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14. ABSTRACT:

The wavelet transform has emerged over the last decade as a powerful new tool for statistical signal processing. The wavelet domain provides a natural setting for many applications involving real-world signals and images, especially those rich in singularities (edges, ridges, and other transients). In this project, we extended wavelet transform modeling and processing algorithms to handle multidimensional signals that are smooth save for singularities along lower-dimensional manifolds. The key building block is a new quaternion wavelet transform (QWT) that generalizes the complex wavelet transform to higher dimensions using a multidimensional Hilbert transform. The QWT has a quaternion magnitude-phase representation that encodes image shifts in an absolute (x,y)-coordinate system and thus provides a theoretical framework for analyzing the phase behavior of 2-D image shifts. We conducted a thorough analysis of the QWT phase around edge regions and thereby developed efficient multiscale edge localization and flow/motion estimation algorithms for image registration based on the QWT phase.

15. SUBJECT TERMS
Wavelets, multiscale analysis, detection, classification, recognition, compression

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Final Report

Multiscale Analysis, Modeling, and Processing of Higher-Dimensional Geometric Data

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Objectives

The wavelet transform has emerged as an exciting new tool for statistical signal processing. The wavelet domain provides a natural setting for many applications involving real-world signals and images, especially those rich in *singularities* (edges, ridges, and other transients).

This research aims to extend the wavelet transform and modeling and processing algorithms to handle multidimensional signals that are smooth save for singularities along lower-dimensional manifolds. The basic building blocks are (1) a new *hypercomplex wavelet transform* (HWT) that generalizes the complex wavelet transform to higher dimensions using a multidimensional Hilbert transform and (2) adaptive dyadic partitions based on *wedgelets*, which are atomic edge segments. From the HWT, we will develop new classes of nearly shift-invariant wavelet frames that are oriented along lower-dimensional subspaces. Finally, we will construct statistical models for these new transforms and partitions that extend the hidden Markov tree (HMT) model developed under previous AFOSR support to accurately, realistically, and efficiently represent singularity structure.

Accomplishments

Efficient image geometry encoding using wavelets and wedgelets

We developed a prototype *image compression algorithm* based on marrying wavelets and wedgelets that encodes the image geometry description and the residual texture information jointly in a rate/distortion optimal fashion. The algorithm outperforms the new JPEG2000 by a fairly significant margin (up to 1-2dB in PSNR for "edgy" images; see Figure 1). Indeed, we have proved that for a simple class of "piecewise smooth" cartoon images consisting of smooth

(C^2) regions delineated by smooth (C^2) edges, that this algorithms is *near-optimal in a rate/distortion sense*. That is, the mean-squared error D of image approximation decays with the number of bits R as $D(R) \sim C\ (N/\log N)^{-2}$. Finally, since the compressed bit stream contains explicit geometry information regarding edges, it could prove very useful for efficient image classification and data base searching. This new framework demands further investigation, in particular for adaptive compression and joint compression/classification of geometric targets.

Dual-tree quaternion wavelet transform (QWT)

Encoding and estimation of the relative locations of image features plays an important role in image processing applications, ranging from feature detection and target recognition to image compression. For example, in edge detection, the goal is to locate the boundary of an object in an image. In image denoising or compression, state-of-the-art techniques achieve significant performance improvements by exploiting information on the relative locations of large transform coefficients.

An efficient way to compute and represent relative location information in one-dimensional (1-D) signals is through the phase of the Fourier transform. The Fourier shift theorem provides a simple linear relationship between the Fourier phase and the signal shift. When only a local region of the signal is of interest, the short-time Fourier transform (STFT) provides a local Fourier phase for each windowed portion of the signal.

The discrete wavelet transform (DWT) is more natural than the STFT for signals containing isolated singularities, such as piecewise smooth functions. The locality of the wavelet basis functions leads to a sparse representation of such signals that compacts the signal energy into a small number of coefficients. Wavelet coefficient sparsity is the key enabler of algorithms such as wavelet-based denoising by shrinkage.

While wavelets have been successfully applied to many signal and image processing applications, one of the major problems with conventional real-valued wavelets is their *lack of shift-invariance*. A small shift of the signal results in significant fluctuations of wavelet coefficient energy, making it difficult to extract or model signal information from the coefficient values. There is also no notion of phase to encode signal location information as in the Fourier case. Complex wavelet transforms (CWTs), such as the dual-tree CWT, can remedy these problems. In 1-D, the dual-tree CWT uses complex wavelet basis functions whose real and imaginary parts form a 1-D Hilbert transform pair. The 1-D dual-tree CWT is slightly (two times) redundant, but the magnitudes of its coefficients are almost shift invariant. The CWT phase also contains information on the locations of signal singularities. An important precursor of the dual-tree approach is the complex Daubechies wavelet transform, which is orthonormal but not shift-invariant.

In 2-D images, a single complex phase is not enough to encode image shifts. For example, in conventional 2-D Fourier analysis, as the underlying images translates, the phase value is a linear combination of the horizontal and vertical image shifts, and inference of the shifts from a single phase value is impossible. The same phase ambiguity problem exists in the 2-D CWT, which measures phase shifts only in directions perpendicular to its wavelet orientations.

To remedy this problem, we married the quaternion Fourier transform (QFT) with the dual-tree CWT to develop a new *quaternion wavelet transform* (QWT) for images and other 2-D data. The QWT leverages the notions of 2-D Hilbert transform, 2-D analytic signal, and quaternion algebra. The 2-D Hilbert transforms of a usual 2-D DWT tensor product wavelet (that is, the 1-D Hilbert transform of the wavelet along either or both of the horizontal and vertical directions) plus the wavelet itself constitute the four components of each quaternion wavelet (see Figure 2). The QWT frame, which can be efficiently generated from a dual-tree filter bank, is a 4x redundant tight frame that is stably invertible.

Image localization via QWT

The QWT has a quaternion magnitude-phase representation that encodes image shifts in an absolute (x,y)-coordinate system and thus provides a theoretical framework for analyzing the phase behavior of 2-D image shifts. We have conducted a thorough analysis of the QWT phase around edge regions. We have also developed an efficient multiscale flow/motion estimation algorithm for image registration based on the QWT phase. Figure 3 illustrates our results.

Disparity estimation via QWT

The QWT has a quaternion magnitude-phase representation that encodes image shifts in an absolute (x,y)-coordinate system and thus provides a theoretical framework for analyzing the phase behavior of 2-D image shifts. A thorough analysis of the QWT phase around edge regions has enabled us to develop an efficient multiscale flow/motion estimation algorithm for image registration based on the QWT phase.

The algorithm estimates the *local disparities* between a target image A(x,y) and a reference image B(x,y) by determining the local translations needed to align different regions in two images, that is, the amount of 2-D translation required to move a local region of a target image centered at pixel (x0, y0) to align with the region in a reference image centered at the same location (x0, y0). This problem figures prominently in a range of image processing and computer vision tasks, including video processing to estimate motion between successive frames, super-resolution, etc. The algorithm is competitive with much more complicated algorithms such as the Gradient Correlation and Exhaustive Search algorithms; it offers sub-pixel accuracy, linear computational complexity, and a multiscale structure that is amenable to regularization with noisy data. Figure 4 illustrates our results.

Hypercomplex wavelets in 3-D and beyond

We have also extended the QWT to handle multidimensional signals that are smooth save for singularities along lower-dimensional manifolds. Using a *hypercomplex wavelet transform* (HWT) as a building block, we have constructed new classes of nearly shift invariant wavelet frames that are oriented along lower-dimensional subspaces. The HWT can be computed conveniently and efficiently using a 1-D dual-tree complex wavelet transform along each signal axis. A series of denoising experiments demonstrates that the HWT significantly outperforms both standard tensor product wavelets and complex wavelets when the signals of interest sport a local manifold structure. See Figure 5 for an example of three hypercomplex wavelets in 3-D. Note their strong 1-D orientation, which could be useful for analyzing aircraft trajectories through 3-D space.

Compressive sensing

Compressive Sensing is an emerging field based on the revelation that a small number of linear projections of a compressible signal contain enough information for reconstruction and processing. It has many promising implications and enables the design of new kinds of data acquisition systems, imaging systems, and cameras. We have started to explore a new camera architecture that employs a digital micromirror array to perform optical calculations of linear projections of an image onto pseudorandom binary patterns (see Figure 6). Its hallmarks include the ability to obtain an image with a single detection element while sampling the image fewer times than the number of pixels (sub-Nyquist measuring). Other attractive properties include its universality, robustness, scalability, progressivity, and computational asymmetry. The most intriguing feature of the system is that, since it relies on a single photon detector, it can be adapted to image at wavelengths that are currently impossible with conventional CCD and CMOS imagers. Examples of compressive image acquisition are given in Figure 7.

Personnel Supported

Richard Baraniuk (PI)
Hyeokho Choi (co-PI)
Mike Wakin (graduate student)
William Chan (graduate student)

Publications

All papers are available at the Rice DSP web site at *dsp.rice.edu/publications*

Journal publications

R. Neelamani, M. Deffenbaugh, and R. G. Baraniuk, "Texas Two-Step: A Framework for Optimal Multi- Input Single-Output Deconvolution," *IEEE Transactions on Signal Processing*, 2007.

R. Neelamani, R. G. Baraniuk, and S. Dash, "On Nearly Orthogonal Lattice Bases," *SIAM Journal on Discrete Mathematics*, 2007.

V. J. Ribeiro, R. H. Riedi, and R. G. Baraniuk, "Optimal Sampling Strategies for Multiscale Stochastic Processes," *Proceedings of Second Erich Lehman Symposium – Optimality*, Institute of Mathematical Statistics, Lecture Notes-Monograph Series, Volume 49, 2006.

D. Baron and R. G. Baraniuk, "Faster Sequential Universal Coding via Block Partitioning," *IEEE Transactions on Information Theory*, Vol. 52, No. 4, pp. 1708–1710, April 2006.

V. J. Ribeiro, R. H. Riedi, and R. G. Baraniuk, "Multiscale Queuing Analysis of Long-Range-Dependent Network Traffic," *IEEE Transactions on Networks*, October 2006.

V. Delouille, R. Neelamani, and R. G. Baraniuk, "Robust Distributed Estimation using the Embedded Subgraphs Algorithm," *IEEE Transactions on Signal Processing*, July 2006.

M. Wakin, J. K. Romberg, H. Choi, and R. G. Baraniuk, "Wavelet-domain Approximation and Compression of Piecewise Smooth Images," IEEE Transactions on Signal Processing, 2006.

R. Neelamani, R. de Queiroz, Z. Fan, S. Dash, and R. G. Baraniuk, "JPEG Compression History Estimation for Color Images," to appear in IEEE Transactions on Image Processing, 2006.

I. Selesnick, N. G. Kingsbury, and R. G. Baraniuk, "The Dual-Tree Complex Wavelet Transform – A Coherent Framework for Multiscale Signal and Image Processing," IEEE Signal Processing Magazine, November 2005.

M. Jansen, R. G. Baraniuk, and S. Lavu, "Multiscale Approximation of Piecewise Smooth Two-Dimensional Functions using Normal Triangulated Meshes," Applied and Computational Harmonic Analysis, Vol. 19, No. 1, pp. 92–130, July 2005.

Conference publications

W. Chan, H. Choi, R. G. Baraniuk, "Multiscale Disparity Estimation using the Quaternion Wavelet Transform," *IEEE International Conference on Image Processing* – ICIP-2006, Atlanta, October 2006.

M. B. Wakin, J. Laska, M. Duarte, D. Baron, S. Sarvotham, D. Takhar, K. Kelly, R. G. Baraniuk, "A New Camera Architecture for Compressive Imaging," *IEEE International Conference on Image Processing* – ICIP-2006, Atlanta, October 2006.

S. Sarvotham, D. Baron, R. G. Baraniuk, "Measurements and Bits: Compressed Sensing Meets Information Theory," *Allerton Conference on Communication, Control, and Computing*, Allerton, IL, September 2006.

M. A. Davenport, R. G. Baraniuk, C. Scott, "Learning Minimum Volume Sets with Support Vector Machines," *IEEE International Workshop on Machine Learning for Signal Processing*, Maynooth, Ireland, September 2006.

S. Sarvotham, D. Baron, R. G. Baraniuk, "Sudocodes – Fast Measurement and Reconstruction of Sparse Signals," *IEEE International Symposium on Information Theory* – ISIT, Seattle, July 2006.

M. Wakin and R. G. Baraniuk, "Random Projections of Signal Manifolds," *IEEE International Conference on Acoustics, Speech and Signal Processing* – ICASSP'06, Special session on Statistical Inference on Nonlinear Manifolds, Toulouse, France, May 2006.

M. Duarte, M. Davenport, M. B. Wakin, and R. G. Baraniuk, "Sparse Signal Detection from Incoherent Projections," *IEEE International Conference on Acoustics, Speech and Signal Processing* – ICASSP'06, Toulouse, France, May 2006.

J. Tropp, M. B. Wakin, M. Duarte, D. Baron, and R. G. Baraniuk, "Random Filters for Compressive Sampling and Reconstruction," *IEEE International Conference on Acoustics, Speech and Signal Processing* – ICASSP'06, Toulouse, France, May 2006.

M. Davenport, C. Scott, and R. G. Baraniuk, "Controlling False Alarms with Support Vector Machines," *IEEE International Conference on Acoustics, Speech and Signal Processing* – ICASSP'06, Toulouse, France, May 2006.

M. Wakin, J. N. Laska, M. F. Duarte, D. Baron, S, Sarvotham, D. Takhar, K. F. Kelly, R. G. Baraniuk, "Compressive Imaging for Video Representation and Coding," *Picture Coding Symposium*, special session on Next Generation Video Representation, Beijing, April 2006.

M. Duarte, M. Wakin, D. Baron, and R. G. Baraniuk, "Universal Distributed Sensing via Random Projections," *International Symposium on Integrated Processing in Sensor Networks* (IPSN), Nashville, April 2006.

R. Wagner, R. G. Baraniuk, Shu Du, D. B. Johnson, A. Cohen, "An Architecture for Wavelet Analysis and Processing in Sensor Networks," *International Symposium on Integrated Processing in Sensor Networks* (IPSN), Nashville, April 2006.

D. Baron, S. Sarvotham, R. G. Baraniuk, "Coding vs. Packet Retransmission in Lossy Communication Systems," *Conference on Information Sciences and Systems* (CISS), Baltimore, March 2006.

D. Takhar, J. N. Laska, M. B. Wakin, M. F. Duarte, D. Baron, S, Sarvotham, K. F. Kelly, R. G. Baraniuk, "A New Compressive Imaging Camera Architecture using Optical-Domain Compression," *IS&T/SPIE Computational Imaging IV*, San Jose, January 2006.

M. Duarte, M. Wakin, S. Sarvotham, D. Baron, and R. G. Baraniuk "Recovery of Jointly Sparse Signals from Few Random Projections," *Neural Information Processing Systems* (NIPS), Vancouver, December 2005.

M. Duarte, M. Wakin, S. Sarvotham, D. Baron, and R. G. Baraniuk, "Distributed Compressed Sensing of Jointly Sparse Signals," *39th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, November, 2005.

S. Sarvotham, D. Baron, and R. G. Baraniuk, "Variable-Rate Universal Slepian-Wolf Coding with Feedback," *39th Asilomar Conference on Signals, Systems, and Computers,* Pacific Grove, CA, November, 2005.

D. Baron, M. Duarte, S. Sarvotham, M. Wakin, and R.G. Baraniuk, "An Information-Theoretic Approach to Distributed Compressed Sensing," Allerton Conference on Communication, Control, and Computing, Allerton, IL, September 2005.

S. Sarvotham, D. Baron, and R.G. Baraniuk, "Variable-Rate Coding with Feedback for Universal Communication Systems," Allerton Conference on Communication, Control, and Computing, Allerton, IL, September 2005.

M. Wakin, D. L. Donoho, H. Choi, and R. G. Baraniuk, "The Multiscale Structure of Non-Differentiable Image Manifolds," Wavelets XI in SPIE International Symposium on Optical Science and Technology, San Diego, August 2005.

W. Chan, H. Choi, and R. G. Baraniuk, "Coherent Image Processing using Quaternion Wavelets," Wavelets XI in SPIE International Symposium on Optical Science and Technology, San Diego, August 2005.

R. Wagner, V. Delouille, H. Choi, and R. G. Baraniuk, "Distributed Wavelet Analysis for Irregular Sensor Network Grids," IEEE Workshop on Statistical Signal Processing – SSP'05, Bordeaux, France, July 2005.

S. Palchaudhuri, R. Kumar, R. G. Baraniuk, and D. B. Johnson, "Design of Adaptive Overlays for Multi-scale Communication in Sensor Networks," IEEE International Conference on Distributed Computing in Sensor Systems, Columbus, OH, June 2005.

S. Sarvotham, D. Baron, R. G. Baraniuk, "Non-Asymptotic Performance of Symmetric Slepian-Wolf Coding," Conference on Information Sciences and Systems (CISS), Baltimore, March 2005.

M. Wakin, D. Donoho, H. Choi, and R. G. Baraniuk, "Multiscale Navigation on Non-Differentiable Manifolds," special session on Higher-Dimensional Geometry in Signal Processing, IEEE International Conference on Acoustics, Speech and Signal Processing – ICASSP'05, Philadelphia, March 2005.

R. Wagner, S. Sarvotham, and R. G. Baraniuk, "A Multiscale Data Representation for Distributed Sensor Networks," IEEE International Conference on Acoustics, Speech and Signal Processing – ICASSP'05, Philadelphia, March 2005.

H. Choi and R. G. Baraniuk, "Multiscale Manifold Representation and Modeling," IEEE International Conference on Acoustics, Speech and Signal Processing – ICASSP'05, Philadelphia, March 2005.

N. Ahmed and R. G. Baraniuk, "Delay-limited Throughput Maximization for Fading Channels using Rate and Power Control," IEEE Globecom, Symposium on Wireless Communications, Networks and Systems, Dallas, December 2004.

N. Ahmed and R. G. Baraniuk, "Throughput Maximization for ARQ-like Systems in Fading Channels with Coding and Queuing Delay Constraints," 38th Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November, 2004.

D. Baron, A. Khojastepour, and R. G. Baraniuk, "How Quickly Can We Approach Capacity for the Gaussian Channel?" 38th Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November, 2004.

W. Mantzel, H. Choi, and R. G. Baraniuk, "Distributed Camera Network Localization," 38th Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November, 2004.

D. Baron, A. Khojastepour, and R. G. Baraniuk, "Excess rates of non-asymptotic Slepian-Wolf coding," Forty-Second Annual Allerton Conference on Communication, Control, and Computing, Allerton, IL, September 2004.

W. Chan, H. Choi, and R. G. Baraniuk, "Quaternion Wavelets for Image Analysis and Processing," IEEE International Conference on Image Processing – ICIP-2004, Singapore, October 2004.

Interactions/Transitions

Presentations

2006 Keynote and plenary presentations at the 5th Annual ASEE Global Colloquium on Engineering Education, (Rio de Janeiro, Brazil), 12th IEEE Digital Signal Processing Workshop, EPFL Bernoulli Center Workshop on Wavelets and Applications, TED – Technology, Entertainment, and Design plus the usual presentations at IMA, TI Developers Conference, Google, Michigan State, Boston U., Toledo, LANL, and the AMD Global Vision Conference.

2005 IMA, Fashion Institute of Technology, CMU, UC-Berkeley, MSRI, TI Developers Conference, ExxonMobil, NLII Conference, USC Annenberg Center, ConocoPhillips, Texas Instruments, MIT, UCLA IPAM

Consulting

None relevant to this project.

Transitions

Technology on the Rice compressive imager has been transferred to Texas Instruments for possible incorporation into next-generation imaging systems.

Projects

None directly relevant this project.

New Discoveries, Inventions, Patent Disclosures

Patents filed: (1) Method and Apparatus for Compressive Imaging Device, (2) Method and Apparatus for Optical Image Compression, (3) Method and Apparatus for Reconstruction of Data from Multiple Sources, and (4) Method and Apparatus for Compressive Imaging Device.

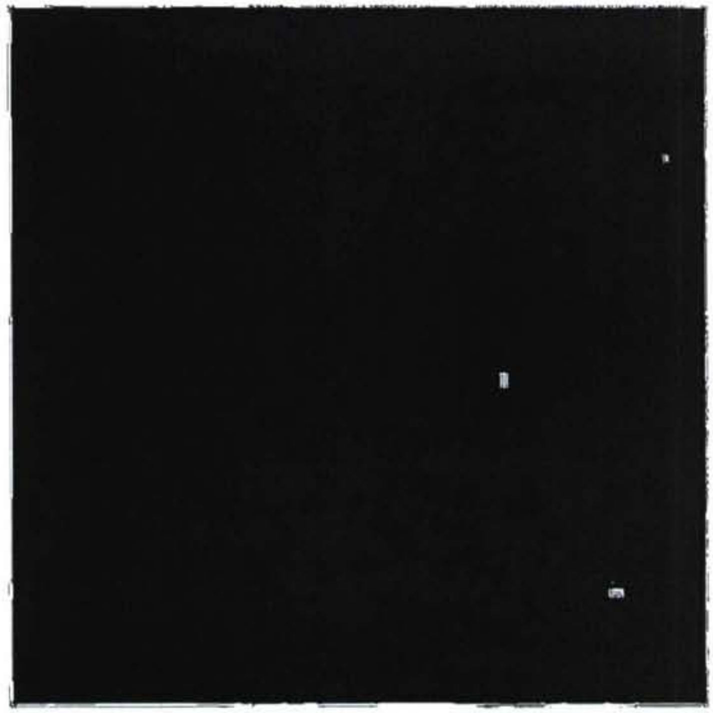
Honors and Awards

2007: Richard Baraniuk received Edutopia Magazine's "Daring Dozen" Education Innovators, MIT Technology Review TR10 Top 10 Emerging Technology for the Rice Single-Pixel Camera, Hershel M. Rich Invention Award (Rice).

2006: Richard Baraniuk received the Tech Museum of Innovation Laureate for Connexions (cnx.org) and the George R. Brown Award for Superior Teaching at Rice (third time) in 2006.

2004: Richard Baraniuk was elevated to the Victor E. Cameron Chaired Professorship in Electrical and Computer Engineering at Rice.

state-of-the-art
SFQ
wavelet image
coder
(zoom)



new
WSFQ
wavelet/wedgelet
image coder
(zoom)

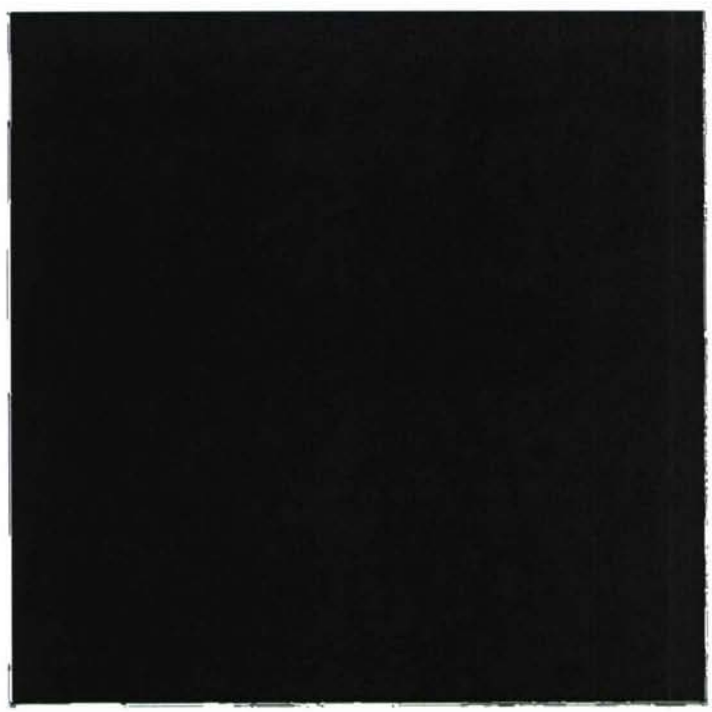


Figure 1: *Comparison of state-of-the-art SFQ wavelet-based image coder versus the new WSFQ coder based on wavelets and wedglets (zoom).*

Figure 2: *Quaternion wavelets and scaling function in 2-D. Each wavelet is 90 degrees phase shifted from the others in either the horizontal, vertical, or both directions.*

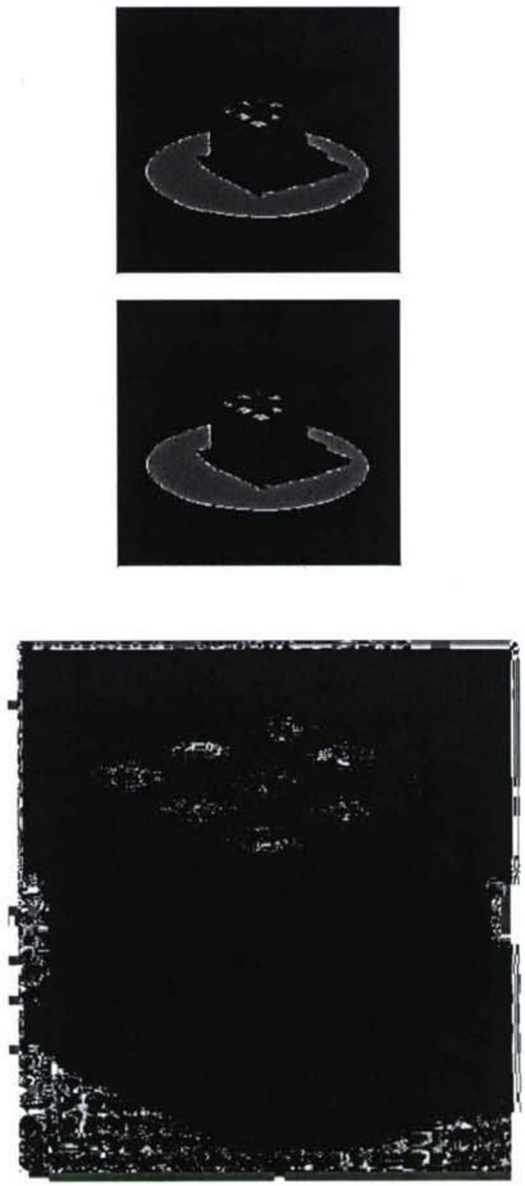


Figure 3: *Quaternion-based image flow estimation. Top: Two images from a video sequence of a spinning cube. Bottom: Multiscale image flow estimate base on the QWT of the images. The algorithm is both efficient and accurate and applicable to a wide range of video processing applications.*

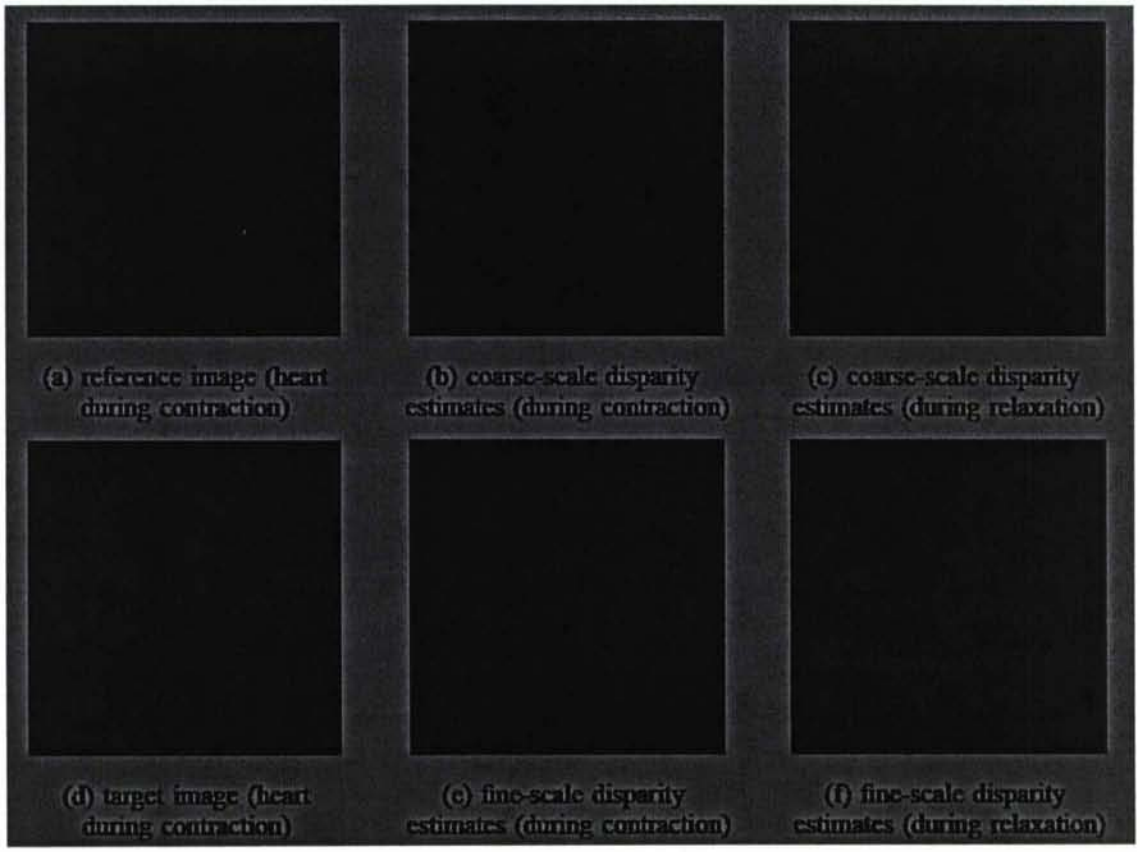


Figure 4: *Quaternion-based image flow estimation. Two images from a video sequence of a beating heart. Multiscale image flow estimate base on the QWT of the images. The algorithm is both efficient (linear complexity in the number of pixels) and accurate (sub-pixel accuracy) and applicable to a wide range of image and video processing applications.*

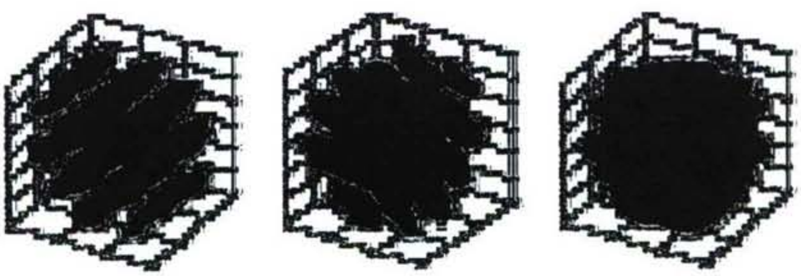


Figure 5: *Quaternion wavelets and scaling function in 3-D. Each wavelet is 90 degrees phase shifted from the others in either the horizontal, vertical, or both directions.*

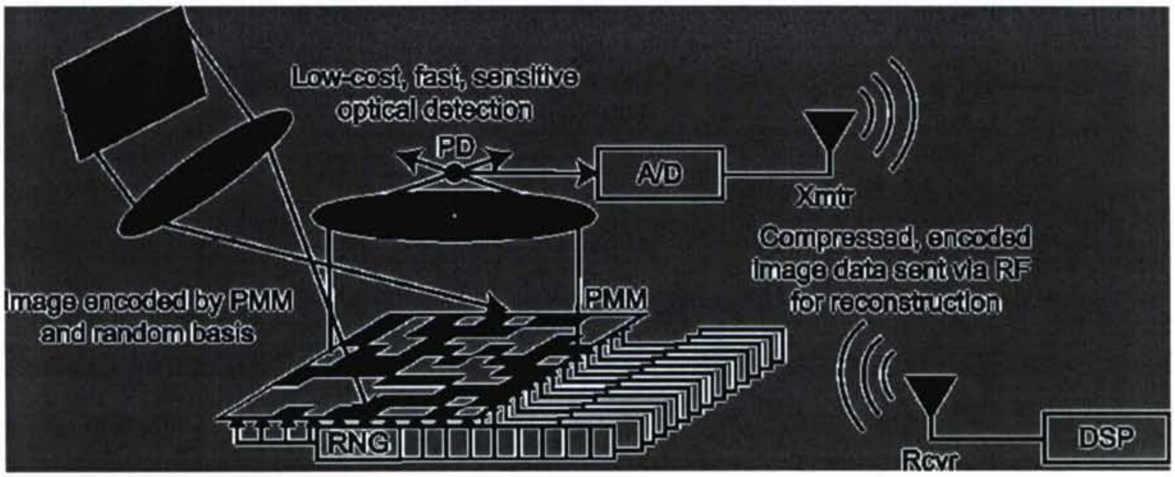


Figure 6: *Compressive sensing, "single pixel" camera architecture that optically computes inner products between the scene being imaged and pseudorandom patterns. Back-end signal processing reconstructs the image from the random measurements.*

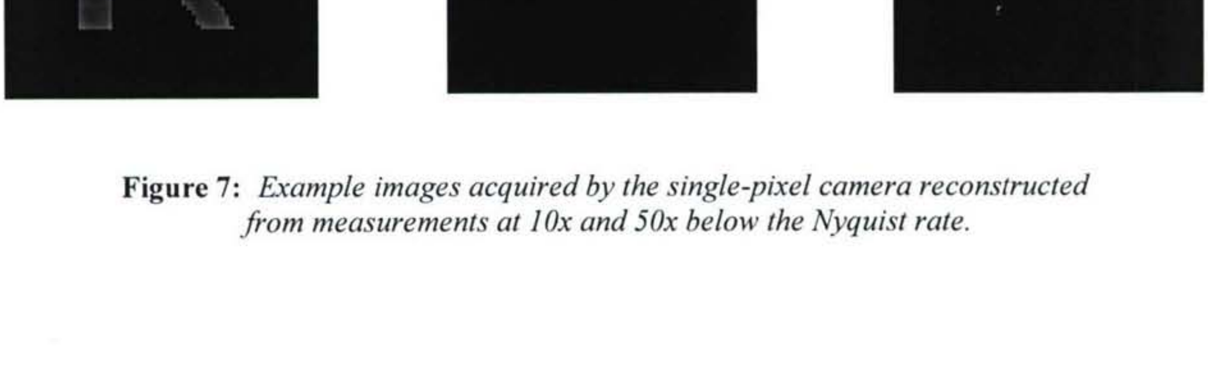


Figure 7: *Example images acquired by the single-pixel camera reconstructed from measurements at 10x and 50x below the Nyquist rate.*