Asymmetric Wargaming: Toward A Game Theoretic Perspective

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Abstract

As we enter the 21st century the art and practice of warfare is radically changing. The US has emerged as the dominant conventional military power only to find its adversaries working their way out of the box. The Defense Advanced Research Agency Information Systems Office (DARPA/ISO) which is seeking new approaches to asymmetric threat modeling, analysis and prediction sponsored this work as well as several related research efforts during FY 2000. This paper enumerates some of the main features of the asymmetric environment and summarizes shortfalls in our current wargame technology. It is argued that contemporary developments in game theory provide a flexible and promising framework in which to efficiently model adversarial motivation and to generate representative asymmetric strategies for improved automation of behaviors in simulations and to support Information Operations analysis and planning. Genetic programming and reinforcement learning are suggested approaches for extraction and refinement of multi-player models from historical data.

Overview

The Defense Advanced Research Projects Agency Information Systems Office (DARPA/ISO) under the guidance of Larry Willis (Wargaming the Asymmetric Environment Program Manager) is investigating technology needs and opportunities for asymmetric wargaming. During FY2000 several efforts were launched to extract predictive models from historical behavioral data, and to explore more powerful and flexible modeling and analysis technology that could be readily applied to asymmetric conflict. This research sponsored by DARPA is presented in four sections:

1. Description and nature of the asymmetric threat
2. DoD wargaming needs with respect to the asymmetric threat
3. Game theoretic and related approaches to modeling and wargaming the asymmetric threat
4. An outline of selected R&D needs for practical and rapid application of game theoretic approaches.

Asymmetric Threat: Nature and Challenge

The term, asymmetric, as applied to asymmetric threat or asymmetric warfare has several meanings, but they are not often carefully distinguished in common parlance. Fundamentally, asymmetry leverages the offensive/defensive equilibrium to the perpetrator's perceived advantage by exploiting defense vulnerabilities or offense restraints with unconventional, relatively inexpensive, methods. An asymmetric attack is much less expensive to wage than it is to defend against. Conversely, it is more difficult (expensive) to penetrate an asymmetric defense tactic than it is to set one up. For example in the UK's Kosovo Lessons from the Crisis¹ we read:

"The Kosovo campaign was notable for the wide use of asymmetric (that is to say non-conventional) tactics by the Yugoslav/Serbian forces. Examples included: the location of tanks and other military equipment in the middle of villages and in other locations where the Yugoslav/Serbian forces knew that our concern to minimise collateral damage would prevent us from targeting them; at least one case of the use of human shields was documented by Human

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The original document contains color images.
In this example the Yugoslavian and Serbian forces understood and took advantage of NATO and Allied forces offensive restraint and were able to win either way. Declining the inevitable collateral damage and loss of innocent civilian life would preserve the Serbian war machine; callously attacking the human shielded targets would be a double loss, first morally and second by the public relations crisis such an event would certainly develop through the inevitable news coverage. The Serbs skillfully controlled news exposure of events during this campaign. They were masters of camouflage, decoys, and deception as well (skills that were developed and retained from WWII). Tank turrets replaced atop destroyed tank hulls repeatedly fooled battle damage assessment (BDA) effectively multiplying the cost of the average tank kill.

Admiral James O. Ellis, Commander in Charge (CINC) US Naval Forces in Europe (CINCUSNAVEUR), Commander Allied Forces Southern Europe, and commander of Joint Task Force (JTF) Noble Anvil during Operation Allied Force says that the allied Operations Plan (OPLAN) focused on brief, single-dimension combat\(^2\). Consequently, deception, diversion & feint opportunities were lost; we failed to adequately plan for branches and sequels. In an age when national decision making and commitment is driven more by public opinion than by policy principles and leadership we are particularly vulnerable to enemy information operations (IO) and propaganda which are generally considered to be tools in the asymmetric war chest. As a consequence, modern asymmetric conflict tends to simultaneously expand the dimensionality of conflict and amalgamate concerns, decisions and actions conventionally separated into strategic, operational and tactical categories. All of the following slowed the Decide-Act side of our own Observe-Orient-Decide-Act\(^3\) (OODA) loops and reduced our control of the operational tempo (OPSTEMPO):

- The NCA / NAC target approval processes
- Our poor OPSEC posture (NATO and US)
- Our inability to wage full IO campaign
- Our self-suspension on cluster munitions
- Our standards for limiting Collateral Damage
- Our aversion to US casualties and ground combat
- Our reactive vs. proactive Public Info & Public Affairs

Admiral Ellis states that IO has "incredible potential…must become our asymmetric point of main effort". In his judgement, properly executed IO could have halved the length of the Operation Allied Force campaign.

The acquisition, operation and maintenance of our Command, Control, Communications, Computers, and Intelligence (C4I) infrastructure opens yet another venue for asymmetric attack. Many Department of Defense (DoD) and Intelligence Community (IC) advances are accrued by embedding or integrating commercial off the shelf (COTS) technology. As a product of the global economy, the origin, composition and distribution of COTS are not entirely under US control. Substantial reliance on COTS\(^4\) poses a risk management challenge: enemy exploitation of discovered or planted COTS security breaches vs. state of the art capability and performance information technology (IT) at much reduced cost.

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\(^4\) Eliminating COTS reliance would not necessarily eliminate vulnerabilities, and even secure systems are subject to denial of service attacks. Homogeneity and social engineering are two primary vulnerabilities of large-scale systems. For more details on this see: [http://niap.nist.gov/presentations/Hacking99.ppt](http://niap.nist.gov/presentations/Hacking99.ppt)
While the DoD/IC C4I infrastructure presents a key target for enemy exploitation it is dwarfed by the domestic commercial infrastructure. White House National Security Council staff coordinator for security, infrastructure protection and counter-terrorism, Richard Clarke, warns that several countries are carrying out "electronic reconnaissance today on our civilian infrastructure computer networks." Richard Perle, former assistant US secretary of defense for international security policy said US authorities had detected intrusions into US networks from North Korea, and that North Korean hackers had left behind a malicious code designed for possible activation as a kind of Trojan horse.

Asymmetric targeting is yet another dimension. Terrorism often intentionally strikes civilian or other non-combatant targets of opportunity, for the purpose of creating panic and shaking confidence in the competence of the ruling power or otherwise damaging social stability and welfare. From an account of the recent atrocities in Kosovo, we read:

“Forcing the refugees over the borders, NATO intelligence experts believe, served another purpose: overwhelming NATO troops stationed in Macedonia with an unmanageable relief crisis, calculating that the task of feeding, housing and caring for hundreds of thousands of refugees would consume the alliance's energies and divert it from preparing a military campaign."

'It was the first use of a weapon like this in modern warfare,' a NATO intelligence officer said. 'It was like sending the cattle against the Indians.'

Electric power generation and distribution, food, water, sewage, banking, financial networks, communications, and transportation systems are all potential targets in asymmetric war. Our asymmetric adversaries are not constrained by Judeo-Christian morality; in their game, the end justifies the means. Suicide bombing, chemical, biological and nuclear attacks against civilian populations are admissible options for many terrorist organizations. Currency manipulation and trade imbalances may also be used effectively in an asymmetric war. US adversaries plan to execute combinations of asymmetric attacks anticipating a composite effect of sufficient pain, damage, chaos and demoralization to achieve their objectives.

Contemporary Chinese thought on future warfare such as Qiao Liang's and Wang Xiangsui's Unrestricted Warfare propose 8 principles of warfare:

1. Omni directionality - all factors bearing on the desired outcome of a war are considered: military, political, economic, cultural, religious, psychological
2. Synchrony - a change of emphasis from sequencing and phasing to completion of actions within the same period of time in order to bring about the greatest impact, not unlike Col. John A. Warden's notion of parallel warfare
3. Limited Objectives - choosing objectives wisely, not over reaching capacity to act effectively
4. Unlimited Measures - all means are considered, the filtering process is limited only by concerns about their potential effects vis a vis the limited objectives
5. Asymmetry - refusal to confront the main force of the opponent, seek out and strike the weak spots which will cause the greatest psychological shock to the adversary

6 http://www.nytimes.com/library/world/europe/052999kosovo-atrocities.3.html
6. **Minimal Consumption** - economy of consumption is guided by maximization of means (unlimited measures), consumption is governed by the form of combat, rational choice of means should dominate thrift

7. **Multidimensional Coordination** - refers back to coordination of (1) and (2)

8. **Adjustment and Control of the Entire Process** - effective feedback and control for what is expected to become a shorter and more dynamic process.

War has always been hell; unrestricted war promises to open the gates of hell far and wide. Our enemies are restrained only by utilitarian self-interest, are increasing their ability to strike us and are in many cases composed of small transnational organizations. Military targets are avoided while soft civilian targets are savaged. Barbaric atrocities are common place as we enter the 21st century; we witness the steady regress of civilization. How do we resist, analyze, plan and train to reverse this process? Clearly the answer does not lie in military force alone, but it is also clear that military engagements in the era of unrestricted (asymmetric) war will require much more sophisticated and agile response than is possible today.

Wargames are used for both training and analysis as well as in mission planning and rehearsal. How we can efficiently synthesize sophisticated and agile C² decision-making models for wargaming the asymmetric environment will be the focus of the rest of this paper. In the next section we review and summarize the current state of wargaming in the DoD. Following the overview, we introduce basic concepts of game theory and outline some R&D directions for wargaming the asymmetric threat.

**Overview of Wargaming**

Contemporary US strategy and doctrine are based on joint and coalition operations. DoD operational wargames typically involve multi-echelon (blue) participants in manual role-playing with enemy (red), controller or arbitrator (white), and possibly a number of neutral, friendly or coalition teams. Depending on the purposes (training, analysis, rehearsal, etc.) and size of the wargame, considerable background support and infrastructure may be involved as well. Virtual simulations are used in training and exercise wargames to stimulate the C² equipment of trainees actually in the field, significantly augmenting the training environment with synthetic red or blue forces as needed. The need for valid & realistic simulated component behavior has long required labor intensive scenario development and setup and, depending on scope, a sizeable support team to steer or correct simulation behaviors that have gone off-track during the course of the run. High-resolution, multi-echelon constructive simulations are particularly susceptible to aberrant or irrational displays of behavior.

**Limitations of Current DoD Wargames**

Current wargames tend to focus on attrition modeling of symmetric force on force employment. Attrition is an important factor and much of our political posturing and commitment is dependent on estimates of attrition for proposed military actions but it is not the only factor. A number of shortfalls exist in the current generation of DoD wargames:

- **Narrow Operational Spectrum** - Existing models do not portray the full range of military operations such as Operations Other Than War (OOTW) and IO

- **Low Fidelity Interaction** - Modeling & simulation (M&S) systems that simulate functions such as transportation, logistics, intelligence, space, and special operations do not interact with desired resolution and fidelity with combat models

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• **Decoupled Strategic Effects** - Existing simulations do not reflect the strategic effects of military operations and require excessive intervention and tedious workarounds to inject effects of strategic attack

• **Poor Adversarial Automation** - Existing simulations provide task organization and equipment for foreign powers but authentic or effective strategy and tactics depend on manual role playing

• **Labor Intensive** - Scenario development can take many staff-months of effort and the necessity of human role players to provide creditable performance in training exercises exacerbates the problem

• **Lagging Visualization** - DoD wargames have not kept up with commercial games in terms of their graphics and performance characteristics but remains focused on geographic and physical environment.

### DoD Responses to Wargaming Needs

Emerging wargames need to incorporate behaviors and combined effects of both major and minor nation states as well as a host of non-state, non-governmental organizations (NGOs), transnational and international terrorist organizations operating in the asymmetric environment as well as corporate and criminal entities with significant business interests. We examined two operational requirements documents (ORDs) for major DoD Joint Level Wargames currently in design and development, Joint Simulation System (JSIMS) and Joint Warfare System (JWARS), and a National Simulation Center (NSC) system known as SPECTRUM. Here is a summary:

• **JSIMS** will be used by unified commands, other joint organizations, and the Services for: training, education, developing doctrine and tactics, formulating and assessing operational plans, and assessing warfighting situations. Efficiency in operational and technical responsiveness in presenting a training, education, or mission rehearsal environment is essential. JSIMS must reduce the personnel and time required providing training, education, or mission rehearsal events. JSIMS must also be tailorable in order to:
  • Provide a Joint Synthetic Battlespace (JSB) representing all warfare domains and applicable functions at a level of resolution appropriate for the training, educational, or mission rehearsal simulation event.
  • Incorporate the effects of non-military factors on mission critical tasks.
  • Provide the capability to modify JSIMS objects so that new warfighting concepts or equipment can be simulated.

• **JWARS** will be used by the Services, Combatant Commands, Joint Staff and Joint Task Force Commanders and Staffs, to support force assessment, planning and execution, system effectiveness and trade off analysis, as well as concept and doctrine development and assessment. In the mid-term JWARS will develop a suite of models, including a true, joint warfare analysis model and in the far-term provide an authoritative representation for analysis.
  • JWARS will be a constructive simulation of multi-sided (Blue including Coalition, Red, and Neutral forces), joint, theater-level wargame for analysis
  • JWARS will assist implementation of Joint Vision (JV) 2010 by providing a vehicle to assess current and future military capabilities within the four now standard operational concepts:
    • dominant maneuver
    • precision engagement
    • focused logistics
    • Full-dimensional protection.

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10 ibid, pages 6-8.
• **SPECTRUM** was developed in the mid-90s in response to a tasking for the NSC to develop a simulation in support of OOTW training. The system is used to provide stimulus for assessing effectiveness of simulated interactions with other forces and the general population in custom developed scenarios.

• SPECTRUM is used in exercises and seminars from tactical to strategic level and in training exercises by incorporating political, sociological, economic and cultural wildcards into the military decision making process.

• It represents government and non-government agencies, military, and para-military forces.

• A Regional Analysis Model (RAM) is embedded in SPECTRUM that provides semi-automated, macro-level interaction between political, economic, and social factors. The RAM represents the region’s constituents by defining social groups, institutions and outside actors.

• There are 21 macro-level regional indicators that model a region politically, economically, and sociologically. A Delphi process was used to determine the variables and then assign a value of 1-100 to the variables.

• There are 28 micro-group indicators, many of which are identical to the macro-level regional indicators. The major difference in the micro indicators is defining the group characteristics in terms of communalism, cohesiveness, leadership, ambition, aggressiveness, protest level, and population.

• A third matrix is developed which defines 24 factors for issues and concerns that relate to the population's satisfaction and importance of these 24 factors.

The effects of actions taken over time are then reflected in SPECTRUM's modeled population’s factor values. The system is polled periodically during the course of an exercise and it emits modeled events according to the current value of these factors thus leading to more chaotic and riotous behavior as population satisfaction levels decline.

• **DEXES**\(^{12}\) is an integrated collection of dynamics models governing the time evolution of

  • Economic
  • Social
  • Political
  • Public health variables.

The Land Information Warfare Activity (LIWA) is currently supporting development and porting of this model to the Windows/NT platform for IO support. This model was originally developed during 1995 - 1997 in support of OOTW for US Southern Command (J5 Plans, Analysis and Simulation Division). Although we requested detailed documentation on the internal design and operation of this model it was never transmitted to us so the modeling technology reported here is based only on publicly available documentation and a presentation of the model given by Dr. Ted Woodcock. In the words of Dr. Loren Cobb, primary developer of DEXES,

> "The DEXES family of causal models brings to the wargame environment a political - social - medical - cultural simulation of the effects of military, governmental, and NGO actions on a society in the aftermath of a major disaster or civil war. Equally important, the DEXES model shows the effects of failures to take action. The DEXES model of society is deliberately and realistically unstable, so that incorrect, omitted, or tardy actions on the part of the players can result in negative consequences, up to and including the sudden failure of the mission through societal collapse or the outbreak of civil war".

DEXES appears to encapsulate and automate many of the features modeled in SPECTRUM with the addition of a causal set of dynamical models governing the societal variables. It would certainly be worth taking a more detailed look at the underlying theory and scope of each of these models to

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\(^{12}\) [http://www.aetheling.com/models/MOOTW/DEXES.html](http://www.aetheling.com/models/MOOTW/DEXES.html)

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determine if a more accurate or comprehensive model could be attained by a convergence of the two, but this was beyond the scope of our investigation.

Discussion on Modeling the Asymmetric Environment

JSIMS, JWARS, SPECTRUM and DEXES are a representative sample of DoD efforts to close gaps in contemporary wargames. A heavy emphasis is placed on building validated behavior by increasing model fidelity in the hopes that this will assure validity. This approach ultimately leads to emulation rather than simulation. We are rich in information on our own forces' strategy, doctrine, tactics, task organization, equipment and weapons. We have sources such as DIA and the National Ground Intelligence Center (NGIC) Land Capabilities Spectrum Model (LCSM)\textsuperscript{13} that capture and profile the spectrum of adversarial military capabilities. In addition, we have allies who perform the same assessments on foreign military powers and US so that authoritative normalized profiles of military capability are relatively accessible\textsuperscript{14}.

DoD wargame developers, eager to rapidly construct authentic and accurate representations, have readily used this kind of information to develop and parameterize their models. Although models have been implemented in a wide range of coding styles with representations in procedurally coded task frames, finite-state machines, rules and constraints the resulting behaviors are typically insensitive to opportunities and impending disasters. It appears that modeling priority has generally been on task organization, weapons and equipment first, sensors and physical environment second, adjudication and attrition third, instrumentation and after action review (AAR) fourth, with communications and automated $C^2$ decision making near the end. The answer to keeping automated wargames on track and rapidly adapting wargame simulations to changes in scenario, strategy, environment, and combatants may lie in a reverse of the apparent modeling priorities.

The more we try to reconcile behaviors by increasing resolution and fidelity the greater the knowledge-engineering burden in development and the smaller the range of alternatives that can be explored during execution. For example, LIWA apparently profiles\textsuperscript{15} selected IO targets and key decision makers by collecting a wide spectrum of information including: education, politics, employment, military service, family, accessibility, religion, political goals, motivation, predisposition, psychological disorders, health, special relationships, advisors, international experience, foreign travel, biases, ambition, upbringing, birth place, age & sex, and heritage. Any individual behavioral model based on a large number of variables such as would likely emerge from a quantitative articulation of this profile would introduce an enormous error budget for the overall model. Maintaining such a model over its life cycle would be problematic, as the interactions between the collected factors would likely evolve as the factors themselves evolve. Even if predictive computational models currently existed that accounted for all of these variables (and they don't) scalability and maintainability would remain challenges. Modeling the asymmetric environment demands a broad spectrum of actors whose venues and methods of attack are not easy to adjudicate and are not limited to attrition.

Commercial Off the Shelf (COTS) Technology

The commercial game industry continues to push the envelope on photo-realism and scene rendering and is moving into massively distributed game playing. The commercial game sector is primarily focused on entertainment and market capture. Remarkable progress has been made in anatomical modeling and rendering of natural movement\textsuperscript{16} and gesture. Emotional response is modeled in several first person shooter and role playing games. These advances may be of some value in visualization and distributed training for DoD wargames, commercial games but have little to offer in addressing the problems of: narrow

\textsuperscript{13} http://www.fas.org/irp/agency/army/tradoc/usaic/mipb/1996-2/schlus.htm
\textsuperscript{14} A fairly comprehensive unclassified overview of US forces, doctrine and equipment is maintained online by the Air War College at http://www.au.af.mil/au/awc/awcgate
\textsuperscript{15} Stakeholder Modeling and Simulation of Asymmetric Environments (Draft Report), The MITRE Corporation, Lee Scott Ehrhart, Ph.D. January 2000, 12.
operational spectrum, de-coupled strategic effects, poor adversarial automation, and labor intensive scenario development.

Some special purpose COTS products do exist that directly address coupling of strategic effects. For example SIAM\textsuperscript{17}, a tool for building influence diagrams (networks of causal, or more accurately, probabilistic links between decision variables and utility measures based on user provided expert opinion). Underlying SIAM's visual influence diagram a Bayesian network enforces consistency and propagates the effects of updated influences and event outcomes. Similar Bayesian or belief network tools such as Hugin\textsuperscript{18}, Genie and Smile\textsuperscript{19} also exist but SIAM has actually been put into operational use for military intelligence applications.

There are emerging third party vendors\textsuperscript{20, 21} who supply, or very soon intend to market, building block technologies for the artificial intelligence (AI) components that could assist in adversarial automation. But the game developer's market does not show any sign of moving toward a decision analytic framework. A straw poll conducted on a game developer's web-page\textsuperscript{22} reveals the top ten AI modules most valued by game developers to be in the areas of group movement control (path-finding, tactics, and social flocking), A*-search, multi-agent control (reactive, rule-based expert system, finite-state machine, and fuzzy inference). COTS game developers are interested in realistic graphics and convincing but efficient behavior. Exploration of alternatives and intelligent rational behavior fall pretty low on the list of market demands.

High Power Knowledge-Based Technology

Do the answers lie in continued investment in knowledge-intensive technology such as planning, scheduling, high-powered knowledge bases? Perhaps in the long run, but these technologies are dogged by two persistent challenges: (1) they demand a heavy upfront investment in knowledge-engineering and development of domain specific heuristics for their strength and (2) once constructed, knowledge-based C\textsuperscript{2} for simulations, such as SOAR\textsuperscript{23} are computationally challenging and do not scale well.

New DARPA efforts in rapid knowledge base formation promise to mitigate the knowledge-engineering bottleneck (1) by development of tools and methodologies for declarative knowledge representation and capture, and ontology design, development, and reuse, but these tools are nascent. Given the current state of knowledge-engineering technology, a new modeling approach may be in order: a shift of emphasis from engineering doctrinal procedures and sophisticated symbolic reasoning toward simpler decision mechanisms based on maximization of expected utility. This approach has been used in econometrics and social sciences for decades and as the foundation for most military strategic analyses, yet it is rarely seen in military simulations as a driver for behavior. However researchers are beginning to explore the efficacy of utility optimization as a practical basis for behavior modeling in computer generated forces. Booker\textsuperscript{24, 25}, et al. have demonstrated improved robustness (more effective and responsive) control of tactical movement in ModSAF with a computationally efficient control theoretic decision formulation known as the DRK-

\textsuperscript{18} http://www.hugin.dk
\textsuperscript{19} http://www2.sis.pitt.edu/~genie
\textsuperscript{20} A French group, MASA has several potential application areas outlined for its products DirectIA and NetworkEvolver (see http://www.animaths.com)
\textsuperscript{21} http://www.cse.buffalo.edu/~goetz/AI/API/gaibody.html#strips
\textsuperscript{22} http://www.cse.buffalo.edu/~goetz/AI/API/gaibody.html#strips
\textsuperscript{23} http://bigfoot.eecs.umich.edu/~soar/main.html
\textsuperscript{24} Toward Motivational Control of Tactical Behaviors, Lashon B. Booker, Ph.D., Carl D. Burke, Gregory M. Whittaker Proceedings of the 7\textsuperscript{th} Computer Generated Forces and Behavior Representation, May 1998.
\textsuperscript{25} Motivational Control of Tactical Behaviors: Interim Report, Lashon B. Booker, Ph.D., Carl D. Burke, James D. Hughes, Proceedings of the 8\textsuperscript{th} Computer Generated Forces and Behavior Representation, May 1999 (http://www.sisostds.org/cgf%2Dbr/8th/docs/papers/8th%2Dcgf%2D058.doc).
motivational\textsuperscript{26} model. Control theory per se is limited to a single controller's perspective. The multi-player perspective is fundamentally game theory. In both control and game theory, action selection is based on environmental values either dynamically sensed or known a priori. When used to model the actions of an individual decision-maker, the focus is on motivation or volitional rather than reasoning or logical causation. We avoid the burden of constructing an explicit reasoning process at the cost of creating a preference relation between modeled actions available to the decision-maker and his perceived environmental state. There are a number of benefits to be gained by simplifying the model structure to that proposed in the game theoretic literature.

1. Operational cost is substantially reduced because computation, synthesis and selection of strategy can largely be done ahead of runtime
2. As the model evolves to cover additional states of the environment or as actors and actions are added, knowledge-engineering consists of extending and refining the preference relations (numerical payoff value or utility measure) in the modeled utility function
3. Integration of any modeled player and consequent behavior changes is completed upon solution of the game, a purely computational process.

Classical AI approaches to action control or planning depend on calculation and maintenance of many distinct pre-conditions and effects while game theory reduces action effects to payoff and focuses on optimizing expected payoff. It is difficult to know in a complex set of productions if something hasn't happened because of a knowledge-engineering oversight, because of a logical conflict, or simply because of an unmet set of conditions. Rule-based systems tend to channel behavior along preconceived lines of attack that sooner or later become very predictable. As we shall see, game theoretic solutions give rise to rational yet non-deterministic behavior which can lead to a much broader exploration of alternative courses of action then is common practice today.

Moving toward a game theoretic framework entails a lessening of model emphasis on detailed how-to information and more emphasis on how much and when. In this framework, every player has a set of options and every combination of player options has a value for each player. By reducing the effects of options to their value or utility and by modeling adversarial decision making as utilitarian self-interest a potentially enormous knowledge-engineering burden is eliminated. This payoff or utility is constant and known in advance in classical game theory but extensions such as Bayesian game theory and stochastic game theory have penetrated the world of incomplete information and dynamic payoff states.

Recent efforts to apply a game theoretic perspective to wargame simulation have been focused at the tactical level. Booker's work mentioned above and some work by Katz and Butler\textsuperscript{27} investigating game theory for C\textsuperscript{2} focused on ModSAF as the target. Earlier work by Shubik and Weber recalls so-called Colonel Blotto Games\textsuperscript{28} that address operational allocation of forces and extends this differential game theoretic approach to strategic issues of complementarity between targets and cost tradeoffs between system defense and asset hardening. A very recent DARPA Advanced Simulation Technology Thrust (ASTT) proof of concept study\textsuperscript{29} has successfully applied an extensive form game representation to a ground combat scenario by combining dynamic programming and game theoretic techniques.

In the following section we introduce some of the basic elements of the game theoretic framework and illustrate the simplest methods of game solution in order to introduce the concept of equilibria. So-called Zero-Sum games model pure conflict, one player's gain is another player's loss. Most wargames fall into

\textsuperscript{27} Game Commander-Applying an Architecture of Game Theory an Tree Look Ahead to the Command and Control Process, A. Katz, B. Butler (found in transactions of the IEEE 1994).
this genre, but there are many cases in asymmetric conflict where the objective is to manipulate or form an alliance or coalition. Shapely and others as long ago as 1953 were laying the theoretical foundations of coalition games and cooperative game theory.

With our dependency on information systems and information operations in the conduct of OOTW (Peace Making, Peace Keeping, Humanitarian Relief) there is a natural match between game theory and the asymmetric environment. After an introduction to the basic concepts of classical game theory and a brief indication of the more advanced recent developments, we will address some of the shortfalls and challenges of applied game theory in analysis and prediction.

Game Theory Fundamentals

In a finite game we have an enumeration of N players \( \{ i \ | \ i \leq N \} \), each of whom has available some finite collection of \( J \) discreet actions, denoted as the list \( A_i = \{ a_1, a_2, \ldots, a_{J_i} \} \), where \( \#A_i \) (cardinality of \( A_i \)) = \( J_i \).

We define as the set of all possible plays of the game generated by the N-fold Cartesian product of the \( A_i \), \( \Pi = \{ (p_1, p_2, \ldots, p_n) \ | \ A_1 \times A_2 \times \ldots \times A_n \} \). For each player \( i \) we are also given a utility function \( u_i : \Pi \rightarrow \mathbb{R} \). We call the value, \( u_i (\omega) \) for \( \omega \in \Pi \), the payoff to player \( i \) for the play \( \omega \). Players must simultaneously choose an action from among their respective options at each play of the game. Classical game theory assumes that each player knows every other's action set and utility function as well as his own (complete information). It is also assumed that all players are rational, that is, they choose actions in order to optimize their expected utility. A player's rule for choosing among his possible actions is called a strategy. (In following sections we discuss modern developments of game theory that handle incomplete or uncertain information.)

A pure strategy continually chooses the same action at every play of the game; a mixed strategy non-deterministically chooses an action according to a probability distribution over all possible actions. A pure strategy is therefore just a special case of the mixed strategy. In either case a strategy for player \( k \) may be represented as a vector \( S_k = < \xi_1, \xi_2, \ldots, \xi_{J_k} > \) in the face of the \( J_k \) dimensional unit simplex \( \sigma_k \) where each \( \xi_i \) is the probability that action \( a_i \in A_k \) will be played. The simplex constrains the components of \( S_k \) so that we have all \( \xi_i \geq 0 \) and \( \xi_i = 1 \). The \( J_k \) dimensional manifold \( \Xi = \sigma_1 \times \sigma_2 \times \ldots \times \sigma_N \) is global strategy space for a given finite game, and we denote the \( J_i \) dimensional sub-manifold \( \Xi_i \) as the restriction of \( \Xi \) to \( \sigma_i \). A solution to a game is a global strategy \( S \in \Xi \) that optimizes the expected utility for each player \( i \) on \( \Xi_i \).

Game theoretic algorithms generate strategies (prescriptions for option choices) that simultaneously account for all players by finding equilibrium points in the strategy space, that is, points where no player can benefit by changing strategy assuming all other players maintain their respective Nash equilibrium strategies. There are actually several variations on the theme of equilibrium for example:

- **Dominant Strategy** - For any player \( i \), strategy \( S_i \) dominates if its corresponding expected value is greater than or equal to the expected value of any other strategy for player \( i \)
- **Nash Equilibrium** - For any player \( i \), any change from Nash equilibrium strategy \( S_i \) will yield lower expected value for player \( i \) given that all other players maintain their respective Nash equilibrium strategies
- **Strong Pareto Optimality** - There is no other equilibrium strategy that gives any player a higher expected payoff
- **Weak Pareto Optimality** - There is no other equilibrium strategy that gives all players equal or greater expected payoff

The variations on equilibrium characterization reveal the game theoretic focus on finding the best possible decision strategy. Table 1 (below) illustrates a simple example of dominant strategy for the case of a two-player game where each player has two options. The values are indicated by pairs (value to player A, value to player B). By inspection, we see that option 1 is a dominant strategy for player A, since no matter what option player B chooses the outcome is better than for option 2. From player B's perspective option 1 is also a dominant strategy.
The coincidence of the dominant strategies leads to a saddle-point, or equilibrium point in strategy space. Both players, if they are motivated by self-gain, and are cognizant of the outcomes, will play the pure strategy of option 1 every time they encounter this game. The game has an equilibrium value of 4 for player 1 and 1 for player 2. Obviously, not all games are this simple, table 2 illustrates a variation on the theme with different payoff values.

<table>
<thead>
<tr>
<th>Player B Option 1</th>
<th>Player B Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player A Option 1</td>
<td>(4, 1)</td>
</tr>
<tr>
<td>Player A Option 2</td>
<td>(1, 4)</td>
</tr>
<tr>
<td>Player A Option 1</td>
<td>(2, 0)</td>
</tr>
<tr>
<td>Player A Option 2</td>
<td>(0, 2)</td>
</tr>
</tbody>
</table>

Table 1. Dominant Strategy Saddle Point (Option 1, Option 1)

In this case, the equilibrium strategy for player A is a mixed strategy choosing option 1 50% of the time and option 2 the remaining 50%. Player B divides his strategy 9/14 (64.29%) for option 1 and 5/14 (35.71%) for option 2. The game value is -1/2 for player 1, and +1/2 for player 2. These two simple examples illustrate the normal form, sometimes called strategic form, of game representation. The game in table 2 is also a zero-sum game, that is, the payoffs add to zero in every outcome; one player's gain is another player's loss. In two-player zero-sum games solution of the game can be found using the maxi-min algorithm.

The maxi-min algorithm calculates the maximum of the minimum utility player 1 can guarantee given the value of his options. This is better described with a calculation and corresponding graph. We must calculate our utility as a function of our strategy for each pure option of our opponent, player 2. We know we must play some combination of our options, either a pure strategy for option 1 or a pure strategy for option 2 or some mixture of the two. We express our unknown strategy as a frequency vector (or convex combination) of our options, ((1-p), p).

<table>
<thead>
<tr>
<th>Player 2 Chooses</th>
<th>Player 1 Strategy is: (1-p) for option 1, p for option 2</th>
<th>Expected Payoff is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option 1</td>
<td>(1-p)* (-3) + p (2)</td>
<td>5p-3</td>
</tr>
<tr>
<td>Option 2</td>
<td>(1-p)* (4) + p (-5)</td>
<td>-9p+4</td>
</tr>
</tbody>
</table>

Table 3. Calculating Maxi-Min Payoff for Player 1

Figure 1 graphs player 1’s payoff for each of player 2’s options and indicates an intersection at the point where:

\[ 5p-3 = -9p+4 \]
\[ p = .5 \]
This is the value of $p$ that maximizes the minimum expected utility for player 1. If player 1 chooses to play option 1 less than 50% of the time, then player 2 can profitably respond by increasing his frequency for his own option 1 to something greater than 9/14 and correspondingly decreasing his play of option 2 to less than 5/14. (The calculation shown above and graphed in figure 2 is done in a completely analogous way for player 2.) In a zero-sum (or generally any constant sum) game, the payoff is sometimes represented as a single value (implicitly player 1’s payoff) for each entry in the payoff matrix instead of the vector of values used in our example matrices (Tables 1 and 2). In this case, player 1 is sometimes referred to as the maximizing player, and player 2 is the minimizing player. The standard approach for solving two-player constant sum games is to look for dominating pure strategies and if none exist apply the Maxi-Min algorithm for the maximizing player and the Mini-Max algorithm for the minimizing player. It is possible for an arbitrary game to have more than one or even an infinite number of equilibria. The simple example given here is only intended to give an intuition of the nature of equilibrium points. Since equilibrium strategies are computed by considering the objectives of all players simultaneously; there is no problem of out-guessing the opponents estimate of your own behavior before you can estimate his, as is the case when using purely decision theoretic analysis.

We have used what is called the normal form or strategic form game representation in our examples. When it is necessary to analyze a particular sequence of moves because information is changing, or new options and new constraints are added at later stages, then it is necessary to use the extensive form representation. Extensive form is a rooted tree graph representation composed of labeled nodes and branches; each branch represents the alternative actions available for the player at that node of the game. Figure 2 illustrates a simplified poker game in extensive form. Player nodes are indicated as ovals labeled in decimal format with the player number before the decimal and the player’s information set label after the decimal. Terminal nodes are indicated by rectangles that include the vector of payoff values for the players. In this illustration, the root node (0.0) represents the 50-50 chance of player 1 drawing a red or black card. When player one chooses to Raise or Fold, he has knowledge of the card he has drawn as indicated by the presence of distinct node labels 1.0 (information set 0 player 1 has drawn a red card) and the node labeled 1.1 (player 1 has drawn a black card). Player 2 does not have this advantage, as indicated by the linked nodes labeled 2.0 and must choose to See or Pass without knowing what color card was drawn by player 1. Since it is possible to convert from extensive form to the equivalent normal form all of the algorithms for finding equilibria can be applied to games in extensive form as well.

Figure 2. Maxi-Min \([(1-p)\text{Option1} + (p)\text{Option2}]\) sets

$p=0.5, \text{ player 1 game value } = -0.5$
We will not describe the much more complicated methods for finding equilibria in multiplayer games; algorithms do exist for multiplayer zero-sum and non-zero sum games. Research and development continues on various approaches to efficiently compute equilibria such as relaxation algorithms\textsuperscript{30} based on the Nikaido-Isoda\textsuperscript{31}, or Simplicial subdivision based on the Lyapunov\textsuperscript{32} function or Quantal Response\textsuperscript{33} logistical function. Most of these equilibrium finding algorithms for multi-player games have been cataloged\textsuperscript{34,35} and there are freely available implementations for R&D. The Gambit\textsuperscript{36} GUI and Gambit Control Language (GCL) are excellent examples. Risk aversion of adversaries, neutrals and friendly players can also be modeled in modern game theory. Game theory is a mature field that is going through a renaissance in evolutionary\textsuperscript{37} game theory, decision theoretic and game theoretic agents\textsuperscript{38}.

\textsuperscript{30} Relaxation Algorithms in Finding Nash Equilibria, Steffan Berridge, Jacek B. Krawczyk (Working Paper version 2.3) Victoria University of Wellington, New Zealand. (Jacek.Krawczyk@vuw.ac.nz).
\textsuperscript{32} A Lyapunov Function Function for Nash Equilibria, Richard D. McKelvey (California Institute of Technology), 1991.
\textsuperscript{34} Computation of Equilibria in Finite Games, Richard D. McKelvey (California Institute of Technology) and Andrew McLennan (University of Minnesota) June 30, 1996.
\textsuperscript{35} Representations and Solutions for Game Theoretic Problems, Daphne Koller and Avi Pfeffer (Stanford University) April 16, 1997.
\textsuperscript{38} Proceedings from the Workshop on Decision Theoretic and Game Theoretic Agents, Fifth European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty, University College, London, UK 5 July 1999, Simon Parsons and Michael J. Wooldridge (editors). More recently, the 2nd Workshop on Decision Theoretic and Game Theoretic Agents was held on 7 July 2000 in Boston, MA. Proceedings from the 2nd workshop, edited by Simon Parsons and Piotr Gmytrasiewicz may be found online at http://www.csc.liv.ac.uk/~sp/events/gtdt/gtdt00/proc.html

\textbf{Figure 2. Extensive Form Representation}
Limitations of Classical Game Theory

The assumption of complete information is probably the greatest impediment to the practical application of classical game theory. An asymmetric information game where players have incomplete information on either the payoffs or options or both are much more typical of the real world situation. In 1966, R. J. Aumann and M. Maschler introduced games of incomplete information. By 1968, John C. Harsanyi had built the theoretical foundations used in modern analysis of information games in a series of three papers, *Games of Incomplete Information Played by "Bayesian" Players (Parts I, II, and III)*. The characterization of incomplete information is interpreted as the lack of full information in terms of the normal form of the game in precisely 3 different ways:

1. Ignorance of the physical outcome function $Y$ of the game which specifies the physical result of each tuple of strategies available to the N players, $y = Y(s_1, s_2, s_3, \ldots s_n)$
2. Ignorance of utility functions $u_i$ which map a physical outcome $y$ to the utility to player $i$
3. Ignorance of the strategy spaces, $\Xi_i$ (set of all pure and mixed strategies, $\sigma_i$ for player $i$) available to each player $i$.

Incomplete information is very carefully distinguished from imperfect information. Imperfect information refers to certainty about the history of the game, lack of perfect recall for previous moves of oneself, other players or nature. The general approach to analysis of games of incomplete information, which we shall not detail here, is transformational in nature. Games of incomplete information are transformed into theoretically equivalent games with complete, but imperfect information. The key assumption in this approach is that every player will assign a subjective probability distribution $P_i$ to all unknown independent variables (variables not dependent on the player's own choice of strategy). Every player will try to maximize his own payoff $u_i$ in terms of his $P_i$. This is known as the Bayesian hypothesis.

We began this introductory overview with the assumption that a game will involve indefinitely repeated plays or at least for some unknown random number of plays. In the case that we know the exact number of plays or stages in a game and have a well defined goal state, dynamic programming is a candidate method to identify the best sequences of plays. Dynamic programming regresses one stage at a time from a specification of the goal state expanding least cost transition paths, constructed from the players’ options, until the initial stage is reached.

Another challenge is of course computational tractability, a problem to which both dynamic programming and the classical discreet game solution methods are vulnerable. This challenge arises as the number of players in the game increases, or their average number of potential options increase. In the case of dynamic programming the problem is exponential in the number of stages to be regressed.

We also assumed in our opening treatment of game theory that the payoff function, $u_i$, is constant. How should our strategy evolve as payoff evolves? Differential game theory introduces state variables and replaces actions with control variables and a set of kinematics equations that link a player’s control variable settings to his traversal of state space. A single-player differential game essentially reduces to an optimal control problem. Every game begins with each player in some initial state space location. The play of the game moves the players through state space according to their control strategy. An equilibrium solution corresponds to an optimal solution for a given objective function. For the most part differential game theory is practical in zero-sum two-player contexts. An optimal control problem with independently steered sets of control variables is a good analogy for differential game theory except multiple players give rise to equivocal surfaces, that is bifurcations in strategy that are equally effective.

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Modern hybrids such as Hyper-Gaming\textsuperscript{41,42} and Decision Theoretic Planning\textsuperscript{43} offer some new roads connecting valuated spaces to knowledge-intensive frameworks where hierarchical decomposition, formal description theories, effects based planning and Markov decision processes may be used to model the necessary dynamics and construct the details of a needed plan. Hybrid game theoretic extensions leverage hierarchical direction of search, reasoning or planning based on strategies developed within the game-decision theoretic domain before turning over control to the lower level domain specific search or planning mechanisms.

Linguistic Geometry\textsuperscript{44} (LG) is yet another related approach to construction of mathematical models for knowledge representation and reasoning about large-scale multi-agent systems. A number of such systems including air/space combat, robotic manufacturing, software re-engineering, Internet CyberWar, etc. can be modeled as abstract board games. These are multi-player adversarial games whose moves can be represented by means of moving abstract pieces over locations of an abstract board. The adversaries, dimensions of the abstract board, mobility of pieces, simultaneity or sequencing of moves can all be tailored to the situation. The purpose of LG is to provide strategies to guide the participants of a game to reach their global goals. Traditionally, finding such strategies required searches in giant search trees. LG dramatically reduces the size of the search trees, by using expert heuristics that replace search by capture of emergent strategies from modeled agents (bombers, space interceptors, etc.) pursuing their local goals. The formalized expert strategies yield efficient algorithms for problem settings whose dimensions may be significantly greater than those for which the experts developed their strategies. Moreover, these formal strategies allow application to problem domains beyond the areas originally envisioned by the experts. To formalize the heuristics, LG employs formal linguistics as well as geometric structures over the abstract board thus it was named Linguistic Geometry.

\textbf{Game theoretic R&D needs for Wargaming}

While game theory has a mature base of research and technical progress upon which to build, there are some particular areas in need of development if game theory is to be usefully applied as a tool in wargaming the asymmetric environment. We outline four areas, there are undoubtedly other areas as well, but progress in these would go a long way toward the realization of game theoretic wargaming.

\textit{(1) Synthesizing the game from the situation and historical data}

How do we automatically enumerate relevant players, their options, and estimated payoffs? (automatic pressure point analysis?) Are tools such as Antecedent, Behavior, Consequent (ABC) databases, text summarization or rapid knowledge bases rich enough for this task? How do we interactively combine expert judgements about consequences and payoff? Can we automatically update games to similar situations thus reusing previous expert assessments on payoffs and previous solution strategies? What must we monitor in order to determine when the situation has changed sufficiently to render the current game invalid, or in need of adaptation? Can game updating keep up with typical situation dynamics?

\textsuperscript{41} Merging AI Game Theory in Multiagent Planning, Russell Vane, Paul Lehner, Kathryn Laskey, 1990 IEEE Transactions, 853-857.
\textsuperscript{42} Using HyperGames to Select Plans in Competitive Environments, Russell Richardson Vane III, Dissertation submitted in partial fulfillment of Ph.D. in Information Technology, George Mason University, Spring 2000.
\textsuperscript{44} Linguistic Geometry From Search to Construction, Boris Stillman, Kluwer Academic Publishers, 2000, LC QA76.9.C65 S76 2000 003'.3--dc21. Per personal communication with the author a number of prototypes of LG systems and commercial products have been developed at Lockheed Martin Corp., GIS Solutions, Sandia National Laboratories, US Air Force Phillips Laboratory, University of Denver, and University of Colorado at Denver. A team of 3 universities (Wayne State, Cornell, and Univ. of Colorado at Denver) led by Rockwell Science Center is applying LG in the Joint Force Air Component Commander (JFACC) Project Agile Symbolic Mission Control and Hostile Counteraction for DARPA.
(2) Finding and applying optimal strategies
Techniques exist for finding some equilibria for multi-player games, but under what conditions can we or must we find all equilibria? How do static equilibria compare to dynamic equilibria arrived at by learning agents of limited capability and intelligence (bounded rationality)? How can efficiency of equilibrium finding algorithms be improved and under what conditions? It has been suggested that efficient implementation of the Lemke-Howson algorithm for multiple player games (n > 2) is possible, (GAMBIT has implemented this for two player games); this and/or an efficient implementation of the Lyapunov approach would be good targets for development of more complete strategy formation. How can we use expert knowledge of psychological factors of personality to select the most likely strategies of other players? (Degree of risk aversion?)

(3) Directed modification of the game
Given that some equilibrium solution is seen to hold in the real world situation, what techniques are available for us to optimally adjust payoffs (when this is possible by virtue of transferable utility or cooperation) to induce a transfer to another more politically desirable equilibrium? Can we equivalently induce such transfer via strategy adjustment? How do we systematically valuate induced asymmetries in the information sets of various players and incorporate such options in our strategy?

(4) Visualization of the Game Space
We need some techniques for making the situation and various solutions intuitive and palpable to the non-technical user. Though not really a game theoretic challenge, this effort would free the user from the current focus on the physical level and open the vistas of strategy space and valuation contours by identifying key intuitive 2d and 3d projections of the hyper-dimensional game environment. An example may be player value surfaces as functions of selected or especially sensitive strategy option variables.

Grounding Analysis & Models in Reality

Given a game theoretic perspective, how do we connect it to the operant reality of any given situation? Where do we begin in the process of formulating players, options, and payoffs? Figure 3 illustrates the concept of lifting a hypothetical game from an ABC database of historical events. An ABC database, as its name implies, includes selected antecedents to historical events, behaviors or options actually executed by the collected targets, and a valuation of the degree of success or value achieved by the target's action (consequent) for a given set of antecedents and behaviors.

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45 Computation of Equilibria in Finite Games, Richard D. McKelvey (rdm@hss.caltech.edu) and Andrew McLennan (mclennan@walley.econ.umn.edu) June 30, 1996.
Figure 3. Bootstrapping the Hypothetical Game

Figure 3 is actually a high-level indication of the process outlined in research area (1) above. A detailed elaboration is beyond the scope of this overview paper, but the fundamental notion is that de facto mixed strategy vectors are implicit in the ABC history for each target or player. The process outlined here extracts the implicit strategy vectors and incorporates available intelligence on player ideology, worldview, beliefs, knowledge, capabilities and objectives to generate a plausible set of payoffs. The combination of implicit strategy vectors, plausible payoff matrix and individual player information sets, constitute the initial hypothetical game. Refinement of the initial hypothesis could be directed by reduction of uncertainty in payoff and information estimates and options available to players over time. The evolving estimate of the game would in turn serve as the analytical basis for developing our own strategy (section 2 research techniques) and directing modification of payoff where possible in accord with research results from section (3) above.

Similar work in machine learning has been performed to evolve negotiation strategies in three party coalition games\textsuperscript{46} and to evolve neural network models from ABC databases to predict behaviors from antecedents\textsuperscript{47}. The advantages that hypothetical game estimation offer are semantic transparency and a theoretical foundation for strategy development. The values assigned to the estimated game payoffs can be inspected and provide a plausible explanation of the otherwise apparently irrational behaviors observed in the history of terrorism. The payoff estimates are the basis for motivation for the players and how we may best respond, but it is an interesting fact that equilibria are invariant under affine translations of payoff values. So we are not constrained to a particular absolute set of payoff values to get a particular equilibrium behavior. On the other hand when using a game theoretic formulation for predictive purposes, we have the problem of selecting the most probable equilibrium from the many possible equilibria. The field of reinforcement learning\textsuperscript{48} is beginning to address the problem of independent agents converging to a common equilibrium\textsuperscript{49}.

\textsuperscript{46} On Automated Discovery of Models Using Genetic Programming in Game-Theoretic Contexts, Garrett Dworman, Steven O. Kimbrough, and James D. Laing, Proceedings of the 28\textsuperscript{th} Annual Hawaii International Conference on System Sciences 1995.

\textsuperscript{47} Evolutionary Models of Terrorist Threat, Final Report DI-MISC-80711 David B. Fogel, Natural Selection, Inc. Sponsored by DARPA on ARPA Order D611/70 Issued under contract no. DAAH01-00-C-R044, June 13, 2000.

\textsuperscript{48} Reinforcement Learning: A Tutorial, Mance E. Harmon and Stephanie S. Harmon

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Conclusion

The broad and dynamic scope of asymmetric war demands flexible and adaptive wargaming and analytical tools and models; tools that can be readily applied to a wide variety of situations, vulnerabilities and threats. The purpose of the Wargaming the Asymmetric Environment Program is to develop underlying technologies and prototypes that will significantly advance US preparation for, and ability to respond and pro-actively mitigate or eliminate asymmetric threats.

This paper outlined what is understood as asymmetry and asymmetric threat and outlined a game theoretic perspective on supporting strategy development and C2 decision support in the asymmetric environment. It is envisioned that game theoretic formulations, analyses and solutions would greatly benefit next generation wargames particularly in the coupling of strategic effects on operational outcomes and development of effective strategies to form or block targeted coalitions. Evaluation and modeling of multiple game equilibria provides a theoretically sound basis for generating rational course of action exploration either for off-line analysis or in automated C2 decision making so sorely needed in wargame simulations.

There remain many unaddressed and unanswered issues that I did not have time to explore in this paper. Two seem most pressing:

1. Can equilibrium calculation or estimation be kept practical in complex games? We know that the number of all possible totally mixed equilibria is in general an exponential function of number of players and actions available per player. How do these multiple equilibria group, disperse, relate on the simplex?

2. How should we profile the rationality bounds of our adversaries? Even if we are able to estimate their utility function how do we characterize their own cognizance of options, their ability to calculate and follow a best response to their particular circumstance.

Developing answers to these and other issues of modeling the asymmetric environment await future research.

http://www-anw.cs.umass.edu/~mharmon/rltutorial/noframes.html

Rational Learning of Mixed Equilibria in Stochastic Games, Michael Bowling and Manuela Veloso

http://www.cs.cmu.edu/~mhb/papers/00-rational.ps.gz
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