Efficient Segmentation of Geophysical Field Images on Basis of Independent Component Analysis

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Abstract—In the paper, we consider a new method of geophysical fields image analysis and sensitive segmentation. The method has a high sensitivity in comparison with other well known techniques of geophysical field image analysis and consists of the following main steps: expand the informational features using zero-space imaging method, processing by Independent Component Analysis, data fusion based on Kohonen’s self-organizing map.

Keywords—sensitive segmentation; space-zero imaging; ICA; SOM; geophysical field;

I. INTRODUCTION

The problem of sensitive segmentation of geophysical fields is traditionally the most essential in the theory and practice of remote sensing.

Visual analysis of geophysical field images has a several nuances, connected with their features of psycho physiological perception. Especially, the brightness jumps of image areas are less than 1%, and it can’t be perceived visually. So, one of the main tasks for visual analysis is the increasing of sensitivity for revealing the low contrast areas.

From the practical point of view the most widely used methods of clusterization are two ones.

1) Fuzzy C-mean clusterization method, which demands a priori setting of clusters’ number, that’s very complicated in many cases. Besides, the convergence of iterative algorithm also depends on choosing of the fuzzification parameter value, without any theoretical recommendations on that.

2) Adaptive clusterization method, which is based on using of Kohonen’s self-organizing map. This method doesn’t require any priori assumptions about the number of clusters, and is more practically preferable in that meaning. However it’s required more quality of informational features coming on the input of net. These features should be normalized, nonredundant and most preferable statistical independent.

The goal of this work is further development of Zero-Space Complex Imaging Method [1,2,3] by using a new informational basis formed from independent components, with next fusion in the result image by using adaptive Kohonen’s clusterization.

II. METHODOLOGY

A. Zero-space imaging method

A zero-space imaging method involves following steps:

- A moving window (3x3) slides along a geophysical image and compares each pixel (m,n) of analyzed image to unwrapped in a spiral order window, forming a vector a:

\[ \vec{a} = \vec{a}(m,n) = [I(m-k,n-l); k,l = -1,0,1] \] (1)

where, \( I(m,n) \) - brightness of geophysical image into a \((m,n)\) point.

- Coefficients of the vector \( a \) are considered as coefficients of a characteristic polynomial \( H(z) \) of ninth degree

\[ H(z) = z^9 + a_1z^8 + a_2z^7 + \ldots + a_9z + a_9 \] (2)

where \( z \)-transformation operator, which characterizes the order of \( a \) coefficients.

Polynomial \( H(z) \) is completely characterized by its zeros \( z_k \):

Figure 1. Original image of magnetic field of the earth surface.
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\[ H(z) = \prod_{k=1}^{9} (z - z_k^*) \]  

(3)

The heart of zero-space imaging method concludes in analysis of zeros \( z_k^* \); \( k=1,\ldots,9 \). Since every zero is characterized by its magnitude \( |z_k| \) and phase \( \phi_k \), the source image can be extended into 18 new (9 magnitude and 9 phase) synthetic images.

From the practical point of view, an expediency of polynomial \( H(z) \) zeros visualization as a nonlinear fusion operation of \( a_i \) coefficients, follows from a well known property, that a small variation of \( a_i \) coefficients (i.e. local brightnesses of image \( I \)) can correspond to a great variation of complex zeros \( z_k^* \) of a polynomial \( H(z) \). It allows to increase a local sensitivity for low-contrast informational areas detection.

However, the fact of space dimension increasing of informational features shows the necessity of additional processing on multidimensional data to make easier a procedure of the next analysis and problem-solving. It stipulates expediency to proceed to the second part of our approach, based on independent component analysis (ICA). The informational processing of independent component analysis should be performed separately for magnitude \( |z_k| \) and phase \( \phi_k \) characteristics, because of the different nature of their informational potentials. A significance of additional multidimensional data processing based on ICA also follows the fact, that no one of new images \( |z_k| \) and \( \phi_k \) couldn’t be given a preference in comparison with others, because each of them is a “zero”, i.e. has an equal rights from the polynomial factorization point of view.

**B. Multidimensional information fusion**

In our experiments, we used Kohonen’s algorithm [7,8] of self-organizing map for the fusion of the zero-space images, which provides unsupervised fusion and segmentation of multidimensional synthetic image ensembles into a single resulting image. Since SOM algorithm is well known, its mathematical specification wasn’t considered in this paper. It should be noted, that success of applying SOM in our example, to our mind, was stipulated by such thing, as for optimal procedure of segmentation based on SOM, an input data have to be maximally independent each of other in a statistical sense. It was achieved by independent components, obtained from the ensemble of zero-space characteristics.

In contrast to correlation-based transformations such as Principal Component Analysis (PCA), ICA not only decorrelates the signals (2nd-order statistics) but also reduces higher-order statistical dependencies, attempting to make the signals as independent as possible. The technique of ICA is a relatively new invention. So it is important to consider the main aspects of ICA.

**C. Independent Component Analysis**

Our ICA algorithm based on the maximization of nongaussiannity[5]. The most natural information-theoretic contrast function is negentropy. Negentropy is based on the information-theoretic quantity of differential entropy, which is related to the information that the observation of the variable gives. The more “random”, i.e., unpredictable and unstructured the variable is, the larger its entropy. The (differential) entropy \( H \) of a random vector \( x \) with density \( p(x) \) is defined as

\[ H(x) = -\int p(x) \log p(x) dx \]  

(4)

A fundamental result of information theory is that a gaussian variable has the largest entropy among all random variables of equal variance. This means that entropy could be used as a measure of nongaussianity. In fact, this shows that the gaussian distribution is the “most random” or the least structured of all distributions. Entropy is small for distributions that are clearly concentrated on certain values, i.e., when the variable is clearly clustered, or has a p.d.f. that is very “spiky”. To obtain a measure of nongaussianity that is zero for a gaussian variable and always nonnegative, one often uses a normalized version of differential entropy, called negentropy. Negentropy \( J \) is defined as follows:

\[ J(y) = H(y_{gauss}) - H(y) \]  

(5)

where \( y_{gauss} \) is a gaussian random vector of the same correlation (and covariance) matrix as \( y \). Due to the above-mentioned properties, negentropy is always nonnegative, and it is zero if and only if \( y \) has a gaussian distribution. Negentropy has the additional interesting property that it is invariant for invertible linear transformations. The advantage of using negentropy, or equivalently, differential entropy, as a measure of nongaussianity is that it is well justified by statistical theory. In fact, negentropy is in some sense the optimal estimator of nongaussianity, as far as the statistical performance is concerned. The problem in using negentropy is, however, that it is computationally very difficult. Estimating negentropy using the definition would require an estimate of the probability density function(p.d.f).

The problem of restoration a probability density function (p.d.f) is the central problem of mathematical statistics. According to the definition, the p.d.f. \( P(t) \) is connected with cumulative density function (c.d.f.) \( F(z) = P(t \leq z) \) by integral relation:

\[ \int_{-\infty}^{z} P(t) dt = F(z) \]  

(6)

we can rewrite it as:

\[ \int_{-\infty}^{z} Q(z-t) P(t) dt = F(z) \; \text{where} \; Q(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases} \]  

(7)

For continues densities there is only one solution of (7). Now let’s determine a fitted c.d.f: \( F(z) = \frac{k}{l} \), if \( z \) is more than \( k \)-elements of sampling \( z_1, \ldots, z_l \):

\[ F_i(z) = \frac{1}{l} \sum_{i=1}^{l} Q(z_i - z) \]  

(8)

The Grivenko-Kantelly theorem states, that with increasing amount of sampling \( l \), the fitted c.d.f. is uniformly coming to the true c.d.f.
Now, we shall try to find the solution of (7) in the situation where the unknown c.d.f. $F(z)$ is substituted by the fitting c.d.f. $F_l(z)$. It should be noted, that approximation of p.d.f. is achieved by solution of ill-conditioned problem of numerical differentiation where the right side is specified inaccurately. Nevertheless, the solution of (7) will not be any continuous function, but the positive-definite function $P(t)$ so as:

$$\int_{-\infty}^{+\infty} P(t) dt = 1$$  \hspace{1cm} (9)

Thus, p.d.f. $P(t)$ is a solution of integral equation of Fredholm order I (7). Approximation of p.d.f. $P(t)$ is searched in the form of expansion of trigonometrical functions:

$$\varphi_k(t) = \sqrt{\frac{4}{\pi}} \cos\left(\frac{2k-1}{2} \pi t\right), \quad t \in [0,1]; \quad k = 1, 2, \ldots$$  \hspace{1cm} (10)

i.e.

$$p(t) = \sum_{i=1}^{N} \lambda_i \varphi_i(t)$$  \hspace{1cm} (11)

The problem of choosing the degree of complexity of estimation (i.e. the number of expansion terms $N$) is solved by the method of structural risk minimization.

In this paper we proposed two-phase algorithm for p.d.f estimation [1,4]. Thus, in this algorithm we realized two stages of problem solution – the rough estimation of distribution and accurate estimation of the probability density function. Obviously, these two stages are fundamentally necessary. If you look on the well-known traditional methods, e.g. histogram method, Parzen-Rozenblat method, you can see that here you have to use two stages also.

We have tested this method on the synthetic data, and compared results with Parzen-Rozenblat method. The proposed method gave much more accurate data than Parzen-Rozenblat method, especially for multimodal p.d.f. Thus the two-phase algorithm gives a very good approximation for unimodal and for multimodal probability density functions.

A fundamental approach to ICA is given by the principle of nongaussianity. The independent components can be found by finding directions in which the data is maximally nongaussian. Nongaussianity is measured by entropy-based measures. Estimation of the ICA model can then be performed by maximizing such nongaussianity measure - negentropy. Several independent components can be found by finding several directions of maximum nongaussianity under the constraint of decorrelation.

The main mathematical aspects of ICA are widely considered in a great number of papers[5,6]. But, it should be noted, that main efforts were focused on the increasing of computational speed, which is necessarily affected on the accuracy of independent component achieving. Our main objective was to reveal very fine effects into analyzed geophysical field images, so we had to develop a novel, more accurate method for computation of independent components of synthetic image ensembles, based on more accurate probability density function approximation.

### Results

In our experiments we considered image of magnetic field of the earth surface (Fig. 1). The image of magnetic field is a typical example of low contrast image with margin fuzziness. In consequence of this, the visual analysis is very difficult problem.

Following our method, we achieved nine complex zero-space characteristics which are not shown there. We use magnitude and phase of these characteristics separately for our future processing. On the ICA step we reduce the number of components to three (precisely on PCA stage), so we achieve 3 independent components(ICs) from magnitude of the characteristics and 3 independent components from phase of the characteristics. Three ICs from the magnitude of zero-space characteristics are shown below.

![Figure 2. First IC achieved from magnitudes of zero-space characteristics.](image1)

![Figure 3. Second IC achieved from magnitudes of zero-space characteristics.](image2)
The result of three independent components fusion into a single resulting image, using SOM is presented in Fig.5 and Fig. 6.

After applying our methods, we achieved very promising result, which shows a very sensitive segmentation of magnetic field. Considering Fig. 5 and Fig.6, you can see that the proposed method of geophysical image analysis based on independent components allows to make very sensitive segmentation of the image of magnetic field. It should be once more emphasized that a direct applying of SOM algorithm to the zero-space characteristics didn’t give any positive results. So the proposed method is supposed to be a very promising and very useful in such an important problem as geophysical field segmentation.

IV. CONCLUSION

In summary, we can assert following:

- Application of zero-space imaging method discovers a potential capability to increase a local sensitivity visual analysis of geophysical images.

- The results of our researches confirmed that imaging of zero space information, based on independent components by applying SOM algorithm for data fusion of synthetic image ensembles into a single resulting one is necessary for increasing a sensitivity of low-contrast areas detection.

- Proposed method of ICA ensures the required accuracy of independent components computations, which is necessary for sensitive segmentation. Considered ICA algorithm is based on a novel and very accurate approximation of probability density function.

- Discussed method of geophysical field analysis has a considerable potential for future development of its informational capabilities.

REFERENCES


