

# Evaluating Vehicle Mobility Using Bekker's Equations

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## ABSTRACT

By using M.G. Bekker's equations for vehicle mobility, we have developed an analytical tool that is used for the evaluation of off-road vehicles. This allows for the departure from historical empirical models by developing universal static and transient equations that fit for all vehicles. One of our primary purposes is to develop a model for the evaluation of different mobility platforms of small robotic vehicles. These vehicles are evaluated on conditions of tractive force, tractive effort, drawbar-pull, sinkage, safe weight pressures, ground pressures, and slippage. This paper attempts to compare the mobility characteristics of wheeled vs. track vehicles for different size, weight, and terrain based off of aforementioned conditions.

BDTM was originally established to give a first pass general evaluation of robotic vehicle mobility performance for different types of running gear configurations. It is a simple, linear one-degree of freedom (1-DOF) model that has been created in a spreadsheet format. The model assumes that the soil is homogenous and the loading effects are linear. The model can simulate both tracked and wheeled vehicles. These vehicles are evaluated using tractive force, tractive effort, soil sinkage, drawbar pull, and tractive coefficients (DP/W).

## INTRODUCTION TO SOILS

The mechanical behavior of soils varies considerably under a wide variety of environmental conditions. For example composition, moisture levels, porosity, temperature, etc., affect bulk soil mechanical behavior relative to vehicle/terrain dynamics. It is also well known that for the same amount of mechanical loading, a tracked vehicle may cross soft terrain without considerable slippage, whereas wheeled vehicles may slip considerably, or simply spin. The amount of slip varies with soil type and terrain conditions.

To understand this behavior, a close examination must be conducted of soil properties. It is generally assumed and accepted by the scientific and engineering community that the relationship between the normal stress ( $\sigma$ ) and shear force ( $s$ ) per area can be represented by *Coulomb's* equation at every point in a soil mass,

$$s = c + p \tan(\phi) \quad [1]$$

The symbol  $C$  represents the cohesive properties of the soil and is measured in pounds per square inch (psi). The quantity  $\phi$  refers to the angle of friction and can also be measured in the laboratory.

A vehicle of adequate power moves across the soil if the strength of the ground can adequately support it, it can overcome the resistances, and it can produce the amount of thrust needed for propulsion. If assumed that the strength of the ground is adequate, homogeneous and ideal (no resistance), maximum thrust calculations can occur. When Eq. 1 is multiplied by the contact area  $A$ , the vehicle thrust or maximum force ( $H$ ) developed between the soil and the vehicle running gear is dependent upon  $A$  its weight ( $W$ ), the coefficient of cohesion ( $C$ ) and the angle of friction ( $\phi$ ),

$$H = cA + W \tan(\phi) \quad [2]$$

When a vehicle operates under soft soil conditions, the treads or grousers become filled with soil in the contact area, which helps to generate a tractive force. When the vehicle develops a thrust, it is the shearing of soil on soil that develops the shearing tractive force. From Eq. 2 it can be seen that this generation of thrust can be isolated to two extreme cases:  $c = 0$  and  $\phi = 0$ .

In the case where the angle of friction is zero, the soil is plastic and cohesive. In this case the soil does not yield easily to external forces, and its mechanical properties are comparable to glue. These soils include wet clay and wet snow. The amount

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of thrust generated by the vehicle is independent of its weight and is proportional to the size of the contact area as can be seen in Eq 3,

$$H = cA. \quad [3]$$

In the case of zero cohesion, the soil can be described as grainy and its mechanical properties are analogous to sand. It is loose in texture, and individual grains move across one other with some frictional forces. The maximum thrust generated is a function of vehicular weight and independent of contact surface area. Heavier vehicles generate larger force as can be seen from Eq. 4,

$$H = W \tan(\phi). \quad [4]$$

In the case of most soils, their mechanical properties consist of both cohesive and frictional components. To calculate the total vehicle thrust, the relationships between vehicle weight, soil contact area, and cohesive and frictional soil properties must be known in some details.

### SAFE WEIGHT

Gregory Bekker stated in his book titled *Off-The-Road Locomotion* that the safe load  $W_s$  for plastic equilibrium at the vehicle/terrain interface is determined by equation 5,<sup>2</sup>

$$W_s = A(cN_c + \gamma z N_q + 0.5\gamma b N_\gamma). \quad [5]$$

The variables  $N_c$ ,  $N_q$ , and  $N_\gamma$  are bearing capacity constants dependent on  $\phi$  and are derived in the book, *Theoretical Soil Mechanics* by Terzaghi.<sup>3</sup> Additional important parameters are the width or minimum dimension of the contact area ( $b$ ), initial plate sinkage ( $z$ ) in inches, and the unit weight of soil ( $\gamma$ ) in lb/cu inch. The safe weight is also dependent on the coefficient of cohesion.

Equation 5 is important because it gives empirical values for vehicle "floatation," which is the condition,  $z = 0$ . When Eq. 5 is normalized by  $A$ , the results are expressed as critical pressure and shown in Eq. 6,

$$p_s = cN_c + 0.5\gamma b N_\gamma. \quad [6]$$

This equation shows that floatation can not be considered in terms of ground pressure alone, but must also be a function of the soil bearing capacity, cohesion coefficient, and the width  $b$  of vehicle contact surface area. The following example illustrates the importance of this concept. Assume there are two robotic track vehicles. One has a track width of 10 inches and the other 5 inches while both systems have the same ground pressure. Each vehicle operates in sandy loam soil, which has a density equal to 0.06 lb/cu in, a coefficient of cohesion of 0.2, and a frictional angle equal to 35 degrees. From Fig. 1,  $N_c = 58$  and  $N_q = 42$  and

$$p_{Vehicle1} = (0.2)58 + (0.5)(0.06)10(42) = 24.2 \text{ psi},$$

$$p_{Vehicle2} = (0.2)58 + (0.5)(0.06)5(42) = 17.9 \text{ psi}.$$

One might think that equal ground pressure for the two cases would produce equivalent floatation pressures. This example shows how misleading it is to compare vehicle mobility performance only on the basis of ground pressure.

Figure 1 shows the functional dependence of  $N_c$  and  $N_q$  upon critical angle obtained from Terzaghi's *Theoretical Soil Mechanics*. An updated calculation for  $N_\gamma$  was obtained from Witlow's *Basic Soil Mechanics*.<sup>4</sup>

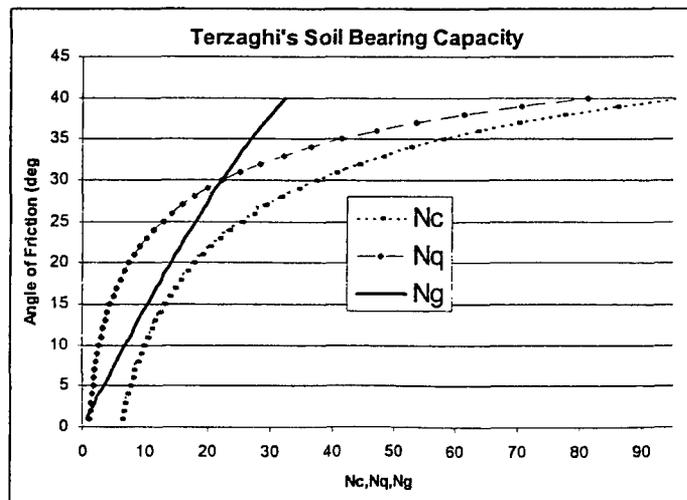


Figure 1 Terzaghi's Bearing Constants

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## SINKAGE

However, under normal conditions vehicles will sink into the terrain. Bekker developed an equation to determine this parameter<sup>1</sup>,

$$z = \left[ \frac{p}{k_c / b + k_\phi} \right]^{1/n} \quad [7]$$

The variables  $k_c$  and  $k_\phi$  are modulus of soil deformation for cohesive and frictional components of soil's strength and  $n$  is the exponent of soil deformation. These three parameters are empirical quantities of the soil and can be readily found in soil property books. Equation 7 indicates that the sinkage,  $z$ , is related to the width of the contact surface area. Larger values of  $b$  lead to greater sinkage,  $z$ , for the same ground pressure. Using the same vehicle as in the first example with  $k_c = 16$  and  $k_\phi = 7$  yields:

$$z_{10} = \left[ \frac{20}{16/10 + 7} \right] = 2.32 \quad \text{and} \quad z_5 = \left[ \frac{20}{16/5 + 7} \right] = 1.96. \quad [8]$$

The wider track clearly causes a significantly greater soil sinkage  $z$ . This phenomenon is not well understood but from field experience has been consistently valid. From these examples it can be seen that there is a trade off between sinkage and floatation. Wider tires can increase the allowable safe pressure of soil but causes larger amounts of sinkage.

## SLIPPAGE

The Bekker model uses the relationship between certain physical soil characteristics and shearing strength to predict vehicle cross-country mobility. Bekker considers wheels and tracks as simple loading surfaces having similar forms, but different lengths and widths. He extrapolates the analogy between soil shear produced by laboratory crawlers to track vehicles as shown in Fig. 2a.<sup>5</sup> When the blocked track is moved relative to the soil mass in the laboratory shear box, the maximum shearing force is not developed instantaneously with the initiation of relative motion. Instead the soil must be compacted to some extent before reaching the final steady state mechanical shearing stress. Thus the track grousers begin slipping before reaching the point of maximum vehicle traction. This transient condition is the basis for Bekker's simple 1-DOF model for vehicle trafficability.

The shear stress is the ratio between the vehicle traction force, which is parallel to the soil surface, and the area of the track normal to the surface. The soil resistance opposes this tractive force as the grousers slip during the shearing process. The normal loading force of the vehicle compacts the soil, which affects the resistance it exudes against the grousers as the track rotates on the vehicle. In effect the track forces push against the soil and generate a soil resistance that is determined by soil type and composition. Vehicle weight generates ground pressure, which further compacts the soil and alters the soil resistance.

When imagining a tracked vehicle in motion. A grouser on the track first comes into contact with the ground at some position. At the moment of first contact, no shearing or relative motion to the soil has occurred. As the vehicle moves forward, a shearing force is developed in the lateral direction parallel to the motion of the vehicle. The position of the grouser relative to the terrain begins to slip, pushing back against the soil and causing a soil distortion ( $S$ ). As the vehicle continues to move, the amount of soil distortion increases.<sup>4</sup>

For modeling purposes, it is critical to come up with a general equation describing the physical behavior of soil under these various conditions. When the empirical data gathered is graphed, it is identical to the displacement ( $x$ ) and natural time frequency ( $\omega t$ ) of an aperiodic or highly damped sinusoidal vibration:

$$x = A_1 e^{(-b + \sqrt{(b^2 - 1)})\omega t} + A_2 e^{(-b - \sqrt{(b^2 - 1)})\omega t}, \quad [9]$$

where  $b$  is the coefficient of damping. To write a formula in terms of soil stress ( $\tau$ ) and soil deformation ( $S$ ), let  $\tau = x$ ,  $K_1 S = \omega t$ , and  $K_2 = b$  where  $K_1$  and  $K_2$  are coefficients of slippage to arrive at the following result:

$$\tau = A_1 e^{(-K_2 + \sqrt{(K_2^2 - 1)})K_1 S} + A_2 e^{(-K_2 - \sqrt{(K_2^2 - 1)})K_1 S}. \quad [10]$$

After solving for  $A_1$  and  $A_2$ ,<sup>6</sup> a general equation for shear can be formulated as follows:

$$\tau = \frac{(c + p \tan \phi)}{y_{\max}} \left[ e^{(-K_2 + \sqrt{(K_2^2 - 1)})K_1 S} - e^{(-K_2 - \sqrt{(K_2^2 - 1)})K_1 S} \right]. \quad [11]$$

The value  $y_{\max}$  in Eq. 11 is the largest value for the expression within the brackets.

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The slip ( $S_m$ ) is assumed to be proportional to a constant slip velocity

$$S_m = v_s t. \quad [12]$$

However, the slip speed of the soil is equal to the rotation speed of the tire or track surface minus the vehicle speed,  $v=d/t$ , where  $d$  is the distance along the terrain where  $S_m$  has occurred. The amount of soil distortion that takes place at an arbitrary point relative to the front of the ground contact area is equal to

$$S = i_o x. \quad [13]$$

Equation 13 implies a relationship between tractive force and slip. Previous papers published by the authors on this subject talk more in detail about this relationship. The main reason is to develop an equation for soil thrust in soil as a function of slippage. The final result is seen below.

$$H_i = \left( \frac{b(c + p \tan \phi)}{K_1 \cdot i \cdot y_{\max}} \right) \left( \frac{-1 + e^{(-K_2 + \sqrt{K_2^2 - 1})K_1 i d}}{-K_2 + \sqrt{K_2^2 - 1}} + \frac{1 - e^{(-K_2 - \sqrt{K_2^2 - 1})K_1 i d}}{-K_2 - \sqrt{K_2^2 - 1}} \right). \quad [14]$$

When the vehicle slips, the maximum tractive force developed begins to change. Figure 2 represents the maximum peak of the curve as a function of slip percent. An equation proposed by the authors in equation 15 was used to generate the charts. For the graph on the left, a tracked M1A2 tank was used in the model. For the graph on the right, a wheeled HMMWV was used in the model. Each graph has three types of soil used. Clay and sand represent the far left and far right of the soil spectrum as explained earlier in this paper and previous papers by the authors.<sup>7</sup> Sandy loam was picked as a soil type having intermediate properties of cohesive and frictional components. By viewing the M1A2 graph, it can be seen that as the tank begins to slip in all soils that the maximum soil thrust that can be obtained by the vehicle decreases. At slips of 50 percent, the track vehicle is experiencing magnitudes of difference relative to no slip. The wheeled HMMWV on the other hand is not quite the same scenario. The wheeled vehicles maximum soil thrust that it can achieve seems to be near 50 percent slip in sand and loam and near 20 percent slip in sand.

$$S_m = \frac{\ln(-K_2 - \sqrt{K_2^2 - 1}) - \ln(-K_2 + \sqrt{K_2^2 - 1})}{2K_1 \sqrt{K_2^2 - 1}}. \quad [15]$$

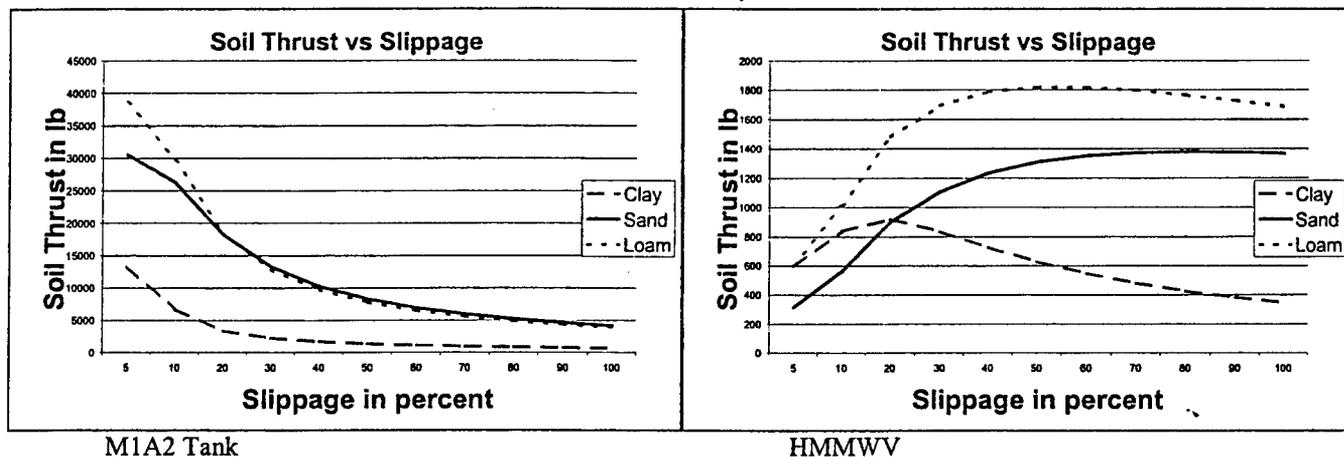


Figure 2 Soil Thrust vs. Slip Percent

It is easy to look at these curves and come to the conclusion that wheeled systems are superior to track systems when the vehicles are operating with slip. However, the tracked system has an advantage of being able to generate most of its tractive effort with no slip. Tracked systems usually have lengths that are 10 times the width. Wheeled vehicles tend to have contact surface areas of close to one to one comparisons. This greater length than width ratio creates a greater max tractive effort ratio at lower slip values. When a tracked vehicle begins to move in soft soil, its ability to overcome the resistive forces are great as can be seen from the M1A2 chart in Figure 2. The HMMWV from a starting position in soil at low slips has a much lower ability to overcome the resistive forces of the soil. More power applied to the wheel can easily cause digging the tire into the soil. Once this occurs, more resistive forces are added due to additional sinkage and a no go situation has developed.

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## RESISTANCES

Not all soil thrust can be accounted for in the production of useful work. Some of the soil thrust is lost in the form of energy expended through vehicle resistance to soil deformation. Soil compaction, bulldozing, and dragging cause this resistive energy loss. It has been shown that the vehicle energy expended for overcoming compaction resistance may be expressed by

$$R_c = \left( \frac{1}{(n+1)(k_c + bk_\phi)} \right) \left[ \frac{W}{l} \right]^{\frac{n+1}{n}} \quad [16]$$

where  $W$  is the vehicle weight in pounds and  $l$  is the length of the tire or track in contact with the ground. Bulldozing is the accumulation of soil mass in front of a vehicle. For most scenarios, the resistances originating from bulldozing, soil trapping and dragging it are neglected. However, the equation used for this model is taking into account the external resistance of the vehicles experience in the soil. Internal resistances are different for dissimilar vehicles and seem to play higher percentages in tracked vehicles. As of yet, no internal resistance is quantified in this model, but will be incorporated in the near future.

## DRAWBAR PULL

The drawbar pull (DP) is the total thrust minus the total resistance to vehicle motion. It is customary to view DP as the vehicle's ability to move both itself and a payload. If DP becomes zero or negative, then the vehicle locomotion will stop. In BDTM, there are three different methods to calculate DP. The first considers soil thrust developed purely from soil parameters without track grousers. The second formula for DP includes the additional thrust that is created by the action of grousers or tire treads.

$$H = blc(1 + 2h/b) + W \tan \phi \left( 1 + 0.64 \left[ (h/b) \cot^{-1} (h/b) \right] \right) \quad [17]$$

where  $b$  is the width,  $l$  is the length,  $h$  is the height of the grouser or tire tread,  $c$  is the coefficient of cohesion, and  $\phi$  is the angle of friction. The third calculation for DP includes the total tractive force as a function of slippage and contact area. A common parameter to compare vehicle performance is to normalize DP by vehicle weight, which is often called the traction coefficient.

## ROLLOVER<sup>6</sup> IN SOIL

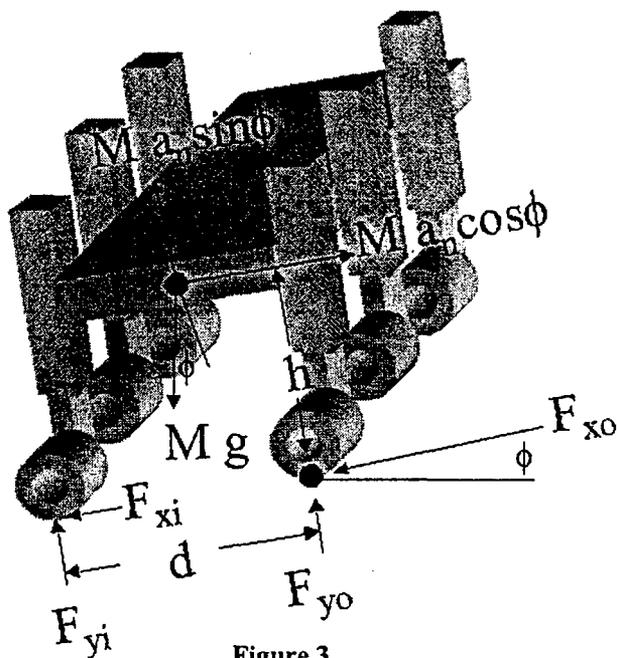


Figure 3

Another way to evaluate vehicles in soil is to understand the fundamental forces involved in rollover. In Figure 3 is a robotic vehicle that is performing a cornering maneuver. We treat this problem as a static problem by using D'Alembert's principle. The normal acceleration ( $v^2/R$ ) is converted into a force acting in the opposite sense with components in the x and y-axes. The lateral forces in the ground plane act to counterbalance this lateral acceleration force. The difference in position of the forces develops a moment, which we will refer to as the roll moment. This causes a roll to the outside of the turn as denoted by the subscript o.

For simplicity, let's assume that the vehicle undergoing the turn is rigid, and is experiencing no angular acceleration. In essence we are assuming a slip condition to simplify the analysis. If the vehicle is on unlevelled ground, a cross-slope ( $\phi$ ) needs to be accounted for where a positive angle represents downward slopes. This cross-slope can also help to counterbalance the acceleration in the lateral direction and is very useful when designing roadways. Taking moments around the outside tire as shown in Figure 3 yields:

$$Ma_n h \cos \phi - Mhg \sin \phi + F_{yi} d - Mg(d/2) \cos \phi - Ma_n (d/2) \sin \phi = 0 \quad [18]$$

Solving for  $a_n/g$  in equation [18] yields:

$$\frac{a_n}{g} = \frac{d/2 \cos \phi + h \sin \phi - \frac{F_{yi}d}{Mg}}{h \cos \phi - \frac{d}{2} \sin \phi} \quad [19]$$

Taking a small angle approximation gives us:

$$\frac{a_n}{g} = \frac{d/2 + h\phi - \frac{F_{yi}d}{Mg}}{h - \frac{d}{2}\phi} \quad [20]$$

On a roadway with no cross-slope, the equation is satisfied when the load of the inside wheels are  $\frac{1}{2}$  of the weight of the vehicle (makes sense). As the lateral acceleration becomes greater, the loads on the inside wheels need to diminish to compensate for the roll moment of cornering. The limit is reached when the load on the inside tire ( $F_{yi}$ ) is zero. At this point rollover occurs because of the vehicle's inability to counterbalance the forces. So with no cross slope and no  $F_{yi}$ , equation [20] becomes:

$$\frac{a_n}{g} = \frac{d/2}{h} \quad [21]$$

This simple measure of rollover threshold is used for the first order estimate of a vehicle's tendency to rollover. It is extremely convenient because it only needs two variables: distance between the tires or tracks and the height of the CG. Now we must check our assumption that the vehicle is slipping by comparing rollover threshold to the frictional coefficient we calculate from Bekker's model.

Let's assume for simplicity that the roll moment

Summing the forces in x and y (when  $F_{yi}$  goes to zero,  $F_{xi}$  goes to zero):

$$F_{yo} = Ma_y \quad [22]$$

$$F_{zo} = Mg - Ma_n \phi \quad [23]$$

If we could assume that the component acceleration in y is small we could simplify [23]

Slip condition:

$$F_{zo} > \mu F_{zo} \quad [24]$$

Substituting [22] and [23] into [24] and solving for  $a_n/g$ :

$$\frac{a_n}{g} \geq \frac{\mu}{1 + \mu\phi} \quad [24]$$

When we graph equation 20 and 24 on the same plot, we will see where the vehicle will rollover or slip. If the value of  $a_n/g$  calculated by equation 20 is greater than the right hand side of equation 24, the vehicle will slip. Otherwise the vehicle will rollover. As seen in Figure 4, the T1 robotic vehicle will slip in sand and loam and rollover in clay at zero cross-slope. As the angle of the cross-slope increases positively, there becomes an angle where the vehicle will slip in all soils before rollover. However, if the angle is decreased in a negative direction, there is a point where the vehicle is predicted to rollover in all three soil types.

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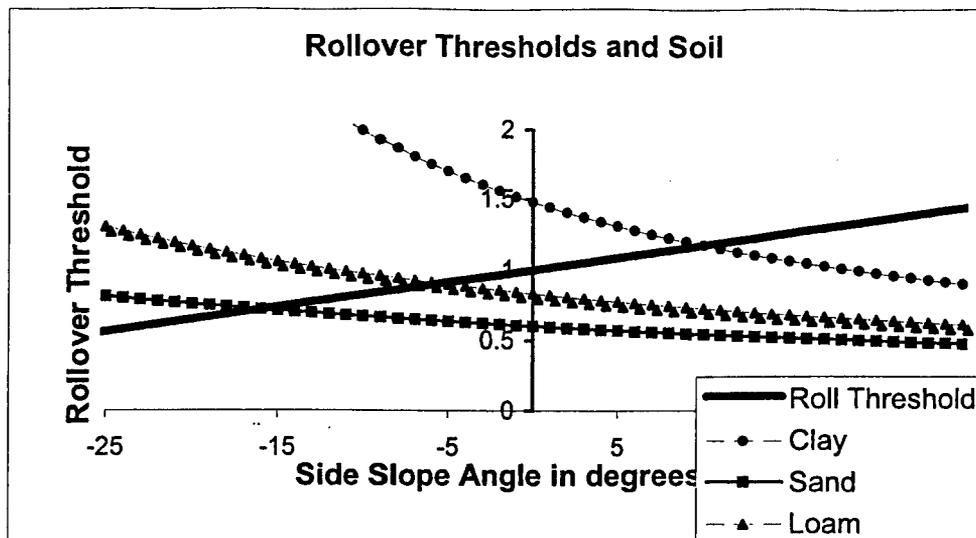


Figure 4

### USING THE MODEL

As in all modeling there comes a time when it is necessary to make use of the model. The BDTM was developed to evaluate vehicles on the premises of comparisons. Gregory Bekker's equations along with others are used as a baseline for these evaluations. The model was created as one particular way to evaluate robotic vehicles in Grant Gerhart's new robotic lab facility located at TACOM in Warren, MI. In order to make sense of the equations, pre-existing army vehicles are evaluated as a learning curve to see how useful the program can be.

Chart 1 has seven vehicles that are used for the purpose of evaluation. M1A2 has two 25"x190" tracks with a weight of 9500lbs. M113 is another tracked military vehicle with two 15"x114" tracks and a weight of 120,251 lbs. The Oshkosh PLS has 10 wheels with a ground contact surface area of 16"x16". Its weight is 88,000 lbs. The HMMWV is a four-wheel vehicle with 10"x10" prints and weighs 10,800. The GM Sonoma truck weighs in at 2300 lbs and has a print 6"x6". The Chevy Cavalier has tire prints of 5"x5" on the ground and weight of 2000 lbs. Finally the T1 is a robotic vehicle that has been constructed by Utah State University as a project for the U.S. Army. It has a weight of 120 lbs. And forms a 2"x2" print for each of its six wheels.

The soil thrust correlates with vehicle weight as seen in column 2 of chart 1 where the type of soil is dry sand. However, because soil sinkage is dependent upon the width of the contact area, the wheeled vehicles experience significantly higher soil sinkage. The long, thin surface contact of the M1A2 and M113 allow for smaller sinkage rates. This allows for smaller external resistance due to compaction which results in greater drawbar pull percentages. As is generally the case, tanks have a smaller ground pressure that they exert on the ground due to their larger contact surface area. In the particular case of sand in the first set of columns, the tracked vehicles are able to maintain the floating pressure or safe weight that is needed to obtain floatation on the surface of the soil. This means that they are under the maximum ground pressure that causes the surface of the soil to break down and not be able to support the vehicle. In all cases of wheeled vehicles, this criteria was not kept and brings understanding to the large amounts of sinkage that these vehicles are predicted to encounter.

Some other interesting observations in sand are that the maximum tractive effort is obtained at different slip percentages for the different vehicles. As the vehicles go down on the chart the size of the tires decrease. As the tire's contact area is decreased the effective surface area is decreased by the square due to the usually standard measurement of tires for wheeled vehicles. As this ratio decreases, the maximum amount of slip required to reach the maximum tractive force is increased. The cars ability to maneuver at these higher values of slip are questionable due to a higher probability of digging into the soil. This digging creates a bulldozing effect or a hole thus creating a much higher resistance to overcome.

Clay on the other hand is a different story. The M1A2 tank has the largest contact surface area as compared to the other vehicles and as expected generated the largest soil thrust as shown in column 9 of chart 1. A surprising result from column 9 is that the Oshkosh PLS wheeled vehicle produced more soil thrust than the M113 track vehicle, which had a greater surface

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area, but at a much lower percentage than in sand. This is easily explained by breaking apart the tractive effort formula into its two parts: frictional and cohesive. For the cohesive such as clay, the fundamental variable is the vehicles contact surface area. The PLS can approach a track like contact surface area by adding additional tires and in this type of soil would benefit dramatically.

All vehicles had ground pressures under the maximum safe soil pressure, however, the wheeled vehicles had larger soil sinkage values, which leads to a much lower normalized drawbar pull (DP/W) value for all these systems. The M113 did have a remarkable performance in this type of soil, because it sinks relatively little in this type of soil and could pull 1.29 times its own vehicle weight.

The last comparison to be viewed is the Roll number. The last column in Chart 1 has the roll numbers that determines its ability to roll. The higher the roll number, the least likely it is going to roll. Our analysis using Gillespie equations, determines that M1A2 is the most unlikely vehicle to experience a roll where as the Oshkosh PLS is the most likely to roll over while experiencing a turn. All turns are assumed to occur at no cross-slope and are approximated by rigid bodies. In both the two cases, sand and clay, the friction component of the soil is placed under the heading  $u$ . This frictional component is dependent on the shear of the soil and the normal force of the vehicle. In sand all vehicles have about the same frictional component of 0.6. In clay however this is not the case. In clay the effect is more non-linear and seems to have greater frictional components at lower ground pressures and can be ranked accordingly. The T1 has the least amount of ground pressure and highest frictional component where as the opposite is true for the PLS. The purpose of these numbers is to determine what is the vehicle going to do when rollover threshold is met. If the roll number is greater than the frictional component then the vehicle will experience spin out and not roll. If the opposite is true then the vehicle will rollover probably causing a large amount of damage to the vehicle. Experiments in this type of research are currently going on in joint research with the T1. Techniques are being done by Utah State that allows a robotic vehicle to change its orientation around a curve that increases its d distance, which increases its Roll number. This decreases the probability of rollover.

Vehicle	Sand							Clay							Roll
	H eq2	P	Ws/A	z	DP/W	MaxS	u	H eq2	P	Ws/A	z	DP/W	MaxS	u	
M1A2	2257	12.6	25.5	3.8	0.53	0	0.6	82953	12.6	63.5	3.8	0.667	0	0.69	1.07
M113	1869	5.81	14.5	1.4	1.586	0	0.6	25713	5.81	62.9	1.4	1.29	0	1.3	0.92
Oshkosh PLS	52875	34.4	38.4	10	0.22	10	0.6	32157	34.4	64.3	10	0.22	10	0.36	0.71
HMMWV	6489	27	20.3	6.5	0.047	20	0.6	4504	27	63.6	6.5	0.047	20	0.42	1.01
GM Truck	1381	16	12.4	2.8	0.318	30	0.6	1342	16	62.9	2.8	0.318	30	0.58	0.89
GM Cavalier	1202	20	13.1	3.2	0.136	40	0.6	1003	20	63	3.2	0.136	40	0.5	1.05
T1	638	5	8.1	0.3	1.41	100	0.6	22	5	62.5	0.3	1.41	100	1.47	1

### CONCLUSION

A primary objective of this paper is to evaluate Bekker's formalism as a tool to perform mobility tradeoffs between wheeled and track vehicles. BDTM is able to predict the percent slip and drawbar-pull characteristics of track vehicles for several different soil conditions. In addition the model predicts soil distortion at some distance  $x$  behind the front portion of the track, maximum tractive effort, and soil sinkage for various soil conditions.

The fundamental assumptions in Bekker's work are that shear/slip is the primary mechanism for generating tractive vehicle forces, a 1-DOF over-damped, linear model describes vehicle motion, and that soils plastically deform under ground pressure forces. At present BDTM does not contain the internal resistance of the vehicle, which turns out to be significant. This deficiency will be corrected in the future.

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- 3. Classified. Cannot be released, and requires classification and control at the level of \_\_\_\_\_

Security Office (AMSTA-CM-XS):

Concur/Nonconcur [Signature] 14 Aug 00  
Signature Date

Public Affairs Office (AMSTA-CM-PI):

Concur/Nonconcur [Signature] 21 Aug 00  
Signature Date