Model based SAR data compression

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Abstract— In this paper a wavelet based method for SAR data denoising and compression is presented. An unsupervised stochastic model based approach to image denoising is presented. SAR image is modeled in wavelet domain Gauss Markov Random field and noise is considered as Gaussian with unknown variance. The parameters are estimated from incomplete data using mixtures of wavelet coefficients, and expectation maximization algorithm. The expectation maximization algorithm is used to efficiently compute a maximum a posteriori estimate. Observed wavelet coefficient is estimated using inter and intra scale of wavelet coefficients to estimate image and noise model parameters. Presented wavelet based method efficiently removes noise from SAR images. The second step is to design an entropy coder that efficiently codes despeckled image. The texture parameters obtained at the despeckling stage are used in the compression process. The image coder is tested on X-SAR data with and achieves comparable compression results with the wavelet based state-of-the art coders for SAR data compression.

Keywords— gauss markov random field, wavelet transform, mixture coefficients, compression

I. INTRODUCTION

Over the past few years all kinds of data presentation has been intensively investigated. The SAR (synthetic aperture radar) data provided by X-SAR and RADAR-SAT missions have 10-20 meter resolution. SAR radars with 1 meter resolution have been intensively investigated. The Terra-SAR radar, will provide images with 0.7 meter resolution.

In this paper we will introduce a wavelet based denoising for SAR data. Many different wavelet based techniques have appeared in recent times. It is well known, that SAR data are corrupted by multiplicative noise called speckles. If the speckles are removed from the SAR image, higher compression ratios for data compression can be achieved [1]. Many authors have shown that the speckles can be efficiently removed in the wavelet domain. Two different approaches are usually applied for speckle removal; the first one uses maximum a posteriori (MAP) filtering of the image, where the image and speckles are separately modeled [2]-[3], and the second approach is to use MMSE estimator as shown in [4]-[5]. With other words, all authors have introduced a shrinkage factor that predicts the coefficient without noise. If the image model is well chosen, the despeckling algorithm can provide side information about the texture or even the image can be segmented into different areas. When the image is modeled as a Gauss Markov Random field, texture parameters can be derived. Many algorithms for segmentation have been proposed. In [6] a Markov random field is used for image model and expectation maximization (EM) [9] is used to estimate parameters. Bayesian tree structured image modeling using wavelet domain hidden Markov models is proposed in [7]. Multiscale Bayesian segmentation using a trainable context model is proposed in [8]. In [7] and [8] a multi-scale wavelet trees are used at modeling stage. In this paper we introduce wavelet based algorithm for SAR data despeckling. The image is modeled as a Gauss-Markov random field and a noise is considered as a Gaussian noise with unknown variance and mean. A Bayesian approach is used to determine a posteriori and with expectation-maximization parameters are estimated.

II. MODEL BASED DENOISING

Let \( y = (x, z) \) be a set of random data of interest, where \( x \) is observed data and \( z \) is a part of \( y \) that is not observable. Observable data \( x \) is usually called incomplete data and \( y \) is called complete data. Let \( p(x \mid \Phi) \) be the probability density function (pdf) of incomplete data \( x \), where \( \Phi \) is a set of parameters that characterize pdf. and \( p(y \mid \Phi) \) pdf of complete data. The incomplete data problem is to estimate \( \Phi \) based only on the observations presented by incomplete data \( x \). The pdf of incomplete data is then

\[
p(x \mid \Phi) = \int_{y(x)} p(y \mid \Phi)
\]

An iterative maximum likelihood (ML) approach is to find estimate \( \hat{\Phi}_{ML} \):

\[
\hat{\Phi}_{ML} = \arg \max_{\Phi} p(x \mid \Phi)
\]

The most general and effective algorithm for solving this problem appears to be EM algorithm [9]. The EM is iterative algorithm and requires expectation and maximization steps. In the E-step of the EM algorithm we have to compute the functional \( Q(\Phi \mid \hat{\Phi}^{(p)}) \) for the subsequent maximization in the M-step.

\[
Q(\Phi \mid \hat{\Phi}^{(p)}) = \mathbb{E} \left[ \log p(y \mid \Phi) \mid x, \hat{\Phi}^{(p)} \right]
\]

In the M-step a maximization of functional is computed:
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To apply EM algorithm, we start at initial \( \Phi(0) \) and iterate algorithm until algorithm converges: \( \| \Phi^{(p+1)} - \Phi^p \| \leq \epsilon \), for appropriately chosen vector norm and value \( \epsilon \).

### Model selection

In this paper the image is modeled using Gauss-Markov random field:

\[
p(w_i | x_i, \theta) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp \left( - \left( x_i - \sum_j \theta_j w_{i,j} \right)^2 / 2\sigma_s^2 \right)
\]

(S5)

Speckle model can be chosen as a Gaussian process:

\[
p(w_i | x_i) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp \left( - (w_i - x_i)^2 / 2\sigma_n^2 \right)
\]

(S6)

In equations (5) and (6) variances \( \sigma_s^2 \) and \( \sigma_n^2 \) are unknown. The parameter \( \theta \) that provides texture information can be estimated on the various ways. In [2] maximum a posteriori is applied in the image domain. Many authors propose denoising techniques by assuming a Gaussian noise with zero mean and variances \( \sigma_s \) and \( \sigma_n \) in equation (4).

A robust estimate of the noise standard deviation \( \sigma_n \) is obtained in the finest decomposition scale by measured wavelet coefficients as:

\[
\hat{\sigma}_n = \frac{1}{0.6745} \text{MAD}\{\{w_{j,k}\}, 0 \leq k < 2^J}\)
\]

where \( w \) denotes a wavelet coefficient and \( J \) denotes the finest level of wavelet decomposition.

### Expectation maximization algorithm for SAR image denoising

The idea is to estimate unknown parameters \( \Phi = [\theta, x_i, \sigma_s^2, \sigma_n^2] \) using EM algorithm and wavelet decomposition trees. In this case, the complete dataset consists of the joint events \((x_k, z_k), k=1,\ldots,N\) and \( z_k \) takes integer values in the interval \([1, J]\), and it denotes the mixture from which \( x_k \) is generated.

\[
p(x) = \sum_{j=1}^{J} p(x | z_j) P_j
\]

\[
\sum_{j=1}^{J} P_j = 1, \int x p(x | z_j) dx = 1
\]

Using Bayesian inference:

\[
p(x | w, \Phi) = \frac{p(w | x, \Phi) p(x | \Phi)}{p(w | \Phi)} \propto p(w | x, \Phi) p(x | \Phi)
\]

Unknown parameters \( \Phi = [\theta, x_i, \sigma_s^2, \sigma_n^2] \) can be estimated using EM algorithm:

\[
\log p(x | w, \Phi) = \sum_{k=1}^{N} \sum_{i=1}^{J} P(z_i | x_i, \Phi) \left[ \log p(w_i | x_i, \Phi) + \log p(x_i | \Phi) + \log P_i \right]
\]

\[
\log p(x | w, \Phi) = \sum_{k=1}^{N} \sum_{i=1}^{J} P(z_i | x_i, \theta) \left[ \log \frac{1}{\sqrt{2\pi\sigma_s^2}} \left( x_i - \sum_j \theta_j w_{i,j} \right)^2 / 2\sigma_s^2 \right]
\]

\[
+ \sum_{k=1}^{N} \sum_{i=1}^{J} P(z_i | x_i, \theta) \left[ \log \frac{1}{\sqrt{2\pi\sigma_n^2}} \left( w_i - x_i \right)^2 / 2\sigma_n^2 \right] + \sum_{k=1}^{N} \sum_{i=1}^{J} \log P_i
\]

(11)

where \( E[z_i | x] \) is defined in (19)-(20) in \( f(.) \) is Gaussian with 0 mean and variances \( \sigma_s^2 \) and \( \sigma_n^2 \). In equation (14) unknown parameters are \( \Phi = [\theta, x_i, \sigma_s^2, \sigma_n^2] \) and \( \theta_j \). By calculating M-step the estimates (15)-(18) are obtained:

\[
\theta_j(t+1) = \frac{\sum_k P(z_i | x_i, \theta(t)) [x_i - \sum_j \theta_j(t) w_{i,j}]}{\sum_k P(z_i | x_i, \theta(t)) w_k}
\]

\[
x_i(t+1) = \sum_k P(z_i | x_i, \theta(t)) w_k \left[ \sum_j [\theta_j(t) w_{i,j} / \sigma_{s,i,j}(t)] + w_i / \sigma_{n,i,j}(t) \right]
\]

\[
[1 / \sigma_{s,i,j}(t) + 1 / \sigma_{n,i,j}(t)] \sum_k P(z_i | x_i, \theta(t))
\]

\[
\sigma_{s,i,j}(t+1) = \sum_k P(z_i | x_i, \theta(t)) w_k \left[ \sum_j [\theta_j(t) w_{i,j} / \sigma_{s,j,i}(t)] + w_i / \sigma_{n,j,i}(t) \right]
\]

\[
[1 / \sigma_{s,i,j}(t) + 1 / \sigma_{n,i,j}(t)] \sum_k P(z_i | x_i, \theta(t))
\]

\[
\sigma_{s,j,i}(t+1) = \sum_k P(z_i | x_i, \theta(t)) [x_i - \sum_j \theta_j(t) w_{i,j}]^2
\]

\[
\sum_k P(z_i | x_i, \theta(t))
\]

(14)
\[ \sigma^2_{j}(t+1) = \frac{\sum_{i} P(z_i | x_i, \theta(t)) [w_i - x_i]^2}{\sum_{i} P(z_i | x_i, \theta(t))} \]  

(15)

\[ P_j(t+1) = \frac{1}{N} \sum_{i=1}^{N} P(z_i | x_i, \theta(t)) \]  

(16)

\[ P(z_j | x_j, \theta(t)) = \frac{p(x_j | z_j, \theta(t)) P(t)}{p(x_j, \Phi(t))} \]  

(17)

\[ p(x_j, \Phi(t)) = \sum_{j=1}^{J} p(x_j, \Phi(t)) P_j(t) \]

The algorithm repeats until \( \| \Phi(t+1) - \Phi(t) \| \leq \epsilon \).

The dependence parent, children in the dyadic wavelet decomposition are shown in Fig. 1. Each parent has four children in lower decomposition level.

The context selection is different for variance estimation \( \sigma_{x}^2 \) and \( \sigma_{n}^2 \). For estimation of variance \( \sigma_{x}^2 \) in equation (17), wavelet coefficients that surround observed coefficient are considered. If for example diagonal wavelet coefficient on second level is observed, horizontal, vertical, left and right neighbor coefficients are considered and the same technique is used for coefficient in vertical and horizontal subband at the same level. The observed coefficient is predicted using:

\[ x_i = \theta_1 w_{diag} + \theta_1 (w_x + w_y) + \theta_2 (w_y + w_p) + \theta_2 (w_{hor} + w_{ver}) \]  

(18)

For variance estimation \( \sigma_{n}^2 \) corresponding children wavelet coefficients that are 2 or one 1 level lower as observed coefficient are considered. In Fig. 1 a particular example is shown, where coefficients 1 level lower of observed wavelet coefficient are considered.

### III. COMPRESSION OF DENOISED IMAGE

Space Frequency Quantization (SFQ), and a fuzzy context based modeling. The Space Frequency Quantization (SFQ) is an effective coding technique, which uses a zerotree pruning of wavelet coefficients. After the Space Frequency quantization is applied to the wavelet coefficients the non-pruned coefficients are quantized using context modeling. The context based quantization technique uses the partitioning proposed by the Trellis Coded Quantization and the fuzzy logic to predict the next TCQ state from the context. Quantized indices are entropy coded using an arithmetic coder. The probability estimation is based on the observation of the past coded bits.

The texture parameters obtained at the despreening stage are used in the compression process. Texture parameters also defines different areas of the image (urban scene, agricultural scene, etc.) and are used to predict an observed wavelet coefficient from the past wavelet coefficients. This approach enabled un-uniform quantization of different scenes in the single image. The prediction is reduced from the observed wavelet coefficient and prediction error is coded using image fuzzy based coder.

### IV. EXPERIMENTAL RESULTS

#### A. Denoising results

The X-SAR image 256×256 with 2-looks, shown in Fig.2(a) is transformed with dyadic wavelet transform using "symlets4" filter bank. The image is denoised using proposed method as shown in Fig. 2(b). 3 levels of wavelet decomposition were used.

Figure 1. Original X-SAR image.

Figure 2. Denoised image using proposed method.

#### B. Compression results

The denoised image and original image are compressed using compression scheme proposed in Section III. Reconstructed images using lossy compression method are depicted in Fig. 3-6.
In this paper despeckling and compression algorithms based on wavelet transform are presented. Experiments showed that the expectation maximization algorithm can be used in the process of denoising SAR images. Only Gaussian model is considered. It is well known that wavelet subbands does not have Gaussian distribution thus other models should be tested. Visual impressions of compressed despeckled images and original-speckled images at higher compression ratios are almost the same.

REFERENCES


