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Revised title: ___________________________________________________________________________

Presented in (input and Bold one): (WG 2, CG___, Special Session ___, Poster, Demo, or Tutorial):

This presentation is believed to be:

UNCLASSIFIED AND APPROVED FOR PUBLIC RELEASE
Time-Adaptive Sampling of a Chemical Hazard Area

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and

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73rd MORS Symposium, 21-23 June 2005
**Report Documentation Page**

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Standard Form 298 (Rev. 8-98)  
Prescribed by ANSI Std Z39-18
Overview

- Background
- Problem statement and assumptions
- Methodology
- Illustrations
- Conclusions
- Future Work
Background

- Chemical and biological WMDs are a current threat to the United States
  - 2001 U.S.: anthrax attacks
  - 1998 Iraq: “cocktail” of weapons killed 5,000+
  - 1995 Tokyo: sarin nerve gas, killed 12, injured thousands

- Terrorist groups are willing to use asymmetric measures
  - Easy manufacturing, storing, and transportation appeal to terrorists
Problem Dynamics

- A chemical agent weapon is released over a fixed operational site
  - Entire site enters the highest level of MOPP
  - Contamination from secondary vapors is the main concern

- Reduce mission oriented protective posture (MOPP)
  - MOPP is cumbersome
  - High levels of MOPP can reduce work efficiency
Problem Dynamics

Figure 1. Mission-oriented protective postures.

- MOPP 0, MOPP 1, MOPP 2, MOPP 3, MOPP 4, MOPP Alpha
- Progressively add gear for increased safety
Problem Statement

• Develop an optimal sampling strategy
  - Route a search crew
    * Reach as many locations as possible (to identify maximum number of areas below the vapor concentration threshold)
    * Time constraint
  
• Provide a framework for future work
  - Using sensor data
  - Predicting future hazard areas
Model Assumptions

- Rectangular region with a finite number of “critical” areas
- Single crew that samples vapor concentrations
- Static, deterministic, and symmetric travel times
- Travel at constant velocity with zero delays
- Fixed amount of time allotted for the search
Model Assumptions

- Chemical agent/characteristics are known

- Only one instrument reading is required, consuming a fixed amount of time

- Known fixed threshold indicating contamination/no contamination

- Secondary vapor concentrations evolve spatially
Optimization Model

- Model the site and its critical areas as a network
- Develop a technique for optimally searching the site
- Desired outcome: Identify areas where secondary vapor levels have decreased (below the fixed vapor concentration level $v^*$) so MOPP can be safely reduced at those locations
Consider the following notional site

Figure 2. Graphical depiction of areas on an installation.
Network Model

Definitions:

• $G = (\mathcal{N}, \mathcal{A})$ describes the graph with:
  
  − $\mathcal{N} \equiv \{1, 2, ..., N\}$, where $N$ is the number of critical areas
  
  − $\mathcal{A} \equiv$ set of arcs $(i, j)$ for $i, j \in \mathcal{N}$
Network Model

- $N_i \equiv \text{set of nodes adjacent to node } i$

- $t_{i,j} \equiv \text{constant time required to travel from node } i \text{ to node } j$
  
  - $t_{i,j} > 0, \forall (i,j) \in A$
  
  - $t_{i,j} = t_{j,i}$

- $v_j(t) \equiv \text{nonnegative vapor concentration at node } j \in N \text{ at time } t$

- $r_j(t) \equiv \text{binary reward received from searching node } j \text{ at time } t$
Figure 3. Example of the network representation for a 4-node site.

- $\mathcal{N} = \{1, 2, 3, 4\}$
- $\mathcal{A} = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (4, 3)\}$
Well-known Network Models

- Shortest Path Problem (SPP)
  - Path from source to sink
  - Not all nodes must be reached

- Knapsack Problem
  - Maximize a value with a constraint on the resource
  - Order does not matter
Well-known Network Models

- Travelling Salesperson Problem (TSP)
  - Minimize tour length
  - Must reach every city
  - Start and end at the origin
Methodology

We consider four distinct cases:

- Static and deterministic vapor concentrations
- Static and stochastic vapor concentrations
- Dynamic and deterministic vapor concentrations
- Dynamic and stochastic vapor concentrations
Dynamic and Deterministic

- Deterministic:
  - Assume vapor level concentration at each node can be calculated deterministically

- Dynamic:
  - Vapor levels depend on time, $v_j(t)$, for all $j \in \mathcal{N}, t \geq 0$
Dynamic and Deterministic

Objective: Maximize reward:

\[
\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_j(t)x_{i,j}
\]

- Time constraint

- Backtracking is allowed

- Vapor concentrations are dynamic ⇒ rewards are dynamic

- Possibly not all nodes will be reached
Algorithm

Initialization:

\[ \mathcal{N} = \{1, 2, \ldots, N\}; \mathcal{N}_i = \{j : i \rightarrow j\}; \]
\[ R = \emptyset; \psi = \emptyset; \]
\[ t \leftarrow t_0; \]
\[ i = 1; \]

Calculate current vapor level at node \( i \), \( v_i(t) \)

If \( v_i(t) < v^* \)

\[ r_i(t) \leftarrow 1; \]
\[ R \leftarrow \{i\}; \psi \leftarrow \psi \cup \{i\}; \]

Else

\[ r_i(t) \leftarrow 0; \]
\[ \psi \leftarrow \psi \cup \{i\}; \]

End
Algorithm

Step 1

Calculate $v_j(t + t_{i,j}) \forall j \in \mathcal{N}_i$

If $v_j(t + t_{i,j}) < v^*$

\[ r_j(t + t_{i,j}) \leftarrow 1; \]

Else

\[ r_j(t + t_{i,j}) \leftarrow 0; \]

End
Algorithm

Step 2

For each \( j \in \mathcal{N} \) such that \( r_j = 1 \)

Choose \( j \) such that \( v_j(t + t_{i,j}) = \arg \min_{j \in \mathcal{N}_i} \{ v^* - v_j(t + t_{i,j}) \} \)

\( R \leftarrow R \cup \{ j \} \);

\( \psi \leftarrow \psi \cup \{ j \} \);

End

If \( r_j(t + t_{i,j}) = 0 \ \forall \ j \in \mathcal{N}_i \)

Choose \( j \) such that \( t_{i,j} = \min_{j \in \mathcal{N}_i} \{ t_{i,j} \} \forall j \in \mathcal{N}_i \)

\( \psi \leftarrow \psi \cup \{ j \} \);

End

\( t \leftarrow t + t_{i,j} \);
Step 3

If \( t \geq T \)

STOP

Else

\( i \leftarrow j; \)

Return to Step 1

End
Dynamic/Deterministic

Result: iterative process yields a time-adaptive policy

• Future decisions depend on arrival times at nodes

• Vapor concentrations (rewards) drive the solution
Dynamic/Deterministic

Network Configuration

![Network Diagram]

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Dynamic/Deterministic

Example: Iteration 1: $t_0 = 7$

$v^* = 0.0006$

$v_3(13.88) = 0.0036$
$v_2(13.71) = 0.0035$
$v_2(14.31) = 0.0036$
$v_8(13.16) = 0.0054$
$v_8(13.16) = 0.0064$
Iteration 2:

$v^* = 0.0006$

$v_6(20.34) = 0.0048$

$v_9(21.10) = 0.0080$

$v_{10}(18.73) = 0.0070$
Iteration 3:

\[ v^* = 0.0006 \]

\[ v_4(27.86) = 0.0035 \]

\[ v_7(26.16) = 0.0056 \]

\[ v_5(26.16) = 0.0056 \]

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Dynamic/Deterministic

Final Solution:

$v^* = 0.0006$

$t = 91.77$

$t = 81.72$

$t = 73.78$

$t = 68.21$

$t = 70.78$

$t = 53.78$

$t = 60.78$

$t = 91.77$
Table 1. Vapor concentrations and rewards for nodes in $\psi$.

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<tr>
<th>Node</th>
<th>$t$ (min)</th>
<th>$v_j(t)$</th>
<th>$r_j(t)$</th>
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<td>5</td>
<td>7.00</td>
<td>0.000650</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>13.16</td>
<td>0.005400</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>18.73</td>
<td>0.007000</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>26.16</td>
<td>0.005600</td>
<td>0</td>
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<tr>
<td>5</td>
<td>32.92</td>
<td>0.002800</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>4</td>
<td>53.78</td>
<td>0.000410</td>
<td>1</td>
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<td>7</td>
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<td>8</td>
<td>73.78</td>
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</tr>
<tr>
<td>9</td>
<td>81.72</td>
<td>0.000059</td>
<td>1</td>
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Table 2. Vapor concentrations and rewards at termination.

<table>
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<th>Node</th>
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<th>$r_j(\tau^*)$</th>
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</tr>
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<tr>
<td>10</td>
<td>0.000057</td>
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Dynamic and Stochastic

- Time-variant probability distribution for each node (e.g., $V_j(t) \sim \exp(\mu_j(t))$ for all $j \in N$)

- Objective: Maximize reward - The number of areas searched where the vapor concentration has most likely decreased below $v^*$
  
  - Reward is dynamic and computed from the expected value
    
    - If $E[V_j(t)] < v^*$, $r_j(t) = 1$, otherwise $r_j(t) = 0$. 

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Algorithm

Initialization:
\[ N = \{1, 2, \ldots, N\}; \quad N_i = \{j : i \rightarrow j\}; \]
\[ \mathcal{R} = \emptyset; \quad \psi = \emptyset; \]
\[ t \leftarrow t_0; \]
\[ i = 1; \]

Obtain realization of vapor level \( v_i(t) \)

If \( v_i(t) < v^* \)
\[ r_i(t) \leftarrow 1; \]
\[ \mathcal{R} \leftarrow \mathcal{R} \cup \{i\}; \quad \psi \leftarrow \psi \cup \{i\}; \]

Else
\[ r_i(t) \leftarrow 0; \]
\[ \psi \leftarrow \psi \cup \{i\}; \]

End
Algorithm

Step 1

Calculate $\pi_j(t + t_{i,j}) \equiv P\{V_j(t + t_{i,j}) < v^*\}$ $\forall j \in \mathcal{N}_i$

Step 2

Choose $j$ such that $\pi_j(t) = \max_{j \in \mathcal{N}_i} P\{V_j(t + t_{i,j}) < v^*\}$

Obtain instrument reading at this node.

If $v_j(t + t_{i,j}) < v^*$

$r_j(t) \leftarrow 1; \mathcal{R} \leftarrow \mathcal{R} \cup \{j\}; \psi \leftarrow \psi \cup \{j\};$

$t \leftarrow t + t_{i,j};$

Else

$r_j(t) \leftarrow 0;$

$\psi \leftarrow \psi \cup \{j\};$

$t \leftarrow t + t_{i,j};$

End
Step 3

If $t \geq T$

STOP

Else

\[ i \leftarrow j; \]

Return to Step 1

End
Dynamic/Stochastic

Result: iterative process yields a time-adaptive policy

- Future decisions depend on arrival times

- *Probability* a vapor concentration is below the threshold $\nu^*$ drives the solution
Dynamic/Stochastic

Example: Iteration 1:

\[ v^* = 0.0006 \]

\[ P\{ V_1(14.31) < v^* \} = 0.989 \]

\[ P\{ V_2(13.71) < v^* \} = 0.757 \]

\[ P\{ V_3(13.88) < v^* \} = 0.726 \]

\[ P\{ V_4(14.31) < v^* \} = 0.989 \]

\[ P\{ V_5(13.76) < v^* \} = 0.955 \]

\[ P\{ V_6(13.16) < v^* \} = 0.155 \]

\[ P\{ V_7(13.76) < v^* \} = 0.955 \]

\[ P\{ V_8(13.16) < v^* \} = 0.155 \]

\[ P\{ V_9(14.31) < v^* \} = 0.989 \]

\[ P\{ V_{10}(14.31) < v^* \} = 0.989 \]
Dynamic/Stochastic

Iteration 2:

\[ v^* = 0.0006 \]

\[ P\{ V_4(20.83) < v^* \} = 0.816 \]

\[ P\{ V_2(20.60) < v^* \} = 0.811 \]
Table 3: Vapor concentrations for nodes in $\psi \ (v^* = 6.0 \times 10^{-4})$.

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<th>$t$ (min)</th>
<th>$v_j(t)$</th>
<th>$r_j(t)$</th>
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<td>5</td>
<td>7</td>
<td>0.0011</td>
<td>0</td>
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<td>14.31</td>
<td>0.000012</td>
<td>1</td>
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<td>4</td>
<td>20.83</td>
<td>0.000106</td>
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<td>7</td>
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<td>44.15</td>
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<td>5</td>
<td>81.3</td>
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<tr>
<td>1</td>
<td>88.61</td>
<td>0.000001</td>
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Summary/Conclusions

- Ignoring dynamics may lead to under- or over-estimation of the number of safe areas (in these examples)

- Spatiotemporal characteristics are critical in developing the sampling strategy

- Want to minimize Type II error (i.e., accept $H_0$ that area is safe given it is not)

- Data was assumed to exist for illustrative purposes, however...

- Real problem presents significant data requirements
Future Work

- Relax assumptions
  - Consider non-deterministic travel times
  - Multiple search crews
  - Estimate probability distributions

- Incorporate real-time information
  - Real-time concentration readings from sensors
  - Road closures/openings
  - Weather changes (e.g., wind velocity, temperature, humidity, etc.)
Questions?
Backups
The following parameters must be known to employ advection diffusion equation to compute $v_j$:

$x, y, z \equiv$ coordinates in the direction of the mean wind, horizontal cross-wind, and upwards vertical direction.

$k_x, k_y, k_z \equiv$ eddy diffusivities in m$^2$sec$^{-1}$

$q \equiv$ the total mass release in kg

$h \equiv$ instantaneous gas release height above the ground in m

$u \equiv$ wind velocity in m/sec
\[ v_j = \frac{q}{8\pi^{\frac{3}{2}}(k_x k_y k_z)^{1/2} t_0^{3/2}} \exp\left[ -\frac{(x - ut_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0} \right] \times \]

\[ \left( \exp\left[ -\frac{(z - h)^2}{4k_z t_0} \right] + \exp\left[ -\frac{(z + h)^2}{4k_z t_0} \right] \right). \]  \tag{1} \]

Equation 1 can be simplified to

\[ v_j = \frac{q}{8\pi^2 (k_x k_y k_z)^{1/2} t_0^{3/2}} \exp \left[ -\frac{(x - ut_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0} \right] \times \left( 2 \exp \left[ -\frac{h^2}{4k_z t_0} \right] \right), \quad (2) \]

since \( z = 0 \) for our numerical illustrations.
Table 6. Rate parameters chosen for the exponential distributions used for example 2.

<table>
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<tr>
<th>Node</th>
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<td>1.50</td>
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<td>2</td>
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<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>4347.83</td>
<td>2.30</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2222.22</td>
<td>4.50</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>909.09</td>
<td>11.00</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>7692.31</td>
<td>1.30</td>
<td>1</td>
</tr>
</tbody>
</table>
Case 1: Static and Deterministic

- Deterministic:
  - Assume vapor level concentration at each node is calculated via a deterministic formula immediately after the attack

- Static:
  - Assume for each $j \in \mathcal{N}$, $v_j$ does not evolve over time
Case 1: Static/Deterministic

Objective: Minimize time required to reach as many areas as possible to obtain the maximum reward (i.e., maximum number of areas not requiring protective gear).

- Time constraint implies it is possible that not all areas will be sampled

- No backtracking unless necessary (i.e., there is no reward for returning to an area)

- No subtours ($\mathcal{S} \subset \mathcal{N}$ ≡ set of all possible subtours)
Figure 4. Example of subtour in a 4-node site.
Case 1: Static/Deterministic

\[
\begin{align*}
\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_{j} x_{i,j}(\text{opt}), \quad \min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j}
\end{align*}
\]

subject to

\[
\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j} < T
\]

\[
\sum_{j \in \mathcal{N}} x_{s,j} = 1 \text{ for } s \in \mathcal{N}
\]

\[
\sum_{i \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } j = 1, \ldots, N; j \neq i
\]

\[
\sum_{j \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } i = 1, \ldots, N; i \neq j
\]

\[
x_{i,j} + x_{j,i} \leq 1 \text{ for all } (i, j) \in \mathcal{A}
\]

\[
\sum_{i \in S} \sum_{j \in S} x_{i,j} \leq |S| - 1 \text{ for } S \in \mathcal{N}, 2 \leq |S| \leq N - 1
\]

\[
x_{i,j} \in \{0, 1\}
\]
Case 1: Static/Deterministic

Example 1: Consider the following 10-node network

\[ r_j = 1 \text{ for } j = 1, 2, 3, 6, 8, 10; \quad r_j = 0 \text{ for } j = 4, 5, 7, 9 \]
Case 1: Static/Deterministic

Solution:

- Total time of search: $\tau^* = 62.85$ minutes
- Optimal path: $\psi = [5, 8, 10, 7, 4, 1, 2, 3, 6, 9]$
  Total reward: $r^* = 6$ from nodes 1, 2, 3, 6, 8, 10
Case 2: Static and Stochastic

- **Stochastic:** Assume vapor level concentration at each node is a random variable $V_j$, for all $j \in \mathcal{N}$, with an associated probability distribution.

- **Static:** $P\{V_j \leq v^*\}$ does not change with time, nor does $E[V_j] \ \forall j \in \mathcal{N}$.
Case 2: Static/Stochastic

Objective: Minimize time required to reach as many areas as possible to obtain the maximum reward

- Same formulation as Case 1

- Rewards are found from expected vapor concentrations
  
  - E.g., $V_j \sim \exp(\mu_j)$ for all $j \in \mathcal{N}$
  
  - $E[V_j] = \frac{1}{\mu_j}$
  
  - If $E[V_j] < v^*$, $r_j = 1$, otherwise $r_j = 0$. 
Case 2: Static/Stochastic

\[
\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_{j} x_{i,j}(opt), \quad \min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j}
\]

subject to

\[
\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j} < T
\]

\[
\sum_{j \in \mathcal{N}} x_{s,j} = 1 \text{ for } s \in \mathcal{N}
\]

\[
\sum_{i \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } j = 1, \ldots, N; j \neq i
\]

\[
\sum_{j \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } i = 1, \ldots, N; i \neq j
\]

\[
x_{i,j} + x_{j,i} \leq 1 \text{ for all } (i, j) \in \mathcal{A}
\]

\[
\sum_{i \in S} \sum_{j \in S} x_{i,j} \leq |S| - 1 \text{ for } S \in \mathcal{N}, 2 \leq |S| \leq N - 1
\]

\[
x_{i,j} \in \{0, 1\}
\]
Case 2: Static/Stochastic

- Total time of search: 62.85 minutes
- Optimal path: $\psi = [5, 8, 10, 7, 4, 1, 2, 3, 6, 9]$
- Total reward: $r^* = 6$ from nodes 1, 5, 6, 7, 8, 10
Comparison: Deterministic Results

Table 4. Comparison of solutions to the static/deterministic and dynamic/deterministic examples.

<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
<td>$r_j$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Time (min)</th>
<th>$r^*$</th>
<th>Total Time (min)</th>
<th>$r^<em>(\tau^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.85</td>
<td>6</td>
<td>81.72</td>
<td>5</td>
</tr>
</tbody>
</table>
Comparison: Deterministic Results

- Static/deterministic vapor concentration case
  - Search each node exactly once
  - Less amount of time

- Dynamic/deterministic vapor concentration case
  - Searches only critical nodes
  - Utilizes time allotted
  - Total reward value accounts for dynamic nature of concentrations
Comparison: Deterministic Results

Main result of comparisons: Incorporating temporal evolution reduces risk of overestimating/underestimating the number of areas safely operating without protective gear.

- **Solution 1: Static**
  - 60% of the areas are determined to be safe
  - 33% of those will become unsafe at later times

- **Solution 2: Dynamic**
  - 50% of areas are determined to be safe
  - 3 of these were previously unsafe
## Comparison: Stochastic Results

**Table 5.** Comparison of solutions to the static/stochastic and dynamic/stochastic examples.

<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Node</strong></td>
<td><strong>$r_j$</strong></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Total Time (min)</strong></th>
<th><strong>$r^*$</strong></th>
<th><strong>Total Time (min)</strong></th>
<th><strong>$r^<em>(\tau^</em>)$</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>62.85</td>
<td>6</td>
<td>88.61</td>
<td>5</td>
</tr>
</tbody>
</table>
Comparison: Stochastic Cases

- **Static/stochastic vapor concentration case**
  - Search each node exactly once
  - Less amount of time
  - Reward based on *expected* vapor concentrations

- **Dynamic/stochastic vapor concentration case**
  - Search is driven by probability a node will be less than the threshold $v^*$
  - Rewards determined from *expected* values and rate parameters are time-dependent
Comparison: Stochastic Cases

Main result: Reduce risk of overestimating/underestimating safe areas in dynamic case. Stochastic elements account for randomness of the real problem.

- Solution 1:
  - 60% of areas are determined to be safe
  - Following this path declares safe areas prematurely
  - 2 of the areas would likely not be safe at later times
Comparison: Stochastic Cases

• Solution 2:
  
  – 50% of areas are determined to be safe

  – 1 of the unsafe nodes in the previous case becomes safe at a later time