HARDWARE REALIZATION OF A TRANSFORM
DOMAIN COMMUNICATION SYSTEM

THESIS

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THESIS

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Abstract

The purpose of this research was to implement a Transform Domain Communication System (TDCS) in hardware and compare experimental bit error performance with results published in literature. The intent is to demonstrate the effectiveness or ineffectiveness of a TDCS in communicating binary data across a real channel. In this case, an acoustic channel that is laden with narrowband interference was considered. A TDCS user pair was constructed to validate the proposed design using Matlab® to control a PC sound card. The proposed TDCS design used the Bartlett method of spectrum estimation, the spectral notching algorithm found in TDCS literature, quadrature phase shift keying, and minimum mean square error transverse equalization to mitigate the effects of noise and intersymbol interference. Water-filling was evaluated as an alternative to spectral notching for performing waveform design and is shown to perform equivalently. Validated software was migrated to code suitable for use onboard a Digital Signal Processor Starter Kit (DSK). Two DSK boards were used, one for transmission and reception, and bit error performance results were obtained. Bit error analysis reveals that the TDCS hardware performs approximately the same as literature suggests.
To my Son and Daughter,

my two best reasons for getting out of bed in the morning
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Marshall E. Haker
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HARDWARE REALIZATION OF A
TRANSFORM DOMAIN COMMUNICATION SYSTEM

I. Introduction

1.1. Background

Transform Domain Communications System (TDCS) research has been conducted for over a decade in a joint effort between the Air Force Institute of Technology (AFIT) and the Air Force Research Laboratory (AFRL) Sensors Directorate [1]-[8]. This research was spurred by the notion that it may be possible to communicate using digital signaling while both avoiding interfering signals in the RF spectrum bandpass of interest [9] and ensuring a low probability of detection, intercept, and/or exploitation (LPD/LPI/LPE) [8, 10]. Over the course of the last decade, this research has involved efforts to determine the feasibility and effectiveness of a TDCS in signal acquisition and synchronization, application in a multiple access network, and the impact of multipath and intersymbol interference on TDCS signaling. The intrinsic capability of a TDCS to avoid interference has been studied as well. While this research has been published and reported in venues such as the IEEE Communications Magazine [8], and has even at least partially motivated the development of a digital communications signaling framework [11], there has never been a formal attempt to implement a TDCS in hardware. Rather, all previous research has involved simulating TDCS signaling between a user pair in various environments. The intent of this research was to take the next
logical step in TDCS research: the realization and study of a TDCS hardware implementation.

1.2. Problem Statement

There has never been a formal attempt to implement the TDCS architecture in hardware. To do this, the design and construction of the traditional transmitter-receiver user pair, as well as functions such as synchronization and equalization, must be considered. Furthermore, several different design decisions must be made in order to fill gaps between a communications transceiver or network model used in simulations and a realized, functional hardware implementation. This thesis represents the first attempt at realizing a TDCS in hardware, with the perspective taken that this realization is intended to serve as a proof-of-concept. This being the case, the TDCS hardware realization uses an acoustic channel that is intended to serve as a scale model of an RF channel, complete with noise and interference.

1.3. Goals

The overarching goals of this research effort are:

1. To compare the bit error performance of a TDCS hardware implementation with the expected values expressed in literature.

2. To assess the capability of a digital signal processor (DSP) in hosting either a TDCS transmitter or receiver.

3. To examine the utility of applying a method of communications waveform sculpting, other than the previously used spectral notching technique [1-8], such as water-filling, through the use of Matlab®-based simulations.
1.4. Scope

This research is restricted to examining an implementation of a TDCS in hardware using transform domain quadrature phase shift keying (TD-QPSK) and a minimum mean square error (MMSE) equalizer in an undefined channel. This channel is assumed to consist of additive white Gaussian noise (AWGN), various interfering signals (both controlled and uncontrolled), and intersymbol interference due to multipath propagation effects. Given the channel at hand, the ability of a TDCS hardware user pair to conduct trained channel estimation via an MMSE equalizer is intrinsically studied. There is no attempt made to implement other modulation schemes, such as cyclic shift keying (CSK) or any form of $M$-ary PSK other than QPSK, nor is there an attempt made to optimize the equalization or synchronization algorithms, either in terms of efficiency or effectiveness. There is no attempt to compare the performance of a TDCS user pair with simulated results in terms of the ability of the TDCS receiver to acquire a TDCS transmission. Synchronization is and must be performed in order to process received communications symbols, but the effectiveness of synchronization is not explicitly studied.

1.5. Assumptions

The following assumptions are made in this research:

1. The MMSE equalizer implemented in the user pair is assumed to mitigate all residual effects due to multipath, so the assumption is made that the matched filter correlation process used to demodulate received TDCS signals is optimal.
2. The hardware-implemented TDCS user pair is assumed to be perfectly synchronized.

1.6. Methodology

TDCS software simulations are used to initially validate the TDCS algorithm used in the hardware user pair. Once this validation is complete, the TDCS software is implemented in a hybrid software/hardware PC-based prototyping platform to conduct validation of the software in an acoustic channel consisting of a common office or home environment. This hybrid prototyping platform consisting of a single PC with multiple instances of Matlab® running. One instance of Matlab® code is written to generate and transmit a TDCS signal through an acoustic channel, using PC speakers. The other instance of code is written for receiving the TDCS signal using a PC microphone and for processing the received signal. Finally, the TDCS software is migrated to a format suitable for a DSP, the software is transferred to the DSP cards, and experiments are conducted. These experiments, conducted using four scenarios, result in calculation of empirically-obtained bit error rates, which are then compared with the analytic QPSK curve, as well as bit error rates from other scenarios.

1.7. Materials

All simulations and the initial software/hardware hybrid prototyping is performed using Matlab® Student Version 7.0 (R14). Simulations and prototyping are run on stand-alone PCs, including a Dell Pentium-based laptop and a Dell Pentium-based desktop computer. The DSP-based TDCS hardware user pair is implemented using two Spectrum Digital TMS320C6416 (1 GHz) DSP Starter Kits (DSK). The DSK uses the
Texas Instruments TMS3206416 fixed point DSP operating at 1 GHz. The DSK is programmed using the Texas Instruments Code Composer Studio Version 3.1, included with the DSK. Code Composer Studio uses C-based scripts to control the DSK. Two Cyber Acoustics AC-54 PC desktop microphones, one for each DSK, are used to perform signal reception and environmental sampling for spectrum estimation. Acoustic signal transmission is performed using one Labtec LCS-1070 PC desktop speaker. Narrowband interfering signals are generated using Matlab® to control two Labtec LCS-1050 PC desktop speakers.

1.8. Overview

This thesis is organized into six chapters. Chapter II, the Literature Review, provides a historical perspective on TDCS and discusses literature on several topics of interest used to make the design decisions involved in this research effort. These topics include discussion of the framework for spectrally modulated, spectrally encoded (SMSE) digital communications; interference avoidance and its association with coexistence and LPD communication; how waveform design, or “sculpting,” thus waveform agility, enables interference avoidance; equalization of received digital communications signals; and background into the DSK implementation. Chapter III outlines the architecture of the TDCS system being implemented, and includes several illustrations to convey graphically some of the ideas behind the theory. Chapter III also discusses the tradeoffs involved in the design used to obtain results with the hardware user pair. Chapter IV provides some simulation results used when studying water-filling as an alternative to spectral notching when performing TDCS communications symbol waveform design. Water-filling is
studied because literature indicates that water-filling may more optimally fit within the managed spectrum than spectral notching. Simulated bit error performance curves resulting from this research are provided in Chapter IV. Chapter V provides the bit error results obtained with DSK hardware. These results are then compared with the analytic QPSK bit error performance curve and with previous results in TDCS literature. Finally, Chapter VI contains the research conclusion and provides some recommendations for future TDCS research.
II. Literature Review

2.1. Transform Domain Communication System (TDCS) History

There have been several theses written by graduate students at AFIT on various aspects of TDCS. In all cases, the research approach in studying TDCS consists of, unless stated otherwise in the context of a specific research effort, conducting a literature review on digital communications enterprise-wide approaches to resolving the obstacles in fielding a TDCS user pair or network, as well as obviously all literature on previous TDCS research efforts. Once the literature review is completed, design approaches are integrated into the TDCS transceiver architecture. The integration into the TDCS design model of the algorithm in question (such as Roberts' integration of synchronization algorithms into the TDCS architecture [3], or Klein or Lee's integration of wavelet-domain processing in the TDCS spectrum estimation algorithm [4, 5], for example) is then performed using \texttt{Matlab®}, and the results obtained via simulations using the \texttt{Matlab®} models are then compared with expected results from literature.

Initially, the TDCS research line was initiated with AFRL-contracted efforts that led to a technical report [9] and the filing of a patent [10]. These technical reports were then used as foundational works by Radcliffe in his groundbreaking proposal of the TDCS, which is outlined in his thesis [1]. The original block diagram for the TDCS transmitter and receiver proposed by Radcliffe still largely stand as written [1]. Radcliffe demonstrated that in an AWGN channel with interference of varying types (tone, swept tone, limited instances of partial band, and interference across the bandwidth of the TDCS signaling) present, the TDCS interference avoidance algorithm demonstrates better
performance (at a signal energy to channel noise ratio ($E_b/N_0$) of 4 dB, antipodal signaling exhibited 12.7 dB improvement, while binary orthogonal modulation exhibited a 6.8 dB improvement [1]) than direct sequence spread spectrum. Radcliffe's research is limited to cases involving a single stationary transmitter and a single stationary receiver operating as a user pair that shares a channel that contains only AWGN and various types of interfering signals (thus no multipath) [1]. Radcliffe assumed that the TDCS user pair operates with a flat frequency magnitude response over the bandwidth of interest, that the propagation delay between transmitter and receiver is zero, that synchronization between the transmitter and receiver is perfect, and that there is a perfect consensus reached between the transmitter and receiver in all decisions on which areas of the spectrum are impacted by interference, thus which parts of the spectrum to avoid [1]. Radcliffe provided the TDCS architecture on which all other AFIT and AFRL research efforts are based.

Swackhammer followed up Radcliffe's work by studying the potential for multiple TDCS users working in a multiple access network, and found that code division multiple access (CDMA) algorithms could be used to achieve multiple access capability [2]. This CDMA coding is integrated into the TDCS waveform design algorithm, and an asynchronous multiple access network of up to eight channels is simulated. The bit error performance of a single user pair within this network is studied for $E_b/N_0$ ranging from 0 to 9 dB [2]. Swackhammer then compared simulation results with bit error performance estimates using cross-correlation calculations and concluded that TDCS signaling could be employed practically in a multiple access network [2]. Swackhammer's research is
limited to cases involving up to eight stationary transmitters and up to eight stationary receivers operating as user pairs that simultaneously share a channel containing only AWGN and multiple access interference created by user pairs within the network (thus no multipath or interfering signals not associated with users in the multiple access network) [2]. Swackhammer assumed that the TDCS user pair operates with a flat frequency magnitude response over the bandwidth of interest, that the propagation delay between transmitter and receiver is zero, that synchronization between the transmitter and receiver is perfect, and that there is a perfect consensus reached between the transmitter and receiver in all decisions on which areas of the spectrum are impacted by interference [2]. Swackhammer's work justifies further study into TDCS in a multiple access environment, and perhaps the physical implementation of TDCS into a network.

Roberts performed study into synchronization of a TDCS system, and found that a TDCS user pair can indeed be synchronized when signaling in the presence of interference [3]. Roberts studied several different acquisition and synchronization protocols, integrated them into the TDCS architecture, and found that a Direct Time Correlation (DTC) algorithm can be executed to provide both peak and threshold detection [3]. Peak and threshold detection enable synchronization and acquisition, respectively. Roberts examined the use of German's technique [9], and found that specifically for threshold detection above -12 dB signal-to-noise ratio (SNR), German's technique performs better than DTC [3]. Roberts's research is limited to cases involving a single stationary transmitter and a single stationary receiver operating as a user pair sharing channel containing only AWGN and 10% partial band interference (thus no
multipath) [3]. Roberts assumed that the TDCS user pair operates with a flat frequency magnitude response over the bandwidth of interest, that the propagation delay between transmitter and receiver is zero, and that there is a perfect consensus reached between the transmitter and receiver in all decisions on which areas of the spectrum are impacted by interference [3]. Roberts' research into the acquisition and synchronization of a TDCS is used in the various design decisions made in implementing TDCS in hardware, such as the use of DTC for symbol synchronization. Roberts goes much further to contribute to TDCS research in offering a framework for SMSE signaling (in which TDCS is included), but discussion into this subject is reserved for Section 2.5.

Klein integrated wavelet processing techniques into the TDCS framework, yielding a Wavelet Domain Communication System (WDCS) [4]. Lee contributed further by integrating the use of packets into the WDCS construct [5]. The work of neither Klein nor Lee has been used in this research effort, so no further discussion into WDCS is included here.

Radcliffe, Swackhammer, and Roberts all used antipodal and CSK modulation to communicate bit values [1-3]. The use of CSK enabled $M$-ary (as opposed to binary) signaling, but exhibits less spectral efficiency than what can be achieved by other means [6]. Nunez researched the integration of PSK modulation into TDCS signaling, to answer this shortcoming [6]. His design involves implementation of transform domain PSK modulation in the time-domain after performing a discrete Fourier transform on TDCS frequency-domain waveforms. This is done through the use of fixed phase rotations of the pseudorandom phase vector used to spectrally encode the TDCS communications
symbols [6]. Section 2.3.5 goes further to discuss the specifics of how PSK modulation is implemented in TDCS, and Section 2.5 presents modulation in the SMSE framework. Nunez's results demonstrate that TD-QPSK mitigates interference in an AWGN channel with a bit error performance roughly equivalent to spectrally unencoded PSK signaling with no interference present [6]. It is expected that applying the spectral notching algorithm to experiments that use TD-QPSK will yield the same result. Nunez's work also yields the finding that, in the presence of narrowband interference and at bit error rates of less than $10^{-3}$, the use of spectral notching yields a gain in $E_b/N_0$ of greater than 1 dB, and that at bit error rates of less than $10^{-2}$, there should be an appreciable improvement in bit error performance [6]. Nunez's research is limited to cases involving up to 32 stationary transmitters and up to 32 stationary receivers operating as user pairs simultaneously sharing a channel containing AWGN, narrowband interference, and multiple access interference (thus no multipath) [6]. Nunez assumed that the TDCS user pair operates with a flat frequency magnitude response over the bandwidth of interest, that the propagation delay between transmitter and receiver is zero, that synchronization between the transmitter and receiver is perfect, and that there is a perfect consensus reached between the transmitter and receiver in all decisions on which areas of the spectrum are impacted by interference [6]. Nunez's work is used directly in implementing TDCS in hardware, as TD-QPSK is used exclusively in all waveform designs.

In the latest of the AFIT research efforts addressing TDCS as the explicit focus, Gaona was the first to study TDCS performance in the presence of multipath interference...
He integrated a RAKE receiver into the TDCS architecture, and found that TDCS intrinsically mitigates multipath effects [7]. A RAKE receiver exploits multipath propagation to coherently reconstruct a communications signal by processing the multiple received signals resulting from the various propagation paths [16]. Gaona's research is limited to cases involving a single transmitter and a single receiver operating as a user pair sharing a channel containing AWGN, various types of interfering signals, and multipath propagation of the TDCS transmission [7]. Gaona assumed that the TDCS user pair operates with a flat frequency magnitude response over the bandwidth of interest, that synchronization between the transmitter and receiver is perfect, and that there is a perfect consensus reached between the transmitter and receiver in all decisions on which areas of the spectrum are impacted by interference [7]. As discussed in Section 2.6, a RAKE receiver is not implemented in the hardware realization used in this research, as multipath is intended to be mitigated through the use of an MMSE transverse equalizer.

### 2.2. An Aside on Hadamard Multiplication and the Hadamard Product

One of the principle instruments of the matrix mathematics used in describing SMSE signaling is Hadamard multiplication, denoted by the symbol $\circ$. The Hadamard product involves multiplication, on a point-by-point basis, of the elements of equally sized matrices or vectors as follows [12]:

$$ (A \odot B)_{ij} = a_{ij}b_{ij} $$

An example to describe how Hadamard multiplication works for a 3x3 matrix is as follows:
Though this operation is performed using 3x3 matrices, it can be extended to any equally sized matrices.

2.3. Current State of the TDCS Architecture

The TDCS transmitter block diagram in its most recently published state is illustrated in Figure 1.

2.3.1. Estimate Spectrum

The purpose of this block is to characterize the RF spectrum of interest to the TDCS user pair. Through the use of a spectral estimation algorithm, the regions of the spectrum containing interference are identified [8]. Periodogram [1], autoregressive [1], and wavelet-based [4] estimation techniques have previously been used. Parametric spectral estimation techniques could possibly be used to characterize the spectrum, if communications systems are operating cooperatively, as cooperative operation would allow for some assumptions about the behavior of the cooperative communications signals [13]. There are many methods available for use in spectral estimation. One such method is the Bartlett method of periodogram-based estimation. The Bartlett method is designed to reduce the wide variations typically seen in periodogram estimates by dividing $N$ observations into $L = N / M$ subsamples of $M$ observations per subsample, calculating periodograms for each of these $L$ subsamples, and then averaging the periodograms [13]. The Bartlett method is calculated using the following equation [13]:

$$\begin{bmatrix} a_{11} b_{11} & a_{12} b_{12} & a_{13} b_{13} \\ a_{21} b_{21} & a_{22} b_{22} & a_{23} b_{23} \\ a_{31} b_{31} & a_{32} b_{32} & a_{33} b_{33} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

(2)
Figure 1. TDCS Transmitter Block Diagram [8]

\[ \hat{\phi}_B(\omega) = \frac{1}{L} \sum_{j=1}^{L} \hat{\phi}_j(\omega) \]  

(3)

where

\[ \hat{\phi}_j(\omega) = \frac{1}{M} \sum_{t=1}^{M} y_j(t) e^{-j\omega t} \]  

(4)

Note that (4) is simply the equation for a periodogram calculation. The equation for the observation of the \( j \)th subsample is as follows [13]:

\[ y_j(t) = y[(j-1)M + t]; \quad t = 1, \ldots, M; \quad j = 1, \ldots, L \]  

(5)

An illustration of the Bartlett method is provided in Figure 2.

In the case where only the relative values between frequencies of a spectrum estimate are required, as is the case when performing waveform sculpting, the periodograms of subsamples can then be calculated and and summed on a frequency
point-by-point basis. Moses has written *Matlab*® functions that execute the Bartlett method practically [14], and these functions are used in the *Matlab*® simulations and *Matlab*®-based experiments in this research effort.

### 2.3.2. Spectrum Magnitude

The purpose of this block is to construct a frequency-domain communications waveform, referred to by Chakravarthy *et al.* as the Fundamental Modulation Waveform (FMW) [8]. Spectral notching consists of restricting subcarriers from use when the spectrum estimate corresponding with this subcarrier exceeds a hard threshold. This process yields a communications waveform power spectral density (PSD) that contains no power in the portion of the spectrum that has been notched, thus the waveform inherently avoids interference in the spectrum [8]. Execution of the spectrum magnitude function yields a vector of magnitude components, $A'(\omega)$, valued at either 1 or 0 (for frequency components where interference at the given frequency $\omega$ in the spectrum estimate is below or above, respectively, the threshold). An illustration of this is provided in Figure 3. This illustration is intended to convey an intuitive explanation of the TDCS spectral notching algorithm, and does not represent actual simulation results.
2.3.3. Pseudorandom Phase

The purpose of this block is to provide LPI/LPD functionality to TDCS transmissions through the establishment of a complex pseudorandom phase vector $e^{j\Phi(\omega)}$ generated for Hadamard multiplication with the spectrum magnitude component $A'(\omega)$, yielding the variable $\beta_b(\omega)$ in the block diagram in Figure 1. This yields a noise-like time-domain communications waveform, thus the LPI/LPD functionality [8]. TDCS multiple access networks can then be constructed through the Hadamard multiplication of a spreading code with the pseudorandom phase vector $e^{j\Phi(\omega)}$ [6]. The pseudorandom phase vector is to be generated on a symbol-by-symbol basis to ensure TDCS transmissions maintain a noise-like appearance over time, securing the LPI/LPD feature of TDCS. An illustration of how the pseudorandom phase generation block works is provided in Figure 4.
2.3.4. Scaling

Scaling of the complex frequency-domain signal $\beta_b(\omega)$ is performed, yielding the scaled signal $\beta(\omega)$. This ensures an appropriate amount of energy is contained in the communications symbol to be transmitted, and that all communications symbols contain the same amount of energy [8]. Thus, an appropriate bit error rate through the communications channel is achieved, given the mapping between $E_b/N_0$ and bit error rate $P_b$.

2.3.5. IDFT

The spectrally encoded frequency-domain communications symbol $\beta(\omega)$ is inverse Fourier transformed, yielding the time-domain symbol $b(t)$ [8]. A waveform is generated for each data symbol, so if two bits per symbol are to be transmitted, four symbols must be constructed, thus four waveforms are generated. In the case of QPSK, the frequency-domain waveform $\beta(\omega)$ is rotated using Hadamard multiplication with an additional modulating vector to generate symbols that are 90, 180, or 270 degrees away from each other. This is done through the use of a Hadamard multiplication with $e^{j\pi/2}$ for the second
symbol, \( e^{j\pi} \) for the third symbol, and \( e^{j3\pi/2} \) for the fourth symbol (neglecting Gray coding in this description). These time-domain symbols will appear noise-like, thus increasing the potential for LPI/LPD, through the multiplication with the pseudorandom phase vector; and will intrinsically avoid interference, through the use of the spectral notching algorithm.

A hardware implementation problem that has been observed in the use of any modulation scheme that relies on an inverse Fourier transform (such as multicarrier modulation) to generate time-domain communications waveforms is the construction of symbols that contain exceptionally high peak-to-average power ratios (PAPR). Symbols that contain high PAPR will exhibit instantaneous spikes of very large power, due to the constructive addition in phase of subcarriers [16]. These spikes saturate the transmitter power amplifier, force a clipping of the communications symbol, and cause intermodulation distortion. This clipping inadvertently reduces the symbol energy and creates a difference in symbol shape between the received time-domain symbol and the demodulator matched filter reference symbol. Both of these problems increase the end-to-end bit error rate. Several techniques have emerged to mitigate this phenomenon, such as the use of the central limit theorem to limit the bounds of the envelope of the time-domain communications symbol waveform [17].

2.3.6. Buffer

The purpose of this block is simply to provide a storage space for the communications symbols (four symbols in the case of QPSK) for subsequent mapping between data bits and symbol waveforms [8]. The refresh rate for the waveforms, the rate
at which spectrum estimation and spectral encoding occurs, depends on the requirements of the communications system, and the channel variance over time.

2.3.7. Modulation/Transmission

Time-domain communications symbol waveforms are concatenated in accordance with the mapping between data bits and symbol waveforms to generate a discrete-valued time-domain communications signal. This signal is then converted to an analog signal and radiated through the channel.

2.3.8. Receiver Architecture at Large

The TDCS receiver block diagram is illustrated in Figure 5. It contains blocks used to locally generate time-domain reference signals $c_j(T)$, indicated by the dashed line in the figure [8]. These reference signals are used in time-domain matched filter correlation with received symbol waveforms to generate a test statistic $z_j(t)$ that is compared with a maximum likelihood decision rule, yielding estimates of the symbol most likely to have been received. The reference symbol associated with the output test statistic with the highest magnitude becomes the symbol estimate, and is mapped to bits. This is the same matched filter detection technique used in conventional correlation receivers. The difference in receivers between those that use TD-PSK and those that use conventional PSK is the process by which reference signals are generated. As can be seen in Figure 5, five highlighted blocks are integrated to construct local symbol waveforms. These blocks are used in the same manner as blocks of the same name in the transmitter. This will yield identical transmitted signals and demodulator reference signals, assuming both the transmitter and receiver are obtaining identical channel
spectrum estimates and that the pseudorandom phase vector is synchronized between the transmitter and receiver. This creates a problem in implementing TDCS in hardware, as the transmitter and receiver may not be obtaining identical spectrum estimates. The results of simulations and experiments demonstrating the effect of spectrum estimate mismatch are provided in Sections 4.3 and 5.4, respectively.

2.4. Synchronization of Received TDCS Signals

As stated in Section 2.1, Roberts performed the initial study into TDCS synchronization, and concluded that DTC can be used to provide both peak and threshold detection, which in turn yields synchronization and acquisition, respectively [3]. In order to perform DTC, a predetermined synchronization codeword that is shared between the transmitter and the receiver is translated into a preamble waveform [3]. This codeword consists of a predetermined group of symbols that, when used in conjunction with a buffer in the receiver, are used in both threshold and peak detection [3]. In threshold detection, which provides signal acquisition, received signals and noise are continuously
correlated with the predetermined codeword waveform [3]. The result of this correlation, a simple dot product of the received signal and the predetermined codeword waveform, will either be above or below a predetermined threshold. If the result of this dot product is greater than the value of the threshold, than a decision is made within the receiver programming that a TDCS signal is present. If no signal is determined to be present at the receiver, no further processing is conducted by the receiver, beyond the ongoing correlation for signal acquisition. However, if a signal is determined to be present, then peak detection is performed, which yields symbol synchronization [3]. In peak detection, correlation is performed again with the buffered received signal and the predetermined codeword waveform. Correlation for peak detection again yields a simple dot product. The difference in peak detection, however, is that the position in time of the peak value of the correlation, rather than the amplitude of the correlation values, is important. The position (in discrete samples) where the peak value of the correlation occurs should be the position of the last sample of the synchronization waveform codeword. This sample, along with all buffered samples before this sample, is discarded, as the position of the start of the data waveform has been identified to be the next sample. Once this process is complete, synchronization between the transmission and the receiver has been achieved.

2.5. Spectrally Modulated, Spectrally Encoded Signaling

Roberts recently published a digital communications framework for categorizing modulation schemes that rely on SMSE signaling [11]. Roberts defines communications signaling as SMSE when “both 1) data modulation [spectral modulation] and 2) encoding [spectral encoding] are applied as amplitude and/or phase variations on a discrete
The SMSE (or SMSE signaling) framework integrates digital communications schemes that rely on waveform design in the frequency domain into a mathematical framework for a cognitive radio-based software defined radio. Examples of SMSE signaling include TDCS, multicarrier code division multiple access (MC-CDMA), or one of the variants of orthogonal frequency division multiplexing. The SMSE framework leverages modern vector and matrix mathematics to functionally describe and design multicarrier- or subcarrier-based communications symbols (for example, in MC-CDMA or TDCS, respectively). The framework consolidates six waveform design variables to enable analytic unification for SMSE signaling. These six variables, described as vectors by lower-case bold letters, include the code $\mathbf{c}$, the data modulation $\mathbf{d}$, the window $\mathbf{w}$, the orthogonality term $\mathbf{o}$, the assigned frequencies $\mathbf{a}$, and the used frequencies $\mathbf{u}$.[11]

The code vector $\mathbf{c}$ is described as follows [11]:

$$
\mathbf{c} = [c_1, c_2, ..., c_{N_f}] = [c_i]_{i=1}^{N_f}, \quad c_i \in \mathbb{C}
$$

where $c_i$ represents the value, which is complex numbered as denoted by $\mathbb{C}$, of the code vector for the $i$th of the $N_f$ frequency subcarriers used in generating communications symbols. In the case of TDCS signaling, the code vector consists of the series of pseudorandom phase variations [11], as described in Section 2.3.3.

The data modulation vector $\mathbf{d}$ is described as follows [11]:

$$
\mathbf{d} = [d_1, d_2, ..., d_{N_f}] = [d_i]_{i=1}^{N_f}, \quad d_i \in \mathbb{C}
$$

where $d_i$ represents the value, again which can be complex valued, of the data modulation vector for the $i$th subcarrier. In the case of TD-QPSK, the data modulation vector $\mathbf{d}$ for
the communications symbol associated with the two-bit pattern 00 would appear as follows [6]:

\[
d_{00} = [e^{j\theta_0}, e^{j\theta_0}, \ldots, e^{j\theta_0}] = [e^{j\theta_0}]_{i=1}^{N_F}
\]  

(8)

The data modulation vector for the two-bit pattern 01 would appear as follows, where the vector of length \(N_F\) would have elements equal to \(e^{j\pi/2}\) for the first \(N_F/2\) elements and elements equal to \(e^{-j\pi/2}\) for the last \(N_F/2\) elements.

\[
d_{01} = [e^{j\pi/2}, e^{j\pi/2}, \ldots, e^{-j\pi/2}, e^{-j\pi/2}, \ldots, e^{-j\pi/2}]
\]  

(9)

This pattern of the second half of the vector being equal to the conjugate of the first half of the vector holds true for all symbols, to account for the implicit use of Hilbert transforms to describe discrete-valued baseband signals [6, 16]. The data modulation vector for the two-bit pattern 10 would appear as follows [6]:

\[
d_{10} = [e^{-j\pi/2}, e^{-j\pi/2}, \ldots, e^{j\pi/2}, e^{j\pi/2}, \ldots, e^{j\pi/2}] = -d_{01}
\]  

(10)

Note that Gray coding is implemented, so the phase shifts used in generating the symbol associated with bit pattern 01 is opposite of those in the symbol associated with bit pattern 10. Note the typical implementation of Gray encoding, which is intended to reduce a potential two-bit error to a one-bit error in a shift across one decision boundary of the QPSK constellation [16]. The data modulation vector for the two-bit pattern 11 would appear as follows [6]:

\[
d_{11} = [e^{j\pi/2}, e^{j\pi/2}, \ldots, e^{-j\pi/2}, e^{-j\pi/2}, \ldots, e^{-j\pi/2}] = [e^{j\pi/2}]_{i=1}^{N_F} = -d_{00}
\]  

(11)

The windowing vector \(w\), used for spectral shaping, is described as follows [11]:

\[
w = [w_1, w_2, \ldots, w_{N_F}] = [w_j]_{j=1}^{N_F}, \quad w_j \in \mathbb{C}
\]  

(12)
where \( w_i \) represents the value, again which can be complex numbered, of the windowing vector for the \( i \)th subcarrier. In the case of TDCS signaling, the windowing vector consists simply of all elements equal to one, or rectangular windowing [11].

The orthogonality vector \( o \), which is used to induce orthogonality between simultaneously transmitted signals in multiple access applications, is described as follows [11]:

\[
o = [o_1, o_2, ..., o_{N_r}] = [o_i]_{i=1}^{N_r}, \quad o_i \in \mathbb{C}, \quad |o_i| = 1 \quad \forall \ i
\]

where \( o_i \) represents the value, again which can be complex valued, of the orthogonality vector for the \( i \)th subcarrier. For this design vector, each of the individual elements of the vector has unit magnitude, as this vector is only used to rotate the phase of individual frequency-domain subcarriers. In the case of a single user pair, there is no need to use an orthogonality vector, so the elements of the vector are simply equal to one.

The frequency assignment vector \( a \), dictated to individual user pairs by a multi-user network controller to individual user pairs (enabling frequency division multiple access) is described as follows [11]:

\[
a = [a_1, a_2, ..., a_{N_r}] = [a_i]_{i=1}^{N_r}, \quad a_i \in \{0, 1\}
\]

where \( a_i \), valued at either 0 or 1, determines whether a potential frequency subcarrier is allowed by the network controller for use by a user pair. In other words, if a set of frequency subcarriers have frequency assignment vector elements equal to 1, then the bands used by the associated frequency subcarriers are allowed for use by the user pair. If an element associated with a subcarrier is equal to zero, then the frequency subcarrier is
not allowed for use by the user pair. In the scenario where only a single user pair is involved, the elements of the frequency assignment vector are all equal to one.

The frequency use vector $\mathbf{u}$ is determined through a consensus within one user pair to perform functions such as interference avoidance and coexistence (rather than multiple access) assurance. The vector $\mathbf{u}$ is described as follows [11]:

$$
\mathbf{u} = [u_1, u_2, \ldots, u_{N_F}] \triangleq [u_i]_{i=1}^{N_F}, \ u_i \in \{0, 1\}
$$

(15)

where $u_i$ equals 0 or 1 and dictates whether a potential frequency subcarrier is to be used. As with the frequency assignment vector, if a set of subcarriers have frequency use vector elements equal to 1, then the band used by the associated frequency subcarrier will contain energy upon signal transmission. It is worth noting that, practically speaking, the elements of the frequency use vector $\mathbf{u}$ will be equal to or smaller than the elements of the frequency assignment vector $\mathbf{a}$. Expressing this mathematically [11],

$$
u_i \leq a_i^\flat
$$

(16)

In all scenarios where CR-based SDR concepts are applied in a hardware implementation, the frequency assignment vector will vary over time to compensate for environmental conditions, to include the presence of interference in the channel. Discussion of specific examples of the TDCS frequency use vector will be reserved for Section 3.1.2.

These six symbol design variables are consolidated to express the baseband value of the $k$th symbol $s_k[m]$ as a series of $N_F$ terms through the following equation [11]:

$$
s_k[m] = \left[ a_m^\flat u_m^\flat c_m^\flat d_{m,k}^\flat w_m e^{j(\theta_m + \phi_m + \eta_m + \delta_m + \epsilon_m)} \right]_{m=0}^{N_F-1}
$$

(17)
where \( m \) is the index of the frequency component (the index of the subcarrier) and \( a_m, u_m \), \( c_m, \theta_c, d_{m,k}, \theta_d, w_m, \theta_w \), and \( \theta_o \) are the magnitudes (denoted by a letter) and phases (denoted by \( \theta \)) of the six design variables. Using row vector notation in the case of one user pair, or matrix notation in the case of multiple user pairs (where each user pair occupies one row of the matrix), the following equation expresses the time-domain output of (17), \( s_{SMSE}(t) \) [11]:

\[
s_{SMSE}(t) = F^{-1} \{ S_k = A \odot \Theta \odot F \}
\]  

(18)

where \( S_k \) represents one of \( K \) distinct communications symbols that have been concatenated together to form a signal for transmission. The variable \( A \) represents the resulting magnitude vector of the Hadamard products of design variable magnitudes \( c_m, d_{m,k}, w_m \); the variable \( \Theta \) represents the resulting phase vector of the additions of design variables \( \theta_c, \theta_d, \theta_w \), and \( \theta_o \); and the variable \( F \), the frequency component identification vector, represents the product of the Hadamard multiplication of the frequency assignment and use vectors \( a \) and \( u \). While this expression completely describes the time-domain value of an SMSE-based communications symbol, it does not convey that only the real value of a generated signal is radiated by an antenna, thus (18) is restated for practical use as follows:

\[
s_{SMSE}(t) = \Re \{ F^{-1} \{ S_k = A \odot \Theta \odot F \} \}
\]  

(19)

While all symbol concatenation is performed in the time-domain in the TDCS hardware implementation, Roberts asserts an additional variable for concatenating symbols in the frequency domain, which exploits the relationship between delays in time and phase shifts in frequency. All transmission waveforms can then be designed in the
frequency domain, so the final frequency-domain signal can be inverse Fourier
transformed to the time domain for transmission [11]. This approach may reduce latency
due to signal processing time requirements.

In the SMSE framework, a received signal $r_s[n]$ is expressed as [11]:

$$r_s[n] = s_s[n] \ast h_s[n] + \eta[n] \tag{20}$$

where $h_s[n]$ is the impulse response of the channel at time index $n$, $\ast$ indicates a
convolution operator, and $\eta[n]$ is the summation of all noise and interference. (20) is
simply the standard model for an end-to-end communications channel consisting of a
single user pair. Roberts goes further to expand this channel model by incorporating an
RF filter (be it a lowpass or bandpass filter) on the front end of the receiver for noise
reduction. This expansion of (20) for a single user pair that includes an RF filter is as
follows [11]:

$$r_s[n] = (s_s[n] \ast h_s[n] + \eta[n]) \ast h_{rf}[n] \tag{21}$$

where $h_{rf}[n]$ is the impulse response of the RF filter. As discussed in Section 2.9, the
practical design requirement for an RF filter is satisfied within the design of the DSK-
based hardware realization. An FFT is then performed on received signal $r_s[n]$ for
spectral demodulation [11]. As discussed in Section 3.5, the TDCS receiver design does
not use an FFT for processing received signals. Roberts modifies the TDCS receiver
design in his research for the purposes of describing TDCS within the SMSE framework
2.6. Equalization of Received TDCS Signals

Though DTC provides a means by which symbol waveforms can be synchronized, DTC does nothing to mitigate phase offsets in the received signal caused by sample timing offsets between the transmitter and receiver nor phase shifts induced by propagation delays. Furthermore, the channel between the transmitter and receiver may be less than ideal, consisting only of an impulse at time $t = 0$ (thus the impulse has no delay). In a noiseless ideal channel, the received signal exactly equals the transmitted signal [18]. A less than ideal channel, which is expected when constructing a hardware implementation of a digital communications user pair, is one where a transmitted signal incurs a phase delay and a channel consisting of multiple taps. Channel taps can result from communicating in a multipath-laden environment [19]. As stated in Section 2.1, Gaona studied TDCS performance in the presence of multipath propagated signals, including the implementation of a RAKE receiver [7]. Using a RAKE receiver to coherently reconstruct TDCS signals may be practical for a fixed-site user pair (such as would be found in communications between buildings), but clearly isn't practical in mobile communications. The prevalent method for mitigating the effects of a less-than-ideal communications channels is through equalization [19]. Typical solutions for equalizing received signals, in either a deterministic or a statistical manner, are a zero-forcing or an MMSE equalizer, respectively [19]. Because an MMSE equalizer does more to mitigate the effects of noise than a zero-forcing equalizer [19], an MMSE equalizer has been implemented in hardware to address the issues of imperfect channels and phase offsets in the received signal.
The mathematics in implementing an MMSE equalizer consist of several computations based on matrix algebra. Given a received vector of distorted samples \( \mathbf{x} \) of finite length which composes the Toeplitz matrix \( \mathbf{X} \); a reference vector of undistorted samples \( \mathbf{z}[n] \) of finite length, referred to as a training sequence; and a channel response vector \( \mathbf{c} \) of finite length, with individual elements representing channel taps; the following equation expresses how the channel response is computed for use in equalization [19]:

\[
\mathbf{c} = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xz}
\]  

(22)

\( \mathbf{R}_{xx} \), the autocorrelation matrix of the received samples, is calculated as follows [19]:

\[
\mathbf{R}_{xx} = \mathbf{X}^H\mathbf{X}
\]  

(23)

\( \mathbf{R}_{xz} \), the cross-correlation vector, is calculated as follows [19]:

\[
\mathbf{R}_{xz} = \mathbf{X}^H\mathbf{z}
\]  

(24)

The Toeplitz matrix \( \mathbf{X} \) is generated for use in (23) and (24) by placing the values of the received vector \( \mathbf{x} \) in \( \mathbf{X} \) in the following format [19]:

\[
\mathbf{X} = \begin{bmatrix}
  \mathbf{x}[-N] & 0 & 0 & \cdots & 0 & 0 \\
  \mathbf{x}[-N+1] & \mathbf{x}[-N] & 0 & \cdots & \cdots & \vdots \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \cdots & \mathbf{x}[N-1] & \mathbf{x}[-N] \\
  0 & 0 & 0 & \cdots & \mathbf{x}[N] & \mathbf{x}[N-1] \\
  0 & 0 & 0 & \cdots & 0 & \mathbf{x}[N]
\end{bmatrix}
\]  

(25)

The index of the received samples \( n = -N, \ldots, N \), the first column of (25), consists of a vector where the first \( 2N+1 \) elements is the entire received vector \( \mathbf{x} \). The Toeplitz matrix \( \mathbf{X} \) is of size \( m \times n \), where \( m \) is two times the length of \( \mathbf{z} \) minus one, and \( n \) is the length of \( \mathbf{z} \).
When used practically, \( z \) is modified with zero padding both before and after the training values. If \( z \) is originally of length 19, \( z \) is modified with zero padding so that it is of length 37. Zero padding is done with an equal number of elements before and after the training values, so there would be nine elements of zero value both before and after training values.

Once the channel response \( c \) is obtained, the equalized received vector \( y \) is calculated as follows [19]:

\[
y = Xc
\]  

(26)

Equalization is typically performed at the symbol rate, thus the equalizer is symbol-spaced, or at some multiple of the symbol rate in a fractional spacing equalizer [16]. In a symbol-spaced equalizer in particular, the output test statistics from demodulation correlation operations can be used as inputs to the equalizer.

### 2.7. Coexistence

Mitola defines cognitive radio in his PhD thesis as a radio that can autonomously identify the resources (time, bandwidth, power, etc.) needed to effectively communicate, and can both intelligently and autonomously obtain and use these resources effectively [21]. Software-defined and, better still, cognitive radios are to be able to adaptively design communications waveforms. This capability, formally referred to as waveform agility, can be viewed as the ability to adapt waveform shapes to meet user needs. If digital communications waveforms can be varied to use the RF spectrum more optimally than can be done with conventional digital radios, this enables what Popescu and Rose define to be interference avoidance [22]. Simply put, interference avoidance means
transmitting the most signal energy in areas where there is the least interference [22]. It is further suggested here that this definition of interference avoidance holds true spectrally, spatially, temporally, and among other areas of communications dimensionality. This statement is inferred by Popescu and Rose as well [22].

User capacity $C$ (bits per second) using the Shannon-Hartley capacity theorem, is computed as follows, where $W$ is the communications bandwidth (Hertz), $S$ is the average communications signal power (Watts), and $N$ is the average noise power (Watts) [19]:

$$C = W \log_2 \left(1 + \frac{S}{N}\right)$$

(27)

The mapping between SNR and $E_b/N_0$ for baseband signals in an AWGN channel is as follows [19], where $R_b$ is the bit rate (bits/second):

$$\frac{S}{N} = \frac{E_b}{N_0} \frac{R_b}{W}$$

(28)

Given (27) and (28), the capacity equation can be rewritten as follows:

$$C = W \log_2 \left(1 + \frac{E_b}{N_0} \frac{R_b}{W}\right)$$

(29)

Bit error rate $P_b$ for a coherently detected $M$-ary PSK system, given $E_b/N_0$ with Gray coded symbol assignment, is expressed as follows [19], where $M$ is the number of communications symbol waveforms required to transmit a given number of bits per symbol:

$$P_b = \frac{2}{\log_2(M)} Q \left[\frac{2 \log_2(M) E_b}{N_0} \sin \left(\frac{\pi}{M}\right)\right]$$

(30)
Given (29) and (30) specifically for coherently detected $M$-ary PSK, the theoretic transmission rate for a given channel and bit error performance can be associated as follows:

$$C = W \log_2 \left( 1 + \frac{1}{2 \log_2(M)} \left( \frac{1}{\sin \left( \frac{\pi}{M} \right)} \right) \left( \frac{\log_2(M) P_b}{2} \right)^2 \frac{R_b}{W} \right)_{UNCODED}$$

(31)

This association allows for the assertion of a codependent relationship between bit error rate, SNR, bandwidth, and the required transmission rate. This codependence defines one of the fundamental tradeoffs in communications engineering: how best to manage resources to transmit information at a given bit error rate. While admittedly this relationship does not take forward error correction coding into consideration, acknowledgment of this codependent relationship allows the first of the two requirements for cognitive radio to be addressed. This assertion furthermore means coexistence between independently functioning communications resource users (users not operating in a multiple access network) can be addressed in a framework shared by all users of the communications resource. Acknowledgment and management of this codependency enables cognitive communication, as defined by Mitola [21], for all users of the communications medium.

Another way of looking at channel capacity theory is by taking this view of the problem: a given channel will permit a fixed number of information bits per second to be communicated through the channel. This fixed number is divided among all users of the channel. If all users are operating using the same SNR, the same bit rate, the same
forward error correction scheme, the same modulation type, and the same bandwidth and center frequency, capacity theory can be used to determine the maximum number of users operating at a given signal-to-interference plus noise-ratio (SINR) [22]. High data rate modulation types simply take up a larger share of the total channel capacity available to all users. Through being able to avoid interference and by properly identifying the resources required to communicate, cognition is reached. By managing (29) and (31), a multi-user view of communications cognition can be taken. If one can accurately estimate the PSD of a channel over time, manage the codependent communications variables which compose (31), and communicate using agile waveforms to best accommodate a time-varying channel, one truly achieves communications cognition.

The ability to coexist in a manner that more optimally uses communications resources requires that all users of a given spectrum perform interference avoidance to deconflict their transmissions with each other, thus coexist. This is not how a multiple access network behaves, where users of the network operate given a set of predetermined rules; coexistence means there are no rules. This gives way to what is called greedy interference avoidance, where each user greedily maximizes its own SINR given a user pair's ability to provide feedback on interference conditions from the receiver to the transmitter [22]. What Popescu and Rose assert is the counterintuitive conclusion that the use of greedy interference avoidance tends to optimize the use of communications resources (in terms of sum capacity) [22]. What Popescu, Popescu, and Rose have found is that optimization of sum capacity is usually reached through what is called water-filling in general information theory. The use of water-filling means that each user is operating
in its own self-interest to maximize SINR [23]. This gives way to what Clemens and Rose have implicitly identified as the fundamental precept of spectrum management in unlicensed, thus *laissez faire*, bands: cognitive radio (awareness of the environment and the ability to adapt to the environment) will be required to achieve optimal coexistence [24].

2.8. Coexistence and the Pursuit of LPI/LPD Communications

For an LPI/LPD communications system to be effective at avoiding detection, the system must be able to coexist with other communicators. The premise is that if a coexistent user of the spectrum of interest is suffering from degraded performance when the system of interest is transmitting, but doesn't suffer from degraded performance when the system of interest is not transmitting, the coexistent user becomes aware that *something* is using the spectrum. The coexistent user, whether the other user even knows it or not, is detecting the system of interest. This means the system of interest should not be classified as LPD, leading to the conclusion that coexistence is a non-negotiable requirement in an LPD communications system. Therefore, coexistence enables not only interference avoidance, but LPI/LPD capability as well.

Initial TDCS research involved the explicit use of different kinds of jamming signals to simulate interference. This definition can be broadened without any breakdowns in the simulations and mathematics simply by asserting that jammers are, if present in the spectrum, in and of themselves simply the most aggressive interferers. In this context, the goal of TDCS is simply to coexist undetected.
2.8.1. Water-filling

Water-filling in the context of communications resource management is explained by Cover and Thomas as follows:

The vertical levels indicate the noise levels in the various channels. As signal power is increased from zero, we allot the power to the channels with the lowest noise. When the available power is increased still further, some of the power is put into noisier channels. The process by which the power is distributed among the various bins is identical to the way in which water distributes itself in a vessel [25: 253].

This explanation is explicitly stated here because Cover and Thomas are cited when defining water-filling in [22] and [23]. Cover and Thomas provide a definition of water-filling that enables adaptive communications symbol waveform design in a manner that, in the words of Popescu and Rose, yields optimal use of the RF spectrum [22]. Popescu and Rose apply Cover and Thomas provide an equation to define water-filling, but for the purposes of this research, water-filling is taken as a concept, rather than a specific equation, so no specific mathematical equation is used.

A graphical example of water-filling is provided in Figures 6 through 10. Note again that no strict mathematical definition is used to describe water-filling in generating these figures. The PSD of the communications waveform generated via water-filling illustrated in Figure 8 is intended to optimally balance use of the spectrum of interest between the TDCS user pair and two interfering signals. The spectral estimate (calculated using the Bartlett method) [13-15] of the water-filled TDCS transmission coexisting with the two interfering signals demonstrates the spectrally balanced nature of water-filling. The wideband water-filled waveform illustrated in Figure 8 flattens the spectrum around the narrowband interferers, as seen in Figure 9 (thus optimally
Figure 6. PSD of Two Narrowband QPSK Interferers in Noiseless Channel

Figure 7. Spectrum Estimate (via Bartlett Method) of Interferers Theoretically Obtained by Transmitter

occupying the spectrum) [23]. Comparing Figures 7 and 9 provides an understanding of the impact of water-filling. Figure 9 illustrates the resulting PSD of the water-filled TDCS signal summed with the two interfering signals. The water-filled signal has the effect of balancing the spectrum exhibited in Figure 7, satisfying the definition of water-
Figure 8. PSD of Proposed Waveform Generated for Transmission via Water-filling

Figure 9. Spectrum Estimate of Water-filled TDCS Communications Waveform Coexisting with Interferers

filling provided by Cover and Thomas. Figure 10 illustrates the balancing of the spectrum immediately around the higher powered of the two narrowband interferers. Note that while only frequency values at and above 0 Hz are illustrated in Figures 6 through 10, symmetry exists between positive and negative sides of the spectrum.
2.9. Digital Signal Processing Hardware and Programming

A TDCS user pair could be constructed through any number of hardware realizations. Recently, hardware implementations of software-defined radio designs have been realized through the GNU Radio project [26]. The goal of the GNU Radio project is to field a software-defined radio with both an open source architecture and accompanying open source software tools for programming the radio. The problem with this radio, in its current state, is that the development has yet to mature to the point where the radio can be practically fielded [26]. Because of the risks associated with “bleeding edge” software-defined radio packages, it has been decided that a more mature technology should be used for implementing a TDCS architecture in hardware. DSP technology has been around for years, and has been fielded in a mature state in widely varying products, ranging from cellular phones to medical instrumentation devices. Because there is an infrastructure available to support DSP-based development in the academic and commercial
communities, it is believed that a DSP-based implementation is less risky than a more novel hardware technology.

There are several different technical challenges in implementing a digital communications system onboard a hardware platform. Interfacing the AIC23 Coder/Decoder (codec), which contains the analog-to-digital converter (ADC) and the digital-to-analog converter (DAC), with the DSP chip onboard the DSK requires particular attention. In order to obtain input samples of the noisy, interference-laden spectrum and/or the communications signal of interest, the ADC onboard the codec is used [27]. The ADC converts an analog acoustic signal into a digital value. This digital value is then transferred to the DSP chip for processing. The processed outputs to be emitted from the DSP chip are passed back to the codec where they are converted from digital values to analog signal levels through use of the DAC.

When sampling received signals at a rate of 48 kHz using the AIC23 codec, an anti-aliasing filter is automatically applied to the received signal [28]. This anti-aliasing filter serves as the RF filter described in (21).
III. TDCS Architecture Implementation

Implementing a TDCS, as with any other engineering effort, requires the designer to make choices based on informed trade-offs. The approach taken in this research effort is to integrate theory with pragmatic solutions to existing problems. A few areas where trade-offs and design choices are made in this research include:

- Choosing a technique for use in spectrum estimation.
- Designing communications symbol waveforms in the presence of an indeterminate, suboptimal (less than flat) end-to-end spectrum response.
- Managing the need to equalize the received TDCS signal.

This chapter outlines these design choices made in implementing TDCS in hardware, and integrates the notation offered by Roberts' proposed framework for SMSE digital communications [11] discussed in Section 2.5 into the TDCS architecture.

3.1. Transmitter

Figure 1 illustrates the general TDCS transmitter block diagram commonly published in literature [8]. The SMSE framework discussed in Section 2.5 has brought about the need to revise the block diagram to incorporate the notation used in the SMSE framework. Figure 11 illustrates the updated TDCS transmitter block diagram used in the implementation of the DSK-based design.

3.1.1. Spectrum Estimation

Spectrum estimation is performed using the Bartlett method, as discussed in Section 2.3.1. The absolute magnitude of the spectrum estimate at a given frequency is not required, as the spectrum estimate is only used to perform communications symbol
waveform design. This being the case, spectrum estimation is performed by calculating the periodograms of subsequences of samples of the noisy environment. The periodograms are then summed, which yields a relative PSD estimate between various frequencies in the environment. This estimate can then be used to perform waveform design.

Performing accurate spectrum estimation in a hardware implementation that uses a wireless channel is complicated by hardware limitations. While the DSK is capable of a sampling frequency of 48 kHz, with a frequency response that is assumed to be flat throughout the codec passband, the microphone used to sample the acoustic environment may have a much smaller bandwidth than the DSK. The microphone may also have a less than optimal magnitude response within its passband, though no attempt is made in this research to characterize the frequency response of the microphone. In an interference avoiding digital communications design, spectrum estimation of a noisy environment is performed to identify frequency bands that have been compromised by the presence of
interference. If the hardware involved in this process is integrated together in such a manner that the end-to-end signal path between the noisy environment and the digital processor is suboptimal, then areas of the spectrum where the magnitude response of the estimation hardware is relatively high will seemingly exhibit more noise than areas where the magnitude response is relatively low. This phenomenon will drive the uninformed user of the interference avoidant system to avoid areas of the spectrum where the magnitude response of the hardware is highest, as the user operates under the impression that these frequencies are compromised. If the transmitter and receiver use the same hardware for reception of interference avoidant signals as for spectrum estimation, parts of the spectrum that may be most favorable for transmission of signals go ignored in favor of less suitable frequency bands. On the other hand, the interference avoidant waveforms generated through spectrum magnitude calculations will load power in areas of the spectrum where the estimation hardware may have no magnitude response at all. In this case, if the transmitter and receiver use the same hardware for reception of interference avoidant signals as for spectrum estimation, then signal power will be transmitted in areas of the spectrum that may go entirely unreceived. More power may then be wasted through the use of interference avoidance algorithms than would be through conventional waveform design techniques. No interference avoidant communications design is free from this difficulty. As discussed in Section 2.7, Mitola describes cognitive radio as the ability to intelligently and autonomously use communications resources effectively [21]. Therefore, if a hardware implementation of a software-defined radio architecture does not perform estimation of the end-to-end
magnitude response, then the user pair cannot be defined as cognitive. All hardware implementations of a communications architecture labeled as cognitive must operate with *a priori* knowledge of the magnitude response of the hardware used in the channel.

In implementing the TDCS in hardware, the magnitude response of the spectrum estimation hardware used by the transmitter is determined by placing the microphone in as quiet an environment as possible. This exposes the microphone to an environment where it is assumed that the only source of energy available in the spectrum is thermal noise. Thermal noise has a theoretically flat magnitude response from DC to roughly $10^{12}$ Hz, thus it is considered to be a white noise source [19]. This noise provides a reference with which the magnitude response of the spectrum estimation hardware, principally the microphone, can be determined. The estimate serves as a “control” against which a noisy environment can be compared in future spectrum estimation calculations. The Bartlett method, as outlined in Chapter II, is used to obtain this *a priori* magnitude response.

Once the magnitude response of the spectrum estimation hardware is quantified, a sampling of the typical environment in which communications will occur is gathered. This sampling is then processed, again using the Bartlett method, to calculate the PSD of the noisy environment. This estimate of the typical noisy channel conditions is divided by the *a priori* magnitude response, yielding a scaled estimate that accounts for the magnitude response of the spectrum estimation hardware and can theoretically be used to perform interference avoidant waveform design. This approach to spectrum estimation is an engineering design choice, and no assertion is made that this is the theoretically optimal approach to this problem. The final spectrum estimate is expressed as follows:
\[
\hat{\phi}[m] = \left\{ \frac{\hat{\phi}_{\text{Noisy}}[m]}{\hat{\phi}_{\text{White}}[m]} \right\}_{m=0}^{N_f-1}
\] (32)

where \( m \) represents the index of a given frequency domain “subcarrier” and \( N_f \) represents the number of frequency-domain subcarriers in the communications symbol waveform.

The output of the spectrum estimation algorithm for a single realization of noise plus interference, expressed in terms of the variables in (32), is illustrated in Figure 12. This realization is generated using the Matlab® \texttt{wavplay} and \texttt{wavrecord} functions with PC speakers and microphone. A single tone interferer is present in the noisy spectrum at a frequency of 2.000 kHz. This interference appears as it should in the figure, with interference present in the noisy and final estimates at and around 2 kHz. While only frequency values at and above 0 Hz are illustrated in Figure 12, symmetry exists between positive and negative sides of the spectrum.

### 3.1.2. Spectrum Magnitude Calculation

As stated in Section 2.3.2, the spectrum magnitude output consists simply of a vector of ones and zeros, as implied in Figure 3 [8]. This vector serves as the frequency component identification vector \( f[m] \) in the SMSE framework, as described in Section 2.5, and is the Hadamard product of the frequency assignment vector \( a \) and the frequency use vector \( u \), so

\[
f[m] = \left\{ a[m]u[m] \right\}_{m=0}^{N_f-1}
\] (33)

As stated in Section 2.5, the frequency assignment vector is dictated to users by a network controller, so the frequency assignment vector is disregarded. The frequency use vector, on the other hand, provides the means by which the TDCS interference avoidance
algorithm is implemented. The spectrum estimate output, as discussed in Section 3.1.1, is used to determine areas of the spectrum that have been most compromised by the presence of interference or noise other than AWGN. The elements of the frequency use vector associated with areas of the spectrum containing relatively high levels of interference power are assigned a value of zero, while the rest of the vector is assigned a value of one. The elements of the frequency use vector associated with the baseband spectrum frequency values equal to DC and the Nyquist frequency (0 and 24 kHz), are assigned values of zero [6]. Different spectrum estimation/magnitude algorithms have been used in the course of TDCS research [1-8, 11]. In this research, the hardware implementation of the spectrum magnitude block involves locating the seven elements of the spectrum estimate vector with the highest magnitude (disregarding the magnitudes at DC and at the sampling frequency). The corresponding elements in the frequency use vector are then assigned a value of zero, along with the seven elements corresponding to

Figure 12: Single Realization of Bartlett Spectrum Estimation Output for Typically Noisy Channel with Tone Interferer Present at 2.000 kHz
the same frequency on the opposite side of the spectrum. There are several reasons this approach to spectral notching has been taken. First, it allows for an interference algorithm to be incorporated into the design without having to perform scaling on a symbol-by-symbol basis, as described in Section 2.3.4. Second, this approach is relatively simple to implement. Third, there is little processing latency associated with this approach. Fourth, this approach ensures the same number of subcarriers are “notched out” on both the transmit and receive sides of the user pair. Finally, since there is no means by which spectral notching information is being communicated from transmitter to receiver (which would most likely require error correction coding, and is outside the scope of this thesis), spectrum estimation and spectrum frequency use vector mismatches will occur in hardware experiments. This will lead to suboptimal symbol estimation and an increase in bit error curves. Given this is the case, initiating interference avoidance for a small subset of the available subcarriers allows for interference avoidance experimentation on a limited scope without compromising the results as a whole. An illustration of the output vector of the spectrum magnitude operation is provided in Figure 13. This figure is generated using the same scenario used to generate the illustration of the spectrum estimation process provided in Figure 12. Note that 14 subcarriers, plus the frequencies associated with DC and the Nyquist frequency, are “zeroed out” in the frequency assignment vector $f[m]$. The decision to notch out 14 subcarriers (disregarding the subcarriers at DC and the Nyquist frequency) incidentally brings this research effort in line with Roberts' work in SMSE signaling. In the bit error performance curves generated through simulations that involve spectral notching, Roberts used partial band interference
that spanned 10% of the available frequencies [11]. If spectral notching is used with 128 total subcarriers, notching out 14 of these subcarriers reduces the available spectrum by 10.9%. The 14 subcarrier frequencies where the most interference is observed, plus the subcarriers at DC and the Nyquist frequency, will be notched out. While only frequency values at and above 0 Hz are illustrated in Figure 13, symmetry exists between positive and negative sides of the spectrum.

3.1.3. Pseudorandom Phase Generation

As stated in Sections 2.3.3 and 2.5, the pseudorandom phase generation operation consists of establishing a uniformly distributed and complex-valued pseudorandom code vector \( c[m] \) to spectrally encode communications symbol waveforms. The magnitude of the elements of the complex-valued code vector are all of unit magnitude [1-8, 11], and are Hadamard multiplied with the frequency assignment vector \( f[m] \) discussed in Section 3.1.2. Though the TDCS transmitter block diagram in Figure 11 does nothing to express
this, the orthogonality vector $\mathbf{o}[m]$ described in Section 2.5 can be Hadamard multiplied with the code vector to produce communications symbols that are suitable for multiple access applications.

In practice, the code vector is generated by randomly assigning phase angles of positions on the unit circle to elements within the code vector. The phase angles are equally spaced on the unit circle, thus a uniform distribution is achieved. While the code vector is to be generated on a transmitted symbol-by-symbol basis to ensure LPI/LPD functionality is maintained, it is assumed that all code vectors exhibit equivalent bit error performance. It is further assumed that all communications symbols are mutually independent from one another in terms of demodulation (and subsequent bit error performance), so only one realization of a code vector is used in the hardware implementation. This is done to ease the computational burden in the hardware realization. As with (8)-(11), the code vector is actually split in half to account for the implicit use of Hilbert transforms to describe discrete-valued baseband signals [6, 18]. Therefore, the first half of the baseband spectrum is assigned negative phase angles between 0 and $2\pi$ radians, while the second half of the baseband spectrum is assigned a reflection of the first half around the y-axis. The reflection in the second half is then multiplied by -1 to account for the implicit use of Hilbert transforms. The elements of the code vector associated with the DC and Nyquist frequencies are always assigned values of zero, and held out of the pseudorandom phase vector generation process. The values of the code vector at DC and Nyquist frequency element positions is irrelevant, since the frequency assignment vector has values of zero at these two elements, so no power will
be present at these two frequencies during symbol transmission. One realization of a generated code vector is illustrated in Figure 14. The value of $\Theta[m]$ illustrates the nature of the reflection around the y-axis.

3.1.4. **Data (Waveform) Modulation**

Data modulation is conducted using the data modulation vectors $d_{00}[m]$, $d_{01}[m]$, $d_{10}[m]$, and $d_{11}[m]$ expressed in (8)-(11) respectively. In a TD-QPSK design, four distinct communications symbols are required. These four data modulation vectors answer this requirement. The data modulation vectors are Hadamard multiplied with the frequency assignment vector and the code vector. The resulting symbol waveform, $s_k[m]$, is expressed as follows:

$$s_k[m] = \left\{ f[m]c[m]d_k[m] \right\}_{m=0}^{N_F-1}$$

(34)

where $k$ denotes the symbol index.

The reason TD-QPSK is used exclusively in the hardware implementation is to provide a means by which orthogonal, rather than simply antipodal, signaling can be studied. Section 2.5 describes how the data modulation vectors used for this research are constructed.

3.1.5. **Scaling**

As stated in Section 2.3.4, scaling is performed on the complex-valued frequency domain communications symbol waveforms $s_{00}[m]$, $s_{01}[m]$, $s_{10}[m]$, and $s_{11}[m]$ to ensure that a required bit error performance is obtained regardless of the number of elements that are valued at one or zero in the frequency identification vector $f[m]$. If the number of frequency subcarriers activated for use with the frequency identification vector varies,
scaling is performed to maintain the bit energy level required to sustain a given error rate.

Given that a fixed number of elements in the frequency identification vector are set to zero in the hardware implementation, there is no need to account for this function in the hardware implementation.

3.1.6. IDFT

As stated in Section 2.3.5, the complex-valued frequency domain communications symbols \( s_{00}[m], s_{01}[m], s_{10}[m], \) and \( s_{11}[m] \) are inverse Fourier transformed, yielding time domain symbols \( s_{00}[n], s_{01}[n], s_{10}[n], \) and \( s_{11}[n] \), respectively. An FFT function is provided by Chassaing [27] for use onboard the DSK. This “black box” FFT function can be used to perform an IFFT by simply adding a \( 1/N \) scaling factor (where \( N \) is the number of complex-valued points in the FFT computations) and conjugating the twiddle factors used in the FFT algorithm [27].
Section 2.3.5 discusses a potential problem in hardware implementation of multicarrier modulation schemes where constructed time-domain waveforms may exhibit exceptionally high PAPR. This problem is intrinsically addressed, as scaling is performed on the time-domain symbols to raise or lower the peak instantaneous magnitude of all four symbols to the maximum possible magnitude output by the DSK codec. This error-free scaling is enabled through the use of floating-point arithmetic in either the MATLAB®-based TDCS implementation or the DSK-based implementation, and is performed using the same scale factor for all four time-domain communications symbol waveforms used in TD-QPSK. The communications symbol $s_k[n]$ after scaling is expressed as

$$s_k[n] ← G \max \left( \max_{k=1}^{128} |s_{00}[n]|, \max_{k=1}^{128} |s_{01}[n]|, \max_{k=1}^{128} |s_{10}[n]|, \max_{k=1}^{128} |s_{11}[n]| \right)$$

(35)

where $G$ is the maximum magnitude the DSK codec is capable of outputting, thus the peak value of the TDCS signal transmission will be equal to $G$.

Generated communications symbols, representing one realization of TD-QPSK time-domain communications waveforms, are illustrated in Figure 15, assuming a maximum signal output magnitude $G$ equal to one. The four communications symbols illustrated in Figure 15 are concatenated together in a time-domain waveform, in accordance with the mapping of bits to symbols dictated by the data bitstream input to the TDCS transmitter. The noise-like structure of the waveforms, is due to the spectrally encoded nature of the waveforms, and works to enable LPI/LPD. Note also that between the symbols $s_{00}[n]$ versus $s_{11}[n]$ and $s_{01}[n]$ versus $s_{10}[n]$ that the symbols opposite in bit...
Figure 15. Time-domain Communications Signal Waveforms for TD-QPSK Signaling

pattern (00 versus 11 and 01 versus 10) are the opposite symbol reflected around the x-axis. In other words, $s_{00}[n]$ equals $-s_{11}[n]$ and $s_{01}[n]$ equals $-s_{10}[n]$, as is expected with QPSK signaling. Finally, symbols $s_{00}[n]$ and $s_{11}[n]$ are both orthogonal to symbols $s_{01}[n]$ and $s_{10}[n]$, as is expected in TD-QPSK signaling [6]. It is appropriate to point out that the symbol waveforms are illustrated using a “connect-the-dots” plot rather than a stem plot. This is done only to cleanly communicate the appearance of the waveform.

### 3.1.7. Buffer

As stated in Section 2.3.6, the buffer is used to provide a storage for the communications symbols. The DSK-based hardware implementation has buffer space intrinsically built within the design. In the design of the hardware user pair, there is no refreshing of the symbols stored in the buffer within the one-second transmission burst. It is assumed that the spectrum estimate will not vary enough to merit a refreshing.
3.1.8. Modulation (Bits-to-Symbols)/Transmission

As stated in Section 2.3.7, modulation refers to the mapping of bit patterns to communications symbols. Two-bit patterns contained in input bitstreams are mapped to communications waveforms that are concatenated in time in preparation for transmission. An illustration of the constellation used in this TD-QPSK design is provided in Figure 16. Note that Gray coding is used in mapping bit patterns to communications symbols.

As stated in Section 2.3.7, transmission consists of converting the final digital TDCS signal data vector $s[n]$ to analog signal $s(t)$ and the subsequent transmission of the analog signal into the channel.

3.2. Acquisition of Received Signals

Acquisition of TDCS signals is performed by manually directing the transmitting DSK and the receiving DSK to operate simultaneously. This process is initiated by simultaneously toggling dip switches on the transmitting and received boards. The receiving DSK stores the one-second burst output to the channel by the transmitting DSK into a buffer sized to store a two-second recording from the receiver microphone.

3.3. Synchronization of Received Signals

Synchronization of TDCS signals is performed using peak detection, as discussed in Section 2.4, and is conducted using only the DTC method. The minimum $E_b/N_0$ value required to evaluate the bit error performance is 0 dB, the TDCS hardware realization operates at a baseband two-sided bandwidth of 48.000 kHz, and 128 samples/symbol are arbitrarily used. Given these conditions, the lowest SNR used in hardware experiments is approximately -18 dB, per (28). Roberts determined that for an 80% probability of
detecting a TDCS signal for use in acquisition at an SNR of approximately -23 dB, a minimum of 10 symbols should be transmitted for use in acquisition [3]. It is assumed that the same minimum required number of symbols is sufficient to perform symbol synchronization. A 100% error margin is built into the synchronization algorithm design, so 20 symbols are used for all symbol synchronization decisions.

The 20 symbols used in synchronization are generated using the same process as that used to generate a waveform for transmission. In a TD-QPSK modulation scheme, four symbol waveforms are generated and concatenated in a prescribed order that is known by both the transmitter and receiver. This allows the receiver to construct a reference waveform that is convolved with (or correlated with) a received signal. Within the two-second recording stored by the receiver DSK that is obtained during signal acquisition, as described in Section 3.2, the sample yielding the peak convolution (or correlation) output serves as the peak reference. Once this peak is located, the one-
second TDCS transmission burst (as well as noise, interference, and channel effects due to such phenomena as multipath propagation) is identified.

### 3.4. Reception and Demodulation of Received Signals

Figure 5 illustrates the general TDCS receiver block diagram published in the literature. As with the TDCS transmitter, the SMSE framework discussed in Section 2.5 has brought about the need to revise the block diagram to incorporate the notation used in the SMSE framework. Figure 17 illustrates the updated receiver block diagram used in the implementation of the DSK-based design.

The receiver block diagram in Figure 17 includes several common functions with the transmitter diagram illustrated in Figure 11, including “Estimate spectrum,” “Spectrum magnitude,” “Data (waveform) modulation,” “Pseudorandom phase,” and “IDFT.” This highlights the fact that the transmitter and receiver perform the same operations to construct communications symbol waveforms on both the transmit-side and receive-side of the user pair. The only difference is that only two waveforms are required for demodulation (though all the waveforms are generated by the receiver to construct a reference signal for use in synchronization). These two waveforms, expressed in the block diagram as $s_{00}[n]$ and $s_{01}[n]$, serve as inphase and quadrature reference signals when performing TD-QPSK demodulation. Subscripts 00 and 01 are used to convey that the waveform mapped to the two-bit symbol 00 is used in correlation to generate the $X$ statistic, while the waveform mapped to the two-bit symbol 01 is used to generate the $Y$ statistic. The lines with arrows through the reference waveforms $s_{00}[n]$ and $s_{01}[n]$ convey that these waveforms are generated adaptively through the spectrum magnitude function.
shared by both the transmitter and receiver. Once these reference waveforms are generated, a synchronized received signal is demodulated using these waveforms. The output values from the correlation, $X$ and $Y$, are input to a four-quadrant arctangent operation. The output of this operation serves to assign a phase value to the received symbol in question. This phase value is compared with a decision rule used to determine a final symbol estimate, $s_i$. The specific decision rule being used in the implemented demodulator, as stated in Section 3.1.8, is illustrated in Figure 16. An illustration of a received and demodulated TD-QPSK symbol constellation is provided in Figure 18. The constellation for $E_b/N_0 = 9.85$ dB uses the decision boundaries illustrated in Figure 16. Each data point in Figure 18 represents the test statistic output value from a single communications symbol, so there are 375 data points illustrated in Figure 18. Note that the constellation points exhibit a phase offset around the unit circle and that there is some degree of variance around the centroid of each of the clusters of points. This variance is
due to noise, be it AWGN, intersymbol interference, and/or an interfering signal present near the receiver. Demodulation of this specific realization of TDCS signaling yielded three bit errors out of 750 total transmitted bits.

An RF filter, as described in (21), is incorporated intrinsically into the design of the receiver when incorporating the TDCS algorithm onboard the DSK. This RF filter is brought about through the use of the anti-aliasing filter built into the codec, and is described in Section 2.9.

3.5. Equalizing Received Signals

As discussed in Section 2.6, the hardware realization includes an MMSE equalizer that is implemented as described in (22)-(26). The MMSE equalizer requires a reference signal for training, and is used in generating the cross-correlation vector $R_{xz}$. The reference signal is obtained by performing a demodulation operation on the reference waveform used for synchronization. The output test statistics from the correlators are
then placed into a complex-valued format. The first 19 of the 20 symbols used to
construct the synchronization reference waveforms are used in training the equalizer, thus
the equalizer is permitted 19 filter taps. Once correlation is performed on a received
TDCS signal and the test statistics output from the two correlators are available, the first
19 of the received symbol statistic outputs are placed into Toeplitz matrix $X$, as expressed
in (25). The autocorrelation matrix $R_{xx}$ of the received symbol test statistics is then
calculated, as stated in (23). Computation of $R_{xx}$ is performed by multiplying matrices
containing complex-valued elements, so a Hermetian transpose is used. The first 19, as
opposed to 20, symbols are used to generate $z$ because an odd-numbered length is easier
to process. This allows the Toeplitz matrix $X$ to be assembled using elements from $x$ that
range from index $-N$ to $N$, as opposed to having to zero-pad a leading or trailing element
in the $x$ and/or $z$ vector. In the hardware implementation, computing the autocorrelation
matrix, as with all matrix-matrix and matrix-vector multiplication operations, requires
either an extensive effort to manage the computation of complex valued matrices, or the
use of library functions. One such set of functions for performing complex arithmetic in
C-based programs is offered by Press, Flannery, Teukolsky, and Vetterling [29]. These
functions are integrated into the design of the software onboard the receive-side DSK.
Once the autocorrelation matrix is computed, it is multiplied with the cross-correlation
vector, as expressed in (22), to obtain the equalizer coefficients, which requires a matrix
inverse operation. Though literature is explicit in stating that direct matrix inverse
computation is to be avoided [30], the decision was made to perform matrix inverse
operations via the Gauss-Jordan method [20], thus the matrix is inverted directly. While
direct computation of matrix inverses are intensive, the inverse of a 19x19 matrix is not unreasonably expensive. An algorithm performing direct inversion of a complex-valued matrix is embedded into the equalizer design and requires less than one second to compute onboard the DSK. This approach is not recommended for real-time operations, however, and the positive semi-definite nature of the autocorrelation matrix lends itself to a more efficient calculation of the inverse. Regardless, the entire vector of received complex-valued symbol test statistics are input into a Toeplitz matrix, as seen in (25). The Toeplitz matrix is multiplied by the vector of equalizer taps, yielding the equalized channel test statistic estimates used to demodulate the signal. This set of estimates then theoretically represents symbol test statistics in which noise, ISI, and phase offsets are mitigated.

In the design of the hardware user pair, there is no retraining past the initial training of the equalizer. This is because the TDCS signal is transmitted in a one-second burst, and it is assumed that the channel response will not vary enough to merit a refreshing of the equalizer training.

Figure 19 illustrates the effect that equalization can have on received symbol test statistics. Figure 19 is generated using an equalized version of the same realization of received symbol test statistic values as those used to generate Figure 18. As in Figure 18, each data point in Figure 19 represents the test statistic output value from a single communications symbol, so there are 375 data points illustrated in Figure 19. The three received bit errors illustrated in Figure 18 are corrected through equalization. The differences between Figures 18 and 19 are obvious, as the phase offset is corrected and
the variances around the cluster centroids are reduced, relative to the scale of the decision regions. Though this is only one example of equalization, the effectiveness of the MMSE equalizer is visually obvious.

3.6. Computing SNR and $E_b/N_0$

As expressed in (28), there is a linear relationship between SNR and $E_b/N_0$. The bit error curves provided in Chapter V illustrate bit error rates relative to $E_b/N_0$, but in reality, SNR is computed at the receiver and then $E_b/N_0$ is subsequently calculated using (28). SNR is simply the average power $P_S$ of the received TDCS signal $r[n]$ divided by the average power $P_N$ of the noise received simultaneously with the signal. Noise power can be closely approximated by the following relationship [19]:

$$P_N = \sigma^2_N + m^2_N$$  \hspace{1cm} (36)

where $\sigma^2_N$ is the variance of noise samples and $m_N$ is the mean value of noise samples.

Note that (36) assumes that the noise is wide-sense stationary, so the noise has a constant
mean and an autocorrelation as a function of \( n_1 - n_2 \) alone [31]. The assumption is made in the hardware implementation that all noise and interference has zero mean, and that any realizations of received signals have a non-zero mean only because of a bias voltage in the ADC. Given this assumption, the mean value of a received realization \( r[n] \) can then be neglected. The transmitted signal \( s[n] \) has zero mean as a result of the uniformly distributed phase encoding employed in the TDCS design. After synchronization of the received signal, there are \( N = 48000 \) samples in the one second time window within which the TDCS transmission resides. The value of each sample is simply the sum of the received TDCS transmission \( r[n] \) at time index \( n \) plus the sum of all noise and interference present in the received signal at time index \( n \). Given the assumption that noise plus interference has zero mean, the noise power \( P_N \) can be estimated per (36) simply by computing the variance of the noise plus interference. The following expression is used to compute the sample variance of the noise plus interference [31], and together with (36) yields an estimate of the power \( P_N \) of all noise plus interference:

\[
P_N = \sigma^2_N = \frac{1}{47999} \sum_{n=1}^{48000} (r[n] - s[n])^2
\]  (37)

The expression in (37) is normalized by \( N-1 \) rather than \( N \) samples (47999 versus 48000) to perform unbiased estimation of the sample variance. Unbiased estimation of the variance has been chosen because the noise plus interference signal available to the DSK represents a sampling, rather than the population, of the signal. Note that (37) assumes that the noise plus interference samples are independent and identically distributed.

The use of (37) is limited to instances when the magnitude of the received TDCS signal \( s[n] \) is known. Given the received signal is the sum of the received TDCS signal
and all noise plus interference simultaneously present in the environment, the magnitude of the received TDCS signal must be scaled to accurately calculate the noise power. This scaling in turn intrinsically yields an estimate of received TDCS signal power. To perform this estimate, the mean square error (MSE) criterion is used. The function used to estimate the power $P_s$ of the received TDCS signal, using the MSE criterion, is as follows [27]:

$$P_s = K_0 = \arg \min_{K \in [0, \infty)} \left( \sum_{n=1}^{48000} \left( r[n] - \frac{\sqrt{K}}{\sigma_s} s[n] \right)^2 \right)$$

(38)

where $K$ represents a scale factor and $\sigma_s$ is the standard deviation of $s[n]$. The value of $K$ that minimizes the argument, $K_0$, is equal to $P_s$, so all that is required to compute SNR is the power of the sum of noise plus interference, $P_N$. Given the scaled version of the received TDCS signal, and the assumption that noise plus interference has zero mean, it is assumed that the noise plus interference power can be computed as follows:

$$P_N = \frac{1}{47999} \sum_{n=1}^{48000} \left( r[n] - \frac{\sqrt{K_0}}{\sigma_s} s[n] \right)^2$$

(39)

Therefore, (28) can be restated as follows:

$$\frac{E_b}{N_0} = \frac{WK_0}{R_b} \frac{1}{47999} \sum_{n=1}^{48000} \left( r[n] - \frac{\sqrt{K_0}}{\sigma_s} s[n] \right)^2$$

(40)

where $W$ is the bandwidth (Hz) and $R_b$ is the bit rate (bits/second). For practical purposes, (40) accurately expresses how all $E_b/N_0$ calculations are made in generating the results provided in Chapter V.
**IV. Spectral Notching Versus Water-filling**

This chapter outlines research conducted to determine whether or not water-filling is a more optimal means than spectral notching for generating TDCS communications symbol waveforms. TDCS literature is explicit in stating that the spectral notching algorithm described in Section 2.3.2 may not be optimal [3], and no attempt has been made to optimize TDCS waveform designs. As discussed in Section 2.8.1, the optimal means by which available spectrum is occupied is usually a water-filled solution [22, 23]. Though this statement has several conditions placed on it in literature, it suggests that a water-filling waveform design algorithm may provide an advantage over spectral notching. This suggestion motivates the simulations conducted in this chapter. If water-filling indeed usually optimally uses the available spectrum, and if spectral notching performs equivalently to water-filling in any given scenario, then spectral notching must usually optimally use the available spectrum as well.

Formal Matlab® simulations are conducted to determine whether a water-filling algorithm yields better bit error performance than spectral notching. This comparison is reasonable, as both the water-filled and spectrally notched waveforms contain the same amount of energy and occupy the same amount of bandwidth. The water-filling algorithm is used directly in identifying which areas of the spectrum of interest are notched out. The only difference between the water-filled and the spectrally notched waveforms is how power is distributed in various areas of the spectrum. The implication of using a water-filling algorithm, or any sort of frequency-domain waveform sculpting
algorithm other than spectral notching, is that the frequency use vector \( \mathbf{u} \) in the SMSE framework [11], expressed in (15), may need to be redefined to express variations in frequency-domain subcarrier magnitude. Specifically, values other than the discrete on/off toggling asserted in (15) may be required and appear as follows:

\[
\mathbf{u} = [u_1, u_2, \ldots, u_N] = [u_i]_{i=1}^N, \quad u_i \in [0,1]
\]  

(41)

The difference between (15) and (41) is subtle. The frequency use vector, as defined by Roberts, consists of subcarrier “toggling” through elements consisting simply of binary ones or zeros. The use of water-filling or an alternate waveform sculpting algorithm leads to varying the magnitude of subcarriers by allowing a continuous range of values between zero and one to be used in the frequency use vector. This may enable the SMSE framework to be used in describing communications that use water-filling or, more generally, a waveform sculpting algorithm. No rigid mathematical proof is offered here to demonstrate that the SMSE framework can be expanded to include water-filling. The only suggestion made here is that the SMSE framework may provide the extensibility required to capture waveform design techniques other than spectral notching.

Regardless of the impact on the SMSE framework, the following figures and discussion outline simulation results used to steer the direction taken in realizing a TDCS implementation in hardware. It is believed that these results serve future SMSE researchers, as attempts to design waveforms through some sort of sophisticated sculpting algorithm may be less consequential than literature seems to assert. This chapter describes the entire study of water-filling in the context of TDCS.
All $E_b/N_0$ computations are performed at the point of the simulated receiver antenna. Unless explicitly stated, each data point in simulated bit error curves in Figures 20 to 22 are generated using at least 1000 bit errors.

4.1. TDCS Performance: Spectral Notching vs. Water-filling

The comparison of bit error performance of the TDCS user pair involves simulating the transmission of TDCS waveforms through an AWGN channel containing two randomly spread QPSK interferers and then processing the TDCS signals received through the simulated channel. The interferers are spread using a randomly generated CDMA spreading code. Demodulation is performed independently using the water-filling and spectral notching algorithm. The TDCS user pair shares four communications symbols used in a TD-QPSK modulation scheme. The interferers are implemented using a bit energy equal to the bit energy of the TDCS transmission, so $E_b/N_0$ will be equal for all three users of the spectrum. The locations of the frequency components are the same for both the spectrally notched and the water-filled TDCS communications symbols. The water-filling algorithm is used to directly identify frequency subcarriers suitable for use in spectral notching, so the TDCS transmission bandwidth is equal in both the spectral notching and water-filling case. Figure 20 illustrates TDCS bit error performance using spectral notching and water-filling over an interference-laden AWGN channel. The two curves appear to be approximately equivalent; spectral notching and water-filling appear to perform approximately equivalently in this scenario. Furthermore, both simulated curves appear to be roughly equivalent to the analytic curve.
4.2. Coexistence Performance: Spectral Notching vs. Water-filling

The comparison of bit error performance of a user pair operating with conventional QPSK modulation involves simulating the transmission of both the QPSK waveform and a TDCS waveform through an AWGN channel. This is done independently using the water-filling and spectral notching algorithms. The conventional QPSK user uses a bit energy equal to the bit energy of the TDCS transmission, so $E_b/N_0$ will be equal for both users of the spectrum. The TDCS user pair uses a TD-QPSK modulation scheme. The locations of the frequency components used by the TDCS are the same for both the spectrally notched and the water-filled communications symbols, so the TDCS transmission bandwidth is equal in both the spectral notching and water-filling case. Figure 21 illustrates the QPSK bit error performance in an AWGN channel containing one TDCS user operating with either spectral notching or water-filling. The

Figure 20. Bit Error Performance for TDCS using Spectral Notching and Water-filling
two curves appear to be approximately equivalent. Given this is the case, spectral notching and water-filling appear to coexist roughly equivalently with a typical digital communications user pair. Furthermore, the QPSK user pair is performing roughly equivalently with what would be expected given the analytic curve in an AWGN channel with no interference. This being the case, TDCS user pairs appear to coexist with other users, regardless of which waveform design algorithm is used.

4.3. Spectrum Estimate Mismatch: Spectral Notching vs. Water-filling

The comparison of bit error performance of the TDCS user pair involves simulating the transmission of TDCS waveforms through an AWGN channel containing two randomly spread QPSK interferers and then processing the TDCS signals received through the simulated channel. The difference between this scenario and the original study of TDCS bit error performance is that the spectrum estimates used by the TDCS
transmitter and receiver are no longer equal. Independent realizations of noise and interference are generated for use independently by the TDCS transmitter and receiver, though the pseudorandom phase vector is synchronized between transmitter and receiver. This will have the effect of mismatching the TDCS signals received by the TDCS receiver and the reference signals generated by the TDCS receiver for use in matched filter detection. The test statistics output by the demodulator will then be less than optimally computed. This simulation is performed using the water-filling and spectral notching algorithms independently. The TDCS user pair uses a TD-QPSK modulation scheme. The interferers are implemented using a bit energy equal to the bit energy of the TDCS transmission, so $E_b/N_0$ will be equal for all three users of the spectrum. The locations of the frequency components used for transmission are the same for both the spectrally notched and the water-filled TDCS communications symbols, so the TDCS transmission bandwidth is equal in both the spectral notching and water-filling case.

Figure 22 illustrates the bit error performance of a TDCS user pair communicating in an interference-laden AWGN channel when there is a mismatch between TDCS transmitter and TDCS receiver spectrum estimates. Each point along the simulated curves is generated using at least 2000 bit errors. Note that the two curves appear to be approximately equivalent. There is a roughly equivalent degradation in performance in both the spectral notching and water-filling cases, versus a scenario where the spectrum estimates are equal between transmitter and receiver. The loss due to the spectrum estimate mismatch is, in its worst case, almost 1.0 dB, at an $E_b/N_0$ between 0 and 7 dB.
4.4. Interpretation of Simulation Results

In all three simulation scenarios in Sections 4.1 through 4.3, spectral notching and water-filling yield roughly equivalent bit error performance in an AWGN channel. The comparison of TDCS performance using both spectral notching and water-filling (Figure 20) and the comparison of a randomly spread QPSK user pair coexisting with a TDCS transmission (Figure 21) demonstrate that, regardless of the waveform design algorithm, a TDCS user pair coexists with interferers in the spectrum of interest.

The degradation in bit error performance seen in the case where spectrum estimates are mismatched between TDCS transmitter and receiver is less than 1 dB in an AWGN channel at an $E_{b}/N_0$ between 0 and 7 dB.

4.5. Summary

Given the result that spectral notching and water-filling yield roughly equivalent bit error performance, this implies that balancing the PSD of the TDCS signals through
water-filling may not be as important as literature implies [22]. However, one should not infer that a water-filling approach has no value, as water-filling allows us to set a threshold that intrinsically distributes power in the most appropriate manner for a given $E_b/N_0$. In an AWGN channel, the weighing function used to balance power among the frequency components in the spectrum magnitude algorithm may not provide any advantage over spectral notching. Spectral notching is much less computationally intense than water-filling, so using a threshold to generate spectral notches may be the best solution for a TDCS user. There may be a more optimal weighting scheme that can be used to distribute symbol waveform power in the spectrum, but these results lend to a degree of skepticism about this possibility. Water-filling is intended to optimize use of the spectrum in terms of sum capacity, so as data rates are increased among coexistent users of the spectrum, there may be motivation to use water-filling rather than spectral notching.

The degradation in bit error performance in the scenario where the spectrum estimates are mismatched between the TDCS transmitter and receiver is not as severe as might be expected. This implies that, in an AWGN channel, synchronization between transmitter and receiver spectrum estimates may not be critical in all instances to communicate somewhat effectively. In this instance, whether this result has value to the user depends entirely on what the user requires in terms of bit error performance.
V. Hardware Implementation Results

Four different scenarios are examined to address the research objective: to prove, through successful TDCS signaling in a hardware implementation, that the concept of TDCS-based digital communications is viable. For all four scenarios, the number of received bit errors is computed both with and without use of the MMSE equalizer discussed in Section 3.5. These scenarios are as follows:

1. No spectral notching with no narrowband interference added to the environment,
2. No spectral notching with narrowband interference added to the environment,
3. Spectral notching with narrowband interference added to the environment, and
4. Spectral notching with no narrowband interference added to the environment (to examine the effects of potential spectral mismatch).

Figure 23 illustrates the equipment positioning used to generate all results provided herein. There is a separation of approximately 0.30 meters between transmitter speakers and receiver microphone in all experiments. The transmitter speaker volume is adjusted to control the gain of the transmitted signal. The narrowband interfering signal consists of a sinusoidal tone generated via Matlab® and transmitted using PC speakers. The narrowband tone resides at randomly chosen frequencies between 1.500 and 22.500 kHz, and remains at a constant frequency and magnitude throughout the duration of a one-second (750 bits/second) TDCS transmission.

Figures 24 through 31 illustrate the bit error performance observed in each of the four different scenarios. There are 100 empirically obtained points in each of the eight figures. In the cases where an equalizer is not used (Figures 24, 26, 28, and 30), 750
received bits are compared with true values to determine bit error performance. In the cases where an equalizer is used (Figures 25, 27, 29, and 31), 712 received bits are compared with true values to determine bit error performance (750 bits are transmitted and received, with 38 of these bits communicated in the 19 training symbols in the preamble to the message). In all instances, the bit error rate is plotted against the received $E_b/N_0$ calculated as described in Section 3.6. Each of the figures includes the analytic curve for M-ary PSK signaling expressed by (30) for comparison, where $M = 4$ communications symbols required to transmit 2 bits per symbol. In each of the observations, colored noise may be present, so differences between empirically obtained observations and the analytic curve may not be absolute. The curve is intended to serve as a reference point for the observations at large, both within and between figures. Although narrowband interference and other interference sources are colored, $E_b/N_0$ is used in all figures (versus SNR or SINR) to enable comparison with TD-QPSK.
performance curves found in literature, particularly those developed by Nunez [6].

Semilogarithmic plots are not provided, as is typically the case in a bit error performance plot. This is because many empirical observations yielded zero bit errors, which cannot be accurately illustrated in a semilogarithmic plot.

All comparisons are made qualitatively, rather than quantitatively. This is because SNR is deliberately uncontrolled in these experiments, to test the utility of the spectral notching algorithm and the use of the MMSE equalizer in the TDCS architecture. Because of the undefined and dynamic nature of the frequency magnitude and phase responses of the speakers and microphones, it is believed that there is no way to practically construct controlled experiments without the use of more sophisticated hardware and test environments (such as an acoustic chamber). As will be seen, these figures provide results that can be used for qualitative comparison.

5.1. No Spectral Notching with No Interference Added to Environment

Figure 24 illustrates the unequalized bit error performance of the hardware realization when no spectral notching is applied and when there is no deliberately generated interference present in the environment. Figure 25 illustrates the equalized bit error performance using the same realizations of received TDCS signals as those used in generating Figure 24.

The trend of the empirical bit error performance illustrated in Figure 24 appears to coincide with the analytic curve. The trend of the empirical bit error performance illustrated in Figure 25, on the other hand, indicates that bit error performance is improved by the equalizer at an $E_b/N_0$ above approximately -15 dB. The equalizer
Figure 24. *Unequalized* Bit Error Performance Curve of Hardware Realization. No Spectral Notching in an Environment *Without* Additional Narrowband Interference

Figure 25. *Equalized* Bit Error Performance Curve of Hardware Realization. No Spectral Notching in an Environment *Without* Additional Narrowband Interference

actually appears to *degrade* bit error performance below -15 dB. While this could be interpreted to mean that the equalizer should not be used when the received $E_b/N_0$ is below -15 dB, the bit accuracy rate at this low SNR already approaches the coin flip
probability of 0.5, so it is not believed that TD-QPSK would be practically attempted under these circumstances. The reason for the performance improvement on the part of the equalizer, to the point where the performance exceeds the analytical curve, is because the MMSE equalizer mitigates some noise effects by design [19]. Judging from Figures 24 and 25 together, it is believed that the hardware realization is performing as it should, and that the firmware used to control the transmitter and receiver is correctly written.

5.2. No Spectral Notching with Interference Added to Environment

Figure 26 illustrates the unequalized bit error performance of the hardware realization when no spectral notching is applied and when there is deliberately generated narrowband interference present in the environment. Figure 27 illustrates the equalized bit error performance using the same realizations of received TDCS signals as those used in generating Figure 26.

The trend of the empirical bit error performance illustrated in Figure 26 appears to coincide with the analytic curve. The trend of the empirical bit error performance illustrated in Figure 27, on the other hand, indicates that bit error performance is improved by the equalizer above approximately -15 dB, while the equalizer degrades bit error performance below -15 dB. This is the case in Scenario 1 as well. When comparing Figures 25 and 27, the presence of deliberately applied narrowband interference in the spectrum of interest appears to only minimally impact the performance of the TDCS user pair operating with the MMSE equalizer when no spectral notching is used.
5.3. Spectral Notching with Interference Added to Environment

Figure 28 illustrates the unequalized bit error performance of the hardware realization when spectral notching is applied and when there is deliberately generated
narrowband interference present in the environment. Figure 29 illustrates the equalized
bit error performance using the same realizations of received TDCS signals as those used
in generating Figure 28.

The empirical bit error performance illustrated in Figures 28 and 29 both appear
to be different from the performance illustrated in Figures 24 and 26 or Figures 25 and
27, respectively (when comparing the unequalized cases or the equalized cases). In both
Figures 28 and 29, bit error performance above approximately -35 dB appears to be
superior to the analytic curve. This is due to the fact that the $E_b/N_0$ estimation algorithm
outlined in Section 3.6 simply separates the received signal into either the TDCS
transmission or noise plus interference, while the use of $N_0$ implies that the PSD of the
noise is flat. When spectral notching is used, an interfering signal is avoided by the
TDCS user pair, thus that portion of the spectrum is avoided. The noise plus interference
content in that area of the spectrum should not be weighed as heavily as in parts of the
spectrum that are used by the TDCS user pair, in order to obtain true SINR measurements
or $E_b/N_0$ calculations at the receiver. The $E_b/N_0$ estimation algorithm, which yields $E_b/N_0$
estimates observed at the position immediately after signals are output from the receive-
side microphone, does not weigh parts of the spectrum that are disregarded by the TDCS
user pair differently from parts of the spectrum that are used in TDCS signaling. In other
words, the $E_b/N_0$ estimation algorithm simply determines the SNR using the signal
content from all parts of the available spectrum. While the $E_b/N_0$ estimation methodology
is admittedly flawed, it does allow for comparisons between cases when spectral notching
is not used and when spectral notching is used. Comparing Figures 26 and 28 with
Figures 27 and 29, it can clearly be seen that the bit error performance using spectral notching is superior to the bit error performance when spectral notching is not used, in both the unequalized and equalized cases. As discussed in Section 2.1, Nunez showed that an appreciable improvement in bit error performance is expected in cases where
spectral notching is applied, versus cases when spectral notching is not applied, in the presence of narrowband interference. The results obtained experimentally illustrate the same phenomenon.

Nunez's results demonstrate that TD-QPSK mitigates interference roughly equivalently in an AWGN channel to spectrally unencoded PSK signaling with no interference present [6]. Applying the spectral notching algorithm to experiments that use TD-QPSK is expected to yield the same result. Nunez's works also yielded the finding that, in the presence of narrowband interference and at bit error rates of less than $10^{-3}$, using spectral notching yielded a gain in $E_b/N_0$ of greater than 1 dB, and that at bit error rates of less than $10^{-2}$, there should be an appreciable improvement in bit error performance [6].

5.4. Spectral Notching with No Interference Added to Environment

Figure 30 illustrates the unequalized bit error performance of the hardware realization when spectral notching is applied and when there is no deliberately generated narrowband interference present in the environment. Figure 31 illustrates the equalized bit error performance using the same realizations of received TDCS signals as those used in generating Figure 30.

The degradation in bit error performance due to spectral estimation mismatch is discussed in Section 4.3. This scenario has been developed to empirically evaluate this possible degradation. The simulated scenario in Section 4.3 assumed completely random spectral mismatches, while in the experiments used to generate the results illustrated in Figures 30 and 31, the microphones used by the transmitter and receiver for spectrum
Figure 30. *Unequalized* Bit Error Performance Curve of Hardware Realization using Spectral Notching in an Environment *Without* Additional Narrowband Interference

Figure 31. *Equalized* Bit Error Performance Curve of Hardware Realization using Spectral Notching in an Environment *Without* Additional Narrowband Interference

estimation are located closely to each other (less than 20 cm apart). Since the microphones are close to each other, and the number of frequency subcarriers that can be notched out is limited to 16 (including DC and the Nyquist frequency), the spectrum
estimation mismatch obviously has less of an impact in these experiments than that observed in Section 4.3. The results illustrated in Figures 30 and 31 resemble the results in Figures 24 and 25, respectively, in the case where no interference is present, and no spectral notching is used.

5.5. Comparing Unequalized and Equalized Bit Error Performance Results

Figure 32 through 35 illustrate both unequalized and equalized bit error performance results for all four scenarios discussed in Sections 5.1 through 5.4. There are no new results presented in these four figures, as the intention is simply to demonstrate the impact of the MMSE equalizer on bit error performance.

5.6. Summary

Figure 24 indicates that, under the assumption that the $E_b/N_0$ estimates used to generate this figure are correct, the TDCS architecture is properly implemented. A comparison between Figures 26 and 28 demonstrates that the spectral notching algorithm implemented in the TDCS architecture mitigates the effects of at least narrowband interference. Figures 26, 27, 29, and 30 all demonstrate that; at an $E_b/N_0$ above approximately -15 dB; an equalizer will improve bit error performance; when compared with Figures 24, 26, 28, and 30, respectively. Given the results of the experiments conducted in this research, it is concluded that the TDCS architecture can indeed be implemented in hardware successfully. The spectral notching algorithm used to provide interference avoidance functionality to TDCS signaling mitigates the effects of at least narrowband interference.
Figure 32. Comparison of Unequalized and Equalized Bit Error Performance in Scenario with No Spectral Notching and No Narrowband Interference Added to the Environment

Figure 33. Comparison of Unequalized and Equalized Bit Error Performance in Scenario with No Spectral Notching and with Additional Narrowband Interference
Figure 34. Comparison of Unequalized and Equalized Bit Error Performance in Scenario with Spectral Notching and with Additional Narrowband Interference

Figure 35. Comparison of Unequalized and Equalized Bit Error Performance in Scenario with Spectral Notching and No Narrowband Interference Added to the Environment
VI. Conclusion

Given experimental results obtained in this research, it is concluded that the TDCS architecture can indeed be implemented successfully in hardware. The spectral notching algorithm, as used to provide interference avoidance functionality to TDCS signaling, effectively mitigates the effects of narrowband interference (the only interference considered for this work).

There were three objectives for this research effort. The first of these objectives, to compare the bit error performance of a TDCS hardware implementation with simulated results from literature, has been met. This comparison has been completed, and results provided in Chapter V validate the simulated results from literature. The second goal, assessing the capability of a DSP in hosting TDCS transmit/receive functionality, has been met. The DSK used for transmission and reception is indeed capable of hosting the TDCS design. The third goal, examining the utility of applying a method of waveform sculpting other than spectral notching, has been met. This comparison has been completed and results presented in Chapter IV suggest that spectral notching may be as effective as water-filling in providing interference avoidance functionality. Furthermore, spectral notching does not require the computational overhead of water-filling. This result appears to validate incorporation of the SMSE frequency “use” vector, as expressed in (15), to mitigate interference by assigning a one or a zero in all elements of the vector. Collectively, the goals of this research have been successfully met; the TDCS concept can indeed be successfully implemented in hardware. While the system demonstrated under this research is admittedly a scaled version of what would be fielded for practical
applications, none of the results obtained preclude TDCS from future implementation in a wireless communications link.

6.1. Recommendations for Further Research

Executing this research effort led to the uncovering of several areas where further study is recommended. These areas are as follows:

1. The implementation of SMSE+ signaling, as discussed by Roberts [11], onboard a DSP-based hardware platform.

2. The implementation of a wavelet-based spectrum estimation algorithm in hardware. A WDCS could be implemented as a standalone realization, or wavelet-domain spectrum estimation could be integrated into SMSE+ signaling and then implemented.

3. The implementation of a hardware realization in a multiple access network.

4. The use of higher-order PSK constellations when implementing TDCS or SMSE+ in hardware.

5. The implementation of real-time SMSE+ or TDCS processing in hardware, to include the embedding of acquisition and fast equalizer algorithms.

6. Much of the DSK firmware uses floating-point calculations. These are more computationally expensive than fixed-point calculations, and are probably unnecessary. Conversion to fixed-point processing is recommended to better optimize the cognitive user pair. An example of conversion techniques for use with trigonometric functions like sine and cosine are described in literature [32], and are recommended for further study.
Bibliography


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Vita

Captain Marshall E. Haker graduated from Washington High School in Vinton, Iowa in 1993. He enlisted in the United States Air Force soon after graduation from high school, and served as an electronic signals intelligence analyst at the National Air and Space Intelligence Center (NASIC) at Wright-Patterson AFB, Ohio. While assigned to NASIC, he was deployed to Saudi Arabia in 1997 in support of Operation SOUTHERN WATCH and to Europe in 1999 in support of Operation ALLIED FORCE.

After being named to an education and commissioning program in 1999, he pursued undergraduate studies at The Ohio State University in Columbus, Ohio where he graduated with honors with a Bachelor of Science in Electrical and Computer Engineering in August 2002. He was subsequently commissioned through the USAF Officer Training School at Maxwell AFB, Alabama in December 2002.

His first assignment upon commissioning was to the Oklahoma City Air Logistics Center (OC-ALC) at Tinker AFB, Oklahoma. More specifically, he served in the 827th Aircraft Sustainment Group as an electrical systems engineer, where he led the engineering effort on multiple communications and situational awareness avionics upgrade and sustainment efforts onboard the KC-135 airborne refueler. In April 2004, he was moved within OC-ALC to the 76th Commodities Maintenance Group where he served as the lead electrical engineer tasked with the sustainment and upgrading of the B-52, C/KC-135, and E-3 radome electrical test range at Tinker AFB. The success of the four person engineering team charged with ensuring the continued operation of the test range, an effort in which he personally designed the modernized radio frequency signal processing architecture, led to the team being awarded the 2006 U.S. Air Force Chief of Staff Team Excellence Award.

In August 2005, he entered the Graduate School of Engineering and Management, Air Force Institute of Technology (AFIT). Upon completion of his Masters Degree at AFIT, Captain Haker will be assigned to the Air Force Research Laboratory Sensors Directorate, where he will continue his research into cognitive and software-defined wireless communications and non-communications methods.
Hardware Realization of a Transform Domain Communication System

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The purpose of this research was to implement a Transform Domain Communication System (TDCS) in hardware and compare experimental bit error performance with results published in literature. The intent is to demonstrate the effectiveness or ineffectiveness of a TDCS in communicating binary data across a real channel. In this case, an acoustic channel that is laden with narrowband interference was considered. A TDCS user pair was constructed to validate the proposed design using Matlab to control a PC sound card. The proposed TDCS design used the Bartlett method of spectrum estimation, the spectral notching algorithm found in TDCS literature, quadrature phase shift keying, and minimum mean square error transverse equalization to mitigate the effects of noise and intersymbol interference. Water-filling was evaluated as an alternative to spectral notching for performing waveform design and is shown to perform equivalently. Validated software was migrated to code suitable for use onboard a Digital Signal Processor Starter Kit (DSK). Two DSK boards were used, one for transmission and reception, and bit error performance results were obtained. Bit error analysis reveals that the TDCS hardware performs approximately the same as literature suggests.


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