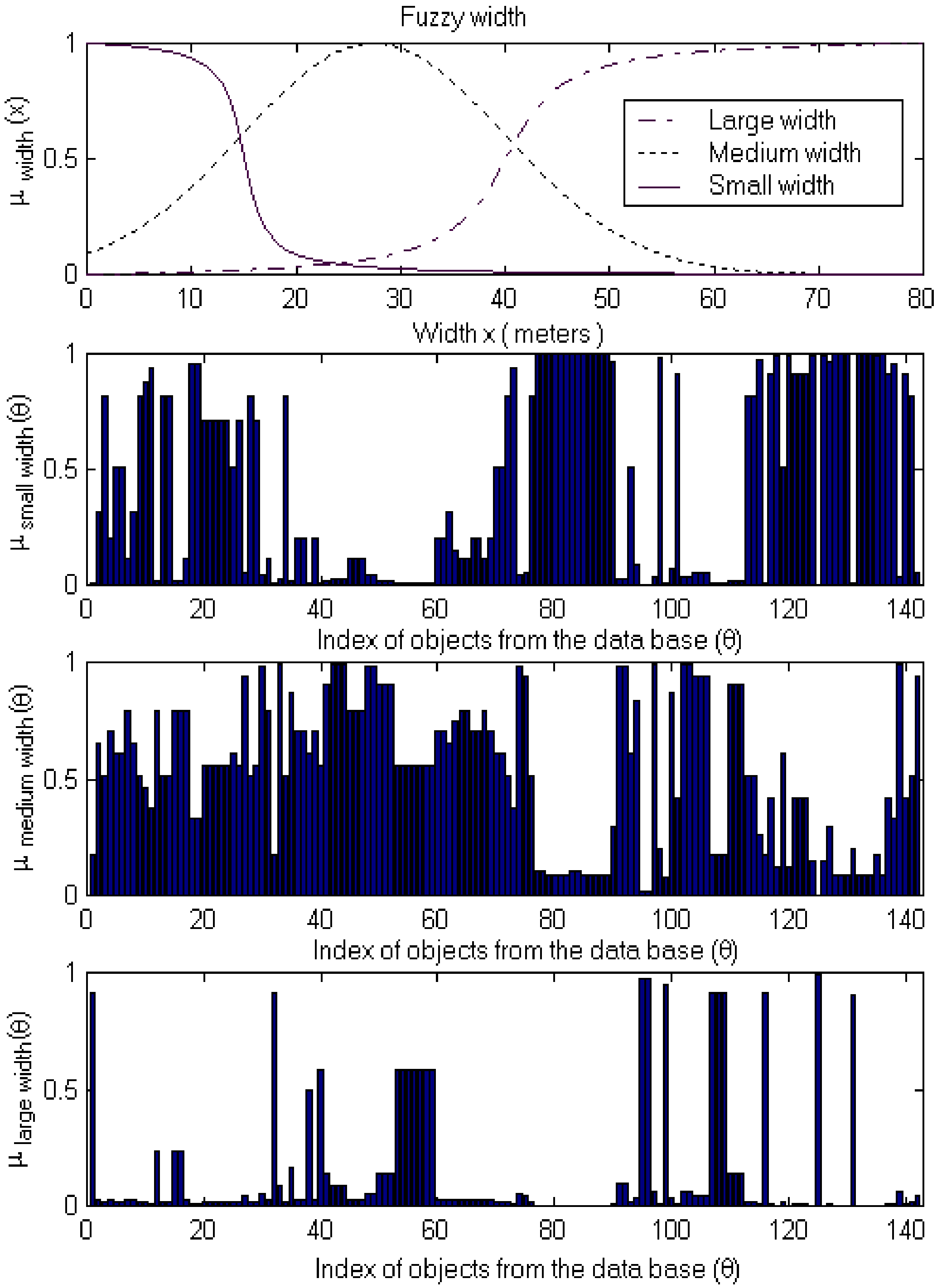

Figure B.2: *Fuzzy width - class small / medium / large. Characterization of the data base by the classes small width, medium width and large width*

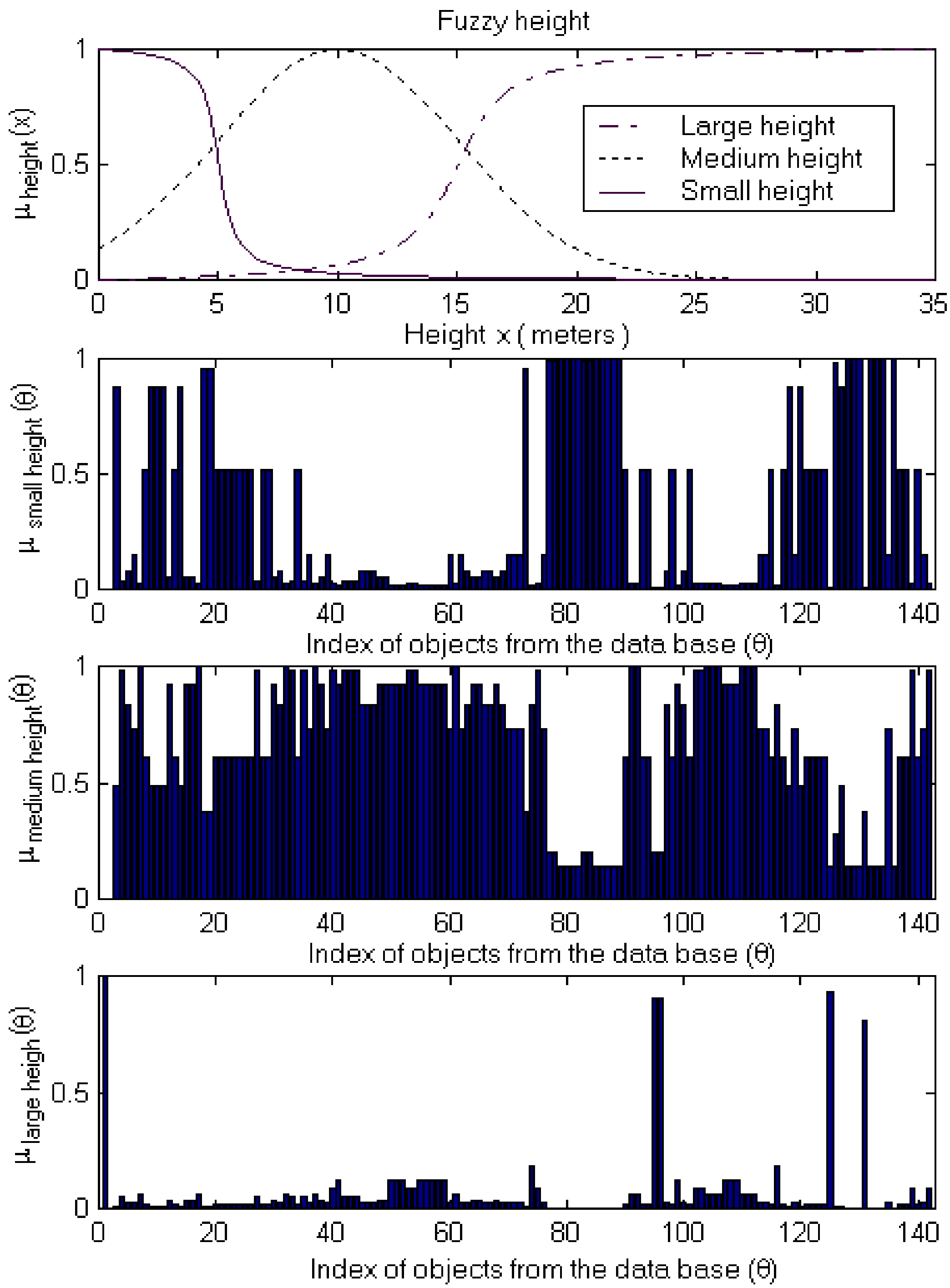

Figure B.3: *Fuzzy height - class small / medium / large. Characterization of the data base by the classes small height, medium height and large height*

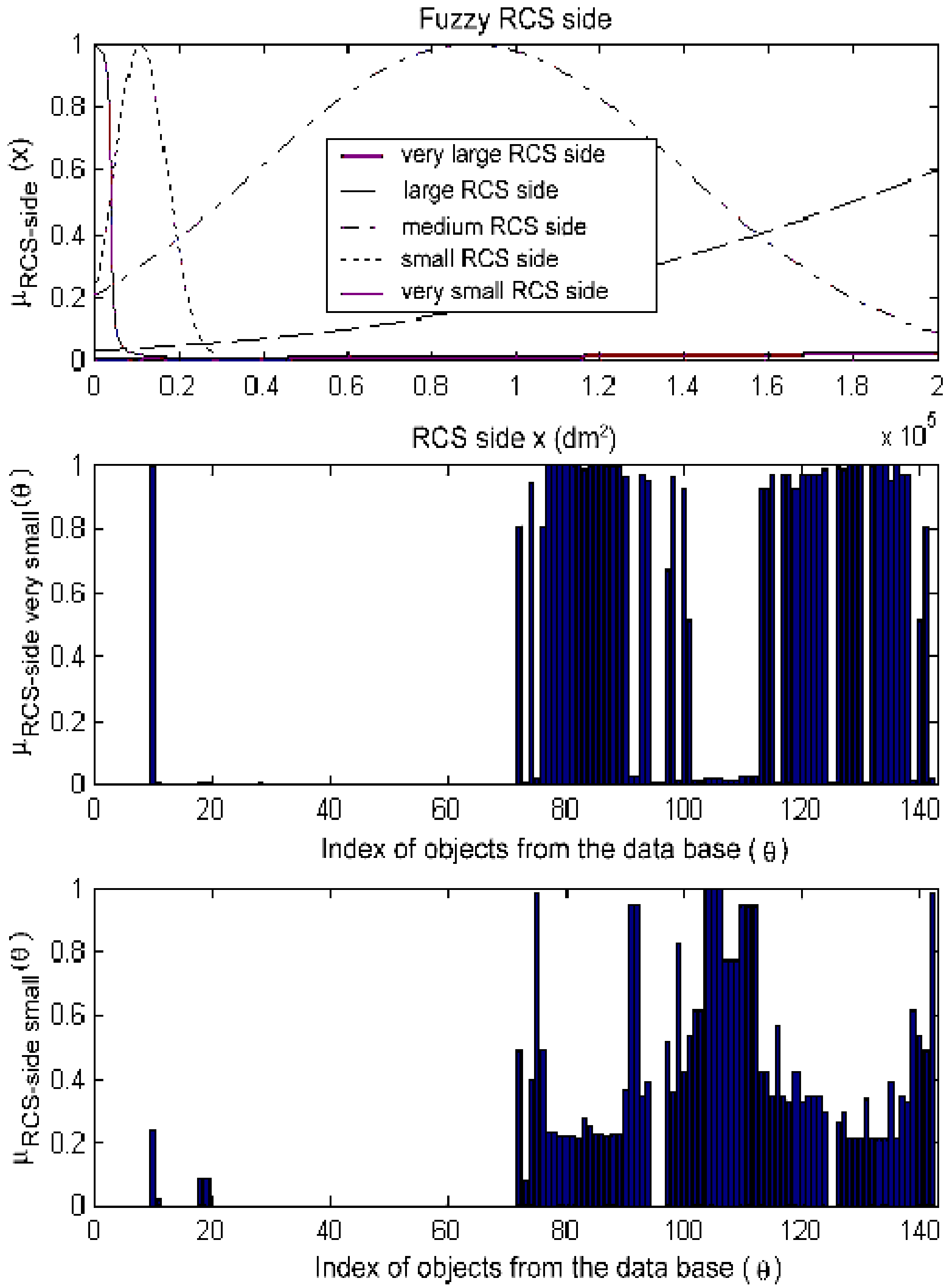

Figure B.4: *Fuzzy side radar cross section - class very small / small / medium / large / very large. Characterization of the data base by the classes RCS_{side} very small and RCS_{side} small*

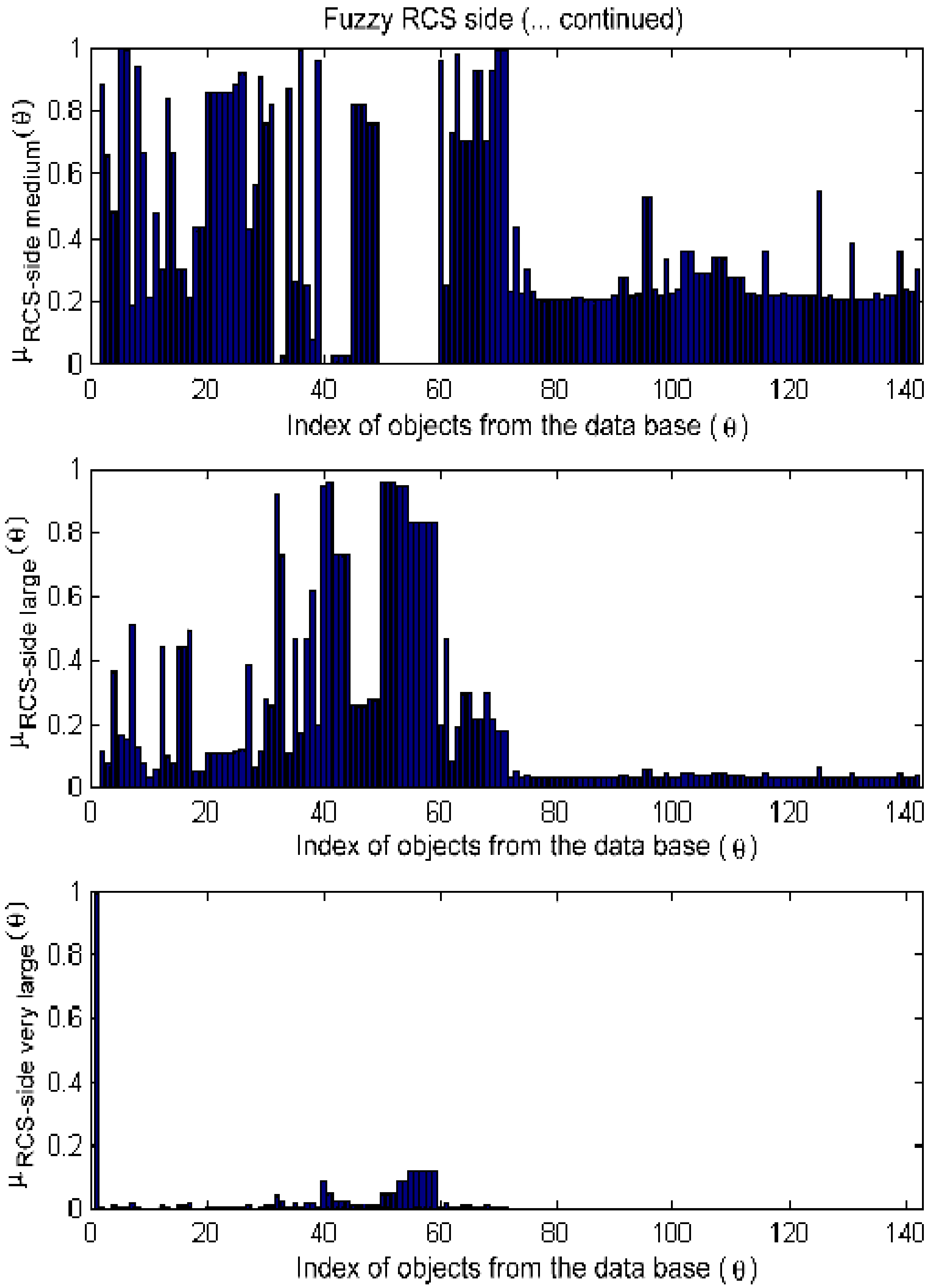

Figure B.5: *Fuzzy side radar cross section - Characterization of the data base by the classes RCS_{side} medium, RCS_{side} large and RCS_{side} very large*

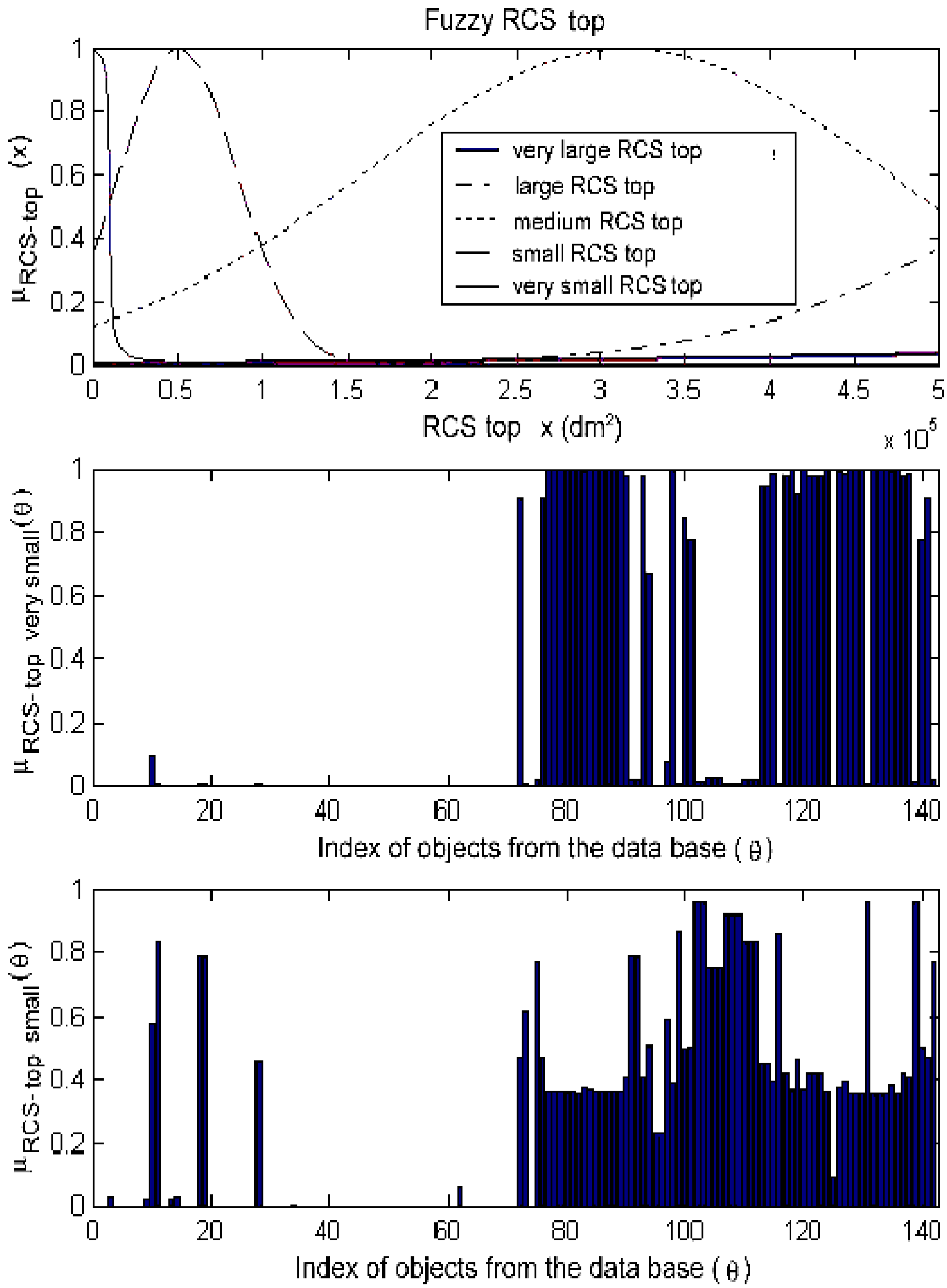

Figure B.6: *Fuzzy top radar cross section - class very small / small / medium / large / very large. Characterization of the data base by the classes RCS_{top} very small and RCS_{top} small*

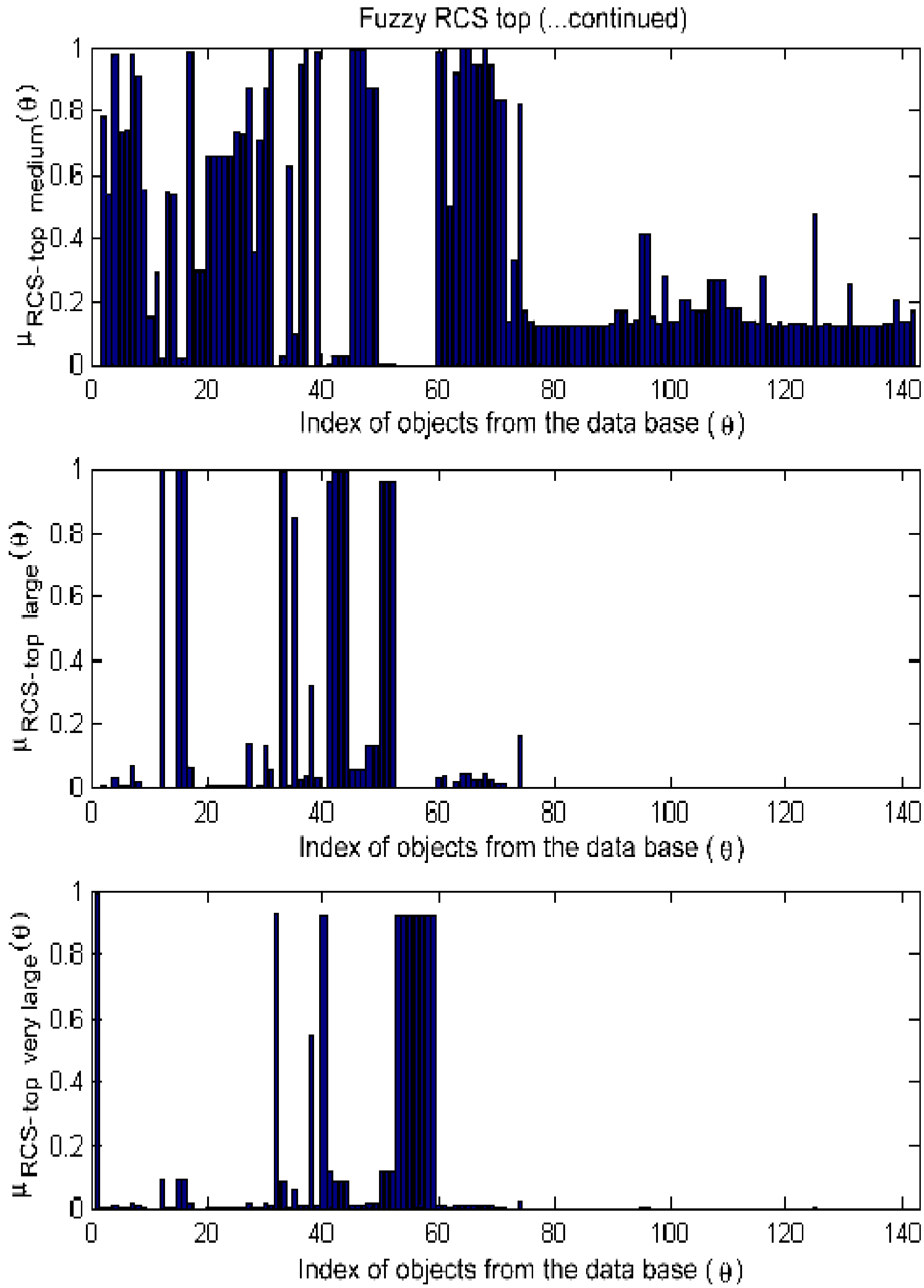

Figure B.7: *Fuzzy top radar cross section - Characterization of the data base by the classes RCS_{top} medium, RCS_{top} large and RCS_{top} very large*

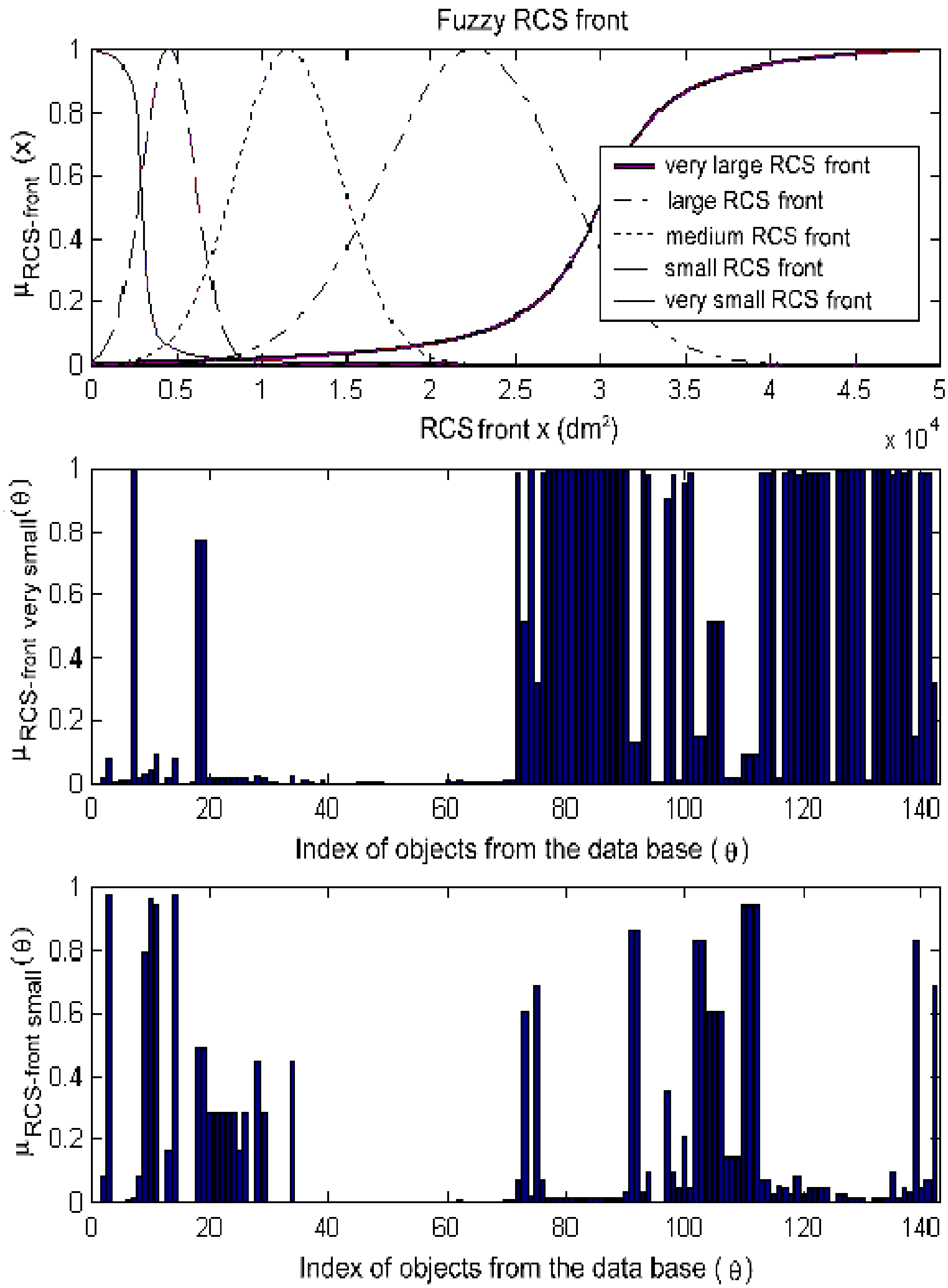

Figure B.8: *Fuzzy front radar cross section - class very small / small / medium / large / very large. Characterization of the data base by the classes* RCS_{front} *very small and* RCS_{front} *small*

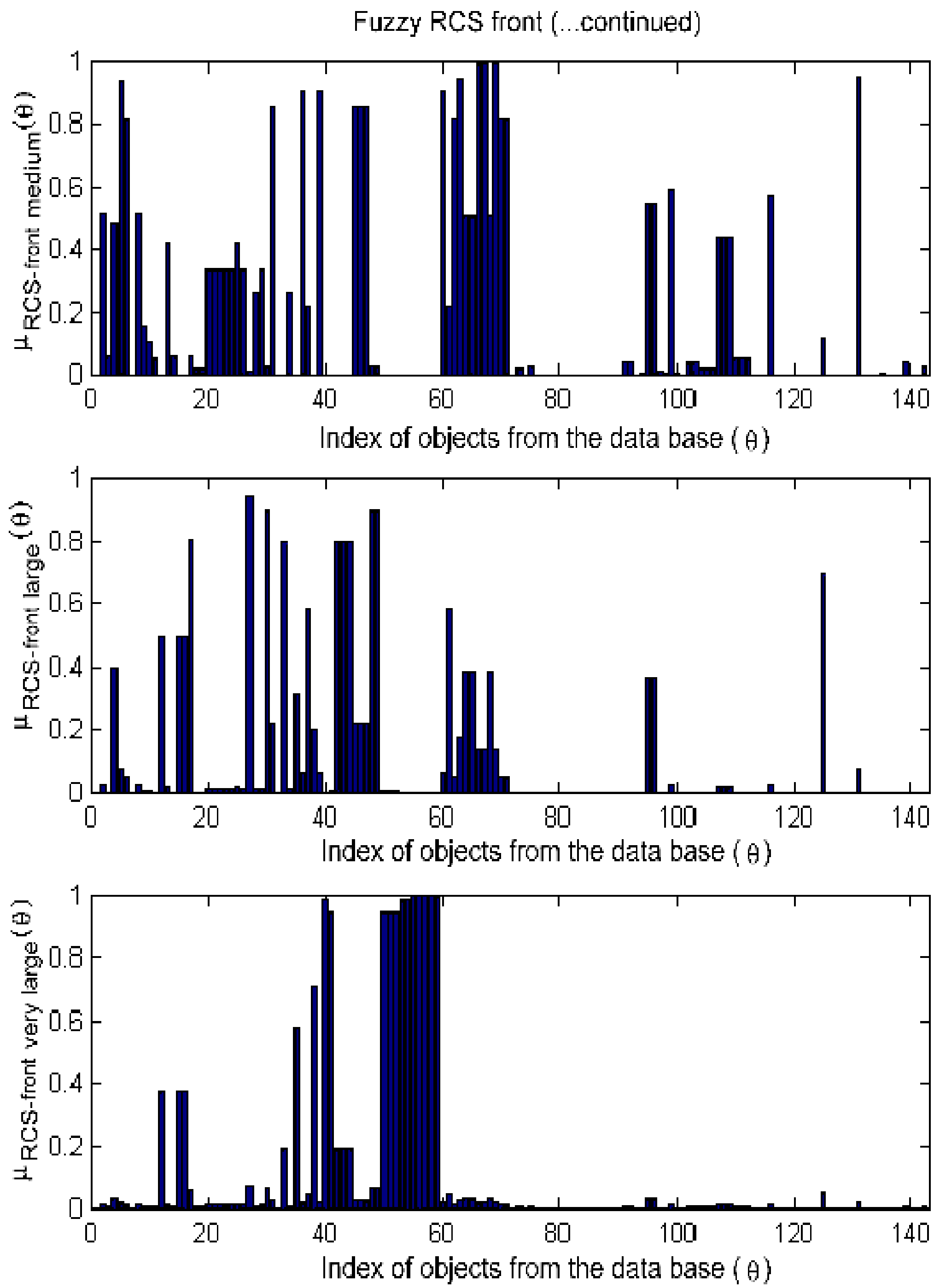

Figure B.9: *Fuzzy front radar cross section - Characterization of the data base by the classes RCS_{front} medium, RCS_{front} large and RCS_{front} very large*

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Annex C: Modeling information using rough sets theory

C.1 The knowledge R_1 (the type and the sub-type), R_2 (the offensiveness classification) and the final knowledge R

$$\begin{aligned}
R_1 = \Big\{ & \{\theta_1\}, \{\theta_2, \theta_3, \theta_5, \theta_6, \theta_9, \theta_{10}, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{24}, \theta_{26}, \theta_{29}, \theta_{70}, \theta_{71}, \theta_{73}\}, \{\theta_{28}\}, \\
& \{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{33}, \theta_{35}, \theta_{36}, \theta_{37}, \theta_{42}, \ldots, \theta_{47}, \theta_{61}\}, \{\theta_8, \theta_{25}, \theta_{34}, \theta_{39}, \theta_{60}, \theta_{64}, \ldots, \theta_{69}\}, \\
& \{\theta_{27}, \theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\}, \{\theta_{12}, \theta_{15}, \theta_{16}, \theta_{32}, \theta_{40}, \theta_{53}, \ldots, \theta_{59}\}, \{\theta_{30}, \theta_{38}, \theta_{48}, \theta_{49}\}, \\
& \{\theta_{72}, \theta_{76}, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{119}, \theta_{49}, \theta_{121}, \theta_{122}, \theta_{123}, \theta_{127}, \theta_{137}, \theta_{138}, \theta_{141}\}, \\
& \{\theta_{74}\}, \{\theta_{75}, \theta_{99}, \theta_{104}, \ldots, \theta_{109}, \theta_{112}, \theta_{116}, \theta_{142}\}, \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \\
& \{\theta_{62}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{91}, \theta_{100}, \theta_{111}\}, \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{95}, \theta_{96}, \theta_{102}, \theta_{103}, \theta_{125}, \theta_{139}\}, \\
& \{\theta_{128}, \theta_{129}, \theta_{130}, \theta_{132}, \theta_{133}, \theta_{134}, \theta_{136}, \theta_{143}\} \Big\}
\end{aligned}$$

$$\begin{aligned}
R_2 = \Big\{ & \{\theta_1, \theta_{28}, \theta_{62}, \theta_{92}, \theta_{95}, \theta_{96}, \theta_{97}, \theta_{101}, \theta_{102}, \theta_{103}, \theta_{110}, \theta_{125}, \theta_{131}, \theta_{139}, \theta_{140}\}, \\
& \{\theta_2, \theta_9, \theta_{12}, \theta_{15}, \theta_{16}, \theta_{26}, \theta_{29}, \theta_{34}, \theta_{36}, \theta_{65}, \theta_{68}, \theta_{73}, \theta_{90}, \theta_{120}, \theta_{124}, \theta_{137}, \theta_{138}, \theta_{141}\}, \\
& \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}, \theta_{25}, \theta_{30}, \theta_{35}, \theta_{38}, \theta_{40}, \theta_{41}, \theta_{48}, \ldots, \theta_{59}, \theta_{63}, \theta_{98}, \theta_{100}, \theta_{111}\}, \\
& \{\theta_4, \ldots, \theta_8, \theta_{17}, \theta_{20}, \ldots, \theta_{24}, \theta_{31}, \theta_{32}, \theta_{37}, \theta_{39}, \theta_{45}, \theta_{46}, \theta_{47}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}, \ldots, \theta_{72}, \\
& \theta_{74}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{104}, \ldots, \theta_{109}, \theta_{112}, \ldots, \theta_{118}, \theta_{121}, \theta_{122}, \theta_{142}\}, \\
& \{\theta_{13}, \theta_{14}, \theta_{27}, \theta_{91}, \theta_{135}\}, \{\theta_{128}, \theta_{129}, \theta_{130}, \theta_{132}, \theta_{133}, \theta_{134}, \theta_{136}, \theta_{143}\}, \\
& \{\theta_{33}, \theta_{42}, \theta_{43}, \theta_{44}, \theta_{61}, \theta_{75}, \theta_{77}, \ldots, \theta_{89}, \theta_{99}, \theta_{116}, \theta_{119}, \theta_{123}, \theta_{126}, \theta_{127}\} \Big\}
\end{aligned}$$

$$\begin{aligned}
R = \Big\{ & \{\theta_1\}, \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{37}, \theta_{45}, \theta_{46}, \theta_{47}\}, \\
& \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{12}, \theta_{15}, \theta_{16}\}, \{\theta_{13}, \theta_{14}\}, \\
& \{\theta_{25}\}, \{\theta_{27}\}, \{\theta_{28}\}, \{\theta_{30}, \theta_{38}, \theta_{48}, \theta_{49}\}, \{\theta_{32}\}, \{\theta_{33}, \theta_{42}, \theta_{43}, \theta_{44}, \theta_{61}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \\
& \{\theta_{35}\}, \{\theta_{36}\}, \{\theta_{40}, \theta_{53}, \ldots, \theta_{59}\}, \{\theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\}, \{\theta_{62}\}, \{\theta_{74}\}, \{\theta_{75}, \theta_{99}, \theta_{116}\}, \\
& \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \\
& \{\theta_{83}, \theta_{87}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}, \{\theta_{91}\}, \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{95}, \theta_{96}, \theta_{102}, \theta_{103}, \theta_{125}, \theta_{139}\}, \\
& \{\theta_{97}, \theta_{110}, \theta_{131}\}, \{\theta_{98}\}, \{\theta_{100}, \theta_{111}\}, \{\theta_{104}, \ldots, \theta_{109}, \theta_{112}, \theta_{142}\}, \{\theta_{118}\}, \{\theta_{120}, \theta_{124}\}, \\
& \{\theta_{119}, \theta_{123}, \theta_{127}\}, \{\theta_{126}\}, \{\theta_{135}\}, \{\theta_{128}, \theta_{129}, \theta_{130}, \theta_{132}, \theta_{133}, \theta_{134}, \theta_{136}, \theta_{143}\} \Big\}
\end{aligned}$$

C.2 Modeling information using classic set theory

$$
\begin{aligned}
A_1 &= \{\theta_1, \theta_2, \ldots, \theta_{70}, \theta_{71}, \theta_{73}\} \\
A_2 &= \{\theta_{11}, \theta_{19}, \theta_{63}\} \\
A_3 &= \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{62}, \theta_{72}, \theta_{73}, \theta_{75}, \ldots, \theta_{127}, \theta_{131}, \theta_{135}, \theta_{137}, \ldots, \theta_{142}\} \\
A_4 &= \{\theta_2, \theta_3, \theta_5, \theta_6, \theta_8, \theta_9, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \ldots, \theta_{31}, \theta_{34}, \theta_{36}, \theta_{39}, \theta_{45}, \ldots, \theta_{49}, \theta_{60}, \\
&\quad\ \theta_{62}, \ldots, \theta_{71}, \theta_{73}, \theta_{95}, \theta_{96}, \theta_{125}, \theta_{131}\} \\
A_5 &= \{\theta_{29}, \theta_{33}, \theta_{35}, \theta_{36}\} \\
A_6 &= \{\theta_2, \theta_3, \theta_8, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{73}, \theta_{77}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{98}, \theta_{101}, \\
&\quad\ \theta_{115}, \theta_{117}, \theta_{118}, \theta_{120}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{137}, \theta_{138}, \theta_{140}\} \\
A_7 &= \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{46}, \theta_{47}, \theta_{63}\} \\
A_8 &= \{\theta_{18}, \theta_{19}\} \\
A_9 &= \{\theta_3, \theta_5, \theta_6, \theta_9, \ldots, \theta_{11}, \theta_{13}, \theta_{14}, \theta_{18}, \ldots, \theta_{26}, \theta_{28}, \theta_{29}, \theta_{34}, \theta_{70}, \ldots, \theta_{73}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \\
&\quad\ \theta_{98}, \theta_{101}, \theta_{113}, \ldots, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141}\} \\
A_{10} &= \{\theta_{18}, \theta_{19}, \theta_{34}\} \\
A_{11} &= \{\theta_{11}, \theta_{18}, \ldots, \theta_{24}, \theta_{30}, \theta_{31}, \theta_{34}, \theta_{35}, \theta_{36}, \theta_{45}, \ldots, \theta_{49}, \theta_{63}, \theta_{67}, \theta_{69}\} \\
A_{12} &= \{\theta_{11}, \theta_{18}, \theta_{19}, \theta_{63}\} \\
A_{13} &= \{\theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}, \theta_{28}, \theta_{73}, \theta_{75}, \theta_{91}, \theta_{92}, \theta_{97}, \theta_{99}, \theta_{102}, \ldots, \theta_{112}, \theta_{116}, \theta_{131}, \theta_{139}, \theta_{142}\} \\
A_{14} &= \{\theta_7, \theta_{18}, \theta_{19}, \theta_{72}, \theta_{73}, \theta_{74}, \theta_{76}, \ldots, \theta_{90}, \theta_{93}, \theta_{94}, \theta_{97}, \theta_{98}, \theta_{100}, \theta_{101}, \theta_{104}, \theta_{105}, \theta_{106}, \theta_{113}, \\
&\quad\ \theta_{114}, \theta_{115}, \theta_{117}, \ldots, \theta_{124}, \theta_{126}, \theta_{127}, \theta_{135}, \theta_{137}, \theta_{138}, \theta_{140}, \theta_{141}\}
\end{aligned}
$$

C.3 Modeling information using rough sets theory

$$\begin{aligned}\underline{R}A_1 =& \Big\{ \{\theta_1\}, \{\theta_2,\theta_9,\theta_{26},\theta_{29},\theta_{73}\}, \{\theta_3,\theta_{10},\theta_{11},\theta_{18},\theta_{19}\}, \{\theta_4,\theta_7,\theta_{17},\theta_{31},\theta_{37},\theta_{45},\theta_{46},\theta_{47}\},\\ &\{\theta_5,\theta_6,\theta_{20},\ldots,\theta_{24},\theta_{70},\theta_{71}\}, \{\theta_8,\theta_{39},\theta_{60},\theta_{64},\theta_{66},\theta_{67},\theta_{69}\}, \{\theta_{12},\theta_{15},\theta_{16}\}, \{\theta_{13},\theta_{14}\},\\ &\{\theta_{25}\}, \{\theta_{27}\}, \{\theta_{28}\}, \{\theta_{30},\theta_{38},\theta_{48},\theta_{49}\}, \{\theta_{32}\}, \{\theta_{33},\theta_{42},\theta_{43},\theta_{44},\theta_{61}\}, \{\theta_{34},\theta_{65},\theta_{68}\},\\ &\{\theta_{35}\}, \{\theta_{36}\}, \{\theta_{40},\theta_{53},\ldots,\theta_{59}\}, \{\theta_{41},\theta_{50},\theta_{51},\theta_{52},\theta_{63}\} \Big\}\\ \overline{R}A_1 =& \Big\{ \{\theta_1\}, \{\theta_2,\theta_9,\theta_{26},\theta_{29},\theta_{73}\}, \{\theta_3,\theta_{10},\theta_{11},\theta_{18},\theta_{19}\}, \{\theta_4,\theta_7,\theta_{17},\theta_{31},\theta_{37},\theta_{45},\theta_{46},\theta_{47}\},\\ &\{\theta_5,\theta_6,\theta_{20},\ldots,\theta_{24},\theta_{70},\theta_{71}\}, \{\theta_8,\theta_{39},\theta_{60},\theta_{64},\theta_{66},\theta_{67},\theta_{69}\}, \{\theta_{12},\theta_{15},\theta_{16}\}, \{\theta_{13},\theta_{14}\},\\ &\{\theta_{25}\}, \{\theta_{27}\}, \{\theta_{28}\}, \{\theta_{30},\theta_{38},\theta_{48},\theta_{49}\}, \{\theta_{32}\}, \{\theta_{33},\theta_{42},\theta_{43},\theta_{44},\theta_{61}\}, \{\theta_{34},\theta_{65},\theta_{68}\},\\ &\{\theta_{35}\}, \{\theta_{36}\}, \{\theta_{40},\theta_{53},\ldots,\theta_{59}\}, \{\theta_{41},\theta_{50},\theta_{51},\theta_{52},\theta_{63}\}, \{\theta_{62}\} \Big\}\end{aligned}$$

$$\begin{aligned}\underline{R}A_2 =& \Big\{ \varnothing \Big\}\\ \overline{R}A_2 =& \Big\{ \{\theta_3,\theta_{10},\theta_{11},\theta_{18},\theta_{19}\}, \{\theta_{41},\theta_{50},\theta_{51},\theta_{52},\theta_{63}\} \Big\}\end{aligned}$$

$$\begin{aligned}\underline{R}A_3 =& \Big\{ \{\theta_{28}\}, \{\theta_{62}\}, \{\theta_{72},\theta_{76},\theta_{93},\theta_{94},\theta_{113},\theta_{114},\theta_{115},\theta_{117},\theta_{121},\theta_{122}\}, \{\theta_{75},\theta_{99},\theta_{116}\},\\ &\{\theta_{77},\ldots,\theta_{82},\theta_{84},\theta_{85},\theta_{86},\theta_{88},\theta_{89}\}, \{\theta_{83},\theta_{87}\}, \{\theta_{90},\theta_{137},\theta_{138},\theta_{141}\}, \{\theta_{91}\},\\ &\{\theta_{92},\theta_{101},\theta_{140}\}, \{\theta_{95},\theta_{96},\theta_{102},\theta_{103},\theta_{125},\theta_{139}\}, \{\theta_{97},\theta_{110},\theta_{131}\}, \{\theta_{98}\}, \{\theta_{100},\theta_{111}\},\\ &\{\theta_{104},\ldots,\theta_{109},\theta_{112},\theta_{142}\}, \{\theta_{118}\}, \{\theta_{119},\theta_{123},\theta_{127}\}, \{\theta_{120},\theta_{124}\}, \{\theta_{126}\}, \{\theta_{135}\} \Big\}\\ \overline{R}A_3 =& \Big\{ \{\theta_2,\theta_9,\theta_{26},\theta_{29},\theta_{73}\}, \{\theta_3,\theta_{10},\theta_{11},\theta_{18},\theta_{19}\}, \{\theta_{28}\}, \{\theta_{62}\}, \{\theta_{75},\theta_{99},\theta_{116}\},\\ &\{\theta_{72},\theta_{76},\theta_{93},\theta_{94},\theta_{113},\theta_{114},\theta_{115},\theta_{117},\theta_{121},\theta_{122}\}, \{\theta_{77},\ldots,\theta_{82},\theta_{84},\theta_{85},\theta_{86},\theta_{88},\theta_{89}\},\\ &\{\theta_{83},\theta_{87}\}, \{\theta_{90},\theta_{137},\theta_{138},\theta_{141}\}, \{\theta_{91}\}, \{\theta_{92},\theta_{101},\theta_{140}\}, \{\theta_{97},\theta_{110},\theta_{131}\},\\ &\{\theta_{95},\theta_{96},\theta_{102},\theta_{103},\theta_{125},\theta_{139}\}, \{\theta_{98}\}, \{\theta_{100},\theta_{111}\}, \{\theta_{104},\ldots,\theta_{109},\theta_{112},\theta_{142}\}, \{\theta_{118}\},\\ &\{\theta_{119},\theta_{123},\theta_{127}\}, \{\theta_{120},\theta_{124}\}, \{\theta_{126}\}, \{\theta_{135}\} \Big\}\end{aligned}$$

$$\underline{R}A_4 = \Big\{ \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_{25}\}, \{\theta_{28}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{36}\}, \{\theta_{62}\} \Big\}$$

$$\overline{R}A_4 = \Big\{ \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{37}, \theta_{45}, \theta_{46}, \theta_{47}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_{25}\}, \{\theta_{28}\}, \{\theta_{30}, \theta_{38}, \theta_{48}, \theta_{49}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{36}\}, \{\theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\}, \{\theta_{62}\}, \{\theta_{95}, \theta_{96}, \theta_{102}, \theta_{103}, \theta_{125}, \theta_{139}\}, \{\theta_{97}, \theta_{110}, \theta_{131}\} \Big\}$$

$$\underline{R}A_5 = \Big\{ \{\theta_{35}\}, \{\theta_{36}\} \Big\}$$

$$\overline{R}A_5 = \Big\{ \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_{33}, \theta_{42}, \theta_{43}, \theta_{44}, \theta_{61}\}, \{\theta_{35}\}, \{\theta_{36}\} \Big\}$$

$$\underline{R}A_6 = \Big\{ \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_{25}\}, \{\theta_{28}\}, \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{98}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{118}\}, \{\theta_{126}\} \Big\}$$

$$\overline{R}A_6 = \Big\{ \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{25}\}, \{\theta_{28}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}, \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{98}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{119}, \theta_{123}, \theta_{127}\}, \{\theta_{118}\}, \{\theta_{126}\} \Big\}$$

$$\underline{R}A_7 = \Big\{ \{\theta_{35}\} \Big\}$$

$$\overline{R}A_7 = \Big\{ \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{37}, \theta_{45}, \theta_{46}, \theta_{47}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{35}\}, \{\theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\} \Big\}$$

$$\underline{R}A_8 = \Big\{ \varnothing \Big\}$$

$$\overline{R}A_8 = \Big\{ \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\} \Big\}$$

$$\begin{aligned}\underline{R}A_9 = \Big\{ & \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_{13}, \theta_{14}\}, \{\theta_{25}\}, \{\theta_{28}\}, \\ & \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}, \{\theta_{98}\}, \{\theta_{118}\}, \\ & \{\theta_{119}, \theta_{123}, \theta_{127}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{126}\}, \{\theta_{135}\} \Big\}\end{aligned}$$

$$\begin{aligned}\overline{R}A_9 = \Big\{ & \{\theta_2, \theta_9, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_{25}\}, \\ & \{\theta_{13}, \theta_{14}\}, \{\theta_{28}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \\ & \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}, \\ & \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{98}\}, \{\theta_{118}\}, \{\theta_{119}, \theta_{123}, \theta_{127}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{126}\}, \{\theta_{135}\} \Big\}\end{aligned}$$

$$\underline{R}A_{10} = \Big\{ \varnothing \Big\}$$

$$\overline{R}A_{10} = \Big\{ \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_{34}, \theta_{65}, \theta_{68}\} \Big\}$$

$$\underline{R}A_{11} = \Big\{ \{\theta_{35}\}, \{\theta_{36}\} \Big\}$$

$$\begin{aligned}\overline{R}A_{11} = \Big\{ & \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_4, \theta_7, \theta_{17}, \theta_{31}, \theta_{37}, \theta_{45}, \theta_{46}, \theta_{47}\}, \\ & \{\theta_5, \theta_6, \theta_{20}, \ldots, \theta_{24}, \theta_{70}, \theta_{71}\}, \{\theta_8, \theta_{39}, \theta_{60}, \theta_{64}, \theta_{66}, \theta_{67}, \theta_{69}\}, \{\theta_{30}, \theta_{38}, \theta_{48}, \theta_{49}\}, \\ & \{\theta_{34}, \theta_{65}, \theta_{68}\}, \{\theta_{35}\}, \{\theta_{36}\}, \{\theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\} \Big\}\end{aligned}$$

$$\underline{R}A_{12} = \Big\{ \varnothing \Big\}$$

$$\overline{R}A_{12} = \Big\{ \{\theta_3, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_{41}, \theta_{50}, \theta_{51}, \theta_{52}, \theta_{63}\} \Big\}$$

$$\underline{R}A_{13} = \Big\{ \{\theta_{28}\}, \{\theta_{91}\}, \{\theta_{97}, \theta_{110}, \theta_{131}\}, \{\theta_{104}, \ldots, \theta_{109}, \theta_{112}, \theta_{142}\} \Big\}$$

$$\begin{aligned}\overline{R}A_{13} = \Big\{ & \{\theta_{2}, \theta_{9}, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_{3}, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_{28}\}, \{\theta_{75}, \theta_{99}, \theta_{116}\}, \{\theta_{91}\}, \\ & \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{95}, \theta_{96}, \theta_{102}, \theta_{103}, \theta_{125}, \theta_{139}\}, \{\theta_{97}, \theta_{110}, \theta_{131}\}, \{\theta_{100}, \theta_{111}\}, \\ & \{\theta_{104}, \ldots, \theta_{109}, \theta_{112}, \theta_{142}\} \Big\}\end{aligned}$$

$$\begin{aligned}\underline{R}A_{14} = \Big\{ & \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \\ & \{\theta_{74}\}, \{\theta_{83}, \theta_{87}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}, \{\theta_{98}\}, \{\theta_{118}\}, \{\theta_{119}, \theta_{123}, \theta_{127}\}, \\ & \{\theta_{120}, \theta_{124}\}, \{\theta_{126}\}, \{\theta_{135}\} \Big\}\end{aligned}$$

$$\begin{aligned}\overline{R}A_{14} = \Big\{ & \{\theta_{2}, \theta_{9}, \theta_{26}, \theta_{29}, \theta_{73}\}, \{\theta_{3}, \theta_{10}, \theta_{11}, \theta_{18}, \theta_{19}\}, \{\theta_{4}, \theta_{7}, \theta_{17}, \theta_{31}, \theta_{37}, \theta_{45}, \theta_{46}, \theta_{47}\}, \{\theta_{74}\}, \\ & \{\theta_{72}, \theta_{76}, \theta_{93}, \theta_{94}, \theta_{113}, \theta_{114}, \theta_{115}, \theta_{117}, \theta_{121}, \theta_{122}\}, \{\theta_{77}, \ldots, \theta_{82}, \theta_{84}, \theta_{85}, \theta_{86}, \theta_{88}, \theta_{89}\}, \\ & \{\theta_{83}, \theta_{87}\}, \{\theta_{90}, \theta_{137}, \theta_{138}, \theta_{141}\}, \{\theta_{92}, \theta_{101}, \theta_{140}\}, \{\theta_{97}, \theta_{110}, \theta_{131}\}, \{\theta_{98}\}, \{\theta_{100}, \theta_{111}\}, \\ & \{\theta_{104}, \ldots, \theta_{109}, \theta_{112}, \theta_{142}\}, \{\theta_{118}\}, \{\theta_{119}, \theta_{123}, \theta_{127}\}, \{\theta_{120}, \theta_{124}\}, \{\theta_{126}\}, \{\theta_{135}\} \Big\}\end{aligned}$$

Annex D: *A priori* data bases

Country code	Allegiance	Country name	Number of official languages	Official languages
USAM	FRIENDL	UNITED STATES	1	ENGLISH
UKRA	NOTFRIE	UKRAINE	2	UKRANIAN, RUSSIAN
TAIW	FRIENDL	TAIWAN	1	CANTONESE
SWED	FRIENDL	SWEDEN	1	SWEDISH
RUSS	NOTFRIE	RUSSIA	1	RUSSIAN
PAKI	NOTFRIE	PAKISTAN	2	OURDOU, ENGLISH
MEXI	FRIENDL	MEXICO	1	SPANISH
LITH	NOTFRIE	LITHUANIA	1	LITHUANIAN
LIBY	NOTFRIE	LIBYA	1	ARAB
KAZA	NEUTRAL	KAZAKHSTAN	2	KAZAKH, RUSSIAN
ISRA	NEUTRAL	ISRAEL	5	HEBREW, ENGLISH, YIDDISH, RUSSIAN, ARAB
IRAQ	NOTFRIE	IRAQ	1	ARAB
INDI	NEUTRAL	INDIA	5	ENGLISH, INDI, GUJRATI, BENGHALI, PUNJABI
GREE	FRIENDL	GREECE	1	GREEK
GERM	FRIENDL	GERMANY	1	GERMAN
FRAN	FRIENDL	FRANCE	1	FRENCH
EGYP	NOTFRIE	EGYPT	1	ARAB
DANM	FRIENDL	DENMARK	1	DANISH
CHIN	NEUTRAL	CHINA	2	MANDARIN, CANTONESE
CANA	FRIENDL	CANADA	2	FRENCH, ENGLISH
BRIT	FRIENDL	UNITED KINGDOM	1	ENGLISH
AUST	FRIENDL	AUSTRALIA	1	ENGLISH
POLA	NOTFRIE	POLAND	1	POLISH
SYRI	NOTFRIE	SYRIA	1	ARAB
PERU	UNKNOWN	UNKNOWN	1	UNKNOWN
N/A-	UNKNOWN	UNKNOWN	1	UNKNOWN
VAR-	UNKNOWN	UNKNOWN	1	UNKNOWN

Table D.1: *Geopolitical data base*

No.	Emitter id.	Function	No.	Emitter id.	Function
6	NAVNAVI	FURUNO	53	NAV2DSU	SPS-55
7	NAVNAVI	URN-25	54	NAVNAVI	SPS-64(V)9
8	IFFINRE	MK-XII	55	NAV2DSU	SLIM-NET
9	NAV2DSU	TYPE-992R	56	NAVFICO	HAWK-SCREECH
10	NAV2DSU	SIGNAAL-DA-08	57	NAV2DSU	SPS-49(V)5
11	NAVFICO	SELENIA-912	58	NAV2DSU	ERICSSON-SG-HC-150
12	NAVNAVI	TYPE-1006	59	NAVFICO	SIGNAAL-VM-25-STIR
13	NAVECMS	SLQ-32(V)3-4	60	NAVECMS	SLQ-503
14	NAVECMS	SLQ-32(V)SIDEKICK	61	NAVNAVI	SPERRY-MK-340
15	NAVFICO	SPG-51D	62	NAV3DSU	HEAD-NET-C
16	NAV3DSU	SPS-48E	63	NAV3DSU	TOP-PLATE
17	NAV2DSU	SPS-67	64	NAVNAVI	DON-KAY
18	NAVNAVI	MARCONI-LN-66	65	NAVNAVI	PALM-FROND
19	NAVFICO	SPG-53F	66	NAV2DSU	PEEL-CONE
20	NAVFICO	SPG-55D	67	NAVFICO	EYE-BOWL
21	NAVECMS	TST-FL-1800	68	NAVFICO	OWL-SCREECH
22	NAVFICO	SIGNAAL-WM-25	69	NAVECMS	BELL-SQUAT
23	NAV2DSU	SIGNAAL-DA-08	70	NAV2DSU	SPIN-TROUGH
24	NAVNAVI	SMA-3-RM-20	71	NAVFICO	KITE-SCREECH
25	NAVECMS	TYPE-670	72	NAV2DSU	SIGNAAL-LW08
26	NAV2DSU	TYPE-967	73	NAVECMS	ELBIT-EA-2118
27	NAVNAVI	TYPE-1007	74	NAVFICO	SPERRY-MK-92
28	NAVFICO	TYPE-911	75	NAVNAVI	RACAL-DECCA-TM-969
29	NAVFICO	TYPE-910	76	NAV2DSU	SPS-502
30	NAV2DSU	TYPE-968	77	NAVNAVI	SRN-15
31	NAVFICO	SPG-60D	78	NAVECMS	SIDE-GLOBE
32	NAVFICO	SPQ-9	79	NAVNAVI	KELVINHUGUES-NUC-2
33	NAV2DSU	SPS-10	80	NAV2DSU	BIG-NET
34	NAVFICO	SPG-53A	81	NAVFICO	SCOOP-PAIR
35	NAVFICO	SPG-55B	82	NAVFICO	PEEL-GROUP
36	NAVECMS	ULQ-6	83	NAVFICO	MUFF-CUB
37	NAV2DSU	SPS-503	84	NAV3DSU	TOP-SAIL
38	NAVNAVI	SPERRY-127E	85	NAVFICO	HEAD-LIGHT
39	NAVFICO	SPG-48	86	NAV2DSU	LOW-TROUGH
40	NAV2DSU	SPS-12	87	NAVFICO	DRUM-TILT
41	NAVFICO	SPG-34	88	NAVECMS	FOOT-BALL
42	NAVNAVI	SPERRY-MK-II	89	NAVNAVI	ROUND-HOUSE
43	NAVNAVI	URN-20	90	NAV2DSU	TOP-STEER
44	NAV2DSU	STRUT-CURVE	91	NAVFICO	CROSS-WORD
45	NAVFICO	POP-GROUP	92	NAVFICO	TOP-DOME
46	NAVFICO	BASS-TILT	93	ATCGCCA	FLY-SCREEN
47	NAVNAVI	DON-2	94	NAVECMS	CAGE-POT
48	NAVECMS	TYPE-675	95	NAV3DSU	SKY-WATCH
49	NAVFICO	TYPE-909	96	NAV3DSU	PLATE-STEER
50	NAV2DSU	TYPE-1022	97	NAV2DSU	STRUT-PAIR
51	NAV2DSU	TYPE-996(2)	98	NAVFICO	TRAP-DOOR
52	NAVFICO	TYPE-909(1)	99	ATCGCCA	FLY-TRAP

Table D.2: *Sensors data base (1)*

No.	Emitter id.	Function	No.	Emitter id.	Function
100	ATCGCCA	CAKE-STAND	145	MISHORA	SUPER-ADAC
101	IFFINRE	SALT-POT-B	146	AIRECMS	ALQ-126B
102	IFFINRE	LONG-HEAD	147	AIRECMS	ALQ-162
103	IFFINRE	HIGH-POLE-B	148	AIRMULT	APG-65
104	IFFINRE	HIGH-POLE-A	149	AIRMULT	APS-134
105	NAV3DSU	HALF-PLATE	150	AIRMULT	APN-510
106	IFFINRE	SALT-POT-A	151	AIRMULT	APS-116-506
107	IFFINRE	UPX-12	152	AIRECMS	SPS-3000
109	IFFINRE	SQUARE-HEAD	153	AIRMULT	APG-68
110	NAV3DSU	SPY-1A	154	AIRFICO	AWG-9
111	NAV3DSU	SPY-1B	155	AIRECMS	ALQ-165
112	NAVFICO	SPG-62	156	AIRMULT	APS-128D
113	NAV2DSU	SPS-52C	157	AIRWEAT	PRIMUS-800
114	NAV2DSU	SPS-40	158	AIRMULT	BLUE-KESTREL
115	NAV2DSU	HUGUES-MK-23-TAS	159	AIRECMS	ALQ-155
116	ATCGCCA	SPN-35	160	AIRECMS	ALQ-172
117	ATCGCCA	SPN-43	161	AIRMULT	B-52-AIRBORNE-RADAR
118	IFFINRE	MK-XV	162	AIRMULT	APS-137
119	NAVNAVI	SPS-53	163	AIRFICO	SKIP-SPIN
121	NAVFICO	RAYTHEON-MK-95	164	AIRFICO	FAN-TAIL
122	NAVECMS	SLQ-29	165	AIRMULT	ORB37-HL
123	NAVECMS	SLQ-17	166	AIRFICO	BOX-TAIL
124	ATCGCCA	SPN-41	167	AIRNAVI	CLAM-PIPE
125	ATCGCCA	SPN-44	168	AIRECMS	GROUND-BOUNCER
126	ATCGCCA	SPN-46	169	AIRMULT	APG-70
127	NAVNAVI	FURUNO-900	170	AIRECMS	ALQ-135
128	NAVFICO	BAND-STAND	171	AIRECMS	ELISRA-SPJ-20
129	NAVFICO	FRONT-DOME	172	AIRMULT	THOMPSON-RDM-RADAR
130	NAV2DSU	SPS-58A	173	AIRMULT	GROUPE-IE-RBE2
131	UNDETER	LIGHT-BULB	174	AIRECMS	THOMPSON-CSF-BAREM
132	NAV2DSU	UNKNOWN-RUSS-NO-1	175	AIRECMS	THOMPSON-CSF-CAIMAN
133	AIRFICO	FLASH-DANSE	176	AIRECMS	DASSAULT-CAMELEON
134	NAV3DSU	TST-TRS	177	AIRMULT	SLOT-BACK
135	NAV2DSU	PHILIPS-9GR-600	178	AIRMULT	SHORT-HORN
136	NAVFICO	PHILIPS-9VL-200	179	AIRMULT	APS-133
137	NAVNAVI	BURMEIS-WES-MIL-900	180	AIRFICO	BLUE-FOX-MK2
138	SUBSUSU	SNOOP-PAIR	181	AIRWEAT	RACAL-DOPPLER-72
139	AIRMULT	DOWN-BEAT	99951	AIRECMS	DOWNBEAT
140	MISHORA	TEXAS-INST-DSQ-28	99952	AIRNAVI	COMMERCIAL
141	AIRNAVI	APN-194	99953	AIRECMS	NO-93
142	MISHORA	MS-2-SEEKER	99954	AIRNAVI	APQ-55
143	MISHORA	KING-FISH-SEEKER	99999	UNKNOWN	UNKNOWN
144	MISHORA	ADAC			

Table D.3: *Sensors data base (2)*

ID	Target name	Type	Sub-type	Offensive degree	Country code	V_{min}	V_{max}	ACC	ALT max	Len.	Hei.	Wid.
1	JAHRE VICKING	SURNOMI	TANKERV	HARMLE	DANM	0	35	999	0	460	33	51
2	HALIFAX CPF	SURMILI	FRIGATE	MEDIOF	CANA	0	35	999	0	130	51	16
3	TARIQ AMAZON	SURMILI	FRIGATE	WEAKOF	PAKI	0	30	999	0	110	4	13
4	BELKNAP	SURMILI	CRUISER	STROOF	USAM	0	38	999	0	167	9	17
5	BREMEN	SURMILI	FRIGATE	STROOF	GERM	0	30	999	0	130	7	15
6	BROADSWORD BATCH 1	SURMILI	FRIGATE	STROOF	BRIT	0	30	999	0	131	6	15
7	CALIFORNIA	SURMILI	CRUISER	STROOF	USAM	0	35	999	0	182	10	19
8	COONTZ	SURMILI	DESTROY	STROOF	USAM	0	35	999	0	156	5	16
9	IMPROVED RES-TIGOUCHE	SURMILI	FRIGATE	MEDIOF	CANA	0	28	999	0	113	4	13
10	MACKENZIE	SURMILI	FRIGATE	WEAKOF	CANA	0	28	999	0	112	4	12
11	GRISHA III (ALBA-TROS)	SURMILI	FRIGATE	WEAKOF	LITH	0	30	999	0	71	4	10
12	INVINCIBLE	SURMILI	CARRIER	MEDIOF	BRIT	0	28	999	0	209	8	36
13	ST LAURENT	SURMILI	FRIGATE	VEWEOF	CANA	0	27	999	0	112	5	13
14	ST CROIX	SURMILI	FRIGATE	VEWEOF	CANA	0	25	999	0	111	4	13
15	INVINCIBLE ILLUS-TRIO	SURMILI	CARRIER	MEDIOF	BRIT	0	28	999	0	209	8	36
16	INVINCIBLE ARK ROYAL	SURMILI	CARRIER	MEDIOF	BRIT	0	28	999	0	209	8	36
17	VIRGINIA	SURMILI	CRUISER	STROOF	USAM	0	35	999	0	178	10	19
18	MIRKA I	SURMILI	FRIGATE	WEAKOF	RUSS	0	32	999	0	82	3	9
19	MIRKA II	SURMILI	FRIGATE	WEAKOF	RUSS	0	32	999	0	82	3	9
20	KRIVAK IA	SURMILI	FRIGATE	STROOF	RUSS	0	32	999	0	124	5	14
21	KRIVAK IB	SURMILI	FRIGATE	STROOF	RUSS	0	32	999	0	124	5	14
22	KRIVAK II	SURMILI	FRIGATE	STROOF	RUSS	0	32	999	0	124	5	14
23	KRIVAK IIIA	SURMILI	FRIGATE	STROOF	RUSS	0	32	999	0	124	5	14
24	KRIVAK IIIB	SURMILI	FRIGATE	STROOF	RUSS	0	32	999	0	124	5	14
25	IROQUOIS	SURMILI	DESTROY	WEAKOF	CANA	0	30	999	0	130	5	15
26	ADELAIDE	SURMILI	FRIGATE	MEDIOF	AUST	0	30	999	0	138	5	14
27	IMPROVED PROVIDER	SURMILI	SUPPORT	VEWEOF	CANA	0	21	999	0	172	9	23
28	QUEST	SURMILI	MISCELL	HARMLE	CANA	0	11	999	0	72	5	13
29	KNOX	SURMILI	FRIGATE	MEDIOF	EGYP	0	27	999	0	134	5	14
30	IVAN ROGOV	SURMILI	ASSAMPH	WEAKOF	RUSS	0	25	999	0	158	8	25
31	KARA KERCH	SURMILI	CRUISER	STROOF	RUSS	0	35	999	0	173	7	19

Table D.4: *Targets data base : 1 - 31*

ID	Type eng.	Nb. cyl.	RCS side	RCS front	RCS top	IR	RP	Blades	Em. nb.	Emitters list
1	2	99	1518000	168300	2346000	1	9		99	
2	3	20	65000	8000	208000	1	2	4 4	8	57 , 58 , 59 , 60 , 61 , 6 , 7 , 8
3	1	0	44000	4200	143000	1	2	99 99	3	10 , 11 , 12
4	4	0	150300	15300	284000	1	2	99 99	9	13 , 16 , 17 , 18 , 19 , 20 , 57 , 7 , 8
5	2	99	91000	10500	195000	1	2	99 99	5	59 , 21 , 22 , 23 , 24
6	1	99	84000	9600	196500	1	2	99 99	4	25 , 30 , 27 , 28
7	4	99	182000	190	346000	1	2	99 99	9	57 , 7 , 13 , 16 , 17 , 18 , 15 , 31 , 32
8	4	99	72000	8000	250000	1	2	99 99	9	57 , 7 , 13 , 16 , 18 , 33 , 34 , 35 , 8
9	4	99	44200	5600	147000	1	2	99 99	7	7 , 58 , 36 , 37 , 33 , 38 , 39
10	4	99	450	5000	13500	1	2	99 99	4	33 , 39 , 40 , 41
11	3	99	28400	4000	71000	1	3	99 99	99	7 , 44 , 45 , 46 , 47 , 103 , 101 , 109
12	1	99	167200	28800	752400	1	2	99 99	5	48 , 50 , 51 , 12 , 49
13	4	99	60000	7500	146000	1	2	99 99	5	33 , 39 , 40 , 42 , 43
14	4	99	44400	4200	144000	1	2	99 99	3	40 , 33 , 39
15	1	99	167200	28800	752400	1	2	99 99	6	48 , 50 , 51 , 27 , 49 , 52
16	1	99	167200	28800	752400	1	2	99 99	5	48 , 50 , 9 , 12 , 49
17	4	99	178000	19000	338200	1	2	99 99	10	13 , 16 , 57 , 53 , 54 , 15 , 31 , 32 , 7 , 8
18	3	99	24600	2700	73800	1	2	99 99	5	55 , 47 , 56 , 103 , 109
19	3	99	24600	2700	73800	1	2	99 99	6	44 , 55 , 47 , 56 , 103 , 109
20	3	99	62000	7000	173600	1	2	99 99	6	62 , 69 , 67 , 45 , 68 , 103
21	3	99	62000	7000	173600	1	2	99 99	6	63 , 69 , 67 , 45 , 68 , 103
22	3	99	62000	7000	173600	1	2	99 99	5	62 , 69 , 67 , 45 , 103
23	3	99	62000	7000	173600	1	2	99 99	8	62 , 69 , 66 , 45 , 71 , 46 , 103 , 101
24	3	99	62000	7000	173600	1	2	99 99	8	63 , 69 , 66 , 45 , 71 , 46 , 103 , 101
25	1	99	65000	7500	195000	1	2	99 99	6	36 , 72 , 23 , 59 , 7 , 8
26	1	99	69000	7000	193200	1	2	99 99	8	7 , 13 , 73 , 57 , 53 , 31 , 74 , 8
27	4	99	154800	20700	395600	1	2	99 99	5	43 , 42 , 75 , 76 , 8
28	2	99	36000	6500	93600	1	2	99 99	1	79
29	4	99	67000	7000	187600	1	2	99 99	6	77 , 19 , 18 , 17 , 14 , 107
30	1	99	126400	20000	395000	1	2	99 99	9	93 , 89 , 103 , 101 , 68 , 46 , 65 , 64 , 62
31	1	99	121100	13300	328700	1	2	99 99	12	78 , 84 , 62 , 64 , 47 , 85 , 45 , 68 , 46 , 93 , 104 , 103

Table D.5: *Targets data base : 1 - 31 (continuation)*

ID	Target name	Type	Sub-type	Offensive degree	Country code	V_{min}	V_{max}	ACC	ALT max	Len.	Hei.	Wid.
32	MODIFIED KIEV	SURMILI	CARRIER	STROOF	RUSS	0	32	999	0	274	10	51
33	KIROV ADM USHAKOV	SURMILI	CRUISER	VESTOF	RUSS	0	35	999	0	252	9	29
34	SAM KOTLIN	SURMILI	DESTROY	MEDIOF	RUSS	0	36	999	0	127	5	13
35	MOSKVA	SURMILI	CRUISER	WEAKOF	RUSS	0	31	999	0	191	9	34
36	KRESTA I	SURMILI	CRUISER	MEDIOF	RUSS	0	35	999	0	156	6	17
37	TICONDEGORA	SURMILI	CRUISER	STROOF	USAM	0	35	999	0	173	10	17
38	TARAWA	SURMILI	ASSAMPH	WEAKOF	USAM	0	24	999	0	254	8	40
39	SPRUANCE	SURMILI	DESTROY	STROOF	USAM	0	33	999	0	172	6	17
40	NIMITZ	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	11	41
41	SACRAMENTO	SURMILI	SUPPORT	WEAKOF	USAM	0	26	999	0	242	12	33
42	KIROV ADM NAKHIMOV	SURMILI	CRUISER	VESTOF	RUSS	0	35	999	0	252	9	29
43	KIROV ADM LAZAREV	SURMILI	CRUISER	VESTOF	RUSS	0	35	999	0	252	9	29
44	KIROV PYOTR VELIKIY	SURMILI	CRUISER	VESTOF	RUSS	0	35	999	0	252	9	29
45	KARA AZOV	SURMILI	CRUISER	STROOF	RUSS	0	35	999	0	173	7	19
46	KARA PETROPAVLOVSK	SURMILI	CRUISER	STROOF	RUSS	0	35	999	0	173	7	19
47	KARA VLADIVOSTOK	SURMILI	CRUISER	STROOF	RUSS	0	35	999	0	173	7	19
48	IVAN ROGOV ALEKSANDR	SURMILI	ASSAMPH	WEAKOF	RUSS	0	25	999	0	158	8	25
49	IVAN ROGOV MITROFAN	SURMILI	ASSAMPH	WEAKOF	RUSS	0	25	999	0	158	8	25
50	CAMDEN SACRAMENTO	SURMILI	SUPPORT	WEAKOF	USAM	0	26	999	0	242	12	33
51	SEATLE SACRAMENTO	SURMILI	SUPPORT	WEAKOF	USAM	0	26	999	0	242	12	33
52	DETROIT SACRAMENTO	SURMILI	SUPPORT	WEAKOF	USAM	0	26	999	0	242	12	33
53	NIMITZ DWIGHT EISENH	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	11	41
54	NIMITZ CARL VINSON	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	11	41
55	NIMITZ THEODORE ROOS	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	12	41
56	NIMITZ ABRAHAM LINCO	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	12	41
57	NIMITZ GEORGE WASHIN	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	12	41
58	NIMITZ JOHN C STENNI	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	12	41
59	NIMITZ HARRY S TRUMA	SURMILI	CARRIER	WEAKOF	USAM	0	35	999	0	333	12	41

Table D.6: *Targets data base : 32 - 59*

ID	Type eng.	Nb. cyl.	RCS side	RCS front	RCS top	IR	RP	Blades	Em. nb.	Emitters list
32	4	99	274000	51000	1397400	1	4	99 99	15	88 , 94 , 95 , 96 , 97 , 65 , 98 , 71 , 46 , 91 , 99 , 100 , 101 , 102 , 106
33	4	99	226800	26100	730800	1	2	99 99	14	77 , 89 , 90 , 65 , 67 , 92 , 84 , 80 , 45 , 71 , 46 , 93 , 101 , 106
34	4	99	63500	6500	165100	1	2	99 99	7	62 , 47 , 86 , 82 , 56 , 87 , 103
35	4	99	171900	30600	649400	1	2	99 99	8	77 , 84 , 62 , 47 , 85 , 83 , 103 , 104
36	4	99	93600	10200	265200	1	2	99 99	9	77 , 80 , 62 , 65 , 81 , 82 , 83 , 46 , 103
37	1	99	173000	17000	294100	1	2	99 99	9	13 , 110 , 57 , 53 , 54 , 32 , 112 , 7 , 8
38	4	99	203200	32000	1016000	1	2	99 99	13	13 , 16 , 113 , 114 , 115 , 17 , 54 , 116 , 117 , 31 , 32 , 7 , 118
39	1	99	103200	10200	292400	1	2	99 99	12	14 , 114 , 115 , 53 , 18 , 119 , 57 , 31 , 32 , 121 , 43 , 8
40	4	99	366300	45100	1365300	1	4	99 99	14	122 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8
41	4	99	290400	39600	798600	1	2	99 99	6	13 , 130 , 33 , 18 , 121 , 7
42	4	99	226800	26100	730800	1	2	99 99	12	88 , 89 , 63 , 65 , 91 , 92 , 84 , 80 , 45 , 71 , 101 , 106
43	4	99	226800	26100	730800	1	2	99 99	12	88 , 89 , 90 , 65 , 91 , 92 , 84 , 80 , 45 , 71 , 101 , 106
44	4	99	226800	26100	730800	1	2	99 99	12	88 , 89 , 63 , 65 , 91 , 92 , 84 , 80 , 45 , 71 , 101 , 106
45	1	99	121100	13300	328700	1	2	99 99	12	78 , 84 , 62 , 64 , 85 , 45 , 92 , 68 , 46 , 93 , 104 , 103
46	1	99	121100	13300	328700	1	2	99 99	12	78 , 84 , 62 , 64 , 47 , 85 , 45 , 68 , 46 , 89 , 104 , 103
47	1	99	121100	13300	328700	1	2	99 99	12	78 , 84 , 62 , 64 , 47 , 85 , 45 , 68 , 46 , 93 , 104 , 103
48	1	99	126400	20000	395000	1	2	99 99	10	93 , 89 , 103 , 101 , 68 , 46 , 45 , 65 , 64 , 62
49	1	99	126400	20000	395000	1	2	99 99	10	93 , 89 , 103 , 101 , 68 , 46 , 45 , 65 , 64 , 105
50	4	99	290400	39600	798600	1	2	99 99	6	13 , 130 , 33 , 18 , 121 , 7
51	4	99	290400	39600	798600	1	2	99 99	5	115 , 33 , 18 , 121 , 7
52	4	99	290400	39600	798600	1	2	99 99	5	130 , 33 , 18 , 121 , 7
53	4	99	366300	45100	1365300	1	4	99 99	14	122 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8
54	4	99	366300	45100	1365300	1	4	99 99	14	122 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8
55	4	99	399600	49200	1365300	1	4	99 99	14	122 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8
56	4	99	399600	49200	1365300	1	4	99 99	14	122 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8
57	4	99	399600	49200	1365300	1	4	99 99	14	13 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8
58	4	99	399600	49200	1365300	1	4	99 99	14	13 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8
59	4	99	399600	49200	1365300	1	4	99 99	14	13 , 16 , 57 , 115 , 17 , 124 , 117 , 125 , 126 , 127 , 54 , 121 , 7 , 8

Table D.7: *Targets data base : 32 - 59 (continuation)*

ID	Target name	Type	Sub-type	Offensive degree	Country code	V_{min}	V_{max}	ACC	ALT max	Len.	Hei.	Wid.
60	SPRUANCE HAYLER	SURMILI	DESTROY	STROOF	USAM	0	33	999	0	172	6	17
61	TICONDEGORA PRINCETO	SURMILI	CRUISER	VESTOF	USAM	0	35	999	0	173	10	17
62	SIR WILLIAM ALEXANDE	SURNOMI	ICEBREA	HARMLE	CANA	0	16	999	0	83	6	16
63	UGRA II	SURMILI	SUPPORT	WEAKOF	RUSS	0	17	999	0	141	7	18
64	UDALOY II	SURMILI	DESTROY	STROOF	RUSS	0	30	999	0	164	8	19
65	UDALOY AND KULAKOV	SURMILI	DESTROY	MEDIOF	RUSS	0	30	999	0	164	8	19
66	SOVREMENNY II	SURMILI	DESTROY	STROOF	RUSS	0	32	999	0	156	7	17
67	SOVREMENNY OSMOTRITE	SURMILI	DESTROY	STROOF	RUSS	0	32	999	0	156	7	17
68	UDALOY SPIRIDONOV	SURMILI	DESTROY	MEDIOF	RUSS	0	30	999	0	164	8	19
69	SOVREMENNY BOYEVOY	SURMILI	DESTROY	STROOF	RUSS	0	32	999	0	156	7	17
70	BROADSWORD BATCH 2	SURMILI	FRIGATE	STROOF	BRIT	0	30	999	0	148	6	15
71	BROADSWORD BATCH 3	SURMILI	FRIGATE	STROOF	BRIT	0	30	999	0	148	6	15
72	MIG31 FOX-HOUND RUSSI	AIRMILI	FIGHTIN	STROOF	RUSS	200	1525	60	20600	23	6	13
73	NIELS JUEL	SURMILI	FRIGATE	MEDIOF	DANM	0	28	999	0	84	3	10
74	TYPHOON	SUBSURF	NUCPSTR	STROOF	RUSS	0	26	999	900000300	165	13	25
75	TU22M2 BACKFIRE B	AIRMILI	BOMBERS	VESTOF	RUSS	200	1080	30	13300	43	11	23
76	MIG31 FOX-HOUND CHINA	AIRMILI	FIGHTIN	STROOF	CHIN	550	1525	60	20600	23	6	13
77	TOMAHAWK 109A/C/D	AIRMILI	SSMISSI	VESTOF	N/A	450	500	999	1000	6	1	1
78	TOMAHAWK 109B	AIRMILI	SSMISSI	VESTOF	N/A	450	500	999	1000	6	1	1
79	HARPOON	AIRMILI	SSMISSI	VESTOF	N/A	500	550	999	1000	4	0	0
80	HARPOON 1D	AIRMILI	SSMISSI	VESTOF	N/A	500	550	999	1000	5	0	0
81	HARPOON SLAM	AIRMILI	SSMISSI	VESTOF	N/A	500	550	999	1000	4	0	0
82	SEA SPARROW	AIRMILI	SAMISSI	VESTOF	N/A	600	650	999	10000	4	0	0
83	AS 6 KINGFISH	AIRMILI	ASMISSI	VESTOF	N/A	500	2000	999	18000	10	1	1

Table D.8: *Targets data base : 60 - 83*

ID	Type eng.	Nb. cyl.	RCS side	RCS front	RCS top	IR	RP	Blades	Em. nb.	Emitters list
60	1	99	103200	10200	292400	1	2	99 99	12	14 , 57 , 115 , 53 , 18 , 119 , 54 , 31 , 32 , 121 , 43 , 7
61	1	99	173000	17000	294100	1	2	99 99	9	13 , 111 , 57 , 53 , 54 , 32 , 112 , 7 , 8
62	2	99	49800	9600	132800	1	2	99 99	99	unknown
63	2	99	98700	12600	253800	1	2	99 99	5	44 , 47 , 83 , 109 , 103
64	1	99	131200	15200	311600	1	2	99 99	9	69 , 97 , 63 , 65 , 128 , 91 , 71 , 93 , 131
65	1	99	131200	15200	311600	1	2	99 99	11	69 , 97 , 65 , 67 , 91 , 46 , 71 , 101 , 106 , 89 , 93
66	4	99	109200	11900	265200	1	2	99 99	6	63 , 132 , 129 , 46 , 128 , 71
67	4	99	109200	11900	265200	1	2	99 99	13	69 , 96 , 65 , 128 , 129 , 71 , 46 , 101 , 106 , 103 , 104 , 102 , 131
68	1	99	131200	15200	311600	1	2	99 99	12	69 , 97 , 63 , 65 , 67 , 91 , 46 , 71 , 101 , 106 , 89 , 93
69	4	99	109200	11900	265200	1	2	99 99	13	69 , 63 , 65 , 128 , 129 , 71 , 46 , 101 , 106 , 103 , 104 , 102 , 133
70	1	99	94800	9600	222000	1	2	99 99	4	25 , 30 , 27 , 28
71	1	99	94800	9600	222000	1	2	99 99	4	25 , 30 , 27 , 28
72	9	99	3500	1000	7500	2	2	26 26	1	133
73	3	20	24900	3000	84000	1	2	99 99	4	135 , 136 , 121 , 137
74	4	99	2500	250	412500	1	2	99 99	1	138
75	9	99	11800	3200	24600	1	2	99 99	2	139 , 166
76	9	99	3500	1000	7500	2	2	26 26	1	133
77	9	99	300	25	300	3	0	nothing	0	nothing
78	9	99	300	25	300	3	0	nothing	2	140 , 141
79	9	99	135	10	135	3	0	nothing	2	140 , 141
80	9	99	170	10	170	3	0	nothing	0	nothing
81	9	99	135	10	135	3	0	nothing	0	nothing
82	9	99	70	5	70	3	0	nothing	0	nothing
83	9	99	1000	100	1000	3	0	nothing	1	143

Table D.9: *Targets data base : 60 - 83 (continuation)*

ID	Target name	Type	Sub-type	Offensive degree	Country code	V_{min}	V_{max}	ACC	ALT max	Len.	Hei.	Wid.
84	SS N 2 STYX	AIRMILI	SSMISSI	VESTOF	N/A	500	600	999	350	6	1	1
85	EXOCET MM38	AIRMILI	SSMISSI	VESTOF	N/A	500	600	999	500	5	0	0
86	EXOCET SM39	AIRMILI	SSMISSI	VESTOF	N/A	500	600	999	500	5	0	0
87	EXOCET AM39	AIRMILI	ASMISSI	VESTOF	N/A	500	600	999	2000	5	0	0
88	EXOCET MM40 BLOCK1	AIRMILI	SSMISSI	VESTOF	N/A	500	600	999	500	6	0	0
89	EXOCET MM40 BLOCK2	AIRMILI	SSMISSI	VESTOF	N/A	500	600	999	500	6	0	0
90	CF18A/B HORNET	AIRMILI	FIGHTIN	MEDIOF	CANA	200	1150	75	15000	17	5	8
91	CP140 AURORA	AIRMILI	PATRSUR	VEWEOF	CANA	120	400	5	25000	35	10	30
92	CP140A ARCTURUS	AIRMILI	RECONNA	HARMLE	CANA	120	400	5	25000	35	10	30
93	F16 FALCON	AIRMILI	FIGHTIN	STROOF	ISRA	999999	1300	90	15000	9	5	15
94	F14A TOMCAT	AIRMILI	FIGHTIN	STROOF	USAM	200	1350	999	999999999	19	5	20
95	BOING 747 400 A	AIRCOMM	JETPROP	HARMLE	VAR	150	550	2	12000	69	19	64
96	BOING 747 400 B	AIRCOMM	JETPROP	HARMLE	VAR	150	550	2	12000	69	19	64
97	CT142 DASH 8	AIRMILI	SUPPORT	HARMLE	CANA	100	300	999	5000	22	7	26
98	EH 101 MERLIN	AIRMILI	MHELICO	WEAKOF	BRIT	0	160	2	999999999	16	5	5
99	B52H STATOFORTRESS	AIRMILI	BOMBERS	VESTOF	USAM	200	525	2	18000	49	12	56
100	S3B VIKING	AIRMILI	PATRSUR	WEAKOF	USAM	100	450	999	11000	16	7	21
101	SR71A BLACKBIRD	AIRMILI	RECONNA	HARMLE	USAM	250	2000	999	30000	33	5	11
102	CONCORDE	AIRCOMM	JETPROP	HARMLE	FRAN	225	1400	999	19000	62	11	26
103	CONCORDE	AIRCOMM	JETPROP	HARMLE	BRIT	225	1400	999	19000	62	11	26
104	TU22K BLINDER	AIRMILI	BOMBERS	STROOF	RUSS	200	900	999	14000	42	10	23
105	TU22K BLINDER	AIRMILI	BOMBERS	STROOF	LIBY	200	900	999	14000	42	10	23
106	TU22K BLINDER	AIRMILI	BOMBERS	STROOF	IRAQ	200	900	999	14000	42	10	23
107	TU95MS BEAR H	AIRMILI	BOMBERS	STROOF	RUSS	175	500	999	12000	50	12	51
108	TU95MS BEAR H	AIRMILI	BOMBERS	STROOF	UKRA	175	500	999	12000	50	12	51
109	TU95MS BEAR H	AIRMILI	BOMBERS	STROOF	KAZA	175	500	999	12000	50	12	51
110	TU16N BADGER	AIRMILI	SUPPORT	HARMLE	RUSS	200	600	999	15000	35	10	33
111	TU16PP BADGER	AIRMILI	PATRSUR	WEAKOF	RUSS	200	600	999	15000	35	10	33
112	TU16K 26 BADGER	AIRMILI	BOMBERS	STROOF	RUSS	200	600	999	15000	35	10	33

Table D.10: *Targets data base : 84 - 112*

ID	Type eng.	Nb. cyl.	RCS side	RCS front	RCS top	IR	RP	Blades	Em. nb.	Emitters list
84	9	99	600	60	600	3	0	nothing	1	142
85	9	99	200	15	200	3	0	nothing	1	144
86	9	99	200	15	200	3	0	nothing	1	144
87	9	99	150	15	150	3	0	nothing	1	144
88	9	99	200	15	200	3	0	nothing	1	144
89	9	99	200	15	200	3	0	nothing	1	145
90	9	99	2100	500	3400	2	2	18 18	4	8 , 146 , 147 , 148
91	9	99	8800	3700	26200	1	4	4 4 4 4	2	8 , 151
92	9	99	8800	3700	26200	1	4	4 4 4 4	3	141 , 149 , 150
93	9	99	1900	550	3400	2	1	13	2	153 , 152
94	9	99	2400	1250	9500	2	2	13 13	2	154 , 155
95	9	99	33000	15000	110000	1	4	38 38	1	99999
96	9	99	33000	15000	110000	1	4	24 24	1	99999
97	9	99	3850	2300	14300	1	2	4 4	2	156 , 157
98	9	99	2000	1250	2000	1	2	5 4	1	158
99	9	99	14700	8400	68600	1	1	23	3	159 , 160 , 161
100	9	99	2800	1850	8400	1	2	99 99	1	162
101	9	99	4000	700	9000	2	8	unknown	99	unknown
102	9	99	17000	3600	40000	2	8	unknown	99	unknown
103	9	99	17000	3600	40000	2	8	unknown	99	unknown
104	9	99	10500	3000	24000	2	2	99 99	2	164 , 139
105	9	99	10500	3000	24000	2	2	99 99	2	164 , 139
106	9	99	10500	3000	24000	2	2	99 99	2	164 , 139
107	9	99	15300	7600	64000	1	4	8 8 8 8	3	166 , 167 , 168
108	9	99	15300	7600	64000	1	4	8 8 8 8	3	166 , 167 , 168
109	9	99	15300	7600	64000	1	4	8 8 8 8	3	166 , 167 , 168
110	9	99	8800	4000	29000	1	2	99 99	99	unknown
111	9	99	8800	4000	29000	1	2	99 99	99	unknown
112	9	99	8800	4000	29000	1	2	99 99	99	unknown

Table D.11: *Targets data base : 84 - 112 (continuation)*

ID	Target name	Type	Sub-type	Offensive degree	Country code	V_{min}	V_{max}	ACC	ALT max	Len.	Hei.	Wid.
113	F15E EAGLE	AIRMILI	FIGHTIN	STROOF	USAM	200	1600	90	20000	19	6	13
114	F15I EAGLE	AIRMILI	FIGHTIN	STROOF	ISRA	200	1600	90	20000	19	6	13
115	YAK38 FORGER A	AIRMILI	FIGHTIN	STROOF	RUSS	0	550	999	12000	15	5	7
116	TU160 BLACKJACK	AIRMILI	BOMBERS	VESTOF	RUSS	200	1300	20	15000	54	13	51
117	MIG29 FULCRUM A	AIRMILI	FIGHTIN	STROOF	SYRI	200	1400	90	20000	15	5	11
118	MI28 HAVOC	AIRMILI	MHELICO	STROOF	RUSS	0	170	30	6000	17	4	2
119	SU27K FLANKER D	AIRMILI	FIGHTIN	VESTOF	RUSS	150	1240	80	11000	19	6	15
120	MI35P HIND F	AIRMILI	MHELICO	MEDIOF	PERU	0	180	5	4500	17	4	2
121	MIG29 FULCRUM A	AIRMILI	FIGHTIN	STROOF	INDI	200	1400	90	20000	15	5	11
122	MIG29 FULCRUM A	AIRMILI	FIGHTIN	STROOF	POLA	200	1400	90	20000	15	5	11
123	MIG29K FULCRUM D	AIRMILI	FIGHTIN	VESTOF	RUSS	200	1400	90	20000	15	5	11
124	KA 25PL HORMONE	AIRMILI	MHELICO	MEDIOF	RUSS	0	120	2	3400	10	5	3
125	ANTONOV 124	AIRCOMM	JETPROP	HARMLE	BRIT	150	470	1	12000	69	20	73
126	KA 50 HOKUM WEREWOLF	AIRMILI	MHELICO	VESTOF	RUSS	0	190	30	4500	16	2	3
127	SEA HARRIER FRS	AIRMILI	FIGHTIN	VESTOF	BRIT	150	800	80	10000	13	4	8
128												
129												
130												
131	C 17A GLOBEMAS-TER	AIRMILI	SUPPORT	HARMLE	USAM	115	500	1	9000	48	17	50
132												
133												
134												
135	CH 47 CHINOOK	AIRMILI	MHELICO	VEWEOF	USAM	0	155	2	3200	16	6	4
136												
137	DASSAULT RAFALE B	AIRMILI	FIGHTIN	MEDIOF	FRAN	150	1300	95	20000	15	5	11
138	DASSAULT MIRAGE 2000	AIRMILI	FIGHTIN	MEDIOF	GREE	150	1400	135	17000	14	5	9
139	CONCORDE AF FAKE	AIRCOMM	JETPROP	HARMLE	FRAN	225	1400	999	19000	62	11	26
140	SR71A BLACKBIRD FAKE	AIRMILI	RECONNA	HARMLE	USAM	250	2000	999	30000	33	5	11
141	MIG31 FOXHOUND FAKE	AIRMILI	FIGHTIN	MEDIOF	RUSS	200	1525	60	20600	23	6	13
142	TU22M2 BACKFIRE FAKE	AIRMILI	BOMBERS	STROFF	RUSS	200	1080	30	13300	43	11	23
143	UNKNOWN	UNKNOWN	UNKNOWN	UNKNOWN	UNKN	0	0	0	0	0	0	0

Table D.12: *Targets data base : 113 - 143*

ID	Type eng.	Nb. cyl.	RCS side	RCS front	RCS top	IR	RP	Blades	Em. nb.	Emitters list
113	9	99	2800	1000	6200	2	2	22 22	2	169 , 170
114	9	99	2800	1000	6200	2	2	22 22	2	169 , 171
115	9	99	1900	400	2600	1	8	unknown	1	163
116	9	99	17500	8300	69000	2	4	99 99	2	99999 , 99999
117	9	99	1900	700	4100	2	2	32 32	1	177
118	9	99	1700	400	850	1	2	5 4	0	nothing
119	9	99	2800	1100	7100	2	2	24 24	2	99999 , 99999
120	9	99	1700	400	850	1	2	5 3	0	nothing
121	9	99	1900	700	4100	2	2	32 32	1	177
122	9	99	1900	700	4100	2	2	32 32	1	177
123	9	99	1900	700	4100	2	2	32 32	1	177
124	9	99	1300	700	400	1	2	3 3	1	178
125	9	99	34500	18000	126000	1	4	33 33	2	99999 , 99999
126	9	99	800	300	1200	1	2	3 3	0	nothing
127	9	99	1300	400	2600	2	1	23	2	180 , 181
128										
129										
130										
131	9	99	20000	10600	60000	1	4	36 36	1	179
132										
133										
134										
135	9	99	2400	1200	1600	1	2	3 3	99	unknown
136										
137	9	99	1900	700	4000	2	2	15 15	2	173 , 174
138	9	99	1700	550	3200	2	2	23 23	3	172 , 175 , 176
139	9	99	17000	3600	40000	2	8	unknown	3	165 , 99952 , 161
140	9	99	4000	700	9000	2	8	unknown	2	99954 , 157
141	9	99	3500	1000	7500	2	2	26 26	1	99953
142	9	99	11800	3200	24600	1	2	99 99	2	99951 , 163
143	0	0	0	0	0	0	0	0 0	0	0

Table D.13: *Targets data base : 113 - 143 (continuation)*

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Fusion of imperfect information in the unified framework of random sets theory: Application to target identification

4. AUTHORS (Last name, followed by initials – ranks, titles, etc. not to be used.)

Florea, M.C.; Jousselme, A.-L.; Bossé, É.

5. DATE OF PUBLICATION (Month and year of publication of document.) November 2007	6a. NO. OF PAGES (Total containing information. Include Annexes, Appendices, etc.) 138	6b. NO. OF REFS (Total cited in document.) 40

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This is a study of the applicability of random sets theory to target identification problems as a technique for fusion of imperfect information. For target identification, several sources of information (radar, ESM - Electronic Support Measures, SAR - Synthetic Aperture Radar, IR images) are available. Since the information provided is always imperfect and several kinds of imperfection may be encountered (imprecision, uncertainty, incompleteness, vagueness, etc.), several theories were developed to assist probability theory (long the only tool available to deal with uncertainty) in data fusion problems. In recent decades fuzzy sets theory was developed to deal with vague information, possibility theory was developed to deal with incomplete information, evidence theory was developed to deal with imprecise and uncertain information, and rough sets theory was developed to deal with vague and uncertain information. These theories have several points in common; here we study random sets theory, which is a unifying framework for all the aforementioned theories. In two simple test scenarios, we demonstrate the effectiveness of this unifying framework for representing and fusing imperfect information in the target identification application.

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