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PATH PLANNING IN THREE DIMENSIONAL ENVIRONMENT USING FEEDBACK LINEARIZATION (PREPRINT)

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14. ABSTRACT
This paper presents a control scheme via feedback linearization for three dimensional cooperative path planning of a class of interconnected systems in general and UAVs in particular. It is shown that the feedback linearization technique along with a distance varied repulsive profile allows UAVs to converge to the invariant set of a known target location without colliding with other vehicles. Lyapunov stability analysis shows the conditions under which such systems are stable. Also a task assignment algorithm, which is a function of distance between the UAVs and the target, is proposed for dealing with multi UAV and multi target scenario.

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put) systems of the form

$$\begin{aligned}\dot{x}^i =& f^i(x_i) + g_1^i(x^i)u_1^i + g_2^i(x^i)u_2^i + \cdots + g_m^i(x^i)u_m^i \\ y_1^i =& h_1^i(x^i) \\ y_2^i =& h_2^i(x^i) \\ &\vdots \\ y_m^i =& h_m^i(x^i)\end{aligned} \tag{1}$$

where $x^i \in \mathcal{R}^n$ represents the states of the i^{th} subsystem, $u^i \in \mathcal{R}^m$ represents the controllers of the i^{th} subsystem and $y^i \in \mathcal{R}^m$ represents the output of the i^{th} subsystem. Assume that f^i, g_j^i and h_j^i are smooth. Define r_j^i to be the relative degree for the j^{th} output of the i^{th} subsystem such that differentiating y_j^i, with respect to x^i, r^j times will result in at least one of the inputs appearing in $y_j^{i\,(r_j^i)}$, i.e.,

$$y_j^{i\,(r_j^i)} = \underbrace{L_{f^i}^{r_j^i} h_j^i}_{\alpha_k^i(x^i)} + \sum_{j=1}^{m} \underbrace{L_{g^i}(L_{f^i}^{r_j^i-1} h_j^i)}_{\beta_{kj}^i(x^i)} u_j^i \tag{2}$$

where $L_{f_i}h$ is the Lie derivative and is defined as $L_{f_i}h = \frac{\partial h}{\partial x_i} f_i$ and $L_{f_i}^2 h = L_{f_i}(L_{f_i}h)$, with at least one of $L_{g^i}(L_{f^i}^{r_j^i-1} h_j^i) \neq 0$. Define an $m \times m$ matrix $B^i(x^i)$ and a $m \times 1$ matrix $A^i(x^i)$ for each subsystem such that

$$B^i(x^i) = \begin{bmatrix} \beta_{11}^i(x) & \beta_{12}^i(x) & \dots & \beta_{1m}^i(x^i) \\ \beta_{21}^i(x^i) & \beta_{22}^i(x^i) & \dots & \beta_{2m}^i(x^i) \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{m1}^i(x^i) & \beta_{m2}^i(x^i) & \dots & \beta_{mm}^i(x^i) \end{bmatrix}$$

$$A^i(x^i) = \begin{bmatrix} \alpha_1(x^i) \\ \alpha_2(x^i) \\ \vdots \\ \alpha_m(x^i) \end{bmatrix} \tag{3}$$

Then equation (2) for the i^{th} subsystem can be written as

$$\underbrace{\begin{bmatrix} y_1^{i\,(r_1^i)} \\ y_2^{i\,(r_2^i)} \\ \vdots \\ y_m^{i\,(r_m^i)} \end{bmatrix}}_{y^{i\,(r^i)}} = A^i(x^i) + B^i(x^i) \begin{bmatrix} u_1^i \\ u_2^i \\ \vdots \\ u_m^i \end{bmatrix} \tag{4}$$

Define $r^i = [r_1^i, r_2^i, \dots, r_m^i]^\top$, $r^i = r_1^i + r_2^i + \cdots + r_m^i \leq n$ for $i = 1, 2, \dots, N$ as the vector relative degree of the i^{th} subsystem. We assume that the relative degree for all the subsystems are equal in the sense $r^i = r^j$ with $r_k^i = r_k^j$ for $k = 1, 2, \dots, m$. However, we allow the actual dynamics of each subsystem (given by f^i, g_j^i, h_j^i) to be different.

We assume that $B^i(x^i)$ is invertible. Then the MIMO feedback linear control is given by

$$u^i = B^i(x^i)^{-1}[-A^i(x^i) + v^i] \tag{5}$$

If $B^i(x^i)$ is not invertible, feedback linearization is still possible using dynamic feedback linearization which is not discussed in this paper. Moreover we assume that each subsystem's zero dynamics are input to state stable.

Define a smooth reference signal R_{ref} as

$$R_{ref} = \begin{bmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \vdots \\ \gamma_m(t) \end{bmatrix} \tag{6}$$

Also define the error between the j^{th} output of the i^{th} subsystem and reference signal $\gamma_j \in R_{ref}$ as

$$\begin{aligned} e_j^i =& k_{1,i}(y_j^i - \gamma_j) + k_{2,i}(\dot{y}_j^i - \dot{\gamma}_j) + \dots \\ &+ k_{r_j^i-1,i}(y_j^{i\,(r_j^i-2)} - \gamma_j^{(r_j^i-2)}) + (y_j^{i\,(r_j^i-1)} - \gamma_j^{(r_j^i-1)}) \end{aligned} \tag{7}$$

so that differentiating the error results in

$$\begin{aligned} \dot{e}_j^i =& k_{1,i}(\dot{y}_j^i - \dot{\gamma}_j) + \cdots + k_{r_j^i-1,i}(y_j^{i\,(r_j^i-1)} - \gamma_j^{(r_j^i-1)}) \\ &+ (y_j^{i\,(r_j^i)} - \gamma_j^{(r_j^i)}) \\ =& k_{1,i}(\dot{y}_j^i - \dot{\gamma}_j) + \cdots + k_{r_j^i-1,i}(y_j^{i\,(r_j^i-1)} - \gamma_j^{(r_j^i-1)}) - \gamma_j^{(r_j^i)} \\ &+ y_j^{i\,(r_j^i)} \\ =& \underbrace{k_{1,i}(\dot{y}_j^i - \dot{\gamma}_j) + \cdots + k_{r_j^i-1,i}(y_j^{i\,(r_j^i-1)} - \gamma_j^{(r_j^i-1)}) - \gamma_j^{(r_j^i)}}_{\chi_j^i} \\ &+ \alpha_j^i(x^i) + \sum_{k=1}^{m} \beta_{j,k}^i(x^i)u_k \end{aligned}$$

Define the error vector for the i^{th} subsystem as $E^i = [e_1^i, e_2^i, \dots, e_m^i]^\top$ and for the composite system as $E = [E^{1\top}, E^{2\top}, \dots, E^{N\top}]^\top$. We define the error dynamics from equation (7) for the i^{th} subsystem as

$$\underbrace{\begin{bmatrix} \dot{e}_1^i \\ \dot{e}_2^i \\ \vdots \\ \dot{e}_m^i \end{bmatrix}}_{\dot{E}^i} = \underbrace{\begin{bmatrix} \chi_1^i \\ \chi_2^i \\ \vdots \\ \chi_m^i \end{bmatrix}}_{\chi^i} + A^i(x^i) + B^i(x^i) \begin{bmatrix} u_1^i \\ u_2^i \\ \vdots \\ u_m^i \end{bmatrix} \tag{8}$$

Substituting the control equation discussed in equation (5), where we select $v^i = -\chi^i + K^i E^i$ with K^i a $m \times m$ matrix having its eigenvalues in the left half plane, we get

$$\dot{E}^i = K^i E^i \tag{9}$$

To study the stability of the error dynamics for each subsystem let us define the Lyapunov function, with a positive definite matrix P^i, the solution of Lyapunov equation $K^{i\top} P^i + P^i K^i = -Q^i$, as

$$V^i = E^{i\top} P^i E^i \tag{10}$$

Differentiating V^i we obtain

$$\begin{aligned} \dot{V}^i =& \dot{E}^{i\top} P^i E^i + E^{i\top} P^i \dot{E}^i \\ =& E^{i\top}[K^{i\top} P^i + P^i K^i] E^i \\ =& -E^{i\top} Q^i E^i \end{aligned}$$

We notice that $\dot{V}^i$ is negative definite thus driving E^i to zero exponentially. To study the stability of the composite systems we define the composite Lyapunov function as

$$V(E) = \sum_{i=1}^{N} V^i(E^i) \tag{11}$$

For any positive symmetric matrix S and vector X we have

$$\lambda_{min}(S) X^\top X \leq X^\top S X \leq \lambda_{max}(S) X^\top X$$

where $\lambda_{min}(S)$ and $\lambda_{max}(S)$ are the minimum and maximum eigenvalues of S respectively. So we have

$$\begin{aligned}\dot{V} =& - \sum_{i=1}^{N} {E^i}^\top Q^i E^i \\ \leq& - \sum_{i=1}^{N} \lambda_{min}(Q^i) ||E^i||^2\end{aligned}$$

$\dot{V}$ to be negative definite thereby regulating the overall error to zero.

A. Interconnection Between the Subsystems

A certain class of interconnected systems does not allow some or all the states of the subsystem to be equal at a time instance. Examples of such class of systems include groups of UAVs with position of each UAV as the state. In order to deal with such type of systems we introduce the interconnectivity among the subsystems via a repulsive force that surrounds each subsystem. We define the force of repulsion F_r^i that surrounds i^{th} subsystem as

$$F_r^i = \sum_{j=1, j\neq i}^{N} \kappa_j \exp\Big[- \frac{||x^i - x^j||^2}{2{r_s^j}^2} \Big] (y^i - y^j + \Psi) \tag{12}$$

where κ_j represents a repulsive constant, Ψ is a design constant whose significance is explained in Section III-C and r_s^j represents the surrounding or spread in which the effect of force is realized. The function has the significance that as the states are far apart from each other the force of repulsion is less as compared to when the states are closer. Also it has the significance that there exists a certain minimum force of repulsion between the states at any time instance. In order to implement this interconnection we introduce F_r^i via v^i for each subsystem so that v^i now becomes $v^i = -\chi^i + K^i E^i + F_r^i$ and equation (9) is updated to

$$\dot{E}^i = K^i E^i + F_r^i$$

To study the stability of the error dynamics for the i^{th} subsystem defined in the above equation we define the Lyapunov function V^i, similar to one defined in equation (10) and calculate its derivative

$$\begin{aligned}\dot{V}^i =& [K^i E^i + B F_r^i]^\top P^i E^i + {E^i}^\top P^i [K^i E^i + F_r^i] \\ =& {E^i}^\top [{K^i}^\top P^i + P^i K^i] E^i + 2{E^i}^\top P^i F_r^i \\ =& - {E^i}^\top Q^i E^i + 2{E^i}^\top P^i F_r^i\end{aligned}$$

To study the stability of the error dynamics defined by the composite systems in the presence of interconnection, we differentiate the composite Lyapunov function defined in equation (11) to obtain

$$\begin{aligned}\dot{V}(E) =& \sum_{i=1}^{N} \Big(- {E^i}^\top Q^i E^i + 2{E^i}^\top P^i F_r^i \Big) \\ \leq& \sum_{i=1}^{N} \Big(- \lambda_{\min}(Q^i) ||E^i||^2 \\ &+ 2||E^i|| \lambda_{\max}(P^i) \exp(-\frac{1}{2}) \sum_{j=1, j\neq i}^{N} (r_s^j + \Psi) \Big) \\ \leq& \sum_{i=1}^{N} \Big(- \lambda_{\min}(Q^i) \Big[||E^i|| - \\ & \frac{2\lambda_{\max}(P^i)}{\lambda_{\min}(Q^i)} \sum_{j=1, j\neq i}^{N} (r_s^j + \Psi) \Big] ||E^i|| \Big)\end{aligned}$$

It is clear that when $||E^i|| - \frac{2\lambda_{\max}(P^i)}{\lambda_{\min}(Q^i)} \sum_{i=1, i\neq j}^{N} (r_s^j + \Psi) > 0$, $\dot{V}(E) < 0$. Define Ω^i as the invariant set for the i^{th} vehicle such that $\Omega^i = \Big\{ E^i \in \mathcal{R}^m : E^i > \frac{2\lambda_{\max}(P^i)}{\lambda_{\min}(Q^i)} \sum_{i=1}^{N} (r_s^j + \Psi) \Big\}$. Hence it is clear that whenever the error vector is outside Ω^i, it is driven to the invariant set Ω^i.

III. Application to UAV case

A. UAV Model

We consider the four DOF model of the UAV with seven states given by

$$\begin{bmatrix} \dot{x}_1^i \\ \dot{x}_2^i \\ \dot{x}_3^i \\ \dot{x}_4^i \\ \dot{x}_5^i \\ \dot{x}_6^i \\ \dot{x}_7^i \end{bmatrix} = \begin{bmatrix} \dot{r}_x^i \\ \dot{r}_y^i \\ \dot{\theta}^i \\ \dot{v}^i \\ \dot{\omega}^i \\ \dot{r}_z^i \\ \dot{v}_z^i \end{bmatrix} = \begin{bmatrix} v^i \cos(\theta^i) \\ v^i \sin(\theta^i) \\ \omega_i \\ 0 \\ 0 \\ v_z \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{J} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \frac{1}{m} F_3 - g \end{bmatrix} \tag{13}$$

where $p^i = [r_x^i, r_y^i, r_z^i]^\top$ represents the x, y and z position of the i^{th} UAV and $s^i = [v^i, v_z^i]^\top$ represents velocity vector where v^i is the linear velocity in $x-y$ direction and v_z^i is the velocity in z direction. θ^i represents the planar orientation of the UAV and ω^i represents the angular velocity of the UAV. The control vector is $u = [u_1, u_2, u_3]^\top$ with $u_3 = \frac{1}{m} F_3 - g$. Equation (13) resembles equation (1) where $f(x^i)$ and $g(x^i)$ can be inferred. Let the sensor of the UAV be pointed at a fixed angle ϕ, $0 < \phi < \pi$, with the horizontal of the plane as shown in Figure 1. Let L_i be the horizontal distance from the sensor footprint to the vertical line joining the sensor and the ground as shown in Figure 1. It is clear that L_i is a function of altitude, $z = r_z^i$, and is given by $L_i = \frac{r_z^i}{\tan\phi}$.

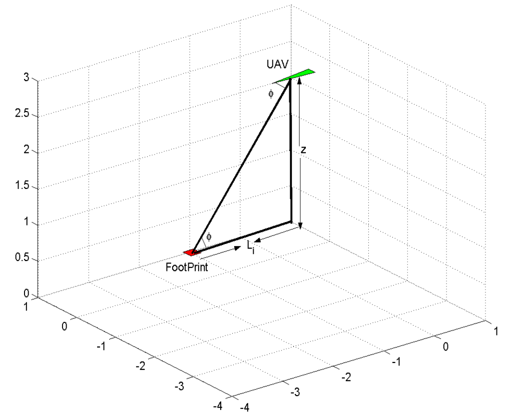

Fig. 1. Figure showing the UAV and sensor arrangement

Since the sensor angle is fixed, let $c = \tan\phi$, a constant. Let the output of the system (13), be given by

$$\Gamma^i = \begin{bmatrix} x_1^i + L_i \cos(x_3^i) \\ x_2^i + L_i \sin(x_3^i) \\ x_6^i \end{bmatrix} \tag{14}$$

where the first two rows represent the x and y position of the sensor footprint respectively and the third row represents the altitude of the vehicle. By defining the output as given in equation (14) we are not only controlling the footprint position of the vehicle but also controlling the altitude of the vehicle. Differentiating equation (14) with respect to time gives

$$\dot{\Gamma}^i = \begin{bmatrix} x_4^i \cos(x_3^i) - L_i \sin(x_3^i)x_5^i + cx_7^i \cos(x_3^i) \\ x_4^i \sin(x_3^i) + L_i \cos(x_3^i)x_5^i + cx_7^i \sin(x_3^i) \\ x_7 \end{bmatrix}$$

Differentiating it again with respect to time yields

$$\ddot{\Gamma}^i = \begin{bmatrix} -x_4^i x_5^i \sin(x_3^i) - L_i (x_5^i)^2 \cos(x_3^i) - 2cx_5^i x_7^i \sin(x_3^i) \\ x_4^i x_5^i \cos(x_3^i) - L_i (x_5^i)^2 \sin(x_3^i) + 2cx_5^i x_7^i \cos(x_3^i) \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{m}\cos(x_3^i) & -\frac{L_i}{J}\sin x_3^i & c\cos(x_3^i) \\ \frac{1}{m}\sin(x_3^i) & \frac{L_i}{J}\cos x_3^i & c\sin(x_3^i) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Since

$$\det \begin{bmatrix} \frac{1}{m}\cos(x_3^i) & -\frac{L_i}{J}\sin x_3^i & c\cos(x_3^i) \\ \frac{1}{m}\sin(x_3^i) & \frac{L_i}{J}\cos x_3^i & c\sin(x_3^i) \\ 0 & 0 & 1 \end{bmatrix} \neq 0$$

the system (13) with output (14) has a vector relative degree of $[2, 2, 2]^\top$ and is input-output feedback linearizable.

B. Presence of a Single Target

Target Model: Here we consider a stationary target whose position $T^\tau = [x_\tau, y_\tau, z_\tau]^\top$ is known a priori either by a global positioning satellite or from the information given by other UAV.

Error signal: We define the error as the sum of the difference between the footprint position and the target position, and its derivatives so that error dynamics defined in equation (7) is nothing but

$$E^i = k_{1,i}(\Gamma^i - T^\tau) + (\dot{\Gamma}^i - \dot{T}^\tau)$$

so that differentiating it with respect to time yields

$$\dot{E}^i = k_{1,i}(\dot{\Gamma}^i) + \ddot{\Gamma}^i$$

The error dynamics for the i^{th} UAV, assuming that all the $k_{i,j} = 1$, is given by

$$\dot{E}^i = \underbrace{\begin{bmatrix} x_4^i \cos(x_3^i) - L_i \sin(x_3^i)x_5^i + cx_7^i \cos(x_3^i) \\ x_4^i \sin(x_3^i) + L_i \cos(x_3^i)x_5^i + cx_7^i \sin(x_3^i) \\ x_7 \end{bmatrix}}_{\chi^i} + \underbrace{\begin{bmatrix} -x_4^i x_5^i \sin(x_3^i) - L_i (x_5^i)^2 \cos(x_3^i) - 2cx_5^i x_7^i \sin(x_3^i) \\ x_4^i x_5^i \cos(x_3^i) - L_i (x_5^i)^2 \sin(x_3^i) + 2cx_5^i x_7^i \cos(x_3^i) \\ 0 \end{bmatrix}}_{A^i(x^i)} + \underbrace{\begin{bmatrix} \frac{1}{m}\cos(x_3^i) & -\frac{L_i}{J}\sin x_3^i & c\cos(x_3^i) \\ \frac{1}{m}\sin(x_3^i) & \frac{L_i}{J}\cos x_3^i & c\sin(x_3^i) \\ 0 & 0 & 1 \end{bmatrix}}_{B^i(x^i)} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

In order to regulate the error we define the feedback linearizing controller u^i to be the one given in equation (5) with $v^i = -\chi^i + K^i E^i$, where χ^i is defined in the above equation and a choice of stable K^i is made. The choice of such a controller results in all the UAV's converging to the target location as shown in Figure 2. But it is evident from Figure 2 that the UAVs collide (blue and green UAV) with each other during the process of convergence.

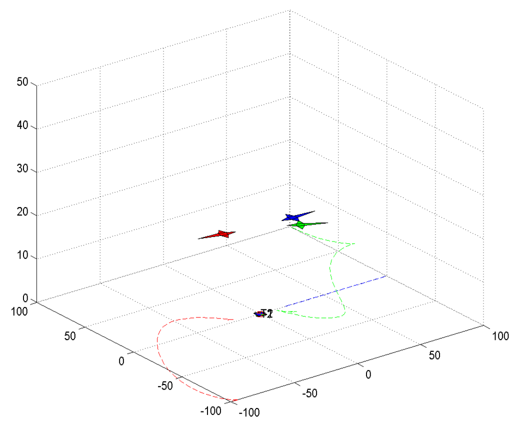

Fig. 2. UAV converge towards the target and collide with each other

C. Force of Repulsion

We define a repulsive profile around each vehicle similar to the one define in equation (12) as

$$F_r^i = \sum_{j=1, j\neq i}^{N} \kappa_j \exp\Big[-\frac{||p^i - p^j||^2}{2{r_s^j}^2}\Big](\Gamma^i - \Gamma^j + \Psi)$$

With this definition of repulsive force it is clear that the repulsive force is a function of all x, y and z positions where the repulsion increases as the positions of the vehicles approach each other, affecting in turn the location of the footprint. The significance of Ψ is that even if the output of the i^{th} and j^{th} vehicle become equal there exists a certain minimum force of repulsion which avoids the vehicles from collision. The output of the vehicles become equal when the angle of sensor footprint, ϕ, for each vehicle is different. But this choice of repulsive function may not seem to be practical all the time. As an example consider the positions of the UAV footprints as shown in Figure 3 when they have a repulsive function in terms of x, y and z position. In this scenario the target is placed at the origin and the vehicles are initially placed at (-100, -100, 20), (100, 100, 20) and (0, 100, 20) initially. The UAVs converge towards the target with their sensor footprint (the rectangular areas in the figure) placed in a invariant set surrounding the target but not exactly on the target. The invariant set, though very small in this example, may increase if there are more UAVs that converge towards the target. The question now arises whether the invariant set can be decreased considerably if the repulsion profile were to be a function of altitude, z, alone. But if the repulsive profile were to a function of z alone the UAV could face a force of repulsion when they are at same altitude but located far away in the $x-y$ plane which is not necessary. In order to deal with such situation we define a repulsive force similar to equation (12) but as function of altitude z and the distance between the vehicles. We define a circle of safe distance S around the UAVs, inside which the UAVs experience a repulsive force that is a function of z alone and outside which the UAVs do not experience any force of repulsion. Thus the force of repulsion between the vehicles can be expressed mathematically as

$$F_r^i = \begin{cases} 0 \quad \text{if} ||p^i - p^j|| \geq S \\ \sum_{j=1, j\neq i}^{N} \kappa_j \Big[e^{-\left(\frac{||r_z^i - r_z^j||^2}{2r_s^2}\right)}\Big](\Gamma^i - \Gamma^j + \Psi)\text{if } ||p^i - p^j|| < S \end{cases}$$

With the introduction of this repulsive force it is evident that the UAV converge towards a smaller invariant set around the target with out colliding with each other as shown in Figure (4).

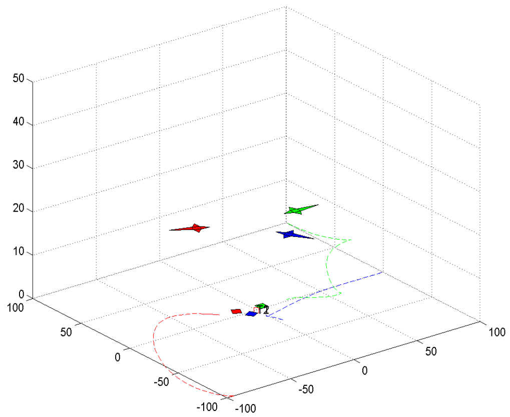

Fig. 3. UAV converge towards target with repulsion among themselves

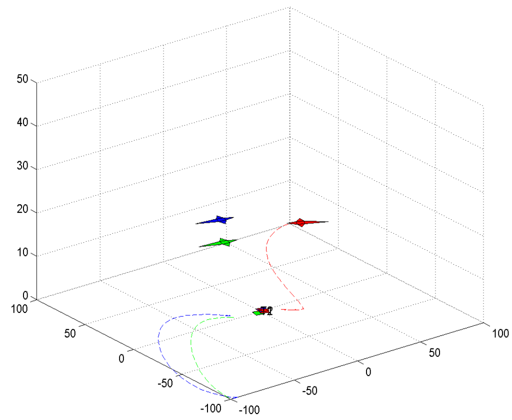

Fig. 4. UAV converge towards target and have a repulsive profile as a function of altitude

D. Presence of Multiple Targets

Consider the case when we have more than one target and to be more specific the case when we have more UAVs than targets. In such instances the task assignments to each UAV plays a vital role. In this section we discuss the problem of task assignment for different UAVs and the scenario for dealing with multiple target case.

Define a closed set D with its center placed at the center of mass of multiple targets. In order to simplify our assumption we assume that D is rectangular invariant set with length l and width w. In practice this set D can be considered as the search area in which the UAVs are interested in looking for target and assume that all the UAVs are initialized outside this search area. The selected invariant set D is such that D is a superset of all the invariant sets Ω^i for $i = 1, 2, \ldots M$ where M is total number of targets.

E. Task Assignment via change of reference signal

Assume that there exists a virtual target whose position is defined as the center of mass of all the target positions. The virtual target position is assigned as the goal for all the UAVs which are outside the set D. This makes the UAVs be attracted towards the center of the search area when they are outside D. Once the UAV enters the set D, task assignment is done based on greedy distance criteria. The task assignment algorithm at present works as follows. Each

UAV calculates its distance to all the target positions and communicates it to all the other UAVs. Therefore at a given time instance all the UAVs know the distance between the UAVs and the targets present in the search area. Based on this information each UAV is assigned the closest target (which is a reference signal in case of general MIMO systems). In the task assignment process all the targets are assigned to UAVs and this assignment changes dynamically. UAVs to which targets are not assigned will converge towards the center of mass of the target. Thus center of mass of the targets acts like a default target for all the UAVs. Figure 5 shows the scheme of how the task assignment works. In the figure there are two targets placed at (40, 0, 0) and (-40, 0, 0) respectively. Two of the UAVs (red and blue) are assigned to two different targets and the third UAV (green) is assigned the center of mass of the targets.

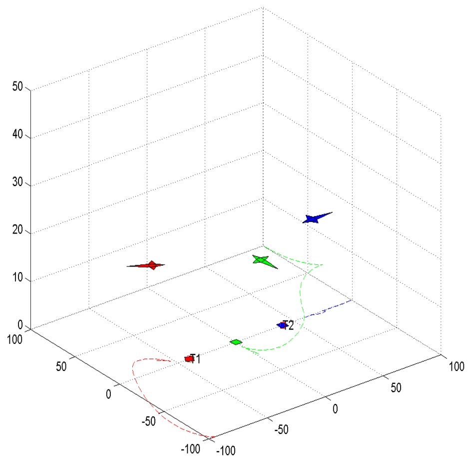

Fig. 5. Task Assignment

IV. Conclusion and Future Research

In this paper we have considered a control scheme for the three dimensional cooperative path planning of inter-connected systems in general and have shown how this theory can be applicable for path planning of autonomous air vehicles to reach their dynamically assigned goal positions. Lyapunov stability analysis is used not only to show the stability of the system but also the conditions under which such systems are stable. Future work involves the stability of such systems under dynamical constraints placed on some states (example saturating the velocity in case of UAVs) and extending it to dynamical environment which involves moving targets and obstacles.

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