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Electrical Engineering and Computer Science Department
The University of Michigan, Ann Arbor, MI, USA


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Experimental and modeling investigation of microwave radiometer noise statistics for Earth remote sensing

Hanh Pham∗, A. W. (Tony) England∗†, Victor Solo∗, and E. J. Kim‡

∗Electrical Engineering and Computer Science Department
The University of Michigan, Ann Arbor, MI, USA
Email: hpham@umich.edu

†Atmospheric, Oceanic, and Space Sciences Department
The University of Michigan, Ann Arbor, MI, USA
Email: england@umich.edu

‡NASA Goddard Space Flight Center
Greenbelt, MD, USA
Email: ed.kim@nasa.gov

Abstract—Recent availability of high-speed analog to digital converters (ADC) has enabled the development of digital radiometers for Earth remote sensing. In the unprotected C band, undesired radio frequency interferences (RFI) have been observed and mitigation efforts are currently being carried out in the research community. They mainly consist of detection and filtering in the time or frequency domain. Because RFI can be very small and mistaken as geophysical signals, an additional approach could rely on the inversion of a model describing internal noise, RFI and geophysical data processed through the radiometer. Such an approach demands a thorough understanding of receiver internal noise statistics. As a first step, we propose to model the system by taking advantage of the time series record available at the digital back end of a receiver. The experimental setup includes an X band benchtop radiometer serving as a standard analog gain chain, and various inputs fed into the radiometer front end. The digital output of an eight-bit ADC following the analog chain was recorded using a logic analyzer. An exploratory data analysis enabled us to select a priori valid data. Statistical signal processing techniques encompassed loglog relationships, classical spectral estimation (smoothed periodogram and windowed autocovariance), and autoregressive modeling.

I. INTRODUCTION

Radio frequency interferences (RFI) have been observed in unprotected spectrum band for passive observations [1] and mitigation efforts are currently being carried out in the research community. As high speed analog to digital converter (ADC) are enabling the development of digital radiometers for Earth remote sensing, the same digitization process allow various mitigation techniques. Mitigation strategies have been first developed in radioastronomy. Among them, those that are actually implemented in Earth Science passive remote sensing include detection and removal in the time domain [2], [3], in the frequency domain [3].

When RFI are very small and and can be mistaken as geophysical signals, a detection in the time domain or the frequency domain may be delicate if the expected time domain or frequency domain response are not well known. Here, we propose to investigate a system model that can actually predict the time series output characteristics and the frequency domain response to various receiver inputs. The model development is solely based on the output process available for analysis and on the assumption of a linear time invariant system with wide sense stationary inputs. Indeed, spectral density can characterize a noise process completely only if the process is stationary, ergodic, and gaussian.

Section II describes the experimental setup and some of the available data. The next section presents an preliminary analysis that includes an exploratory data analysis, autocorrelation function evaluation and classical spectral estimation. Section IV develops an autoregressive model for the receiver tested. Finally, future work to this experimental and modeling investigation are discussed in the last section.

II. EXPERIMENT

A. Laboratory set up

A block diagram of the measurement set up is given in Fig. 1. The device under test consists of an X band benchtop heterodyne receiver followed by a Maxim MAX 104 8 bit analog to digital converter (ADC). A primary and secondary stimuli to the receiver are fed to the receiver via the two SMA ports of the front end directional coupler, and the digitized outputs are recorded with a high speed digital logic analyzer. The analog front end downconverts a 100MHz band from X band to 150MHz. This nominal IF passband of 100-200MHz is filtered by a last stage 450MHz low passfilter. A waveguide network and a noise diode can provide the input excitation. This source can produce output noise temperatures ranging from ambiant 300K to about 14,200K.
III. PRELIMINARY ANALYSIS

A. Exploratory Data Analysis

Exploratory Data Analysis (EDA) [5] allowed to gain qualitative insight into the ensemble of the data set and to preselect a potentially interesting data set. By visual inspection of quantile-quantile plots and lag plots, we saw some slight deviation from gaussianity and whiteness. As expected, no growth trend can be seen in the time series plots. Plots of the histogram of the amplitudes didn’t show any skewness, see Fig. 2.

B. Available data

Sine waves at Xband and Gaussian white noise at various power were fed into port A and port B of the directional coupler. For most of the data recorded, the logic analyzer recorded data on the rising edge of the ADC clock. At 500MHz, the maximum frequency bandwidth that can be captured is 250MHz. With a spectral analyzer plugged before the ADC, we verified that the power level of the signal after 250MHz was low enough so that it cannot be captured by the LSB of the ADC. Hence, no aliasing of the data is expected [4]. In the study presented here, we will work on the data corresponding to an input excitation of about 1200K on the main port of the directional coupler, and a 50Ω load at ambiant temperature on the coupling port. This setting corresponds to the beginning of the ADC saturation as shown on the histogram of the amplitudes of Fig. 2. A 65,536 data record length is acceptable for statistical data analysis.

B. Autocorrelation function

A more quantitative assessment of the data includes plots of the autocorrelation function (ACF), where self correlation can be inspected. A (stochastic) signal is called stationary it its statistical properties do not change with time. We’ll assume that it is the case in the experiment above. For stationary signals, the following can be defined:

- the Autocovariance sequence (ACV). The empirical or sample acv is:

\[ C_{n|s} = \bar{C}_s = \frac{1}{n} \sum_{i=1}^{n-|s|} (x_i - \bar{x})(x_{i+s} - \bar{x}) \quad (1) \]

where \( \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \) is the empirical or sample mean.

- the (normalized) autocorrelation sequence or function (ACF). The empirical or sample ACF is:

\[ \hat{\rho}_s = r_s = C_s/C_0 \quad (2) \]

In Fig. 3, within fewer than 30 lags, the ACF settles down, which suggest a reasonable order if parametric modeling is used.

C. Classical spectral estimation

Moving to the frequency domain, two basic methods are considered classical approaches to spectral estimation. The first one called the direct or periodogram method develops an estimator from the Fourier transform of the sample function, it is based on Parseval’s theorem:

\[ I_n(\omega) = \frac{|\hat{Y}_\omega|^2}{n} \quad (3) \]

The second method, called indirect method of Blackman-Tukey method considers the spectral density function as the Fourier transform of the autocovariance function.

\[ I_n(\omega) = \frac{1}{(n-1)} \sum_{|s|=1}^{n-1} C_s e^{j\omega s} \quad (4) \]

Both of these methods yield a very noisy periodogram as shown in Fig. 4. Several approaches strive to obtain a better
estimate. Any technique that reduces the variance of the estimate also increases its bias and vice versa. Several techniques aim at developing a consistent estimator of the periodogram. We will work on the windowed ACV periodogram and the smoothed periodogram method to gain insight into the actual spectrum.

1) Windowed autocovariance periodogram: This method consists in windowing out the noise estimates of the autocovariance sequence in (4). Using a Bartlett or triangle window, we obtain the plots of Fig. 5.

2) Smoothed periodogram: The idea is to average over neighbouring values to reduce the variance of the periodogram. This introduces bias but the continuity of the spectrum limits this effect. The estimator is:

\[
\hat{F}_n(\omega) = \frac{1}{2m+1} \sum_{-m}^{m} I_n(\omega + \omega_k), \omega_k = \frac{2\pi k}{n}
\]

with \(2m+1\) the spectral window span and \(n\) the number of record points. \(m\) is a tuning parameter that can be found using Lee’s unbiased risk estimator (URE) [6]. The optimal \(m\) is found to be 166 for a sample record of length 15,000. Fig. 6 illustrates the smoothed periodogram for various frequency window span. As \(m\) gets bigger that 166, the periodogram gets smoother, the bias rises and the variance decreases. The periodogram misses some spectral details.

IV. PARAMETRIC SYSTEM MODEL

A. Parametric spectral analysis

Second order statistics of a random process can be represented alternatively by either the autocorrelation function (ACF) of the power spectrum density (PSD), both of which are non parametric descriptions. Classical methods of spectral estimation use Fourier transform operations on either windowed data or windowed ACF estimates as shown in the previous section. Therefore these methods make the implicit unrealistic assumption that the unobserved data or the ACF outside of the window are zero.

We propose in this section to use a parametric approach to spectral estimation. Parametric spectral estimation yields a better spectral estimate by basing it on a chosen model and estimating the parameters of the model from the observations [6]. Parametric spectral estimation involves three steps: model selection, parameters estimation of the assumed model using the available data samples, and spectral estimate.

The autoregressive (AR) spectral estimator or linear prediction spectral estimator is the most popular of the time series modeling approaches to spectral estimation because AR model parameters can be accurately estimated by solving a set of linear equations. Because the observed system is assumed to be causal linear time invariant, an AR model was chosen, keeping in mind that the AR, MA and ARMA models can be related by the Wold decomposition theorem [7].

1) AR model: We assume the system input is gaussian white noise. An AR model can produce an output with structure when its input is uncorrelated.

\[
y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \epsilon_t, t = 1, ..., n
\]

where \(\epsilon_t\) is white noise, with zero mean and variance \(\sigma^2\), \(\phi_i, i = 1, ..., n\) are the AR parameters such that \(|\phi_i| < 0\) to
the transfer function in the spectral domain is:

\[ H_{27}(f) = H(f) = \sum_{m=0}^{27} a_m e^{-j2\pi mf}, (0 \leq |f| \leq 1/2) \]  

Using the system input-output relationship, the power spectrum densities of the input and the output of the system are related by:

\[ S_y(f) = |H(f)|^2 S_x(f) \]  

Therefore, the PSD for the output signal assuming a gaussian white noise input can be obtained by substituting for the noise variance and the system parameters:

\[ \hat{S}_y(f) = \frac{\sigma^2}{|\sum_{m=0}^{27} a_m e^{-j2\pi mf}|^2}, (0 \leq |f| \leq 1/2) \]  

where \( \sigma^2 \) is the variance of \( \epsilon_t \).

V. CONCLUSION

An analog receiver and its back end analog to digital converter were modeled using an autoregressive model, using modern spectral estimation techniques. In the time domain, this AR model enables the simulation of the expected output time series for different brightness temperature inputs, whereas in the spectral domain, fine spectral resolution power spectral density can be simulated. The natural next step is to use the parametric model to find the spectral characteristics of the receiver internal noise, and combine this new element of understanding with the AR model to yield a complete system description.

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REFERENCES