NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

PROBABILITY MODELING OF MULTI-TYPE AUTONOMOUS UNMANNED COMBAT AERIAL VEHICLES ENGAGING NON-HOMOGENEOUS TARGETS UNDER IMPERFECT INFORMATION

by

Themistoklis Papadopoulos

March 2007

Thesis Advisor: Moshe Kress
Second Reader: Kyle Lin

THIS PAGE INTENTIONALLY LEFT BLANK

REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.

1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** March 2007	**3. REPORT TYPE AND DATES COVERED** Master's Thesis
4. TITLE AND SUBTITLE Probability Modeling of Multi-Type Autonomous Unmanned Combat Aerial Vehicles Engaging Non-Homogeneous Targets Under Imperfect Information		**5. FUNDING NUMBERS**
6. AUTHOR(S) Themistoklis Papadopoulos		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000		**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A		**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited	**12b. DISTRIBUTION CODE**

13. ABSTRACT (maximum 200 words)

UCAVs are advanced weapon systems that can loiter autonomously in a pack over a target area, detect and acquire the targets, and then attack them. Modeling these capabilities in a specific hostile operational setting is necessary for addressing weapons' design and operational issues. While much attention has been given to the engineering and technological aspects of UCAV developments, there are very few studies on operational concepts for these weapon systems and their effectiveness and efficiency. This thesis builds probability models (Markov Chains) that describe UCAV operations, defines Measures of Effectiveness (MOEs) for the engagement performance, maps the functional relations between the parameters and the MOEs, and obtains insights regarding the design of the UCAVs and their tactical employment. The models are used to conduct extensive numerical analysis, based on experimental design concepts and traditional sensitivity analysis. The main focus of the analysis is to investigate optimal and robust mixes of UCAVs of different types, with respect to the MOEs. While in most cases, extreme-point solutions are optimal, there are cases where a balanced UCAV mix is better.

14. SUBJECT TERMS Autonomous UCAV, Analytic Probability Modeling, Markov Chain, States, Transitions, Terminating Situation, Feasibility, DOE, Sensitivity Analysis, MOE, Sensitivity, Specificity, Visual Obstruction, Loiter-Search-Detect-Acquire-Strike.	**15. NUMBER OF PAGES** 99
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	**18. SECURITY CLASSIFICATION OF THIS PAGE**	**19. SECURITY CLASSIFICATION OF ABSTRACT**	**20. LIMITATION OF ABSTRACT**
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

THIS PAGE INTENTIONALLY LEFT BLANK

Approved for public release; distribution is unlimited

PROBABILITY MODELING OF MULTI-TYPE AUTONOMOUS UNMANNED COMBAT AERIAL VEHICLES ENGAGING NON-HOMOGENEOUS TARGETS UNDER IMPERFECT INFORMATION

Themistoklis Papadopoulos
Captain, Greek Air Force
Electrical Engineering, Greek Air Force Academy, 1994

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MODELING, VIRTUAL ENVIRONMENTS AND SIMULATION (MOVES)

from the

NAVAL POSTGRADUATE SCHOOL
March 2007

Author: Themistoklis Papadopoulos

Approved by: Moshe Kress
Thesis Advisor

Kyle Lin
Second Reader

Rudolph P. Darken
Chair, MOVES Academic Committee

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

UCAVs are advanced weapon systems that can loiter autonomously in a pack over a target area, detect and acquire the targets, and then attack them. Modeling these capabilities in a specific hostile operational setting is necessary for addressing weapons' design and operational issues. While much attention has been given to the engineering and technological aspects of UCAV developments, there are very few studies on operational concepts for these weapon systems and their effectiveness and efficiency. This thesis builds probability models (Markov Chains) that describe UCAV operations, defines Measures of Effectiveness (MOEs) for the engagement performance, maps the functional relations between the parameters and the MOEs, and obtains insights regarding the design of the UCAVs and their tactical employment. The models are used to conduct extensive numerical analysis, based on experimental design concepts and traditional sensitivity analysis. The main focus of the analysis is to investigate optimal and robust mixes of UCAVs of different types, with respect to the MOEs. While in most cases, extreme-point solutions are optimal, there are cases where a balanced UCAV mix is better.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF FIGURES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

ACRONYMS, ABBREVIATIONS AND SYMBOLS

A. LIST OF ACRONYMS AND ABBREVIATIONS

C2	Command and Control
CAB	Combined Arms Battalion
CTMC	Continuous Time Markov Chain
DoD	Department of Defense
FCS	Future Combat Systems
FoS	Family of Systems
GPS	Global Positioning System
INS	Inertial Navigation System
MOE	Measure Of Effectiveness
NOLH	Nearly Orthogonal Latin Hypercube
NVT	Non-Valuable Target
OR	Operations Research
RSTA	Reconnaissance, Surveillance, Target Acquisition
UA	Unit of Action
UAV	Unmanned Aerial Vehicle
UCAV	Unmanned Combat Aerial Vehicle
VT	Valuable Target

B. LIST OF SYMBOLS

λ_i	detection rate of type-I UCAV, i = A, B
θ_i	failure rate of type-I UCAV, i = A, B
r_i	probability that a type-i UCAV recognizes a NVT as such, i = A, B
q_{ij}	probability that a type-i UCAV acquires a type-j target, given a detection of such a target, i = A, B, and j = 1, 2
p_{ij}	probability that a type-i UCAV kills a type-j target, given an acquisition of such a target, i = A, B, and j = 1, 2
T_{Vi}	the military (operational) value of a type-i target, i = 1, 2
T_i	initial number of type-i targets, i = 1, 2, 3
N_i	initial number of type-i UCAV's, i = A, B
n_i	current number of type-i UCAV's, i = A, B
t_j	current number of type-j targets, j = 1, 2
E_v	expected value of killed targets
Time	expected length of the engagement

ACKNOWLEDGMENTS

I would like to begin by thanking the Lord for watching over me, and providing me with courage and perseverance to successfully complete my studies.

I would also like to thank my great thesis team, Professor Moshe Kress and Professor Kyle Lin for their guidance and constructive comments. One additional faculty member here at NPS contributed to overcoming obstacles along the way in the analysis, to whom, Professor Suzan Sanchez, I want to say thank you.

Finally, I would like to thank my wife Vasiliki for her unfailing support and love.

THIS PAGE INTENTIONALLY LEFT BLANK

I. INTRODUCTION

A. FOREWORD

In this chapter we present some aspects of Unmanned Aerial Vehicles (UAVs) - in particular Unmanned Combat Aerial Vehicles (UCAVs) – and describe the problem addressed in this thesis. We also discuss the methods and techniques used for solving the problem and for making useful inference from the results. Finally, we outline the thesis chapters.

B. UNMANNED (COMBAT) AERIAL VEHICLES

UAVs are mobile airborne machines that do not require an on-board human operator. Typically they are controlled by a remote operator or autonomous control logic (Corner and Lamont, 2004).

The Department of Defense (DoD) defines UAVs as “powered, aerial vehicles that do not carry a human operator, use aerodynamic forces to provide vehicle lift, can fly autonomously or be piloted remotely, can be expendable or recoverable, and can carry a lethal or non-lethal payload.” (Bone and Bolkcom, 2003).

UAVs are a critical part of (future) armed forces, that consists of highly mobile and network enabled systems with integrated sensors and precision munitions. UAVs either provide eyes on the battlefield that trigger the deployment of precision munitions by other platforms, or engage targets themselves (UCAVs) (Sulewski, 2005). In addition to triggering the deployment of precision munitions, and providing situational awareness of the engagement area, UAVs assist in all communication aspects throughout the theater of operations.

The increasingly important role of UAVs in warfare is demonstrated by the U.S. Army’s resolution to have these systems at the core of its FCS FoS (Future Combat Systems Family of Systems). FCS UAVs are broken down into four classes according to their capabilities. Class I UAVs provide RSTA (Reconnaissance, Surveillance, Target Acquisition) capabilities at the platoon level. Class II UAVs provide RSTA capabilities and target designation at the platoon and company level. Class III UAVs provide RSTA capabilities, target designation, communication relay, and mine detection at the CAB (Combined Arms Battalion) level, while class IV UAVs provide similar capabilities at the UA (Unit of Action) level (Sulewski, 2005).

The future military force will be complex: a highly integrated mix of manned and unmanned units. These unmanned units could function individually or within a swarm. The addition of unmanned units will decrease the danger that soldiers face in direct combat. The tendency is to either have a single operator controlling a swarm of UAVs, or to let them operate autonomously with no human supervision. The ability to use autonomous vehicles to perform wartime mission is an important application in future military operations. Technology in the UAV arena is also moving toward smaller and more capable systems.

One of the initial motivations that served as impetus for developing UAVs was that UAVs would be inexpensive. They could be launched into high risk missions without risking a costly manned aircraft and the lives of its crew. Of course as the UAVs continuously grow in complexity and utility, they also increase in cost, and therefore it becomes more crucial for them to be highly combat effective (McMindes, 2005). The effectiveness of UAVs in battle depends on many factors, some of which are addressed in this thesis. Exploring these factors may let us better understand what design characteristics or operational decisions would lead to a more effective (and cost-efficient) use of UAVs.

The use of UCAVs removes the risk of aircrew being killed, injured or captured if the vehicle is shot down or lost due to mechanical failure. Airframe designs can be smaller and lighter than their manned counterparts and can be designed for longer endurance. Also, UCAV platforms are cheaper to buy and operate, and require less expensive testing and training. These might be among the main advantages in future planning (Baggesen, 2005).

Modern UCAVs are navigated and guided by radar, video, infrared cameras, lasers, and Inertial Navigation Systems (INS) and aided by the satellite based Global Positioning System (GPS). The enhancement of sensor systems, processor units, decision making algorithms, and terminal seekers leads to autonomy for target acquisition, recognition, and attack. These capabilities, combined with inexpensive designs and operational opportunities, make UCAVs a disruptive technology on the battlefield. UCAVs enable war fighters to attack targets with weapon systems that can operate in highly defended areas, and cause less collateral damage, due to enhanced precision. There no longer seems to be a trade-off between own casualties and the effect of attacks. This is especially important for a society that is perceived as being less and less tolerant of high-casualty engagements and collateral damage (Baggesen 2005).

Experts worry that the more abstract the use of weapon systems becomes, the more abstract the enemy becomes, and as humans recede from the battlefield as combatants, war will become more likely, not less.

Autonomous technology is still not completely unleashed. For the near future, however, UCAV developers believe that the man-in-the-loop will be the weakest part of the weapon system because humans will be too slow for the decision-making cycle, causing underperformance and collateral damage (Baggesen 2005).

C. PROBLEM STATEMENT

UCAVs are advanced weapon systems that can loiter autonomously in a pack over a target area, detect and acquire the targets, and then engage them. Modeling these capabilities in a specific hostile operational setting is necessary for addressing weapons' design and operational issues.

While much attention is given to the engineering and technological aspects of UCAV developments, there are very few studies on operational concepts for these weapon systems and their effectiveness and efficiency. The wide range of design and operational factors and capabilities of such autonomously acting and interacting weapons will most likely lead to a wide range of engagement performance in various scenarios.

In the present thesis we consider a combat situation involving two types of UCAVs against two types of passive ground targets and we seek answers to the following issues:

- The effect of UCAV design and operational parameters on the end state of the engagement in the presence of imperfect situational awareness.
- Suitable Measures Of Effectiveness (MOEs) for measuring the effect of UCAV design and tactics.
- Sensitivity of the values of the MOE's to the UCAV parameters.
- Given the capabilities of the various types of UCAVs, the best mix of these UCAV's that optimizes certain operational goals.

D. TECHNIQUES AND METHODS

In order to gain insight about real combat situations, we need to first model it using appropriate techniques and then analyze it in the hope of deriving useful conclusions that will help decision makers make better choices when it comes to selecting parameter settings of the weapon systems or developing scenarios and deployment tactics.

For modeling the combat situation addressed in this thesis, a continuous time Markov chain (CTMC) model is developed. This model represents all the UCAV design and environmental parameters and also contains temporal information (i.e., the expected duration of a UCAV operation). The basic idea behind the CTMC is that at each moment in time the system can be described by the state it is in. The state, in general, contains information about the number of UCAVs and targets alive. All of the parameter information is kept in the model and is used in order to calculate the next state and the elapsed time. The outcomes resulting from the model are used to address the issues described above in Section C. In this thesis, two MOEs are calculated, the expected relative effectiveness and the expected time of the operation, both described in Chapter III. The parameter value settings along with their respective MOE outputs are then analyzed according to two techniques. The first one is the

NOLH DOE (Nearly Orthogonal Latin Hypercube Design of Experiments). This method allows for an exploration as broad as the analyst deems necessary, and also allows for exposing potential interactions among the various factors. One product of this type of analysis is a regression equation that can be used for its explanatory power but also as a quick substitute for the Markov model. The second technique is the traditional sensitivity analysis that helps magnify the effects of particular factors when everything else is kept constant. These two techniques complement each other and provide a comprehensive view of the combat situation we are modeling. The statistical package JMP is used for most of the DOE and analysis part, and Excel is used for generating and formatting the various plots during the analysis.

E. THESIS FLOW

In the next chapter we discuss previous UCAV related models that employed different approaches and conveyed certain takeaways. Also, we expand on the Markov chain and DOE concepts. Chapter III describes the combat situation and develops the basic Markov model. It also gives a thorough discussion of the analysis performed using various methods, as well as conclusions and recommendations. Supporting documentation on the programming code is contained in the Appendix.

THIS PAGE INTENTIONALLY LEFT BLANK

II. BACKGROUND – LITERATURE REVIEW

In this chapter we discuss previous Operations Research (OR) models related to UCAVs, continuous time Markov chains, and basic concepts of experimental design related to the analysis presented later on.

A. PREVIOUS UCAV MODELS

Progress on the various technologies of UCAVs is promising, but there are insufficient analytical tools for evaluating the effectiveness of these weapon systems in operational settings. Most of the research on modeling UCAV operations relies on simulation and not on analytic modeling. In (Jacques, 2002) the author presents some basic analytic results on the single UCAV/single target and general multi-UCAV/multi-target cases. It is shown there, that analytically it becomes intractable to develop a mathematical formulation for arbitrary numbers of munitions executing arbitrarily specified search patterns. These are the cases, however, that are most interesting operationally, and the most practical way of performing this more general analysis is by a numerical simulation.

Some work has been done on investigating the possibility of having UCAVs share information and act in a cooperative fashion. Cooperative behavior is being investigated to improve the overall mission effectiveness. The general problem addressed is typically how to best find and engage an unknown number of targets in unknown locations using multiple UCAVs. In (Frelinger, et al, 1998) it is stated that while an individual UCAV may be less capable than conventional munitions, through communication across the swarm of weapons, the group may exhibit behaviors and capabilities that can exceed those of more conventional systems that do not employ communication between weapons. The potential benefits, which come about through shared knowledge include relaxed sensor performance requirements, robustness to increases in target location errors, and adaptivity to attrition and poor target characterization. In (Gillen and Jacques, 2002) an attempt is made to emulate the behavior of UCAVs via simulation, and measure their overall expected performance. One extension to the approach described in (Frelinger, et al, 1998) is taking into account the degradation due to false target attacks. The simulation allows for any number of targets with varying priority levels, as well as non-targets (military or civilian), and it is very flexible in its capabilities to handle a multitude of input parameters and supply multiple outputs, such as total hits or total kills. During this simulation, the UCAVs employ a decision algorithm. It is shown that the selection of the optimal weights of the factors in this decision algorithm are very sensitive to almost all battlefield characteristics, therefore producing no robust conclusions, apart from the fact that cooperative engagement alone is not able to compensate for higher false target attack rates.

In (Kress et al, 2006), several analytic probability models, which range from a simple regenerative formula to a large-scale continuous-time Markov chain, are developed, with the objective to address design and operational issues of UCAVs operating in hostile environments. The focus is on autonomous UCAVs, which are designed to operate as a pack of vehicles that autonomously search, detect, acquire and attack targets. The main idea is that while target detection and recognition capabilities, and weapon accuracy and lethality determine the effectiveness of a single UCAV, two phenomena may affect the performance of the UCAVs as a pack – multiple acquisitions and multiple kills. Also, the impact of memory on the acquisition capabilities of a UCAV is studied in this paper. It is shown that, under reasonable assumptions, memory is a rather redundant design feature in UCAVs, unless we consider time-critical missions. Some other takeaways are that detection rate is a major factor in determining the operation length, that attack coordination among UCAVs is not significant (at least for certain examined scenarios), and that UCAV sensor specificity is more important than sensitivity. The models described in the paper above are limited to homogeneous targets and homogeneous UCAVs. The present thesis extends these concepts to multi-type UCAVs and non-homogeneous targets.

B. CONTINUOUS TIME MARKOV CHAINS

A Continuous Time Markov Chain (CTMC) is used for modeling a probabilistically evolving situation where time is continuous and the time periods between changes in the situation follow an exponential distribution. The situation is fully described by the state it is in. Each state can transition to another feasible state. The selection of the next state is the result of an 'exponential race', that is the next state is determined by the event that happens first, where the time until the next event follows an exponential distribution. Each state can be either transient or absorbing. The model keeps on running as long as the visited states are transient. When an absorbing state is reached, the process stops. The core of our UCAV model is the CTMC 'engine', around which other functions can be built. The CTMC model calculates the values of the various MOEs, as well as the expected duration of the process. The underlying assumption, though, is that the rates of events follow a Poisson distribution. This is not a long stretch, if we consider that the behavior of many actual systems approximately follows this distribution. The exponential assumption, and the consequent ability to use a CTMC gives the analyst an enormous analytical and computational advantage since the math would be very difficult or intractable otherwise.

C. ASPECTS OF EXPERIMENTAL DESIGNS AND DATA ANALYSIS

In this thesis we implement two types of model analysis: an experimental design and a single-factor sensitivity analysis with respect to a base case. A good Design of Experiments (DOE) allows for simultaneously assessing the impact of more than one factor, and identifying potential interactions among the factors. On the other hand, single-factor sensitivity analysis, when all other factors remain fixed at their base-case values, may better reveal the effect of certain parameters in the neighborhood of a realistic base case.

The primary objectives of computer experiments, according to (Sacks et al, 1989) are: predicting the response at untried inputs, optimizing a function of the input factors, or calibrating the computer code to physical data. A more modern approach (Sanchez, 2001) contends that the appropriate objectives should be: developing a basic understanding of a particular system, finding robust decisions, tactics, or strategies, or comparing the merits of various decisions.

There are several DOE structures to choose from. In this thesis we employ Latin Hypercubes and more specifically Nearly-Orthogonal Latin Hypercubes (NOLH) for various reasons discussed in the next section.

1. Latin Hypercubes

The challenge in conducting analysis is in the curse of dimensionality. In general we need L x F design points where F is the number of factors and L is the number of levels of each factor, in order to cover all the possible combinations. This is known as a full factorial design. As we raise the number of factors and desired levels to accommodate the idea of data farming the number of design points quickly gets out of hand.

A NOLH DOE addresses how to sample the design space without looking at all possible combinations. It is beyond the scope of this thesis to explain in detail how and why this works, but we can imagine the NOLH DOE as selecting interior points from the parameter space additionally to the corner points that a factorial would select. Those interior points are selected such that the correlation between factor levels is very low, so that we get a much more complete picture of the landscape from which we are sampling. The low correlation and the large number of design points allow the analysis of both main effects and interactions between factors without sampling at all combinations of levels of each factor. By the application of data farming and NOLH a very broad parameter space can be explored and robust solutions can be found. A robust solution may not be the optimal choice for any given set of parameters, but is a good overall choice given a variety of possibilities (McMindes 2005 and Cioppa 2002) .

NOLH is a very good all-purpose design, particularly when all or most of the factors are quantitative. It is apparently efficient, it has excellent space-filling properties, and it adds flexibility by imposing fewer restrictions on the number of the factors and their levels. Also, it allows us to fit many different types of complex metamodels to multiple MOEs.

THIS PAGE INTENTIONALLY LEFT BLANK

III. THE BASIC MODEL

A. THE COMBAT SITUATION AND ITS MODELING OBJECTIVES

1. Detailed Description of the Combat Situation

The basic combat situation modeled in this thesis is that of a swarm of UCAVs loitering over an area of interest, looking for operationally valuable ground targets, acquiring them, and finally attacking them.

There are two types of UCAVs, A and B, and two types of Valuable Targets (VTs), 1 and 2, in the target area. Also, there are other types of targets of no operational value called type-3 targets. The latter targets, along with all killed VTs of any type, are collectively referred to as Non-Valuable Targets (NVTs).

The type-1 and type-2 targets can be anything valuable to the enemy; for example type-1 targets could be C2 (Command and Control) vehicles, and type-2 targets could be soft-skin military trucks. A mix of UCAVs of both types A and B, attack the target area.

Throughout we assume that the total number of attacking UCAVs is 16 which typically corresponds to two squadrons. The number of targets, of both types, varies according to the scenario that is considered. Nonetheless the number of targets of each type is confined within a certain range.

Each UCAV is disposable (as opposed to retrievable), meaning that the weapon is an integral part of the aerial vehicle; the UCAV searches, detects, and acquires a target, and finally attacks it. Also, once launched, a UCAV cannot return to base, so eventually, if not used, it is wasted.

The UCAVs loiter over the target area and search for targets independently, in a random search pattern. We assume that a UCAV has no memory of any previous detection, but it has the capability to recognize an NVT, although this capability is imperfect; there is some probability for erroneous classifications. Also, there is continuous communication among the UCAVs that leads to perfect coordination, in the sense that simultaneous multiple attacks on a single target do not occur.

The targets are passive in the sense that they do not fight back. So, there is no UCAV attrition due to fire by the targets. The combat situation we are considering has a limited time length which is in all cases small enough compared to the fuel capacity of the UCAVs. Therefore, the UCAVs never run the risk of being wasted due to fuel shortage. Because there is no reinforcement of UCAVs, this combat situation reaches an absorbing state, when all the UCAVs have been disposed.

In summary, each UCAV loiters above the area of interest, until it detects a target which it tries to identify. If the target is identified as a NVT, the UCAV takes no action and keeps on loitering. If the target is identified as a VT, the UCAV promptly acquires and attacks it. The identification may or may not be correct. The attack may or may not be successful. In addition to unnecessary or unsuccessful attacks, a UCAV can also be wasted due to a mechanical failure.

2. The Modeling Objectives and MOEs

a. Objectives

The combat situation involves several parameters that describe the UCAVs' performance (e.g., kill probabilities), and the operational environment (e.g. the number of targets in the area of interest). The values of these parameters are not always known accurately and with certainty. For example, the mission planners cannot be absolutely positive about the kill probabilities of a type-B UCAV, but they may have an idea about the range of possible values for those parameters. In the same manner, the planners cannot accurately predict the number of type-1 targets the enemy will employ, but they might well be able to provide a valid range for it. These issues are discussed in further detail in the experimental design sections later on.

The objective is to build a stochastic model of the combat situation described above, and implement it in order to gain insights and produce useful takeaways on design aspects of the UCAVs as well as on operational issues. We use two different approaches for the analysis. The first approach is by means of a Design Of Experiments (DOE), and the second one is by defining a reasonable base case for all the parameter values and performing sensitivity analysis for each parameter separately.

The main focus of the analysis is to investigate optimal UCAV mixes with respect to appropriate Measures Of Effectiveness (MOEs) defined below, and under uncertainty regarding the values of the parameters.

b. Measures of Effectiveness

In order to measure the mission success, we define the following MOEs:

- Expected value of killed targets
  Assuming that type-1 and type-2 targets have operational values assigned to them, this is the total accumulated expected value of all the killed targets.
- Expected relative effectiveness
  This is the ratio of the expected total number of killed VTs, over the initial total number of VTs.
- Probability to exceed an operational threshold
  This is the probability that the total number of killed VTs, is at least a given percentage of their initial number.
- Expected time length of the engagement
  This is the expected total duration of the engagement – until all UCAVs are disposed.

The analysis to follow will focus on the first and fourth MOEs. The first MOE measures the effectiveness of the UCAVs; the last one measures their time-efficiency, which provides insight about the possible success of time critical missions. Specifically, it might be the case that an otherwise optimal UCAV mission mix, is not optimal when time is of essence.

B. DESCRIPTION OF THE MODEL

1. Markov Chain

The stochastic model for the combat situation described above is a continuous time Markov chain. A step in this chain is defined as either a detection event by a UCAV or a UCAV failure. A state in this model is defined by the number of UCAVs (of each type) alive and the number of targets (of each type) still alive. A transition from one state to the next is a result of one of seven possible events:

- A type-A UCAV kills a type-1 target
- A Type-A UCAV kills a type-2 target
- A Type-B UCAV kills a type-1 target
- A Type-B UCAV kills a type-2 target
- A Type-A UCAV fails, or misses a VT, or is wasted on a NVT
- A Type-B UCAV fails, or misses a VT, or is wasted on a NVT
- A detection is recorded but none of the six events mentioned above occurs; the state remains unchanged

2. Assumptions

We make the following assumptions:

a. Imperfect Recognition

The sensor of each type of UCAV can identify the status of the targets (VT or NVT), but the identification is not perfect; it is accurate with a given probability, which depends on the UCAV type.

b. Communication – Coordination and Zero Attack Time

There is communication among the UCAVs that leads to perfect coordination, in the sense that there are no simultaneous multiple attacks on a single target. Also, because the attack time is short compared to the detection time and, as mentioned above, there are no simultaneous multiple attacks on the same target, we assume that the attack time is zero.

c. Fixed Total Number of UCAVs

The total number of UCAVs is 16. This way we have a convenient (and rational) upper limit on the model complexity, and by determining the number of type-A UCAVs we also determine the number of type-B ones.

d. Unlimited Endurance

UCAVs are assumed to have enough fuel for the purposes of their mission, and therefore they can never crash due to fuel shortage.

e. Disposable UCAVs

As mentioned in the model description, the UCAVs are disposable, and they never run out of fuel. So, even if all the VTs are destroyed, the remaining UCAVs, if any left, keep loitering, until they either mistakenly engage an NVT or crash due to mechanical failure. This can potentially distort the values of our MOE results because the mission duration may be artificially extended, after all the targets are killed. But, this could only be a problem in the event where there is a significant probability that all the VTs are destroyed. As we see later on, this is a very rare event in our analysis. To make this point clearer, consider this example: there are only two targets, and presumably six UCAVs are sufficient for destroying them all. In that case, if we were to employ more than six UCAVs, the duration of the mission would be much longer since the redundant UCAVs would loiter until being disposed, and the Ev would be the same although at a higher UCAV expense. But again, in the DOEs employed in the analysis, this situation is very rare.

On the other hand, note that, if our definition of absorbing states changes to also include the condition $t_1 + t_2 = 0$, this potential distortion of MOE values mentioned above would never be the case, while at the same time, we would no longer have to assume that the UCAVs are disposable. Of course, for that to be true, the target recognition has to be perfect.

f. Passive Targets

The targets do not shoot back. So, there is no UCAV attrition due to enemy fire. This assumption implies that the number of VTs does not affect the failure rate of the UCAVs. Nonetheless, we can always incorporate the effects of an existing air defense into the aggregated UCAV failure parameters.

g. Exponential Detection Rate

The detections follow a Poisson process; therefore the inter-detection times are exponential random variables, of which we need only define the mean. This is a reasonable assumption, because we can think of targets as forming a Poisson field. These targets move randomly in the target area, so that they are always spatially distributed according to a spatial Poisson distribution.

h. Exponential Failure Rate

Mechanical failures follow a Poisson process with a mean that can be estimated from statistical data obtained from controlled experiments. In the presence of enemy air defense, this failure rate could also incorporate the effects of UCAV attrition by the air defense.

i. Fixed Probabilities

All the probabilities in the model are fixed numbers that are independent of the time and state of the operation.

j. No Partial Damage

Damage is not accumulated on a target. If a target has not been destroyed during an attack, it is considered as good as new.

k. Repeated, Random, Independent, Memory-Less Search

UCAVs search independently, in a random search pattern, and they have no memory regarding previous detections. Since we want to model static and moving targets, this assumption is necessary, to avoid implementing a complex tracking algorithm.

3. Notation

a. Detection Exponential Rates

λ_i: the detection rate of type-i UCAV, i = A, B.

b. Failure Exponential Rates

θ_i: the failure rate of type-i UCAV, i = A, B.

c. Specificity

r_i: the probability that a type-i UCAV recognizes a NVT as such, i = A, B.

d. Acquisition

q_{ij}: the probability that a type-i UCAV acquires a type-j target, given a detection of such a target, i = A, B, and j = 1, 2.

e. Kill Probabilities

p_{ij}: the probability that a type-i UCAV kills a type-j target, given an acquisition of such a target, i = A, B, and j = 1, 2.

f. Target Values

T_{Vi}: the military (operational) value of a type-i target, i = 1, 2.

g. Initial Number of Targets

T_i: the total initial number of type-i targets, i = 1, 2, 3.

h. Initial Number of UCAVs

N_i: the initial number of type-i UCAVs, i = A, B.

i. State Variables

- n_i: the current number of type-i UCAVs, i = A, B.
- t_j: the current number of type-j targets, j = 1, 2.

4. Mathematical Formulation

a. Transitions

Letting the initial state be (N_A, N_B, T_1, T_2), and the *current* state (n_A, n_B, t_1, t_2), we have seen in section 1 above that there are seven possibilities for the next state (feasibility permitting).

The transition probabilities for these seven cases, along with a short description of the characteristics of the transitions, are shown below:

- Cases 1 - 4: Type-i UCAV kills Type-j target, i = A, B, j = 1, 2.

$$\frac{n_i\lambda_i}{\sum_{k=1}^{2} n_k(\lambda_k+\theta_k)}\frac{t_j p_{ij} q_{ij}}{\sum_{m=1}^{3} T_m}.$$

- Cases 5 and 6: Type-i UCAV fails, or misses a VT, or is wasted on a NVT, i = A, B.

$$\frac{n_i(\theta_i+\lambda_i)}{\sum_{k=1}^{2} n_k(\lambda_k+\theta_k)}\frac{\sum_{k=1}^{2} t_k q_{ik}(1-p_{ik})}{\sum_{k=1}^{3} T_k}+(\sum_{k=1}^{3} T_k-\sum_{k=1}^{2} t_k)(1-r_i)).$$

- <u>Case 7:</u> State remains unchanged, i = A, B.

$$\sum_{i=1}^{2}\frac{n_i\lambda_i}{\sum_{k=1}^{2}n_k(\lambda_k+\theta_k)}\frac{\sum_{k=1}^{2}t_k(1-q_{ik})+(\sum_{i=1}^{3}T_i-\sum_{i=1}^{2}t_i)r_i}{\sum_{k=1}^{3}T_k}.$$

b. Absorbing States

We define a state as absorbing when $n_A + n_B = 0$. Note that, if a state with $t_1 + t_2 = 0$ is also specified as absorbing, and if $r_i = 1$ (i = A, B), we could drop the disposable-UCAV assumption.

c. Feasibility Conditions

The feasibility condition for the states is:

$N_A\text{-}n_A+N_B\text{-}n_B \geq T_1\text{-}t_1+T_2\text{-}t_2$ (or equivalently $T_1+T_2\text{-}N_A\text{-}N_B \leq t_1+t_2\text{-}n_A\text{-}n_B$) and all 'components' of each state must be non-negative. Thus we can discard all states that do not satisfy this condition.

d. MOEs

The two MOEs to be calculated in the analysis are shown below:

(1) Expected value of killed targets:

E_v = E[#type-1 targets killed]T_{v1} + E[#type-2 targets killed]T_{v2}

(2) Time (expected length of the engagement)

Note that both MOEs are derived from the Markov chain model; there are no explicit analytical formulas for their calculation.

C. IMPLEMENTATION OF THE MODEL

1. General

The continuous time Markov chain model and all the subsequent models associated with the analysis are implemented using the Matlab programming environment.

2. Matlab Code Objectives

The code written, accomplishes many tasks:

- Getting input and assigning values to the parameters
- Finding and counting the feasible states
- Distinguishing between transient and absorbing states
- Mapping and populating the transition probability matrix P
- Deriving results from Markov chain theory, with respect to P
- Calculating the MOE values
- Checking for errors during run-time
- Generating output

3. Code

The Matlab code written for this model implementation appears in the Appendix.

D. BROAD EXPERIMENTAL SET-UP AND DATA ANALYSIS

We apply a DOE scheme for building a regression meta-model, and maximizing the information obtained from a given number of runs of the model. The DOE of choice is NOLH for reasons explained in Chapter II. The basic concepts and terminology used herein are also presented in Chapter II.

1. Factors

a. Decision Factors

We are primarily interested in exploring optimal mixes of the two types of UCAVs. Since we assume a total number of 16 UCAVs, $N_B = 16 - N_A$, and therefore we only have one decision factor, N_A.

b. Environmental and Design Factors, and Their Ranges

(1) Environmental (noise) factors

- $T_1 \in [3, 7]$ [discrete]
- $T_2 \in [3, 7]$ [discrete]
- $T_3 \in [0, 28]$ [discrete]
- $T_{v1} = 1$ [fixed]
- $T_{v2} \in [0, 1]$ [continuous]

The absolute expected total value of the engagement is not important and therefore it can be scaled such that T_{v1} is fixed at the value 1, and only T_{v2} varies. By doing that, we reduce the dimensionality of the model by one, thus making the DOE and the subsequent analysis less cluttered.

(2) UCAV-design (noise) factors

- $q_{A1} \in [0.5, 1]$ (probability that a type-A UCAV acquires a type-1 target, given a detection of such a target)
- $q_{A2} \in \left[\frac{q_{B2}}{1.2}, q_{B2}\right]$ (probability that a type-A UCAV acquires a type-2 target, given a detection of such a target)
- $q_{B1} \in \left[\frac{q_{A1}}{1.2}, q_{A1}\right]$ (probability that a type-B UCAV acquires a type-1 target, given a detection of such a target)
- $q_{B2} \in [0.5, 1]$ (probability that a type-B UCAV acquires a type-2 target, given a detection of such a target)

- r_A є [0.5, 1.0] (probability that a type-A UCAV recognizes a NVT as such)
- r_B є [0.5, 1.0] (probability that a type-B UCAV recognizes a NVT as such)
- p_{A1} є [0.4, 1.0] (probability that a type-A UCAV kills a type-1 target, given an acquisition of such a target)
- $p_{A2} \in \left[\frac{p_{B2}}{1.2}, p_{B2}\right]$ (probability that a type-A UCAV kills a type-2 target, given an acquisition of such a target)
- $p_{B1} \in \left[\frac{p_{A1}}{1.2}, p_{A1}\right]$ (probability that a type-B UCAV kills a type-1 target, given an acquisition of such a target)
- p_{B2} є [0.4, 1.0] (probability that a type-B UCAV kills a type-2 target, given an acquisition of such a target)
- λ_A є [0.2, 1] (detection rate of type-A UCAV)
- $\lambda_B \in \left[\frac{\lambda_A}{1.2}, \lambda_A\right]$ (detection rate of type-B UCAV)
- θ_A є [0.005, 0.03] (failure rate of type-A UCAV)
- $\theta_B \in \left[\frac{\theta_A}{1.2}, \theta_A\right]$ (failure rate of type-B UCAV)

Note that some parameters (e.g., q_{A1} and q_{B1}) are correlated. The reason for doing that is to explore whether reducing the dimensionality of the DOE still conveys similar analysis results or not. If so, fewer varying parameters will be examined in any consequent analysis thereon, without any loss of generality.

Also note that restricting, for example q_{B1}, to be 1 to 1.2 times less than q_{A1}, is equivalent to determining that q_{B1} is 83% to 100% of q_{A1}. The same is true for all the other pairs of correlated parameters. We prefer showing the 1.2 ratio factor instead of a percentage because in the next section we use this factor for narrowing down the DOE (we call it a handicap).

Absent hard data, the ranges for the factor values could only be educated guesses, based on the existing literature. However, we intentionally made these ranges quite broad, because the NOLH DOE (see Ch. II) that we employ for the analysis gives us flexibility, which facilitates exploring a broader range of factor values. In Section F we set up a narrowed down experimental design that significantly reduces the dimensionality of our model, while at the same time, maintains the potential for robust inference, and clear conclusions.

We tried not to give any UCAV a clear overall advantage over the other one (otherwise the optimal UCAV mix is trivial). So, a type-A UCAV is more effective against a type-1 target, and a type-B UCAV is more effective against a type-2 target. Also, a type-A UCAV has a higher detection rate, but is more failure-prone than a type-B UCAV. The targets, too, have some differences. Type-1 targets are more valuable but harder to acquire than type-2 targets.

We seek to identify situations where the mission mix affects the outcome of the engagement, despite this balanced setup.

2. Design of Experiments

We use a 129-NOLH design for the 18 (varying) noise factors and we cross it with the 17 discrete levels of the (unique) decision factor N_A. This gives 129 x 17 = 2,193 design points in total.

3. Batch-Running the Model

Since MATLAB® can't run in command line mode, we cannot use a batch file approach to automate the 2,193 runs. Instead, we feed an Excel® file that contains all the design points into the MATLAB workspace, and the MATLAB code sequentially reads the parameter values corresponding to each design point, runs the continuous time Markov model, and generates the output values which are then saved to another Excel file. All this input and output (I/O) is performed automatically by the MATLAB code without the need for any additional setup.

4. Analysis and Results

a. Course of Analysis

The Markov chain model employs various design, operational, and decision factors. The relations between these parameters and the MOEs are mapped, in order to gain insights regarding the design of the UCAVs and their tactical employment. The statistical package JMP® is used to evaluate the MOE outputs from the model runs based on sets of inputs that are determined according to the NOLH DOE principles. Since we are interested in the optimal UCAV mix, we plot the MOE values as functions of N_A, in order to observe how significant the decision factor is.

Also we employ regression analysis as our main tool for meta-modeling the mathematical model. We construct various regression models of the MOEs, which include linear, quadratic, and second degree interaction terms, for investigating whether a simpler meta-model is more efficient than a more accurate one, by being able to capture most of the modeling situation with only a subset of the factors. If the regression accounts for most of the variability of our model, it can be assigned the role of a 'hard and fast' substitute for the model. This is convenient when the model is not available, or when the time available for running the model is limited. Also, the regression function transparently shows which factors have an effect on the MOE, how big the effect is, and in what direction.

b. Analysis

First we explore how does mission mix (i.e., the N_A decision factor) affect the average value of the E_v MOE. As we see in Figure 1, although the E_v tends to be higher for larger N_A values, the effect is insignificant.

Throughout we follow the convention that plots be accompanied by their respective data tables, which in some cases contain also some additional information. Here, in Table 1, the E_V values are shown along with their standard deviation values which are fairly constant, ratifying the use of regression analysis. Also, we can see the extreme values and the range of E_V values for each N_A. They all moderately increase with N_A, showing that if N_A has any effect on E_V it has to be positive, even though the overall effect appears insignificant in this case. This insignificance is attributed to the design of the 18-noise-facrtor fairly balanced combat situation.

Note that the plot in Figure 1 is drawn on a magnified scale, and the nature of the relation (e.g., quadratic) is revealed, due to the very limited range of E_v values; this is indifferent to our conclusions though. In this case, actually, our interpretation is that the mission mix makes no practical difference.

N_A	Mean(Ev)	StDev(Ev)	Min(Ev)	Max(Ev)	Range(Ev)
0	4.112	1.670	1.177	8.958	7.781
1	4.131	1.643	1.243	8.773	7.530
2	4.147	1.618	1.308	8.587	7.279
3	4.163	1.597	1.362	8.399	7.037
4	4.178	1.580	1.362	8.361	6.999
5	4.191	1.565	1.362	8.470	7.108
6	4.203	1.555	1.362	8.576	7.214
7	4.214	1.548	1.362	8.681	7.319
8	4.223	1.544	1.362	8.784	7.422
9	4.232	1.545	1.362	9.061	7.699
10	4.239	1.549	1.362	9.420	8.058
11	4.245	1.556	1.362	9.764	8.402
12	4.250	1.567	1.362	10.088	8.727
13	4.254	1.581	1.361	10.389	9.028
14	4.256	1.598	1.361	10.662	9.300
15	4.257	1.617	1.361	10.904	9.542
16	4.258	1.638	1.361	11.113	9.752

Table 1. Statistics on the E_V MOE for the different N_A values

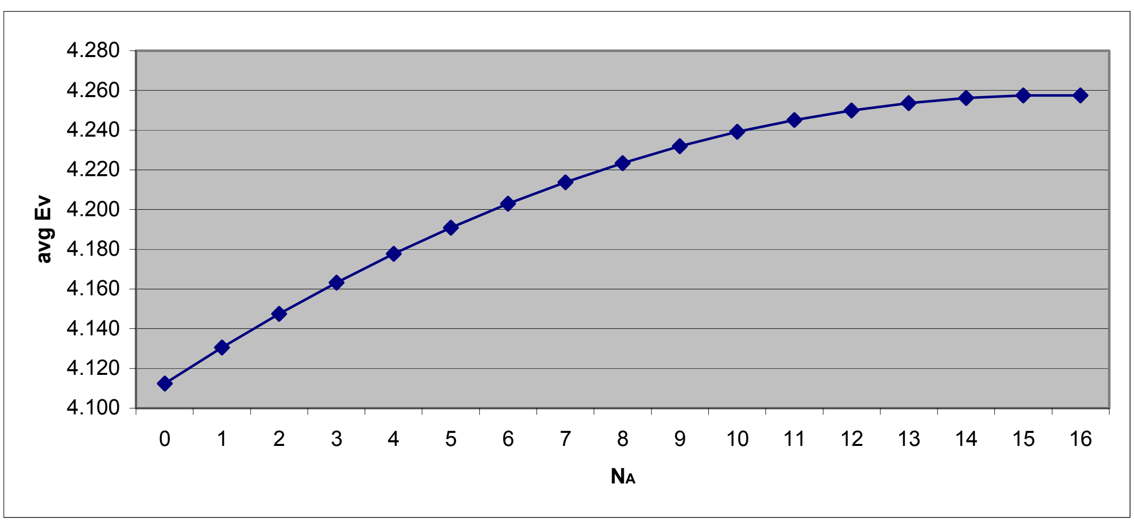

Figure 1. The average E_V MOE values as a function of N_A

Next, we explore how N_A affects the mean time of the operation (second MOE). As we can see in Figure 2, the expected time tends to be shorter for larger N_A values. So, as with E_v, this MOE too, gets a more favorable value as N_A increases. Note though, that the effect of N_A on Time is more significant than it is on E_v.

In Table 2, the Time values are shown along with the respective standard deviation values, which are not constant, making the use of regression analysis for this MOE less appropriate. Also, we can see the extreme values and the range of Time values for each N_A. When N_A is about its middle value, the Time values are more consistent. Because of this behavior, we conclude that although the Time is shorter for larger N_A, it is wiser to choose an intermediate N_A value (like 9, in this case), since this gives a worst case (max) Time value which is almost half the value we would get if N_A was closer to its extreme points (0 and 16).

N_A	Mean(Time)	StDev(Time)	Min(Time)	Max(Time)	Range(Time)
0	22.668	15.434	6.623	117.669	111.045
1	22.581	14.727	6.815	111.031	104.216
2	22.490	14.073	7.002	104.418	97.416
3	22.397	13.482	7.184	97.863	90.679
4	22.301	12.962	7.362	91.403	84.040
5	22.203	12.524	7.536	85.067	77.530
6	22.104	12.178	7.590	78.883	71.293
7	22.005	11.934	7.558	72.875	65.317
8	21.906	11.800	7.526	67.057	59.531
9	21.808	11.786	7.493	61.439	53.946
10	21.711	11.904	7.460	62.108	54.648
11	21.617	12.167	7.427	65.629	58.202
12	21.527	12.592	7.394	69.161	61.768
13	21.439	13.196	7.289	79.896	72.607
14	21.353	13.997	7.124	92.774	85.650
15	21.265	15.003	6.956	107.535	100.579
16	21.170	16.210	6.578	123.934	117.356

Table 2. Statistics on the Time MOE for the different N_A values

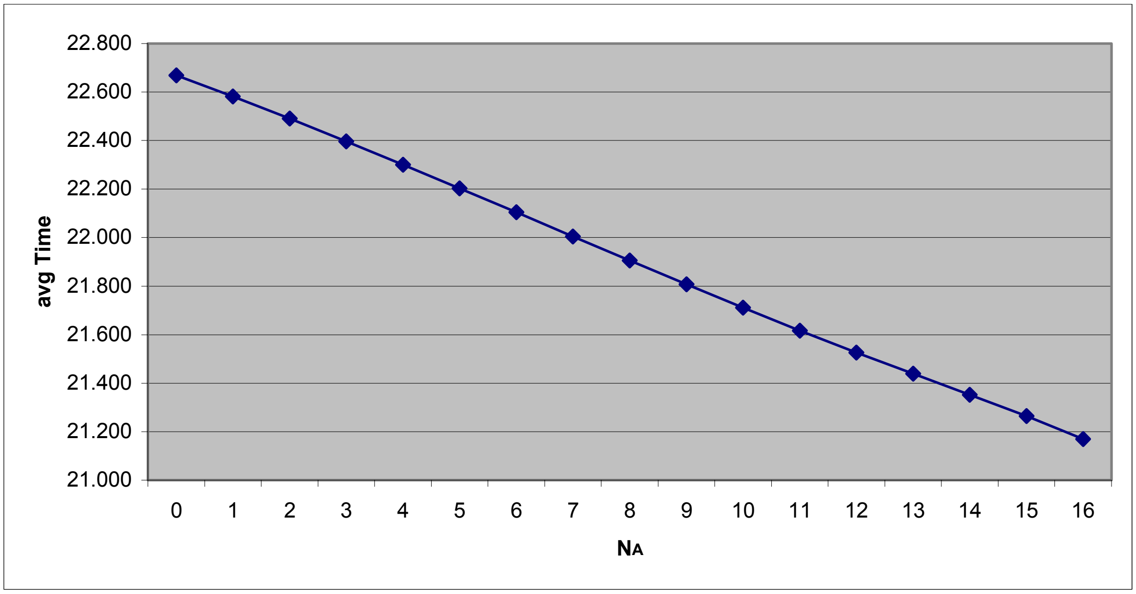

Figure 2. The average Time MOE values as a function of N_A

Next we build three regression meta-models of the mathematical model. They describe the E_v MOE only (since due to heteroscedasticity we deemed Time inappropriate for regression). The first regression only employs the (linear) model factors, the second one additionally employs quadratic terms (i.e., the model factors squared), and the third one additionally employs all possible second degree interactions of the model factors. It is obvious that the third meta-model has the most descriptive power, but we explore the other two cases because simplicity and fewer terms are desirable regression attributes (they aid the interpretation) even when the descriptive power falls a little bit behind.

(1) Regression with linear terms only

Here, the 18 noise factors and the single decision factor are initially added to the regression which sequentially eliminates the factors that are not significant. The R^2 (as well as the adjusted R^2) is about 86%, which is fairly high. In Table 3 we see the values of the parameter estimates for this regression. Notice how small the standard error for each of the 13 estimates is, making the regression more robust. Also, note that N_A is eliminated as a factor (i.e., it is statistically insignificant), as we might have expected from the previous analysis.

Response Ev
Summary of Fit

RSquare	0.858
RSquare Adj	0.857
Root Mean Square Error	0.598
Mean of Response	4.209
Observations (or Sum Wgts)	2193

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-6.911	0.171	-40.44	<.0001
T_1	0.385	0.009	42.55	<.0001
T_2	0.104	0.009	11.59	<.0001
T_3	-0.075	0.002	-47.78	0.0000
T_{V2}	2.910	0.044	66.27	0.0000
q_{A1}	1.245	0.088	14.17	<.0001
q_{B2}	0.458	0.088	5.22	<.0001
r_A	2.545	0.088	28.98	<.0001
r_B	2.631	0.088	29.87	<.0001
p_{A1}	1.325	0.341	3.88	0.0001
p_{B1}	1.618	0.366	4.43	<.0001
p_{A2}	1.748	0.078	22.30	<.0001
λ_A	0.532	0.055	9.70	<.0001
θ_A	-17.883	1.909	-9.37	<.0001

Table 3. Regression with linear terms only output

(2) Regression with quadratic terms also

This time, the squared factors are also considered for the regression. After the elimination process, we are left with 35 estimates. The corresponding terms are: $T_1, T_1^2, T_2, T_2^2 T_3, T_3^2, T_{V2}, T_{V2}^2, q_{A1}, q_{A1}^2, q_{B1}, q_{B1}^2, q_{B2}, q_{A2}, q_{A2}^2, r_A, r_A^2,$ $r_B, r_B^2, p_{A1}, p_{A1}^2, p_{B1}, p_{B1}^2, p_{B2}, p_{B2}^2, p_{A2}, p_{A2}^2, \lambda_A, \lambda_A^2, \theta_A, \theta_A^2, \theta_B, \theta_B^2.$ The parameter estimates for these terms are shown in Table 4. The R^2 value is 90% in this case. That value is not much larger than 86%, and therefore the use of 22 additional terms does not seem to be justified.

Note how the quadratic terms are centered about the mean value of their corresponding linear term. For example, the average T_1 value is 5, and therefore the quadratic T_1 term is $(T_1 - 5)^2$ instead of T_1^2. This, according to regression theory, leads to a less biased regression equation. Besides that, note the elimination of N_A from this regression too.

(3) Regression with all linear, quadratic, and second-degree interaction terms

This regression considers all the factors, their squares, and their possible second degree interactions. After the elimination process the terms left in the regression are 126 which render the use of this regression unreasonable, even though the R^2 has climbed up to 98%.

c. Results

By employing this broad experimental design setup, it is obvious that, due to the balancing of the parameter values, there appears to be no effect of N_A on E_v. And although there is some effect on the other MOE, Time, it does not seem to be that important either. Nevertheless, for Time, we reached the important conclusion that absent hard data it is better to employ a balanced mix of UCAVs, instead of a biased one where most of the UCAVs are of one type.

In the next section, we try to decrease the overall noise, by employing a different experimental setup. This is based on the current one, but it is narrower, by using the 'handicap' concept, and selecting discrete values for the T_{v2} factor.

E. NARROWER EXPERIMENTAL SET-UP AND DATA ANALYSIS

In this DOE, the varying factors are decreased down to 13 as opposed to 19 factors in the previously discussed DOE. The other six factors are correlated to six of the 13 independent factors, according to a handicap that is decided to be equal to 1.2. A smaller handicap wouldn't reveal the various factor effects as articulately, whereas a larger one would just provide for the same insight. Therefore, we can employ a 1.2 handicap without loss of generality on our analysis conclusions.

Response Ev
Summary of Fit

RSquare	0.900
RSquare Adj	0.898
Root Mean Square Error	0.504
Mean of Response	4.209
Observations (or Sum Wgts)	2193

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-7.324	0.155	-47.16	0.0000
T_1	0.384	0.008	50.29	0.0000
$(T_1-5)^2$	0.032	0.008	4.12	<.0001
T_2	0.106	0.008	13.94	<.0001
$(T_2-5)^2$	-0.034	0.009	-3.89	0.0001
T_3	-0.075	0.001	-56.40	0.0000
$(T_3-14.015)^2$	0.0006	0.0002	2.77	0.0056
T_{V2}	2.951	0.037	79.40	0.0000
$(T_{V2}-0.500)^2$	2.398	0.182	13.17	<.0001
q_{A1}	0.392	0.296	1.33	0.1847
$(q_{11}-0.750)^2$	8.661	1.177	7.36	<.0001
q_{B1}	1.045	0.316	3.30	0.0010
$(q_{B1}-0.684)^2$	-4.249	1.254	-3.39	0.0007
q_{B2}	0.775	0.290	2.68	0.0075
q_{A2}	-0.415	0.306	-1.36	0.1753
$(q_{A2}-0.684)^2$	1.788	0.730	2.45	0.0144
r_A	2.531	0.074	34.11	<.0001
$(r_A-0.750)^2$	2.541	0.744	3.41	0.0007
r_B	2.600	0.075	34.74	<.0001
$(r_B-0.750)^2$	3.739	0.702	5.33	<.0001
p_{A1}	-0.075	0.331	-0.23	0.8207
$(p_{A1}-0.700)^2$	6.727	1.168	5.76	<.0001
p_{B1}	3.210	0.358	8.97	<.0001
$(p_{B1}-0.638)^2$	-15.913	1.366	-11.65	<.0001
p_{B2}	1.078	0.326	3.30	0.0010
$(p_{B2}-0.700)^2$	-2.867	1.059	-2.71	0.0069
p_{A2}	0.626	0.353	1.77	0.0761
$(p_{A2}-0.638)^2$	5.258	1.146	4.59	<.0001
λ_A	-0.192	0.415	-0.46	0.6441
$(\lambda_A-0.600)^2$	2.168	0.955	2.27	0.0234
λ_B	0.800	0.455	1.76	0.0790
$(\lambda_B-0.547)^2$	-5.114	1.086	-4.71	<.0001
θ_A	-60.146	14.240	-4.22	<.0001
$(\theta_A-0.017)^2$	5483.747	1331.215	4.12	<.0001
θ_B	47.936	15.561	3.08	0.0021
$(\theta_B-0.016)^2$	-6208.849	1542.055	-4.03	<.0001

Table 4. Regression with linear and quadratic terms output

1. Factors

a. Decision Factor

Again, we only have one decision factor, N_A, which determines the UCAV mix.

b. Environmental and Design Factors, and Their Ranges

(1) Environmental (noise) factors

- $T_1 \in [3, 7]$ [discrete]
- $T_2 \in [3, 7]$ [discrete]
- $T_3 \in [0, 28]$ [discrete]
- $T_{v1} = 1$ [fixed]
- $T_{v2} \in \{0, 0.5, 1\}$ [discrete]

Note that when T_{v2} (which is now discrete) is equal to 0, then the T_{v1}/T_{v2} ratio goes to infinity. Killing type-2 targets adds no value to the military operation. On the other hand, when T_{v2} is equal to 1, that ratio becomes 1, meaning that both types of targets have the same military value and thus their value is not a factor mission-wise.

(2) UCAV-design (noise) factors

This experimental setup has a reduced dimensionality compared to the previous broad setup. For accomplishing this, the correlated parameters (e.g., q_{A1} and q_{B1}) have a fixed relation, called a handicap, instead of a randomly varying relation. So, the handicap is a fixed coefficient by which some parameters are inferior compared to their respective correlated parameters. In the subsequent analysis, we also explore the effect of different handicap values (1.0, 1.1, 1.2) on the inference significance. The default handicap is 1.2 (corresponds to 83% inferiority).

- $q_{A1} \in [0.5, 1]$ [continuous]
- $q_{A2} = \frac{q_{B2}}{1.2}$ [correlated with q_{B2}]

- $q_{B1} = \frac{q_{A1}}{1.2}$ [correlated with q_{A1}]
- q_{B2} є [0.5, 1] [continuous]
- r_A є [0.5, 1.0] [continuous]
- r_B є [0.5, 1.0] [continuous]
- p_{A1} є [0.4, 1.0] [continuous]
- $p_{A2} = \frac{p_{B2}}{1.2}$ [correlated with p_{B2}]
- $p_{B1} = \frac{p_{A1}}{1.2}$ [correlated with p_{A1}]
- p_{B2} є [0.4, 1.0] [continuous]
- λ_A є [0.2, 1] [continuous]
- $\lambda_B = \frac{\lambda_A}{1.2}$ [correlated with λ_A]
- θ_A є [0.005, 0.03] [continuous]
- $\theta_B = \frac{\theta_A}{1.2}$ [correlated with θ_A]

This narrower experimental design above, has a significantly reduced dimensionality compared to the previous broad DOE, while at the same time, as we will see, conveys the same types of results for E_V, when analysis is done. For Time, though, we get different results, which now are more consistent with the results for E_V.

Again, the properties of the UCAVs and the targets, are balanced, enabling the exploration of non-trivial UCAV mixes.

2. Design of Experiments

We use a 33-NOLH design for the 11 (varying) noise factors and we cross it with the 3 discrete levels of the T_{v2} noise factor as well as the 17 discrete levels of the decision factor N_A. This gives 33 x 3 x 17 = 1,683 design points in total, 30% less than in the initial 129 x 17 DOE, while at the same time we have very enhanced resolution (i.e., transparency for the effects of N_A and the T_{v1}/T_{v2} ratio) due to the double crossing in the design. If the design was not crossed, then a possible effect on some MOE would not be clearly attributed to either the decision factor or a noise factor or the T_{V1}/T_{V2} ratio, without additional investigation. This kind of design leads to clearer cause-and-effect conclusions, at the expense of more design points than an equivalent single DOE would have.

3. Analysis and Results

a. Course of Analysis

As in the previous section, we are interested in the optimal UCAV mix, and therefore we plot the MOE values as functions of N_A, in order to observe how significant the decision factor is. We investigate the role of T_{V2} to the significance of the N_A effect. It turns out that T_{V2} affects only E_V, not Time. The results we get are consistent with the previous section analysis results, only more pronounced. So, the use of the computationally less expensive narrowed down DOE is justified.

Also we employ regression analysis as our main tool for meta-modeling the mathematical model. We construct various regression models of the MOEs. Again, arguably, the regression function of choice is the regression with linear terms only.

b. Analysis

Initially, we explore how the mission mix (i.e., the N_A decision factor) affects the value of the E_v MOE, for various handicap values and T_{V1}/T_{V2} ratios. What we first see is that the higher the T_{V1}/T_{V2} ratio the better the effect of N_A on Ev shows. Given a high T_{V1}/T_{V2} ratio, larger handicaps give more noticeable effects. Therefore an initial conclusion here is that we should choose a large enough handicap for the effects to show, and that when the operational values of the two types of targets are close, the effects tend to be masked no matter what the handicap is. In the subsequent analysis, whenever a handicap has to be employed it will take on a value of 1.2. Note that the selection of a 1.2 handicap value is arbitrary (for example it could be larger) but without introducing any loss of generality. The above conclusions are backed up by the plots in

Figures 3 to 5. As always, the plots are accompanied by their respective data tables (Table 5 to 7). Note that Figure 3 has T_{V2} = 0, Figure 4 has T_{V2} = 0.5, and Figure 5 has T_{V2} = 1. T_{V1} is fixed to 1 as per the design explained in Section 1b.

The conclusion here is that a larger N_A is always better for the military value of the operation, but we should only strive for it when the differences between the two UCAV types are substantial and the operational values of the two types of targets are quite distant.

handicap = 1.2		handicap = 1.1		handicap = 1.0	
T_{V2}=0		T_{V2}=0		T_{V2}=0	
N_A	Ev	N_A	Ev	N_A	Ev
0	2.395	0	2.637	0	2.910
1	2.438	1	2.660	1	2.914
2	2.478	2	2.683	2	2.918
3	2.518	3	2.705	3	2.921
4	2.556	4	2.725	4	2.924
5	2.593	5	2.745	5	2.926
6	2.629	6	2.764	6	2.927
7	2.664	7	2.782	7	2.927
8	2.697	8	2.800	8	2.927
9	2.730	9	2.816	9	2.925
10	2.761	10	2.832	10	2.923
11	2.791	11	2.847	11	2.920
12	2.821	12	2.861	12	2.916
13	2.849	13	2.874	13	2.912
14	2.877	14	2.887	14	2.906
15	2.904	15	2.899	15	2.900
16	2.931	16	2.911	16	2.893

Table 5. E_V as a function of N_A when T_{V2} = 0 for three different handicaps.

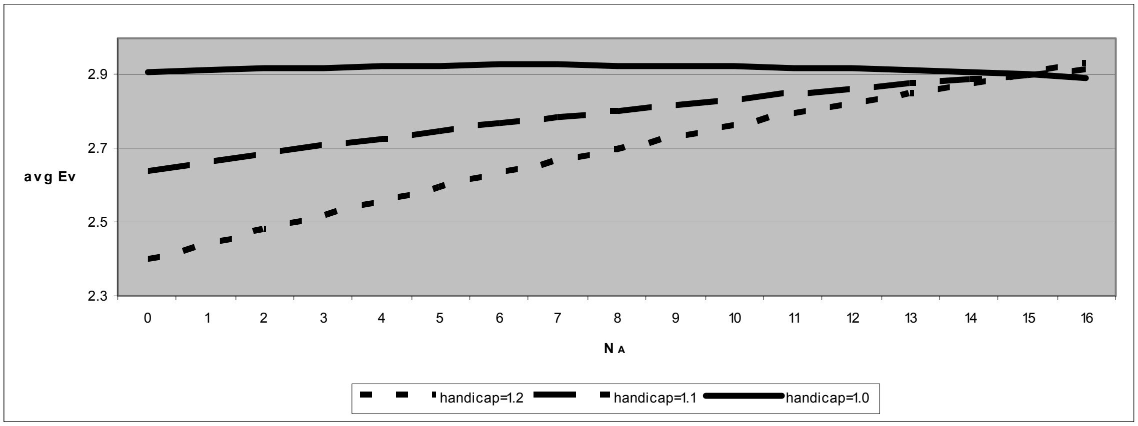

Figure 3. E_V as a function of N_A when T_{V2} = 0 for three different handicaps.

handicap = 1.2	
T_{V2}=0.5	
N_A	Ev
0	3.869
1	3.897
2	3.924
3	3.949
4	3.972
5	3.993
6	4.013
7	4.031
8	4.047
9	4.061
10	4.074
11	4.085
12	4.095
13	4.103
14	4.109
15	4.114
16	4.118

handicap = 1.1	
T_{V2}=0.5	
N_A	Ev
0	4.101
1	4.117
2	4.132
3	4.147
4	4.160
5	4.171
6	4.182
7	4.191
8	4.199
9	4.206
10	4.212
11	4.216
12	4.219
13	4.222
14	4.223
15	4.223
16	4.222

handicap = 1.0	
T_{V2}=0.5	
N_A	Ev
0	4.364
1	4.371
2	4.377
3	4.381
4	4.385
5	4.388
6	4.389
7	4.390
8	4.389
9	4.388
10	4.385
11	4.381
12	4.376
13	4.369
14	4.362
15	4.354
16	4.344

Table 6. E_V as a function of N_A when T_{V2} = 0.5 for three different handicaps.

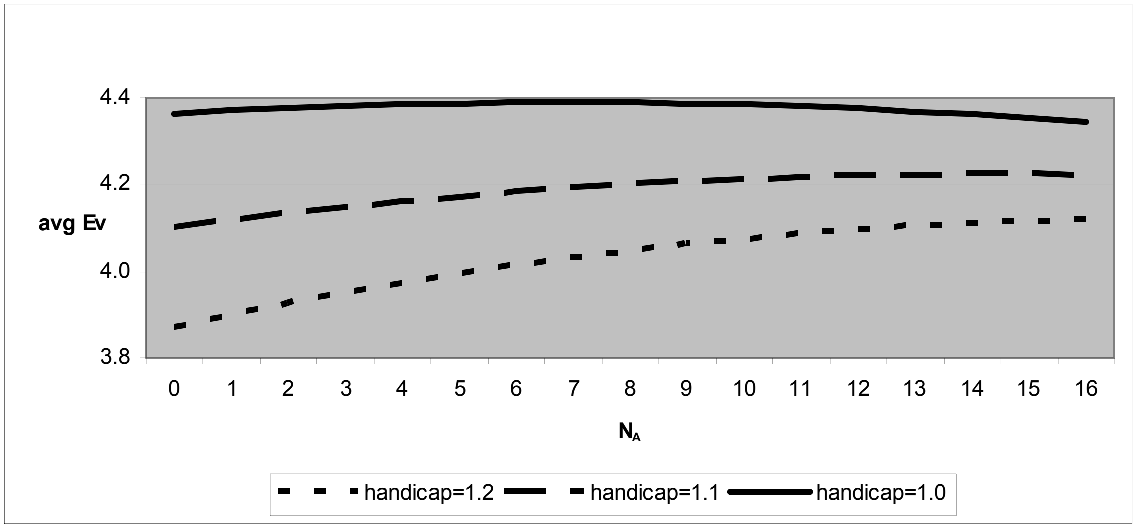

Figure 4. E_V as a function of N_A when T_{V2} = 0.5 for three different handicaps.

handicap = 1.2	
T_{V2}=1	
N_A	E_V
0	5.343
1	5.357
2	5.369
3	5.379
4	5.387
5	5.393
6	5.396
7	5.397
8	5.396
9	5.393
10	5.387
11	5.379
12	5.369
13	5.356
14	5.341
15	5.324
16	5.305

handicap = 1.1	
T_{V2}=1	
N_A	E_V
0	5.564
1	5.574
2	5.582
3	5.589
4	5.594
5	5.597
6	5.599
7	5.600
8	5.599
9	5.596
10	5.592
11	5.586
12	5.578
13	5.569
14	5.558
15	5.546
16	5.533

handicap = 1.0	
T_{V2}=1	
N_A	E_V
0	5.819
1	5.828
2	5.835
3	5.841
4	5.846
5	5.850
6	5.852
7	5.853
8	5.852
9	5.850
10	5.846
11	5.841
12	5.835
13	5.827
14	5.818
15	5.807
16	5.795

Table 7. E_V as a function of N_A when T_{V2} = 1 for three different handicaps.

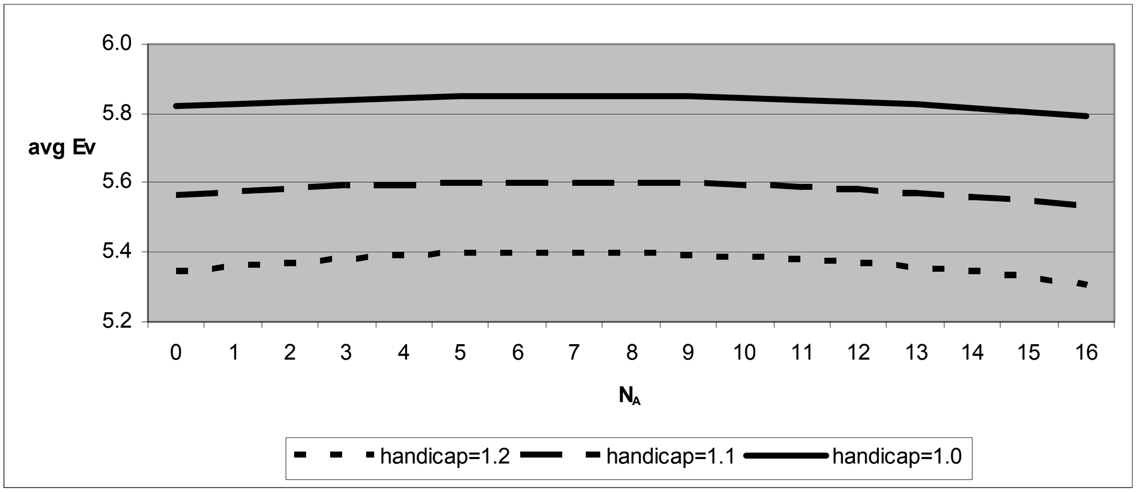

Figure 5. E_V as a function of N_A when T_{V2} = 1 for three different handicaps.

Adopting a handicap of 1.2, this time we estimate the Ev values along with other statistical quantities for different T_{V2} values as a function of N_A. This data is shown in Tables 9 to 11. By examining the standard deviation columns, we observe homoscedasticity and we conclude that a regression would be appropriate on Ev as a function of all the model parameters. This is done and explained later on in this Section.

NA	Mean(Ev)	StDev(Ev)	Min(Ev)	Max(Ev)	Range(Ev)
0	2.395	1.100	0.614	5.614	5.000
1	2.438	1.091	0.646	5.589	4.943
2	2.478	1.084	0.678	5.565	4.887
3	2.518	1.078	0.710	5.541	4.831
4	2.556	1.074	0.741	5.516	4.775
5	2.593	1.072	0.771	5.491	4.720
6	2.629	1.071	0.802	5.467	4.665
7	2.664	1.071	0.832	5.443	4.611
8	2.697	1.072	0.861	5.419	4.558
9	2.730	1.075	0.890	5.395	4.505
10	2.761	1.078	0.919	5.372	4.453
11	2.791	1.083	0.947	5.349	4.402
12	2.821	1.089	0.975	5.326	4.351
13	2.849	1.096	1.003	5.305	4.302
14	2.877	1.104	1.030	5.309	4.280
15	2.904	1.113	1.027	5.413	4.386
16	2.931	1.122	1.013	5.513	4.500

Table 8. Statistics on the E_V MOE for the different N_A values for T_{V2} = 0.

Next, we explore how N_A affects the mean time of the operation (Time MOE). The trend is the same for different handicaps, therefore we display only the findings for a handicap of 1.2. Of course these results are unaffected by the T_{V2} value, since it makes no difference on the Time MOE (it only affects E_V). Time is consistently more favorable for larger N_A values; even the variance gets smaller as N_A grows. So, for time critical missions, sending as many type-A UCAVs as possible, is always a good strategy, no matter how small the difference of the two types of UCAVs (up to a reasonable point) or how small the difference in the operational values of the targets.

NA	Mean(Ev)	StDev(Ev)	Min(Ev)	Max(Ev)	Range(Ev)
0	3.869	1.305	1.772	7.170	5.398
1	3.897	1.287	1.918	7.111	5.193
2	3.924	1.271	2.057	7.050	4.993
3	3.949	1.258	2.092	6.988	4.896
4	3.972	1.249	2.098	6.924	4.826
5	3.993	1.241	2.082	6.858	4.777
6	4.013	1.237	2.065	6.791	4.726
7	4.031	1.235	2.048	6.723	4.675
8	4.047	1.235	2.031	6.653	4.622
9	4.061	1.239	2.013	6.667	4.653
10	4.074	1.245	1.995	6.799	4.804
11	4.085	1.254	1.977	6.929	4.952
12	4.095	1.266	1.958	7.056	5.098
13	4.103	1.280	1.939	7.179	5.240
14	4.109	1.298	1.920	7.299	5.380
15	4.114	1.317	1.900	7.416	5.516
16	4.118	1.340	1.879	7.529	5.650

Table 9. Statistics on the E_V MOE for the different N_A values for $T_{V2} = 0.5$.

N_A	Mean(Ev)	StDev(Ev)	Min(Ev)	Max(Ev)	Range(Ev)
0	5.343	1.686	2.286	8.726	6.440
1	5.357	1.659	2.447	8.632	6.185
2	5.369	1.634	2.601	8.544	5.942
3	5.379	1.614	2.749	8.575	5.826
4	5.387	1.596	2.890	8.603	5.713
5	5.393	1.583	2.937	8.628	5.692
6	5.396	1.573	2.935	8.651	5.715
7	5.397	1.567	2.895	8.670	5.775
8	5.396	1.565	2.855	8.687	5.833
9	5.393	1.567	2.813	8.701	5.888
10	5.387	1.573	2.771	8.747	5.976
11	5.379	1.584	2.728	8.886	6.158
12	5.369	1.599	2.684	9.023	6.339
13	5.356	1.619	2.639	9.157	6.518
14	5.341	1.644	2.593	9.289	6.696
15	5.324	1.673	2.547	9.419	6.872
16	5.305	1.707	2.499	9.546	7.047

Table 10. Statistics on the E_V MOE for the different N_A values for $T_{V2} = 1$

N_A	Mean(Time)	StDev(Time)	Min(Time)	Max(Time)	Range(Time)
0	27.953	27.697	8.205	162.267	154.062
1	27.550	26.162	8.062	153.032	144.970
2	27.133	24.593	7.916	143.425	135.509
3	26.702	23.005	7.767	133.490	125.722
4	26.258	21.415	7.615	123.284	115.670
5	25.803	19.844	7.459	112.885	105.426
6	25.339	18.322	7.301	102.384	95.083
7	24.867	16.880	7.139	91.888	84.748
8	24.391	15.554	6.975	81.516	74.541
9	23.912	14.383	6.809	71.395	64.586
10	23.433	13.404	6.640	61.649	55.009
11	22.956	12.648	6.468	54.610	48.141
12	22.481	12.133	6.295	55.150	48.854
13	22.008	11.862	6.121	55.694	49.573
14	21.534	11.822	5.944	56.241	50.296
15	21.054	11.994	5.767	56.790	51.023
16	20.560	12.358	5.588	57.341	51.753

Table 11. Statistics on the Time MOE for the different N_A values

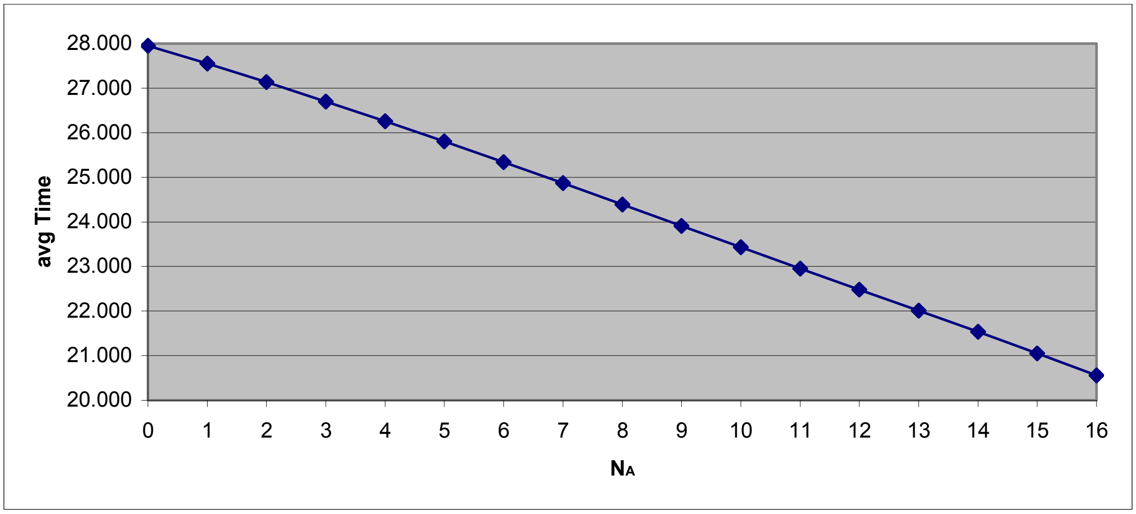

Figure 6. The average Time MOE values as a function of N_A

Next we build three regression meta-models of the mathematical model. They describe the E_v MOE only (since due to heteroscedasticity we deem Time inappropriate for regression). The first regression only employs the (linear) model factors, the second one additionally employs quadratic terms, and the third one additionally employs all possible second degree interactions of the model factors. It is obvious that the third meta-model has the most descriptive power, but we explore the other two cases because simplicity and fewer terms are desirable regression attributes (they aid the interpretation) even when the descriptive power falls a little bit behind.

(1) Regression with linear terms only.

All 13 factors are employed (none is eliminated). The resulting R^2 is about 87%.

Response Ev
Summary of Fit

RSquare	0.874
RSquare Adj	0.873
Root Mean Square Error	0.613
Mean of Response	4.027
Observations (or Sum Wgts)	1683

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-7.069	0.199	-35.50	<.0001
T_1	0.341	0.011	32.38	<.0001
T_2	0.107	0.011	10.09	<.0001
T_3	-0.077	0.002	-42.85	<.0001
q_{A1}	1.548	0.101	15.33	<.0001
q_{B2}	0.591	0.101	5.84	<.0001
r_A	2.485	0.101	24.59	<.0001
r_B	2.649	0.101	26.15	<.0001
p_{A1}	2.894	0.084	34.37	<.0001
p_{B2}	1.373	0.084	16.31	<.0001
λ_A	0.483	0.063	7.67	<.0001
θ_A	-14.807	2.015	-7.35	<.0001
T_{V2}	2.685	0.037	73.28	0.0000
N_A	0.015	0.003	5.07	<.0001

Table 12. Regression with linear terms only output

(2) Regression with quadratic terms also

This time, the squared factors are also considered for the regression. After the elimination process, we are left with 22 estimates. The corresponding terms are: $T_1, T_1^2, T_2, T_2^2 T_3, T_{V2}, q_{A1}, q_{B2}, q_{B2}^2, r_A, r_A^2, r_B, r_B^2, p_{A1}, p_{A1}^2, p_{B2}, p_{B2}^2, \lambda_A, \theta_A, \theta_A^2, N_A.$. The R^2 is increased by about 2% compared to the R^2 of the linear regression above. Thus, this quadratic regression is deemed inefficient compared to the linear one above.

Response Ev
Summary of Fit

RSquare	0.896
RSquare Adj	0.894
Root Mean Square Error	0.559
Mean of Response	4.027
Observations (or Sum Wgts)	1683

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-7.380	0.194	-38.03	<.0001
T_1	0.344	0.009	35.71	<.0001
$(T_1-5)^2$	0.052	0.012	4.37	<.0001
T_2	0.109	0.009	11.27	<.0001
$(T_2-5)^2$	0.029	0.013	2.26	0.0240
T_3	-0.077	0.002	-47.00	<.0001
$(T_3-14.0606)^2$	0.002	0.0005	4.02	<.0001
q_{A1}	1.535	0.092	16.68	<.0001
q_{B2}	0.587	0.092	6.36	<.0001
$(q_{B2}-0.7503)^2$	4.435	1.306	3.40	0.0007
r_A	2.479	0.092	26.91	<.0001

Term	Estimate	Std Error	t Ratio	Prob>|t|
$(r_A-0.7503)^2$	5.358	1.026	5.22	<.0001
r_B	2.656	0.092	28.75	<.0001
$(r_B-0.7503)^2$	12.969	1.368	9.48	<.0001
p_{A1}	2.908	0.077	37.91	<.0001
$(p_{A1}-0.70061)^2$	-9.674	0.976	-9.91	<.0001
p_{B2}	1.362	0.077	17.76	<.0001
$(p_{B2}-0.70061)^2$	-4.939	0.908	-5.44	<.0001
λ_A	0.481	0.057	8.38	<.0001
θ_A	-14.716	1.836	-8.01	<.0001
$(\theta_A-0.0175)^2$	-1023.385	564.542	-1.81	0.0700
T_{V2}	2.685	0.033	80.43	0.0000
N_A	0.015	0.003	5.56	<.0001

Table 13. Regression with linear and quadratic terms output

(3) Regression with all linear, quadratic, and second-degree interaction terms

This regression considers all the factors, their squares, and their possible second degree interactions. After the elimination process the terms left in the regression are 52 which render the use of this regression unreasonable, even though the R^2 has climbed up to 99%.

c. Results

By employing this narrow experimental design setup, we reached the following results:

- A larger N_A is always better for the military value of the operation, but it is only worth the extra cost when the differences between the two UCAV types are substantial and the operational values of the two types of targets are quite apart.
- For time critical missions, sending as many type-A UCAVs as possible, is always a good strategy, no matter how small the difference of the two types of UCAVs (up to a reasonable point) or how small the difference in the operational values of the targets.
- The regression displayed in Table 12 (linear terms only), is a good quick substitute for the model, since it accounts for 87% of the variability, without having to run the model.

F. BASE CASE AND SENSITIVITY ANALYSIS

In this chapter we define a base case for our scenario and then explore the effect of each parameter on the outcome, when the rest of the parameters remain fixed at their base-case values. This type of analysis, which may not reveal possible interactions among factors, complements the DOE setup described in Sections D and E. It might reveal potential relations that were concealed in the DOE analysis, due to averaging and canceling-out phenomena.

1. Base Case

In the previous section it was shown that we can get clearer and more consistent results if we reduce the dimensionality and variability of the DOE, by correlating the six factors q_{A2}, q_{B1}, p_{A2}, p_{B1}, λ_B, and θ_B, to six corresponding factors q_{B2}, q_{A1}, p_{B2}, p_{A1}, λ_A, and θ_A respectively. Therefore, for the sensitivity analysis setup, the varying factors are twelve (the other six factors are dependent). This time, the independence assumption of the varying factors is not required, as was the case with the NOLH DOEs previously discussed.

a. Decision Variable

As in the analyses in the previous sections, we only have one decision variable, N_A, which determines the UCAV mix. The total number of UCAVs is fixed at 16, and in the base case, $N_A = 8$.

b. Scenario and Design Factors

(1) Scenario factors

- $T_1 = 5$
- $T_2 = 5$
- $T_3 = 10$
- $T_{v1} = 1$
- $T_{v2} = 0.5$

(2) UCAV-design parameters

- $q_{A1} = 0.8$

- $q_{A2} = \frac{q_{B2}}{1.2}$ [correlated with q_{B2}]
- $q_{B1} = \frac{q_{A1}}{1.2}$ [correlated with q_{A1}]
- $q_{B2} = 0.8$
- $r_A = 0.8$
- $r_B = 0.8$
- $p_{A1} = 0.9$
- $p_{A2} = \frac{p_{B2}}{1.2}$ [correlated with p_{B2}]
- $p_{B1} = \frac{p_{A1}}{1.2}$ [correlated with p_{A1}]
- $p_{B2} = 0.9$
- $\lambda_A = 1$
- $\lambda_B = \frac{\lambda_A}{1.2}$ [correlated with λ_A]
- $\theta_A = 0.01$
- $\theta_B = \frac{\theta_A}{1.2}$ [correlated with θ_A]

c. Results of the Base Case

Running the model at the base case parameter values, we get results for the two MOE values as functions of N_A, which are displayed in Table 14. These results are graphically depicted in Figure 7 and Figure 8 for E_V and Time respectively. We can see that the optimal mix of UCAVs for the base case is $N_A = 14$ and $N_B = 3$, and that the shortest Time value occurs at $N_A = 16$, so it appears that a larger type-A UCAV presence provides for better effectiveness and time-efficiency.

N_A	E_V	Time
0	3.921	13.940
1	4.110	13.738
2	4.284	13.531
3	4.444	13.322
4	4.589	13.112
5	4.718	12.906
6	4.831	12.706
7	4.928	12.517
8	5.009	12.340
9	5.074	12.179
10	5.123	12.036
11	5.156	11.912
12	5.174	11.810
13	5.177	11.730
14	5.166	11.671
15	5.142	11.634
16	5.104	11.616

Table 14. Both MOE values for the different N_A values.

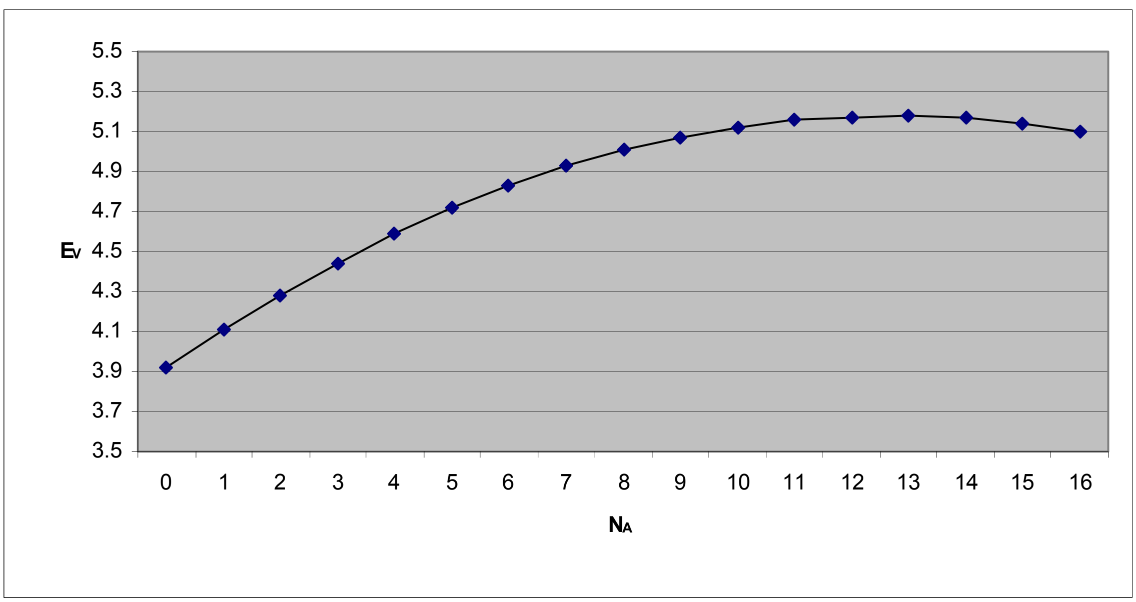

Figure 7. The average E_V MOE values as a function of N_A

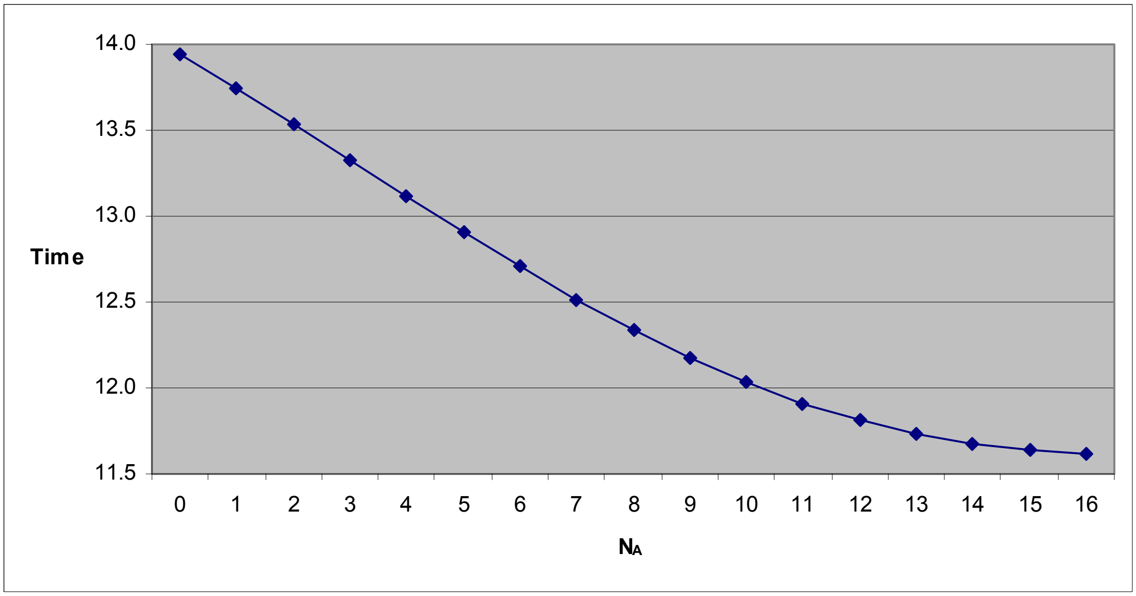

Figure 8. The average Time MOE values as a function of N_A

2. Sensitivity Analysis

a. Course of Analysis

Each one of the twelve varying factors – T_1, T_2, T_3, T_{V2}, q_{A1}, q_{B2}, r_A, r_B, p_{A1}, p_{B2}, λ_A, θ_A - consecutively varies within its range, everything else kept constant. Values of the MOEs are calculated for all 17 N_A values during this process, so that plots of the MOEs as functions of the decision factor can be plotted for the different levels of the examined scenario or design factor.

Additionally, we explore the effect on E_V of the T_1/T_2 ratio and handicap ratios on E_V.

b. Sensitivity Analysis on T_1

T_1 takes on values in the set {3, 5, 7}. The expected value of killed targets consistently increases with T_1, at every N_A value. The increase rate is higher for larger T_1 values. The expected operation time decreases as the number of type-1 targets increases. This might seem counterintuitive, but in fact the more targets there are in the area of interest, the higher the acquisition rate of the UCAVs is, and this leads to a decrease in the expected operation time.

Note that the combat situation is considered terminated when there are no UCAVs left, but there can still be alive targets left in the battlefield.

The overall conclusion here is that the operational benefit increases with the number of type-A UCAVs, but that this effect is even more pronounced as T_1 increases. So, given a high T_1 value, the use of more type-A UCAVs becomes more imperative versus the use of type-B ones. Of course, time critical missions dictate the use of more type-A UCAVs, no matter how many type-1 targets there are.

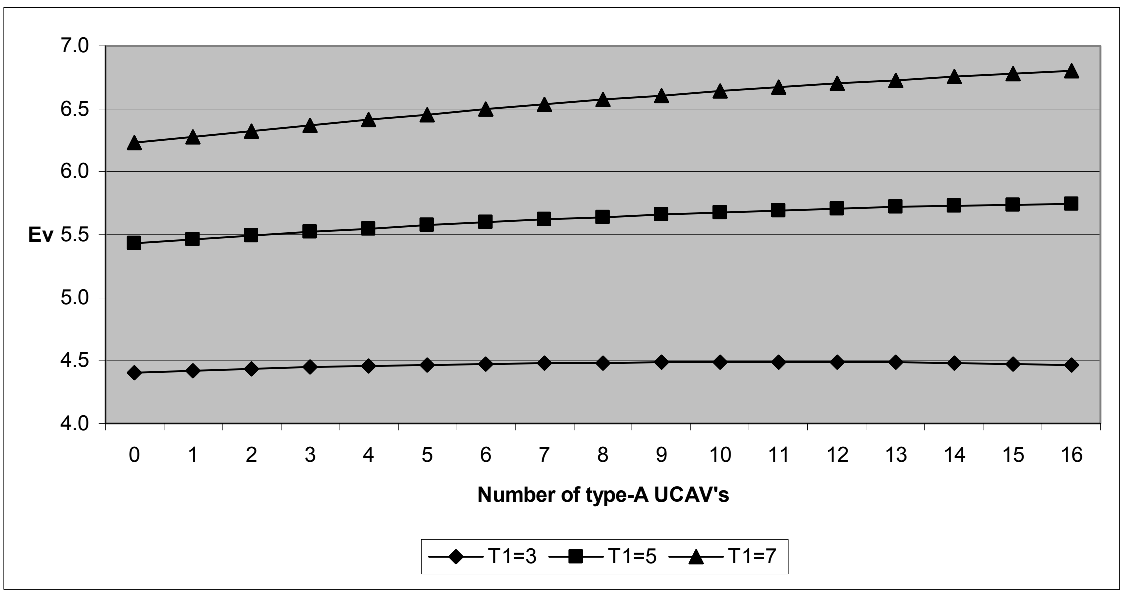

Figure 9. E_V as a function of N_A for different T_1 parameter values

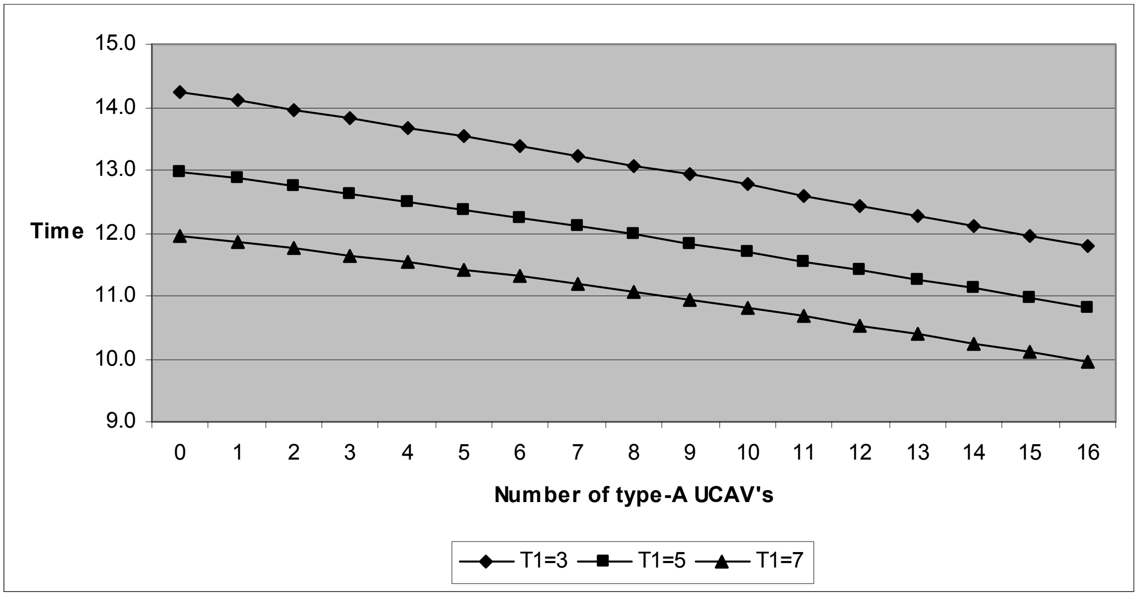

Figure 10. Time as a function of N_A for different T_1 parameter values

c. Sensitivity Analysis on T_2

T_2 takes on values in the set {3, 5, 7}. The plots of E_V and Time as a function of T_2 are similar to the previous plots for T_1, and the same conclusions naturally hold. The fact that the total E_V value is smaller in the present case is attributed to the T_{V1} base case value being twice that of T_{V2}, and so T_1 has a greater effect on the E_V maximum value than T_2 does. But, T_2 seems to have a greater effect on the E_V increase rate. The conclusion here is that a high N_A value is always more desirable, even for small T_2 values.

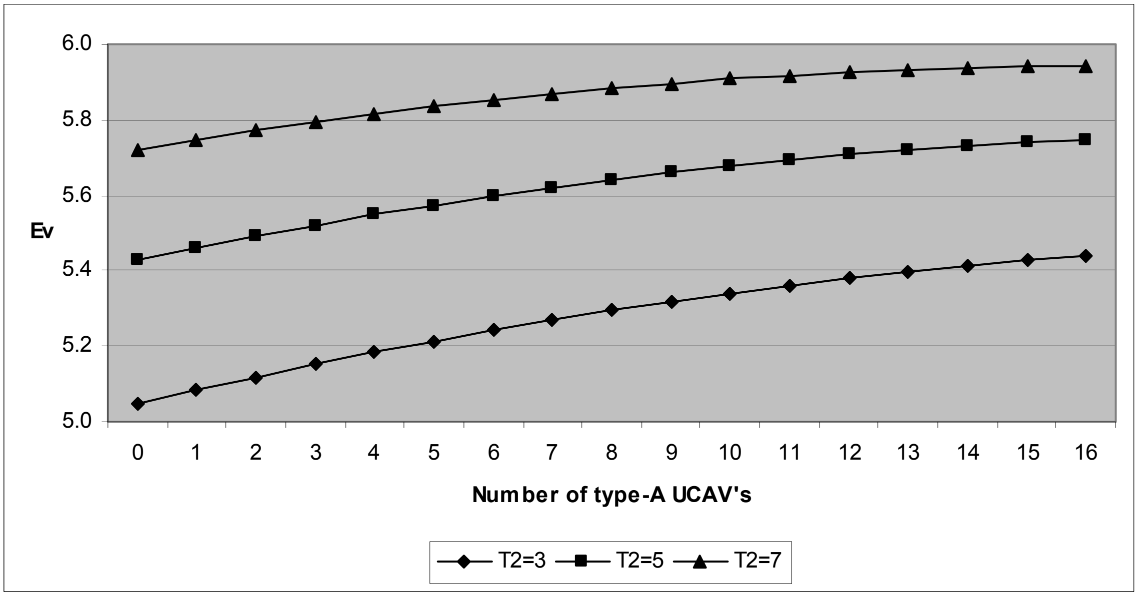

Figure 11. E_V as a function of N_A for different T_2 parameter values

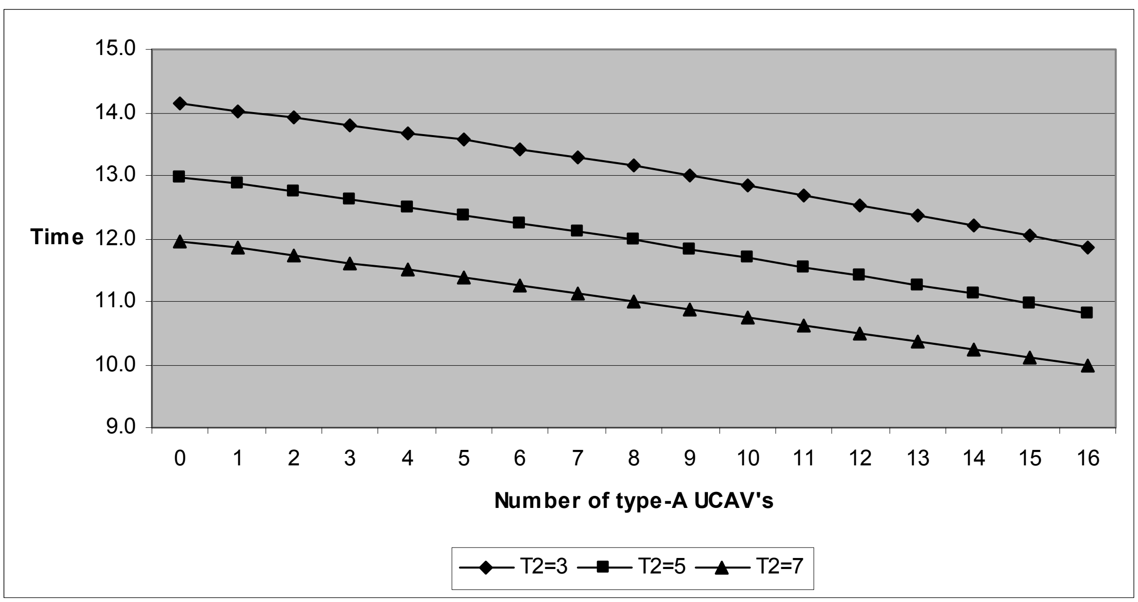

Figure 12. Time as a function of N_A for different T_2 parameter values

d. Sensitivity Analysis on T_3

T_3 takes on values in the set {0, 14, 28}. As the number of active NVTs increases, more UCAVs are attrite without adding any operational value to the combat mission. The Time MOE is also negatively affected by a larger T_3 number. Nevertheless, T_3 is an environmental noise factor, on which we can exercise no control. The conclusion here is that the mission mix is not a crucial factor for the expected military value, but when time is of essence, a larger number of type-A targets gives a considerably smaller length of operation.

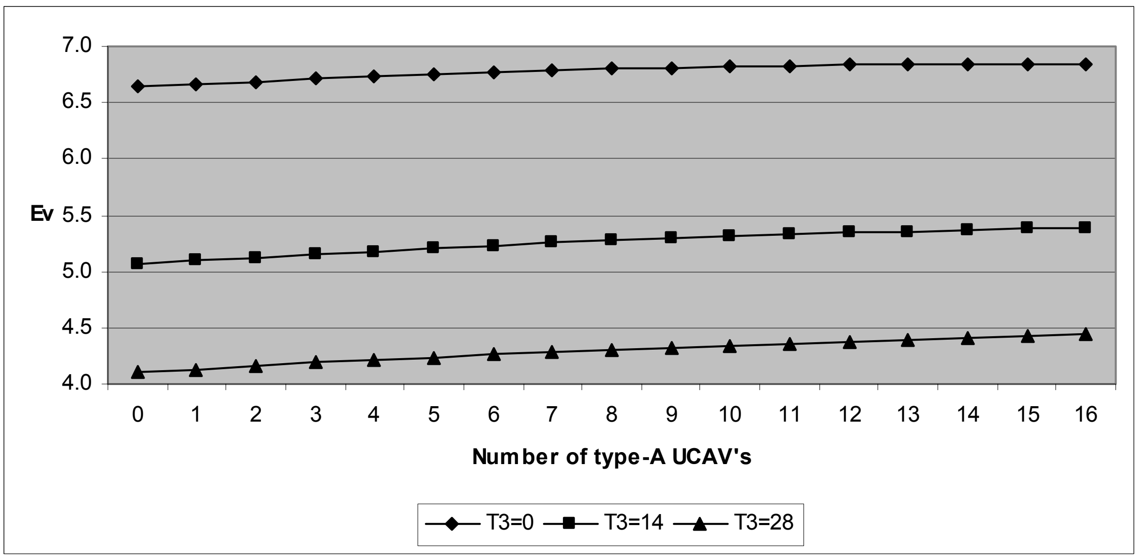

Figure 13. E_V as a function of N_A for different T_3 parameter values

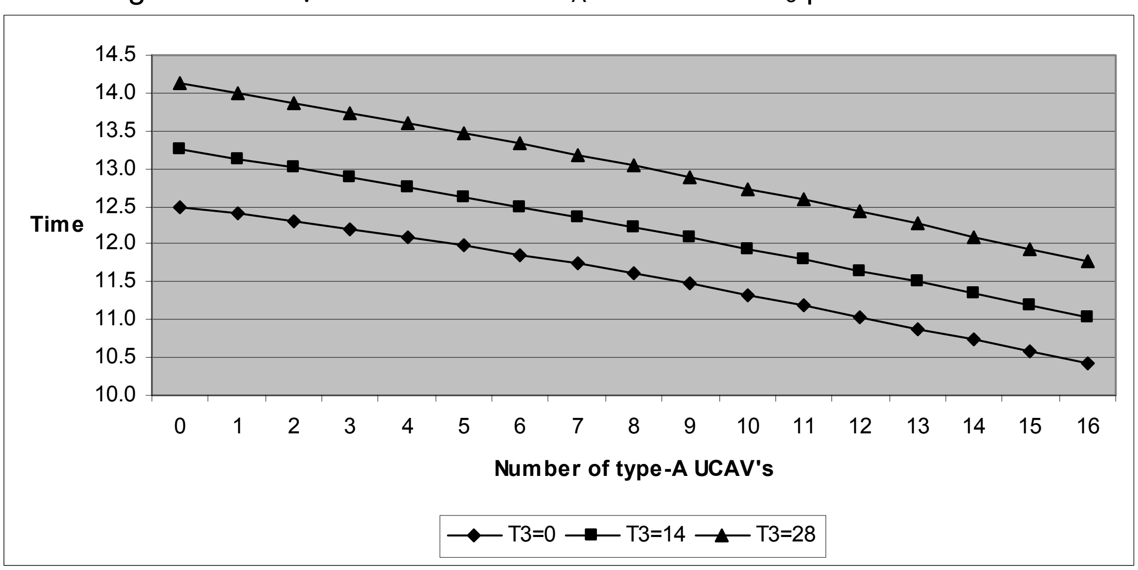

Figure 14. Time as a function of N_A for different T_3 parameter values

e. Sensitivity Analysis on T_{V2}

T_{V2} takes on values in the set {0, 0.5, 1}. Obviously, T_{V2} does not affect Time, but only affects E_V. The conclusion is that for T_{V2} values comparable or equal to T_{V1} values, the mission mix is irrelevant, but for comparatively small T_{V2} values, a larger number of type-A UCAVs generates a higher operational value.

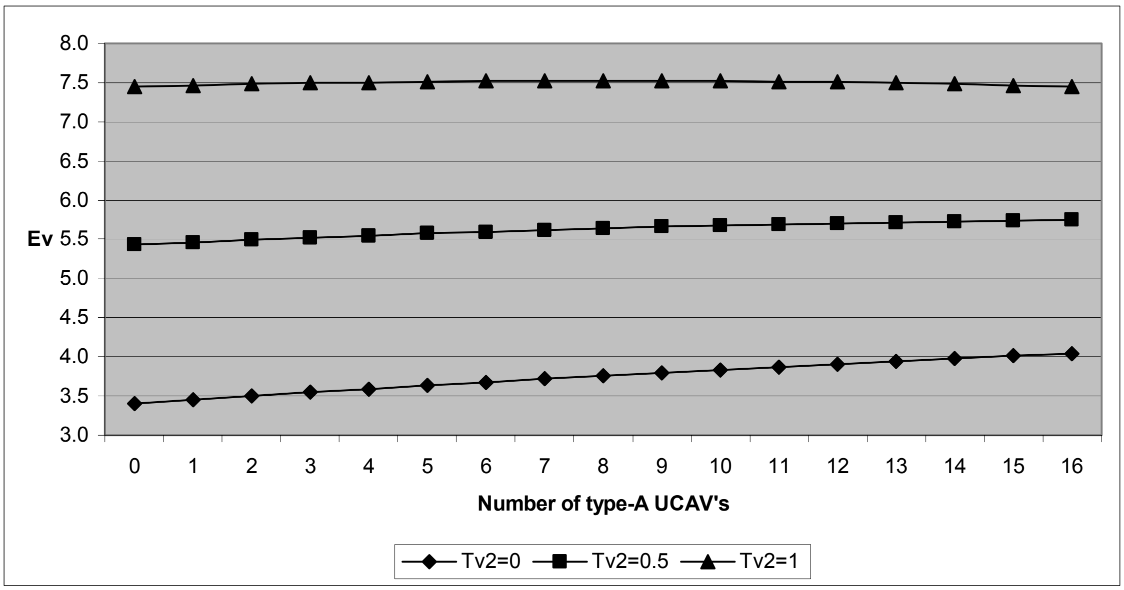

Figure 15. E_V as a function of N_A for different T_{V2} parameter values

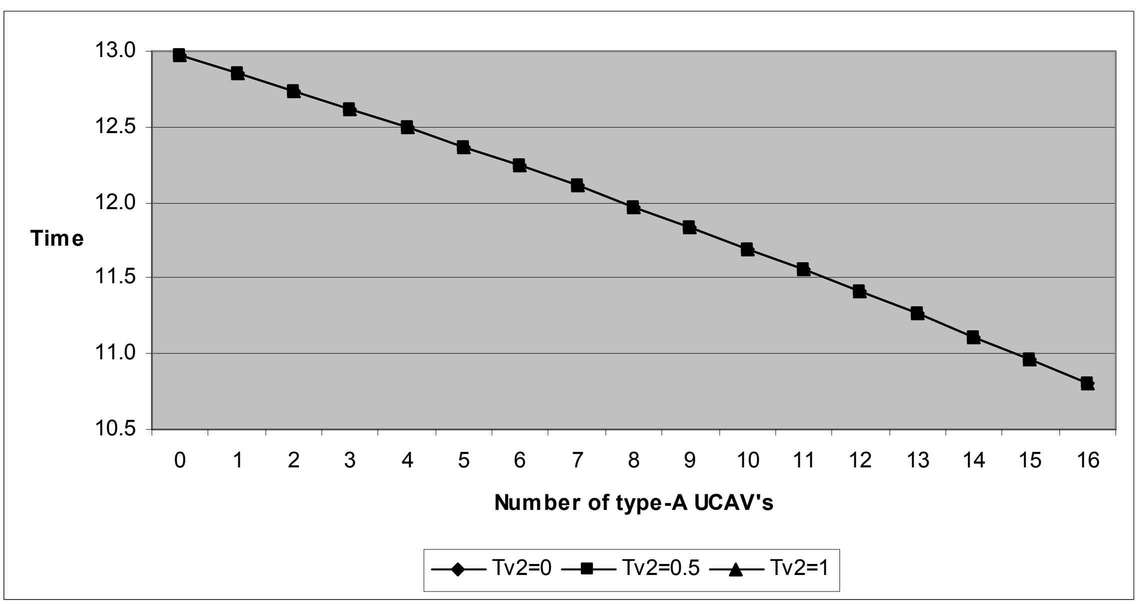

Figure 16. Time as a function of N_A for different T_{V2} parameter values

f. ***Sensitivity Analysis on q_{A1}***

q_{A1} takes on values in the set {0, 0.5, 1}. What we see here is that the unrealistic q_{A1} value of 0, accounts for totally unfavorable MOE results. Nevertheless, cutting the q_{A1} value in half (from its maximum possible value) has only minimal effects on both MOE values. A value of 0.5 is already too low for realistic situations; therefore the conclusion here is that improving the recognition capability of type-A UCAVs should not be a high priority.

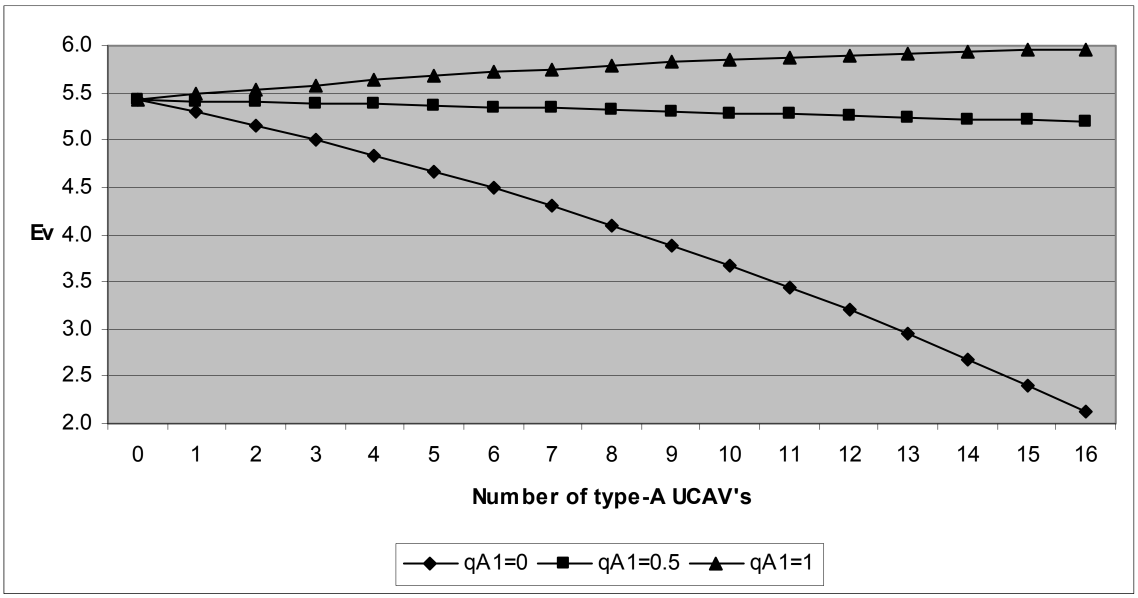

Figure 17. E_V as a function of N_A for different q_{A1} parameter values

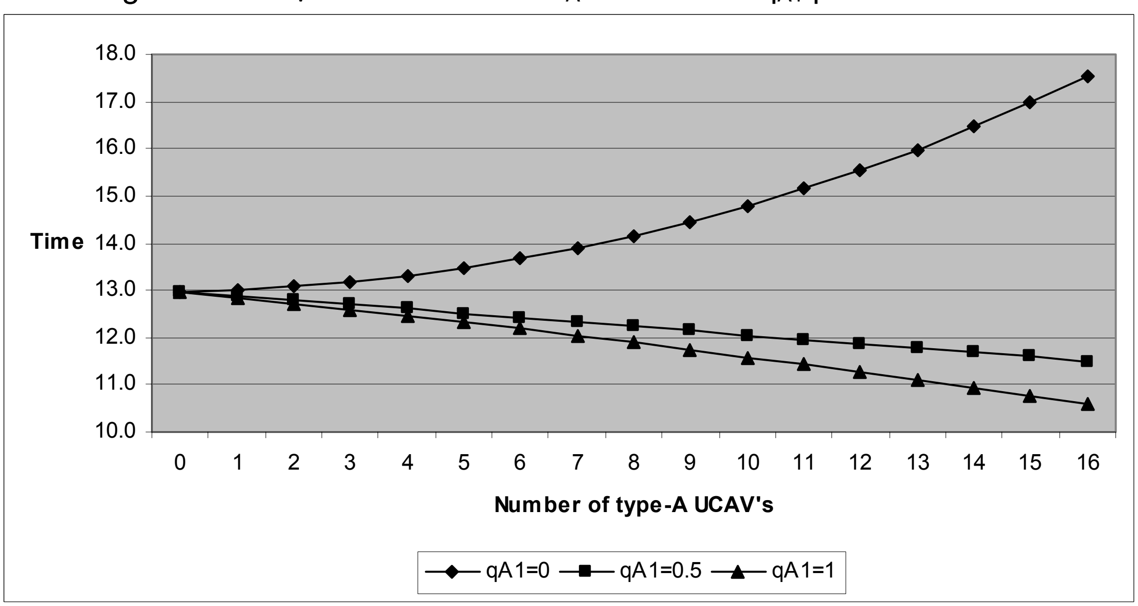

Figure 18. Time as a function of N_A for different q_{A1} parameter values

g. Sensitivity Analysis on q_{B2}

q_{B2} takes on values in the set {0, 0.5, 1}. When q_{B2} varies we observe an identical behavior as when q_{A1} varied. Thus, we should not primarily spend our resources to improve this capability of type-B UCAVs, either.

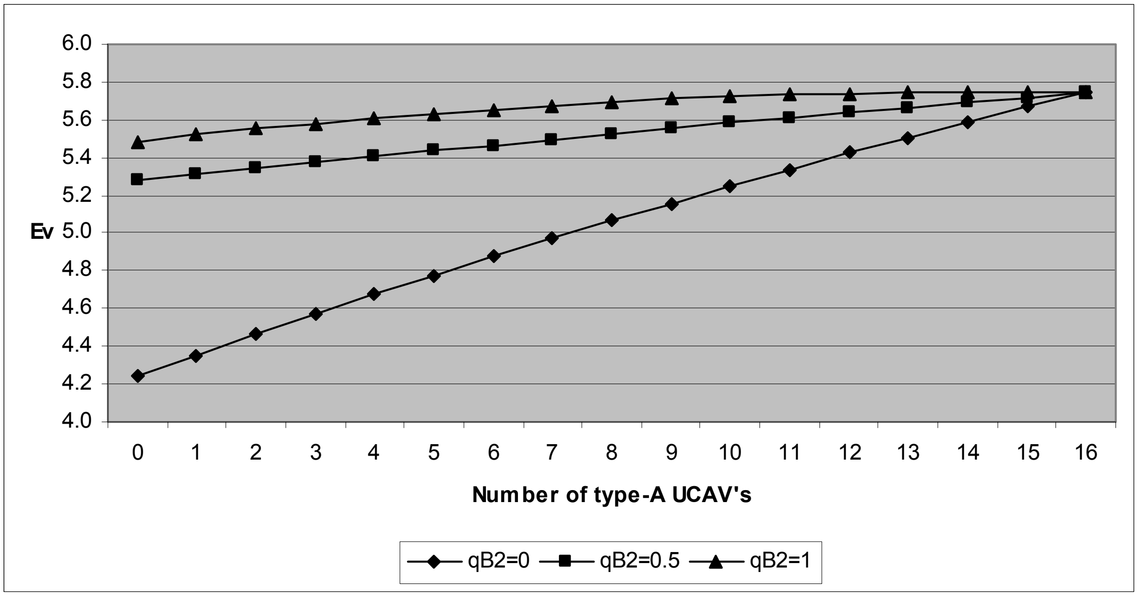

Figure 19. E_V as a function of N_A for different q_{B2} parameter values

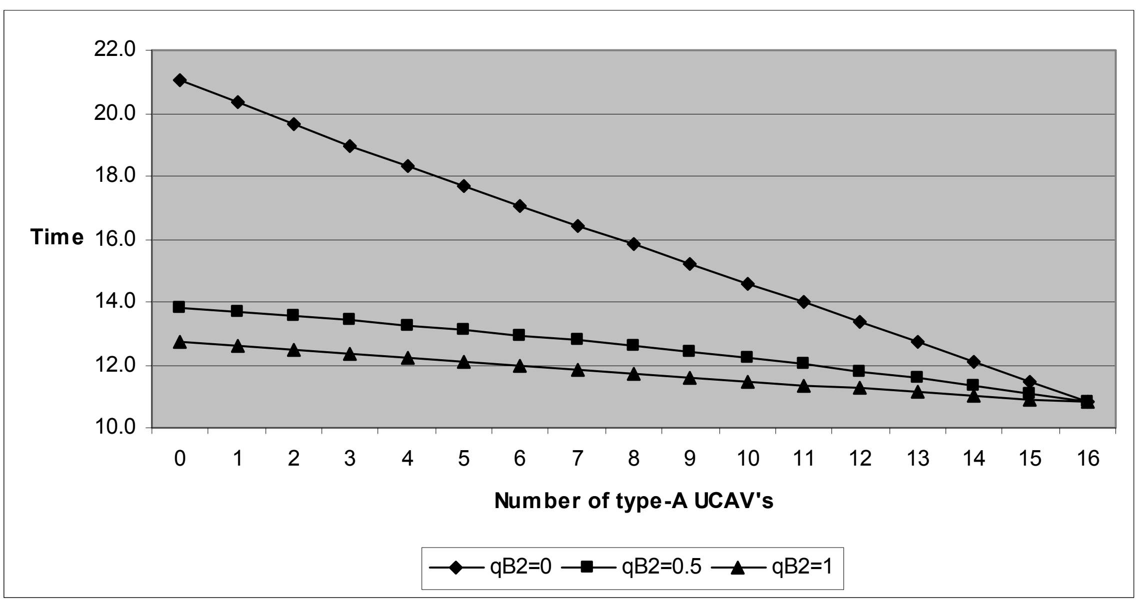

Figure 20. Time as a function of N_A for different q_{B2} parameter values

h. Sensitivity Analysis on r_A

r_A takes on values in the set {0, 0.5, 1}. By observing the following two plots we reach many conclusions. Firstly, if the type-A UCAV has perfect capability to recognize NVTs, then the more the type-A UCAVs the higher the expected operational value. The expected operation time, though, explodes. As r_A approaches 0, smaller N_A values give a more favorable E_V (and make no difference to the (small) expected operation time). If there is no data about the r_A value, then the best strategy is to employ type-B UCAVs only.

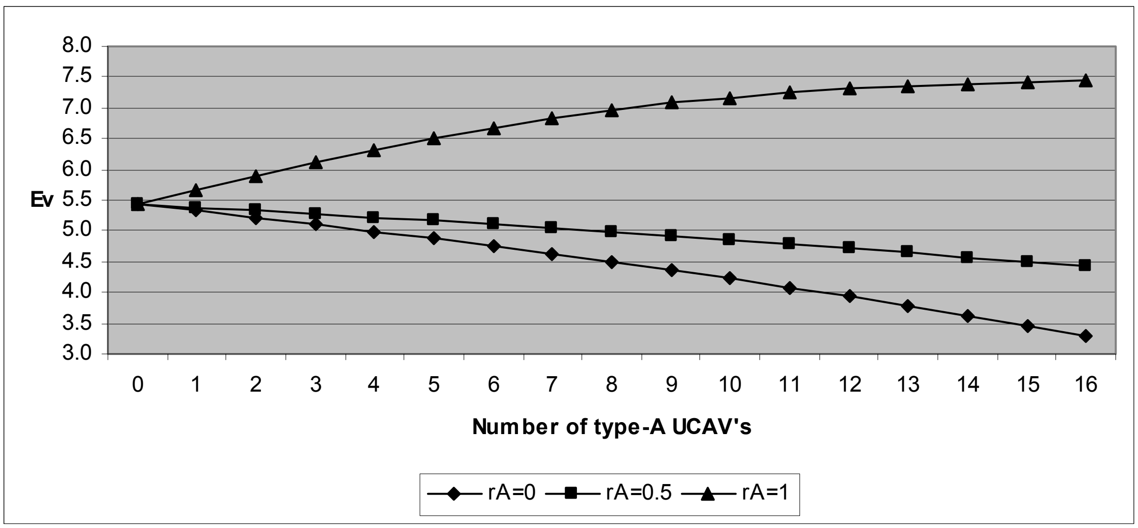

Figure 21. E_V as a function of N_A for different r_A parameter values

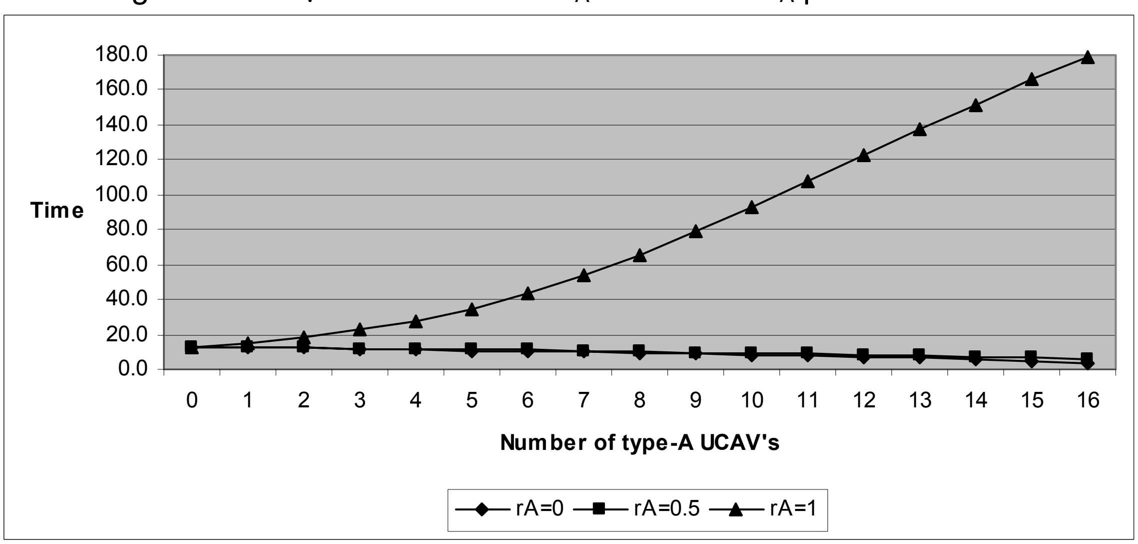

Figure 22. Time as a function of N_A for different r_A parameter values

i. *Sensitivity Analysis on r_B*

r_B takes on values in the set {0, 0.5, 1}. The results here are symmetrical to the results of the previous case. If the type-B UCAV capability to recognize NVTs is close to perfect, then the more the type-B UCAVs the higher the expected operational value. This, also, happens at a high cost in operational time. As r_B approaches 0, larger N_A values give a more favorable E_V (and make no difference to the (small) expected operation time). If there is no data about the r_B value, then the best strategy is to employ type-A UCAVs only.

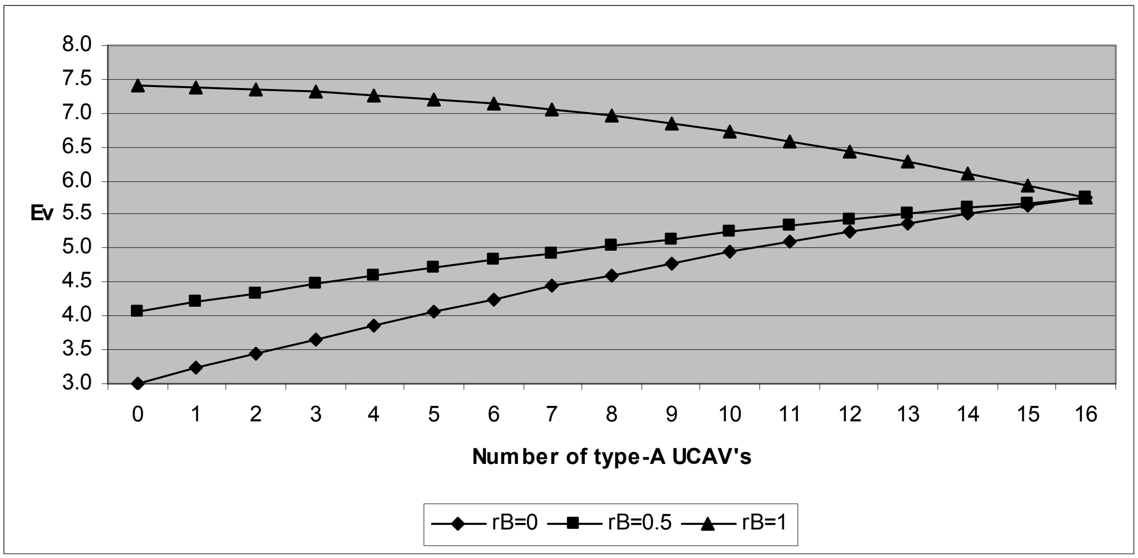

Figure 23. E_V as a function of N_A for different r_B parameter values

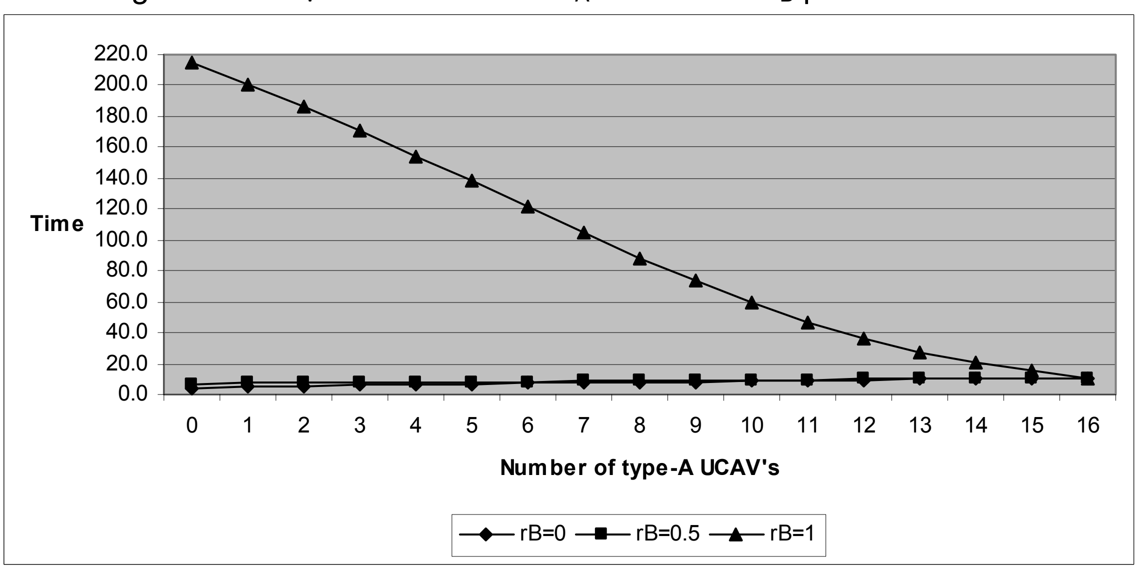

Figure 24. Time as a function of N_A for different r_B parameter values

j. Sensitivity Analysis on p_{A1}

p_{A1} takes on values in the set {0, 0.5, 1}. For p_{A1} = 1, a larger N_A is desirable. But the smaller the p_{A1} value gets the better we are with a smaller N_A. Effects on Time are less dramatic than they are on E_V. So, depending on whether p_{A1} is closer to 1 or to 0, the strategy differs.

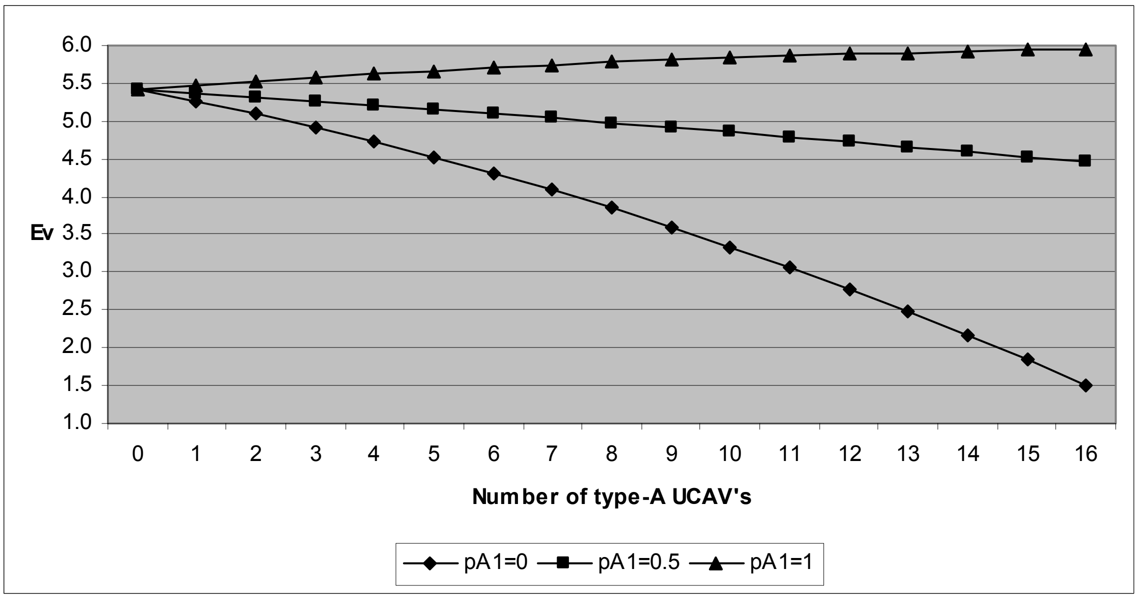

Figure 25. E_V as a function of N_A for different p_{A1} parameter values

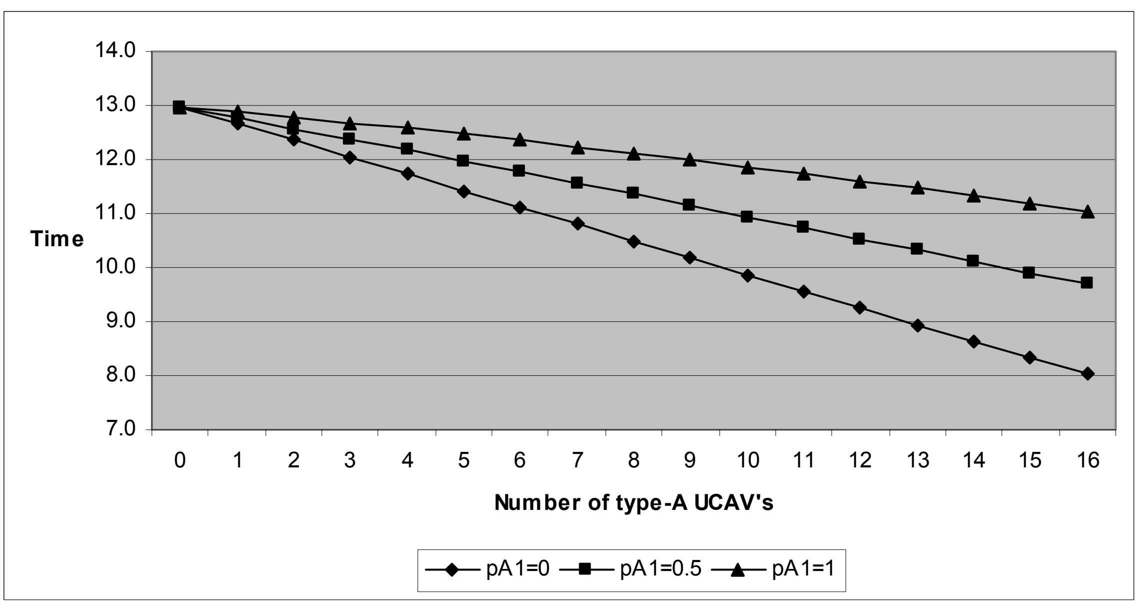

Figure 26. Time as a function of N_A for different p_{A1} parameter values

k. Sensitivity Analysis on p_{B2}

p_{B2} takes on values in the set {0, 0.5, 1}. The conclusion here is that we are always better off with as many type-A UCAVs as possible. Nonetheless, for p_{B2} = 1, the mission mix becomes less important.

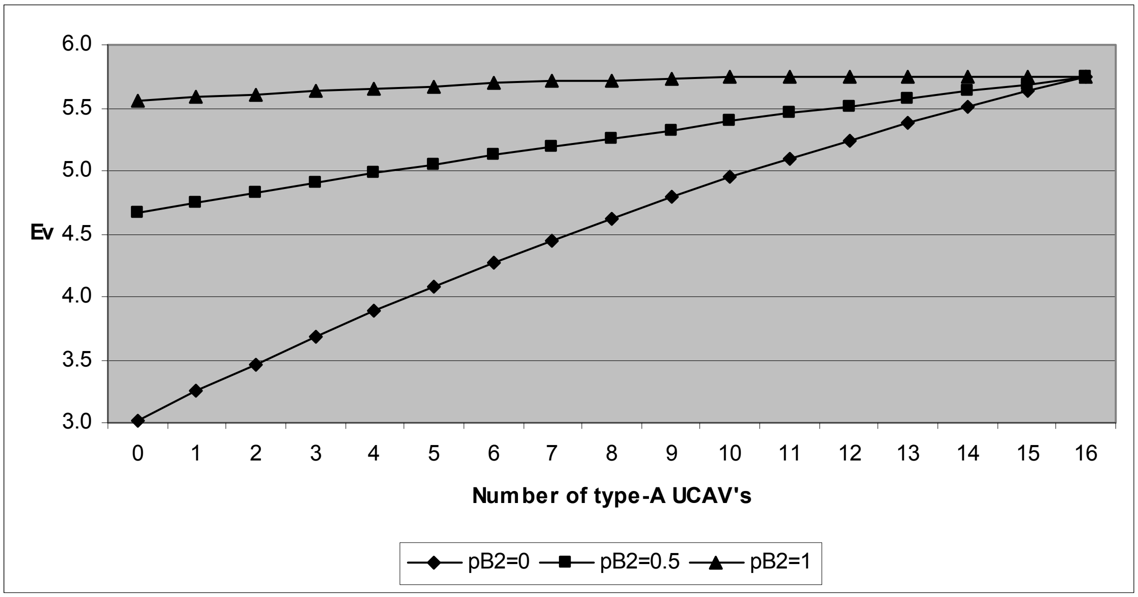

Figure 27. E_V as a function of N_A for different p_{B2} parameter values

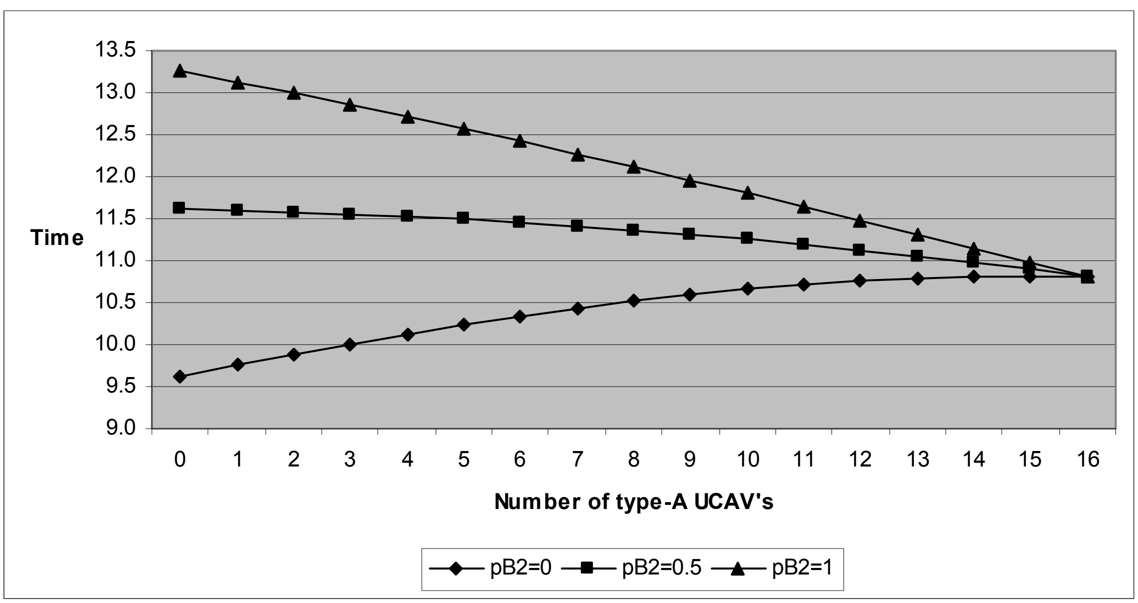

Figure 28. Time as a function of N_A for different q_{B2} parameter values

I. *Sensitivity Analysis on λ_A*

λ_A takes on values in the set {0.5, 1, 2}. The plots for this factor show that E_V is not significantly affected by λ_A, but Time is. We conclude that it is the magnitude of λ_A (and therefore of λ_B too) that determines the best mission mix time-wise. When λ_A is large enough, we should strive for more type-A UCAVs, otherwise the opposite is true.

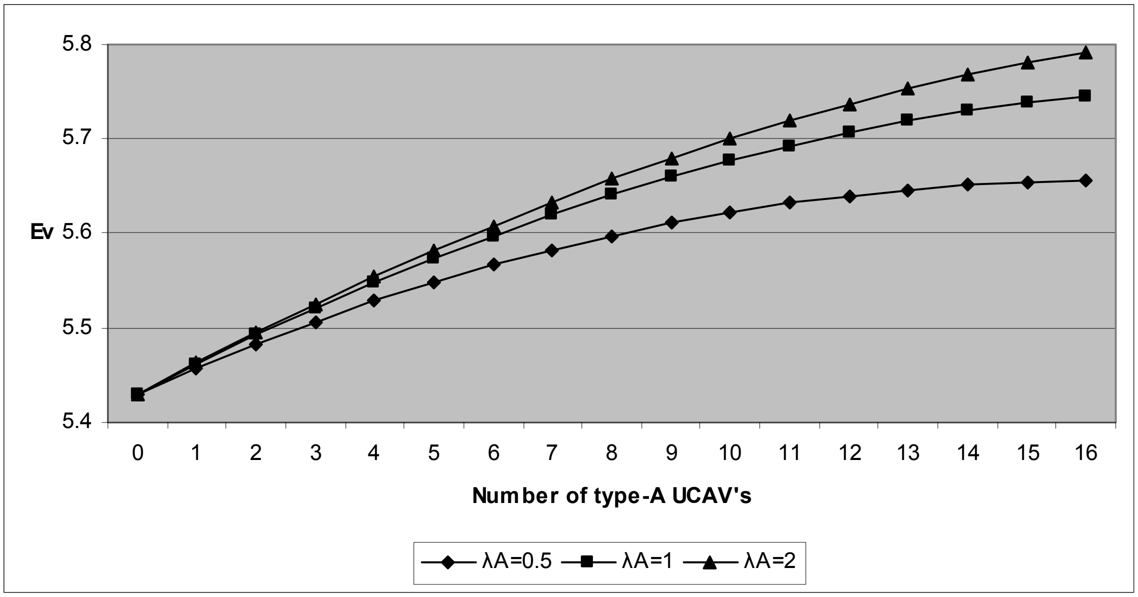

Figure 29. E_V as a function of N_A for different λ_A parameter values

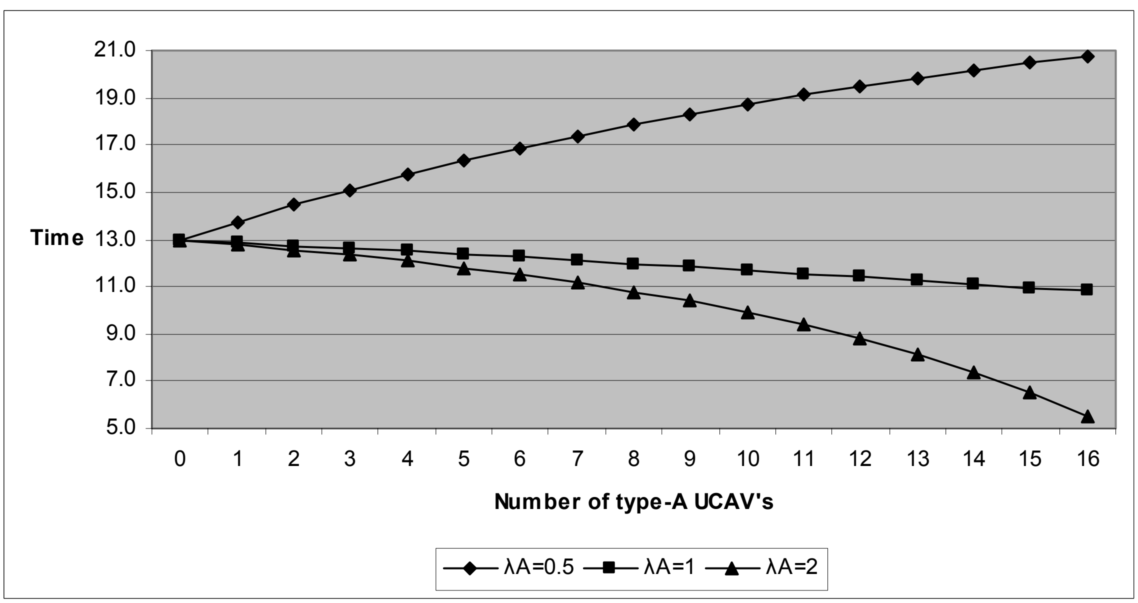

Figure 30. Time as a function of N_A for different λ_A parameter values

m. Sensitivity Analysis on θ_A

θ_A takes on values in the set {0.005, 0.010, 0.020}. The effect of θ_A on E_V is minimal. The same is true for Time. Note that if the operation time is a concern, then the best strategy remains to employ as many type-A UCAVs as possible, regardless of the θ_A value.

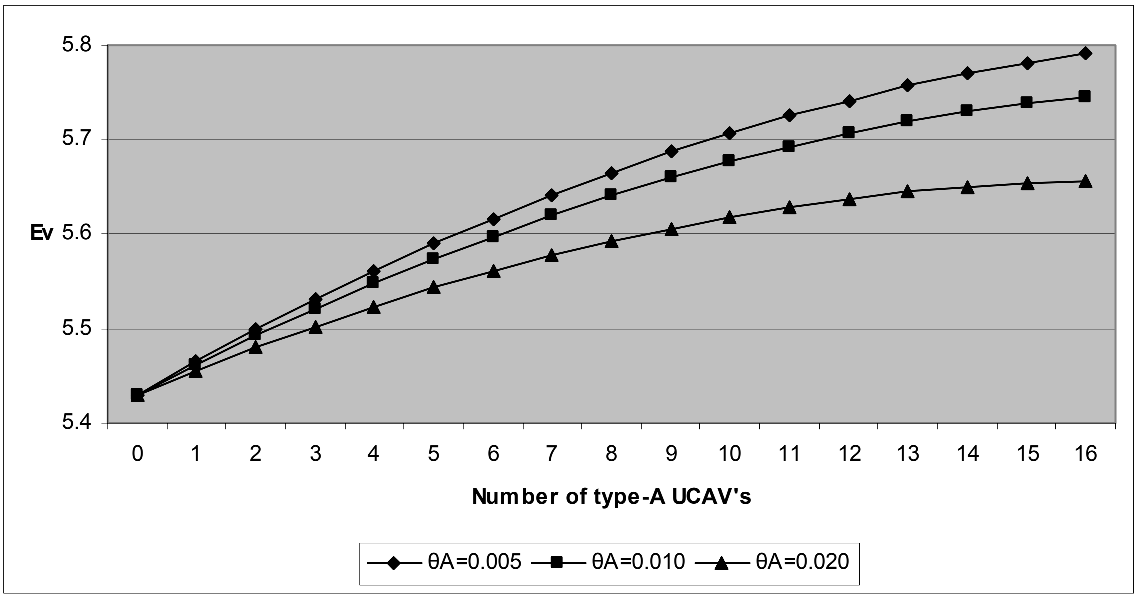

Figure 31. E_V as a function of N_A for different θ_A parameter values

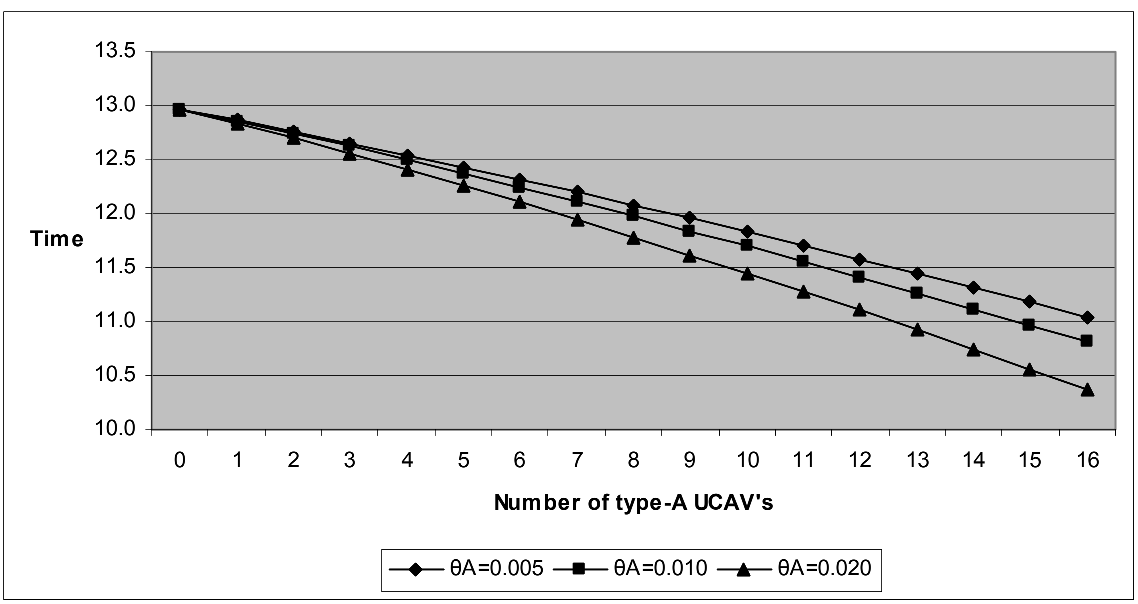

Figure 32. Time as a function of N_A for different θ_A parameter values

n. Additional Exploration

In this section, we explore the effect that the T_1/T_2 ratio and the handicap ratio have on the E_V value. This is one example of the many uses of our model. It demonstrates that by simultaneously varying more than one factor (and more so ratios of factors) we can gain additional insight, due to interactions revealed. This example lies between the NOLH DOE and the strict one-dimensional sensitivity analysis approaches.

The three figures below expose the main concept: if the ratio T_1/T_2 is equal to 1, then the optimal UCAV mix balances in the middle (i.e., $N_A = 8$). When T_1/T_2 is greater than 1, then the optimal mix tends to be displaced to the right (i.e., $N_A > 8$), and when the ratio is smaller than 1, the best mission mix is displaced to the left (i.e., $N_A < 8$). The further the T_1/T_2 ratio is from 1, the further the optimal N_A value is from the middle value (i.e., 8). These effects are more dramatic, for larger handicaps, as is displayed by the three plots in each figure.

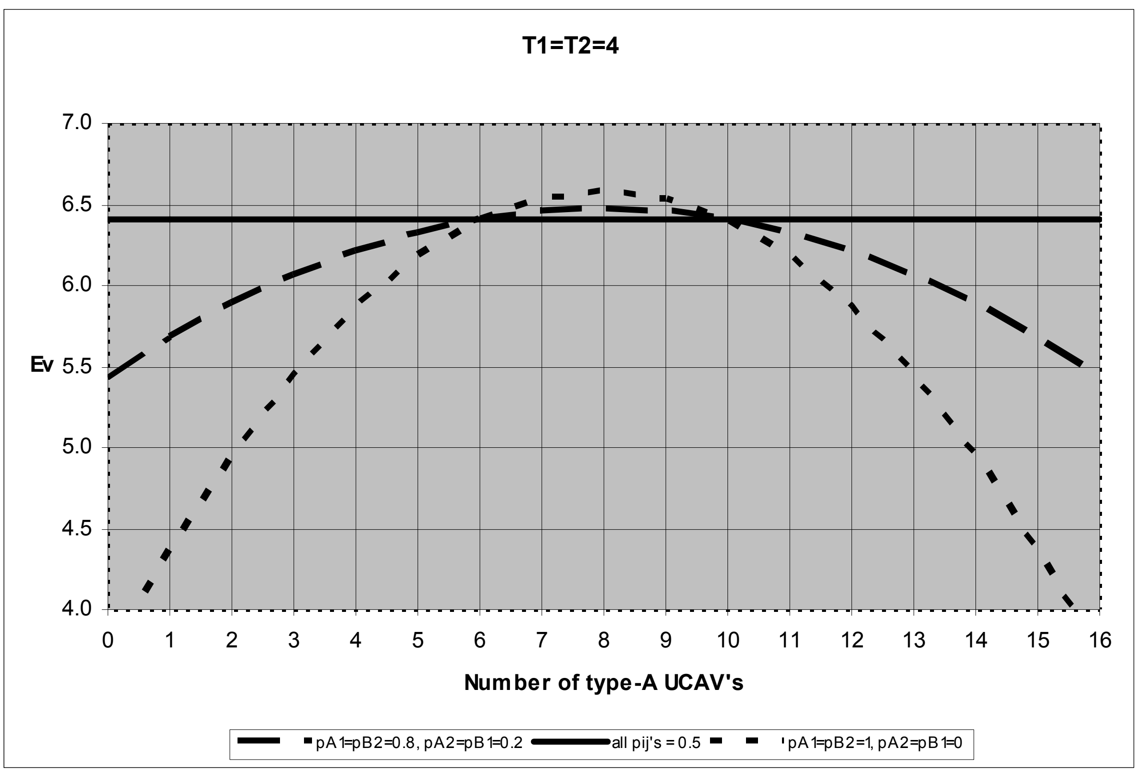

Figure 33. E_V as a function of N_A for different p_{ij} parameter scenarios when T_1=4 and T_2=4

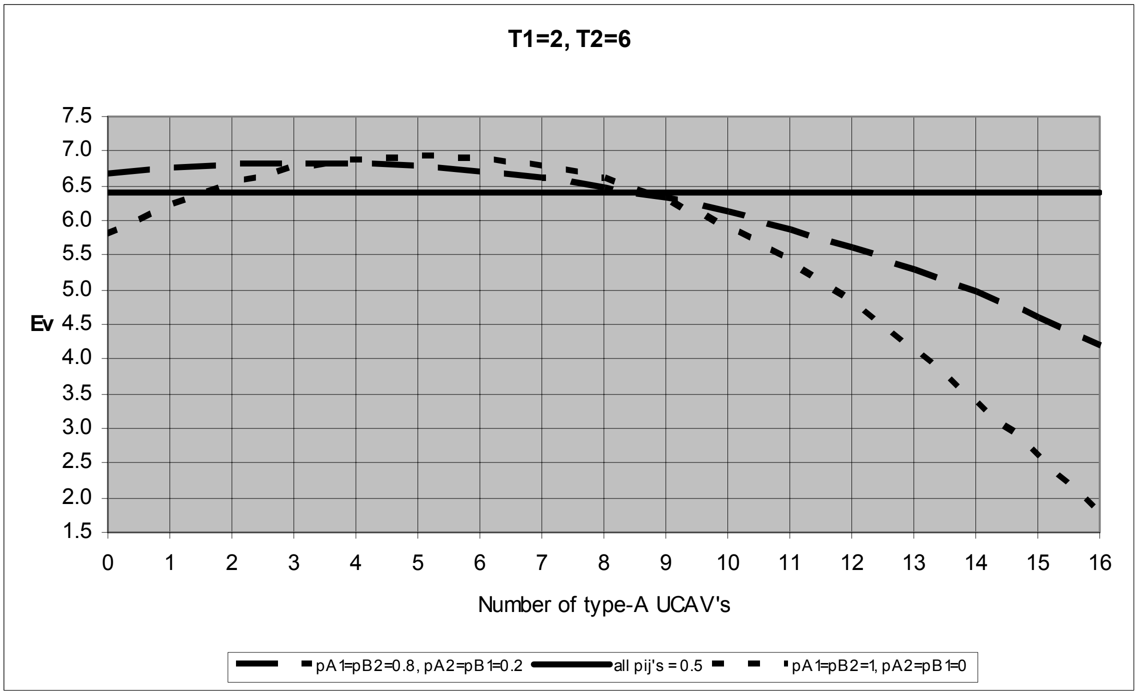

Figure 34. E_V as a function of N_A for different p_{ij} parameter scenarios when T_1=2 and T_2=6

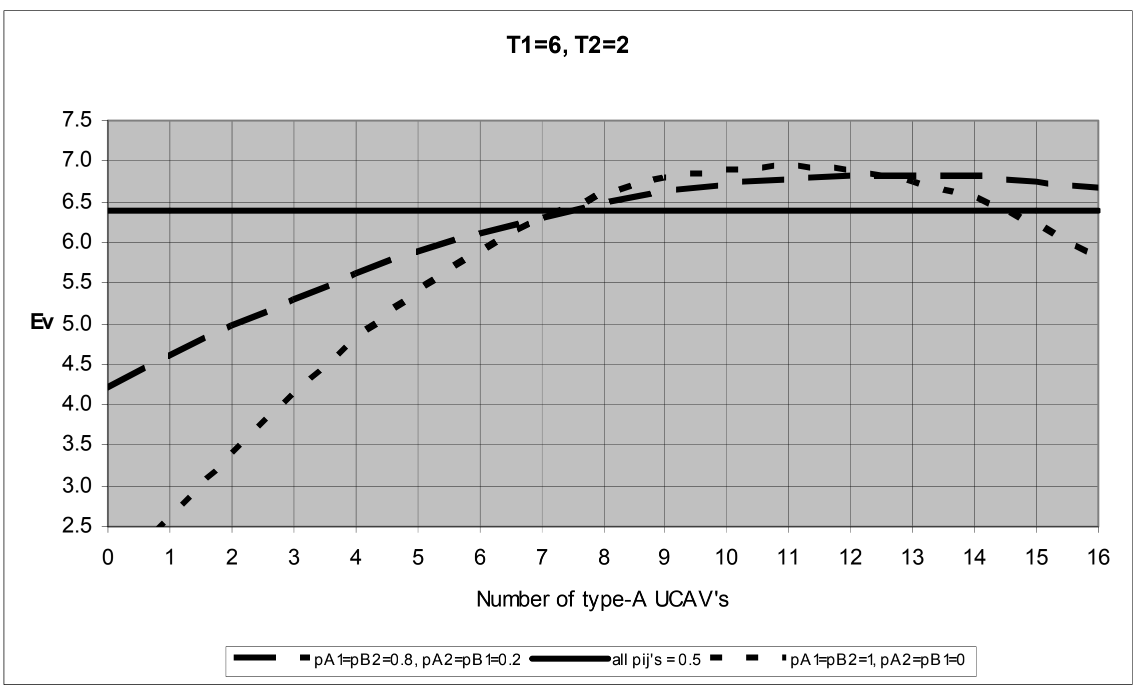

Figure 35. E_V as a function of N_A for different p_{ij} parameter scenarios when T_1=6 and T_2=2

G. CONCLUSIONS

As is often the case with DoD exploratory analysis, the model has very limited predictive power. Indeed, due to a lack of data, the model cannot be empirically validated. Rather, the model is used in a descriptive mode to help us devise new ideas or assess the consequences of certain assumptions. Potential insights gleaned from such exploration usually need to be tested elsewhere, perhaps by field experiments.

In this exploratory analysis we are primarily trying to identify the optimal mission mix, and secondarily the factors that have a strong effect on the MOE values, the directions of those effects, and which, if any, factors interact.

By employing the broad experimental design setup, we observe that N_A (mission mix) does not significantly affect E_V (expected military value). This observation is due to the balancing effect of the factor values. Although there is some effect on the other MOE, Time, it does not seem to be that significant either. Nevertheless, for a more robust Time outcome, absent hard data, it is better to employ a balanced mix of UCAVs of different types, instead of a biased one where most of the UCAVs are of the same type.

By employing the narrow experimental design setup, we decreased the overall design noise, and we reached the following results:

- A larger value of N_A is always better for the military value of the operation, but it is only worth the extra cost (note that a type-A UCAV is more costly than a type-B UCAV) when the differences in design characteristics (like kill probability, detection rate, etc) between the two UCAV types are substantial and the operational values of the two types of targets are quite apart.
- For time critical missions, sending as many type-A UCAVs as possible, is always a good strategy, no matter how small the difference between the two types of UCAVs (note that if there are no differences then type-B UCAVs should be employed exclusively since they are less expensive) or how small the difference in the operational values of the targets.

By running a sensitivity analysis on our model, we reach some conclusions pertaining to the individual parameters as follows:

- Given a high T_1 value (number of type-1 targets), the use of more type-A UCAVs becomes more imperative versus the use of type-B ones. Of course, time critical missions dictate the use of more type-A UCAVs, no matter how many type-1 targets there are.
- A higher N_A value is always more desirable, even for small T_2 values (number of type-2 targets).
- For T_{V2} values (type-2 target military value) comparable to T_{V1} values (type-1 target military value), the mission mix is irrelevant, but for comparatively small T_{V2} values, a larger number of type-A UCAVs generates a higher operational value.
- Improving the recognition capabilities (which are typically good enough already) of UCAVs (of either type) should not be on a high priority (i.e., we should allocate available resources into improving other aspects first).
- If there is no data about the recognition capabilities of a type-A UCAV (r_A), then the best strategy is to employ type-B UCAVs only. Otherwise, if the type-A UCAV capability to recognize NVTs is close to perfect, then the more the type-A UCAVs the higher the expected operational value. In this case, though, the expected operation time gets long. As r_A approaches 0, smaller N_A values give a more favorable E_V (and make no difference to the (small) expected operation time).
- The conclusions for r_B (recognition capability of type-B UCAV) are symmetrical to the conclusions for r_A stated above.
- For high p_{A1} (kill probability of type-A UCAV against a type-1 target) values (i.e., close to 1), a larger N_A is desirable. The smaller the p_{A1} value gets the better we are with a smaller N_A. Effects on Time are less dramatic than they are on E_V. So, depending on whether p_{A1} is closer to 1 or to 0, the strategy differs.

- No matter what the value of p_{B2} (kill probability of type-B UCAV against a type-2 target) is, we are always better off with as many type-A UCAVs as possible. Nonetheless, for very high p_{B2} values the mission mix becomes less important. Note that, as it is implied by the experimental design, the p_{B2} value is correlated (i.e., not far from) to the p_{A2} value, for this conclusion to hold.
- E_V is not significantly affected by λ_A (type-A UCAV detection rate), but Time is, and it is the relationship between λ_A and λ_B (type-B UCAV detection rate) that determines the best mission mix time-wise. When λ_A is greater than λ_B, we should strive for more type-A UCAVs and when λ_B is greater than λ_A the opposite is true.
- The effect of θ_A (type-A UCAV failure rate) on E_V is minimal. The same is true for Time. Of course this is due to the large $\frac{\lambda_A}{\theta_A}$ ratio (by the UCAV design).

APPENDIX. MATLAB CODE FOR THE BASIC MODEL

This Appendix contains the Matlab code for the basic model. By adopting this code and slightly modifying it, we can also implement the extension of the basic model presented in Chapter IV, or even other potential extensions.

```
format compact

format short

A = xlsread('DOEinput.xls');
[row, col] = size(A);

N = 16;  %hard-wired value

u = zeros(2);
u(1,1) = 0;
u(1,2) = 0;
u(2,1) = 0;
u(2,2) = 0;

for j = 1:row

T1 = A(j,1);  %input
T2 = A(j,2);  %input
T3 = A(j,3);  %input

Tvalue = zeros(2,1);
Tvalue(1) = A(j,4);  %input
Tvalue(2) = A(j,5);  %input

q = zeros(2);
q(1,1) = A(j,6);  %input
q(2,1) = A(j,7);  %input
q(2,2) = A(j,8);  %input
```

```
q(1,2) = A(j,9);  %input

r = zeros(2,1);
r(1) = A(j,10);  %input
r(2) = A(j,11);  %input

p = zeros(2);
p(1,1) = A(j,12);  %input
p(2,1) = A(j,13);  %input
p(2,2) = A(j,14);  %input
p(1,2) = A(j,15);  %input

lamda = zeros(2,1);
lamda(1) = A(j,16);  %input
lamda(2) = A(j,17);  %input

theta = zeros(2,1);
theta(1) = A(j,18);  %input
theta(2) = A(j,19);  %input

N1 = A(j,20);  %input
N2 = N - N1;  %derived

n1 = 0;
n2 = 0;
t1 = 0;
t2 = 0;

numFeas = 0;  %counts total number of feasible states
numAbs = 0;  %counts total number of absorbing states

rn = (N1 + 1)*(N2 + 1)*(T1 + 1)*(T2 + 1);

state = zeros(rn, 6);

for n1 = 0:N1
  for n2 = 0:N2
```

```
      for t1 = 0:T1
        for t2 = 0:T2
          if(T1+T2-N1-N2<=t1+t2-n1-n2 & n1>=0 & n2>=0 & t1>=0 & t2>=0)
            numFeas = numFeas + 1;
            state(numFeas, 1) = numFeas;
            state(numFeas, 2) = n1;
            state(numFeas, 3) = n2;
            state(numFeas, 4) = t1;
            state(numFeas, 5) = t2;
            state(numFeas, 6) = 0;
            if( (n1 + n2) == 0 )
                numAbs = numAbs + 1;
                state(numFeas, 6) = 1;  %this flags an absorbing state
             end
          end
        end
      end
    end
  end

  stateAbs = zeros(numAbs, 6);  %temp storage of absorbing states
  numTrans = numFeas - numAbs;  %number of transient states
  stateTrans = zeros(numTrans, 6);  %temp storage of transient states
  nextAbs = 1;  %counter for the next available line of 'stateAbs'
  nextTrans = 1;  %counter for the next available line of 'stateTrans'

  for ix = 1:numFeas
     if( state(ix,6) == 1 )
        stateAbs(nextAbs, :) = state(ix, :);
        stateAbs(nextAbs, 1) = nextAbs;  %allowing for consecutive indices
        nextAbs = nextAbs + 1;
     else
        stateTrans(nextTrans, :) = state(ix, :);
        stateTrans(nextTrans, 1) = numAbs + nextTrans;
        nextTrans = nextTrans + 1;
     end
  end
```

```
state = [stateAbs ; stateTrans];

state = state(1:numFeas, :);

P = zeros(numFeas);

for ix = 1:numFeas
  for iy = 1:numFeas
    %Case 1: from (n1, n2, t1, t2) to (n1 - 1, n2, t1 - 1, t2) state
    if( state(ix,2)-1 == state(iy,2) & state(ix,3) == state(iy,3) ...
         & state(ix,4)-1 == state(iy,4) & state(ix,5) == state(iy,5) )
        P(ix,iy) = 1;
    %Case 2: from (n1, n2, t1, t2) to (n1 - 1, n2, t1, t2) state
    elseif ( (state(ix,2)-1 == state(iy,2)) & (state(ix,3) == ...
         state(iy,3)) & (state(ix,4) == state(iy,4)) & ...
         (state(ix,5) == state(iy,5)) )
        P(ix,iy) = 2;
    %Case 3: from (n1, n2, t1, t2) to (n1, n2 - 1, t1, t2) state
    elseif ( (state(ix,2) == state(iy,2)) & (state(ix,3)-1 == ...
         state(iy,3)) & (state(ix,4) == state(iy,4)) & ...
         (state(ix,5) == state(iy,5)) )
        P(ix,iy) = 3;
    %Case 4: from (n1, n2, t1, t2) to (n1 - 1, n2, t1, t2 - 1) state
    elseif ( (state(ix,2)-1 == state(iy,2)) & (state(ix,3) == ...
         state(iy,3)) & (state(ix,4) == state(iy,4)) & ...
         (state(ix,5)-1 == state(iy,5)) )
        P(ix,iy) = 4;
    %Case 5: from (n1, n2, t1, t2) to (n1, n2 - 1, t1 - 1, t2) state
    elseif ( (state(ix,2)  == state(iy,2)) & (state(ix,3)-1 == ...
         state(iy,3)) & (state(ix,4)-1 == state(iy,4)) & ...
         (state(ix,5) == state(iy,5)) )
        P(ix,iy) = 5;
    %Case 6: from (n1, n2, t1, t2) to (n1, n2 - 1, t1, t2 - 1) state
    elseif ( (state(ix,2)  == state(iy,2)) & (state(ix,3)-1 == ...
         state(iy,3)) & (state(ix,4) == state(iy,4)) & ...
         (state(ix,5)-1 == state(iy,5)) )
        P(ix,iy) = 6;
```

```
    %Case 7: from (n1, n2, t1, t2) to (n1, n2, t1, t2) state (the same)
    elseif ( (state(ix,2)  == state(iy,2)) & (state(ix,3) == ...
        state(iy,3)) & (state(ix,4) == state(iy,4)) & ...
        (state(ix,5) == state(iy,5)) )
       P(ix,iy) = 7;
    else
    %None of the 7 cases; meaning that no transition happens
       P(ix,iy) = 0;
    end
  end
end

[x1,y1] = find(P == 1);
[x2,y2] = find(P == 2);
[x3,y3] = find(P == 3);
[x4,y4] = find(P == 4);
[x5,y5] = find(P == 5);
[x6,y6] = find(P == 6);
[x7,y7] = find(P == 7);

%Case 1
len1 = length(x1);
for ix = 1:len1
  P(x1(ix),y1(ix)) = lamda(1)*p(1,1)*state(x1(ix),2)*state(x1(ix),4)/ ...
    ((lamda(1)+theta(1))*state(x1(ix),2) + (lamda(2)+theta(2)) ...
    *state(x1(ix),3))/(T1+T2+T3)*(q(1,1)+u(1,2)*(1-q(1,1)));
end
%Case 2
len2 = length(x2);
for ix = 1:len2
  P(x2(ix),y2(ix)) = state(x2(ix),2)*theta(1)/((lamda(1)+theta(1)) ...
    *state(x2(ix),2) + (lamda(2)+theta(2))*state(x2(ix),3)) + ...
    state(x2(ix),2)*lamda(1)/((lamda(1)+theta(1))*state(x2(ix),2)+ ...
    (lamda(2)+theta(2))*state(x2(ix),3))/(T1+T2+T3)* ...
    (state(x2(ix),4)*(q(1,1)+u(1,2)*(1-q(1,1)))*(1-p(1,1))+ ...
    state(x2(ix),5)*(q(1,2)+u(1,1)*(1-q(1,2)))*(1-p(1,2))+ ...
    (T1+T2-state(x2(ix),4)-state(x2(ix),5)+T3)*(1-r(1)));
```

```
end
%Case 3
len3 = length(x3);
for ix = 1:len3
   P(x3(ix),y3(ix)) = state(x3(ix),3)*theta(2)/((lamda(1)+theta(1)) ...
      *state(x3(ix),2) + (lamda(2)+theta(2))*state(x3(ix),3)) + ...
      state(x3(ix),3)*lamda(2)/((lamda(1)+theta(1))*state(x3(ix),2)+ ...
      (lamda(2)+theta(2))*state(x3(ix),3))/(T1+T2+T3)* ...
      (state(x3(ix),4)*(q(2,1)+u(2,2)*(1-q(2,1)))*(1-p(2,1))+ ...
      state(x3(ix),5)*(q(2,2)+u(2,1)*(1-q(2,2)))*(1-p(2,2))+ ...
      (T1+T2-state(x3(ix),4)-state(x3(ix),5)+T3)*(1-r(2)));
end
%Case 4
len4 = length(x4);
for ix = 1:len4
   P(x4(ix),y4(ix)) = lamda(1)*p(1,2)*state(x4(ix),2)*state(x4(ix),5)/ ...
      ((lamda(1)+theta(1))*state(x4(ix),2) + (lamda(2)+theta(2)) ...
      *state(x4(ix),3))/(T1+T2+T3)*(q(1,2)+u(1,1)*(1-q(1,2)));
end
%Case 5
len5 = length(x5);
for ix = 1:len5
   P(x5(ix),y5(ix)) = lamda(2)*p(2,1)*state(x5(ix),3)*state(x5(ix),4)/ ...
      ((lamda(1)+theta(1))*state(x5(ix),2) + (lamda(2)+theta(2)) ...
      *state(x5(ix),3))/(T1+T2+T3)*(q(2,1)+u(2,2)*(1-q(2,1)));
end
%Case 6
len6 = length(x6);
for ix = 1:len6
   P(x6(ix),y6(ix)) = lamda(2)*p(2,2)*state(x6(ix),3)*state(x6(ix),5)/ ...
      ((lamda(1)+theta(1))*state(x6(ix),2) + (lamda(2)+theta(2)) ...
      *state(x6(ix),3))/(T1+T2+T3)*(q(2,2)+u(2,1)*(1-q(2,2)));
end
%Case 7
len7 = length(x7);
for ix = 1:len7
   %Distinguishing between absorbing and non-absorbing states
```

```
    if (state(x7(ix),2) == 0 & state(x7(ix),3) == 0)
        P(x7(ix),y7(ix)) = 1;
    else
        P(x7(ix),y7(ix)) = lamda(1)*state(x7(ix),2)/ ...
            ((lamda(1)+theta(1))*state(x7(ix),2) + (lamda(2)+theta(2)) * ...
            state(x7(ix),3))/(T1+T2+T3)*(state(x7(ix),4)* ...
            (1-u(1,2))*(1-q(1,1))+state(x7(ix),5)*(1-u(1,1))* ...
            (1-q(1,2))+(T1+T2-state(x7(ix),4)-state(x7(ix),5)+T3)*r(1)) ...
            + lamda(2)*state(x7(ix),3)/ ...
            ((lamda(1)+theta(1))*state(x7(ix),2) + ...
            (lamda(2)+theta(2))*state(x7(ix),3))/(T1+T2+T3)*(state(x7(ix),4)*...
            (1-u(2,2))*(1-q(2,1))+state(x7(ix),5)*(1-u(2,1))* ...
            (1-q(2,2))+(T1+T2-state(x7(ix),4)-state(x7(ix),5)+T3)*r(2));
    end
end

num1 = 0;  %stores the number of lines that have errors
disp('Checking Transition Matrix for Integrity and Errors')
for ix = 1:numFeas
    if (abs(sum(P(ix,:))) - 1 >= 0.0001)
        disp(['Error in Transition Matrix, line ' num2str(ix)])
        num1 = num1 + 1;
    end
end
disp(['Number of lines containing errors: ' num2str(num1)])

R = P(numAbs+1:numFeas, 1:numAbs);
Q = P(numAbs+1:numFeas, numAbs+1:numFeas);
I = eye(size(Q));
I_Q = I - Q;
I_Q_inv = inv(I_Q);
I_Q_inv_R = I_Q_inv * R;

num2 = 0;  %stores the number of lines that have errors
disp(' ')  %insert an empty line on the screen
disp('Checking Trans-to-Absorb Matrix for Integrity and Errors')
for ix = 1:numTrans
```

```
   if (abs(sum(I_Q_inv_R(ix,:))) - 1 >= 0.0001)
      disp(['Error in Trans-to-Absorb Matrix, line ' num2str(ix)])
      num2 = num2 + 1;
   end
end
disp(['Number of lines containing errors: ' num2str(num2)])

initialState = 0;

for ix = numAbs+1:numFeas
   if( state(ix,2) + state(ix,3) + state(ix,4) + state(ix,5) == ...
      N1 + N2 + T1 + T2)
         initialState = ix - numAbs;
         break;
   end
end

expNumKilled_T1 = 0;
for iy = 1:numAbs
  expNumKilled_T1 = expNumKilled_T1 ...
  + I_Q_inv_R(initialState, iy)*( T1 - state(iy,4) );
end

expNumKilled_T2 = 0;
for iy = 1:numAbs
  expNumKilled_T2 = expNumKilled_T2 ...
  + I_Q_inv_R(initialState, iy) * ( T2 - state(iy,5) );
end

Ev = expNumKilled_T1 * Tvalue(1) + expNumKilled_T2 * Tvalue(2);

Time = 0;
for k = 1:numTrans
   Time = Time + I_Q_inv(initialState,k) / ...
      (state(numAbs+k,2)*(lamda(1)+theta(1))+state(numAbs+k,3)* ...
      (lamda(2)+theta(2)));
end
```

```
A(j, col + 1) = Ev;
A(j, col + 4) = Time;

end

save 129x17_Model_A_NOLH_output.xls A -ascii;

disp(' ')  %insert an empty line on the screen
if( num1 + num2 == 0 )
   disp('Script successfully completed.')
   disp('Check workspace for more results.')
else
   disp('There are flaws in the code. Debugging needed.')
end
```

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF REFERENCES

Baggesen, A., *Design and Operational Aspects of Autonomous Unmanned Combat Combat Aerial Vehicles*, Master's Thesis, Naval Postgraduate School, Monterey, California, September 2005.

Bone, E. and Bolkcom, C., Unmanned Aerial Vehicles: Background and Issues for Congress, Report for Congress, 2003.

Cioppa, T.M., Efficient *Nearly Orthogonal and Space-Filling Experimental Designs for High-Dimensional Complex Models*, Ph.D. Dissertation, U.S. Naval Postgraduate School, Monterey, CA, September 2002.

Corner, J.C. and Lamont G.B., "Parallel Simulation of UAV Swarm Scenarios" paper presented at the Proceedings of the 2004 Winter Simulation Conference.

Devore, J.L., *Probability and Statistics for Engineering and the Sciences*, Sixth Edition, Brooks/Cole, 2004.

Gillen, D.P. and Jacques, D.R., *Cooperative Behavior Schemes for Improving the Effectiveness of Autonomous Wide Area Search Munitions*, Kluwer Academic Publishers, 2002.

Jacques, D.R., "Modeling Considerations for Wide Area Search Munition Effectiveness Analysis" paper presented at the Proceedings of the 2002 Winter Simulation Conference.

Kress M., Baggesen A., and Gofer E., "Probability Modeling of Autonomous Unmanned Combat Aerial Vehicles (UCAVs)" *Military Operations Research*, v. 11, pp. 5-24, number 4 2006.

McMindes, K.L., *Unmanned Aerial Vehicle Survivability: the Impacts of Speed, Detectability, Altitude, and Enemy Capabilities*, Master's Thesis, Naval Postgraduate School, Monterey, California, September 2005.

RAND, *Proliferated Autonomous Weapons*; An Example of Cooperative Behavior, by Frelinger, David, et al, 1998.

Sanchez, S.M., "ABCs of Output Analysis" paper presented at the Proceedings of the 2001 Winter Simulation Conference.

Sulewski, C.A., *An Exploration of Unmanned Aerial Vehicles in the Army's Future Combat Systems Family of Systems*, Master's Thesis, Naval Postgraduate School, Monterey, California, December 2005.

THIS PAGE INTENTIONALLY LEFT BLANK

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California