Performance Analysis of Adaptive Interleaving for MIMO-OFDM Systems

FengYe Hu, ShuXun Wang and Yang Liu
Institute of Communication Engineering, Jilin University
Changchun, Jilin Province, China, 130025
Tel: +86-431-5684201, Fax: +86-431-5691881
E_mail: hufengye@yahoo.com.cn

Abstract-In this paper, an adaptive interleaving scheme for multiple-input and multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems is proposed. The scheme could better exploit the instantaneous channel variation by rearranging the transmitted symbols according to the quasi-instantaneous channel SNR of the OFDM frame. The purpose is to break up a long burst channel error so that overall bit error probability is reduced. The upper bound of pairwise error probability (PEB) of the MIMO_OFDM systems is investigated. The code gain and diversity gain of the proposed scheme are given. The result shows that the major factor affecting performance of the proposed scheme is SNR, channel order and the number of receive antenna. Extensive simulations show that significant SNR can be obtained with the proposed scheme and the MIMO-OFDM case has a performance improvement over the single antenna case. Compared with block interleaving, for 2Tx-2Rx case at a BER of $10^{-2}$, adaptive interleaving offers a 5dB improvement.

I. INTRODUCTION

In wireless communication systems, multi-path fading, that is frequency selective fading, causes performance degradation and constitutes the bottleneck for increasing data rates. It is proven in [1],[2] that the system capacity can be improved by a factor of the minimum value of the number of transmit and receive antennas compared with a single-input single-output (SISO) system with flat Rayleigh fading or narrowband channels when multiple transmit-and-receive antennas are used to form a multiple-input and multiple-output (MIMO) system. The orthogonal frequency division multiplexing (OFDM) is an efficient approach for wideband channels. Therefore, MIMO-OFDM systems can combat the severe inter-symbols interference (ISI) resulting from the frequency selective fading of the dispersive channel and improve the capacity and quality of mobile wireless communication systems. MIMO-OFDM systems has been investigated in [3-8][12].

Interleaving has become an extremely useful technique in all digital cellular systems. Interleaving is used to obtain time diversity in a digital communications system without adding any overhead[9]. The purpose of interleaving is to randomize the burst channel errors. This can greatly improve the performance of the error-correcting codes, which are often good at correcting random errors. In general, interleaving rearranges the order of symbols to be transmitted according to a given rule. At the receiver, the reverse rule is used to restore the original sequence. Traditionally, the error correction codes and block interleaving [9-10] are employed to randomize the burst channel errors because channel errors caused by the mobile wireless channel are usually burst in nature. A novel interleaving technique called adaptive interleaving for SISO-OFDM systems is first proposed in [11]. The adaptive interleaving scheme could better exploit the instantaneous channel variation by rearranging the transmitted symbols according to the channel state information(CSI) of the OFDM frame. The purpose is to break up a long burst of the bad CSI sequence so that overall bit error probability is reduced. [12] proposed an adaptive interleaving scheme for MIMO-OFDM systems. The scheme could better exploit the instantaneous channel variation by rearranging the transmitted symbols according to the quasi-instantaneous channel SNR of the OFDM frame. [13] proposed a new interleaver design for Turbo codes with short block length based on the distance spectrum of the code and the correlation between the information input data and the soft output of each decoder corresponding to its parity bits.

The CSI is desirable at the receiver for adaptive interleaving and detection of the transmitted signal. The CSI could be estimated at the receiver with the aid of pilot symbols. A channel estimator by exploiting the time and frequency domain correlations of the channel parameters is developed in [3],[5-6],[14-15].

In this paper, we utilize the proposed adaptive interleaving scheme for MIMO-OFDM systems and simulate its performance. The upper bound of pairwise error probability (PEB) of the MIMO_OFDM systems is investigated. The code gain and diversity gain of the proposed scheme are given. The result shows that the major factor affecting performance of the proposed scheme is SNR, channel order and the number of receive antenna. The performance between RS(63,47) codes and RS(15,11) codes using adaptive
Performance Analysis of Adaptive Interleaving for MIMO-OFDM Systems

Institute of Communication Engineering, Jilin University Changchun, Jilin Province, China, 130025

See also ADM001846, Applied Computational Electromagnetics Society 2005 Journal, Newsletter, and Conference., The original document contains color images.
interleaving is compared. Extensive simulation shows that significant SNR can be obtained with the proposed scheme.

II. MIMO-OFDM SYSTEM MODEL

The block diagram of MIMO-OFDM system is illustrated in Fig.1. The input information bits are first encoded by an outer Reed Solomon channel encoder. The output of the outer code is then processed by transmitter diversity processor and interleaved into different signals \( \{ c_{k,i} [n,k] : k = 0,\cdots,K-1 & i = 1,\cdots,N_i \} \), where \( K, k, j \) and \( N_i \) are the number of sub-channel of the OFDM systems, sub-channel(or tone) index, antenna index and transmit antennas, respectively. Each stream is then modulated and sent over the corresponding antenna.

![MIMO-OFDM System Diagram](image)

At the receiver, the signal at each receive antenna is OFDM demodulated and de-interleaved. The de-interleaved signals from antennas are then fed into the diversity gain processor followed by the outer Reed Solomon decoder.

The received signal at the \( j \) th receive antenna can be expressed as

\[
r_j[n,k] = \sum_{i=0}^{N_i} H_{ij}[n,k] c_i[n,k] + v_j[n,k]
\]  

where \( H_{ij}[n,k] \) denotes the channel frequency response at the \( k \) th tone of the \( n \) th OFDM block, corresponding to the \( i \) th transmit and the \( j \) th receive antenna. \( v_j[n,k] \) Denotes the additive complex Gaussian noise on the \( j \) th receive antenna and is assumed to be zero mean with variance \( \sigma^2 \) and two-sided power spectral density \( N_0/2 \) per dimension. The noise is uncorrelated for different \( n', k', s \) or \( j' \).

For OFDM systems with proper cyclic extension and timing, it can be shown in [3-6],[14-15] that with tolerable leakage, the channel frequency response can be expressed as

\[
H[n,k] = H(nT, k\Delta f) = \sum_{l=0}^{L-1} h[l,n] W^l_k
\]  

where \( h[l,n] = h(nT, k(T_s/K)) \) and \( W_k = \exp(-j(2\pi l/K)) \). \( T_s \), \( T_f \) and \( \Delta f \) in the above expression are the block length, symbol duration, and tone spacing, respectively. They are related by \( T = 1/\Delta f \) and \( T_f = T_s + T_g \), where \( T_g \) is the duration of the cyclic extension. In (2), \( h[l,n] \)'s, for \( l = 0,1,\cdots,L-1 \), are wide-sense stationary(WSS) narrow-band complex Gaussian processes. \( L \) is the channel length of frequency selective fading channel.

Note that, for systems with transmitter diversity and a constant-modulus signal, the instantaneous SNR ratio at the receiver is defined as

\[
SNR(n,k) = \frac{\sum_{i=0}^{N_i} |H_{ij}[n,k]|^2}{|v[n,k]|^2}
\]  

The decoding of transmit diversity processor use the Maximum Likelihood(ML) decoding algorithm as the following

\[
\hat{c} = \arg \min \sum_{k=0}^{K-1} \sum_{j=1}^{N_j} \left| r_j[n,k] - \sum_{i=1}^{N_i} H_{ij}[n,k] c_i[n,k] \right|^2
\]  

where \( r_j[n,k] \) and \( H_{ij}[n,k] \) are the received signal on the \( j \) th receive antenna and the estimated channel parameter corresponding to the \( i \) th transmit and the \( j \) th receive antenna. \( c_i[n,k] \) is the estimated signal.

Let us define the following matrix at the \( n \) th OFDM block

\[
C = [C(0), C(1), \cdots, C(K-1)]
\]

where

\[
C(k) = [c_1[n,k], \cdots, c_{N_i}[n,k]]
\]

and

\[
R(k) = [r_1[n,k], \cdots, r_{N_j}[n,k]]
\]

\[
V(k) = [v_1[n,k], \cdots, v_{N_j}[n,k]]
\]

\((\cdot)^T\) stand for transpose.

\[
H(k) = \begin{bmatrix} H_{11}[n,k] & \cdots & H_{1,N_i}[n,k] \\ \vdots & \ddots & \vdots \\ H_{N_i,1}[n,k] & \cdots & H_{N_i,N_i}[n,k] \end{bmatrix}
\]

It follows from (1) that

\[
R(k) = H(k) C(k) + V(k)
\]  

which confirms that OFDM system yields parallel \( C(k) \) transmissions over different frequencies. Because each can be thought of as being transmitted using an ST system. It is important to point out that the transmissions of \( c_i[n,k] \) are separable in both time and frequency but not in space[8]. This separability will prove useful when we design our adaptive interleaving scheme.

According to (6), the ML decoding algorithm of (4) can be rewritten as the following

\[
\hat{C} = \arg \min_{C(k)} \| H(k) C(k) - R(k) \|^2
\]  

where \( \| \cdot \| \) denotes the Frobenius norm.

III. ADAPTIVE INTERLEAVING SCHEME

FOR MIMO-OFDM SYSTEMS

A novel interleaving technique called adaptive interleaving for MIMO-OFDM systems is proposed in [12]. The scheme could rearrange the transmitted symbols by exploiting the quasi-instantaneous SNR according to the varying channel condition. The purpose of adaptive interleaving is that the transmitted symbols will result in a more random mix of “Good” and “Bad” state according to the quasi-instantaneous SNR.
From (1), the signal from \( j \) th receive antenna can be expressed as
\[
r[n,k] = \sum_{i=1}^{N_t} H_{ij}[n,k]c_i[n,k] + v[n,k]
\] (8)

For simplicity, the index \( j \) for different receive antennas is omitted from \( H_{ij}[n,k] \), \( r_j[n,k] \) and \( v_j[n,k] \).

From (8), we know that the \( j \)th receive signal is sum of the \( N_t \) transmitted signal. We redefine quasi-instantaneous SNR as the following
\[
\text{SNR}(n,k) = \frac{\sum_{i=1}^{N_t} |H_{ij}[n,k]|^2}{\sigma^2}
\] (9)

We first sort the symbols in ascending order of the quasi-instantaneous SNR so that \( \text{SNR}(t_1) \leq \text{SNR}(t_2) \leq \cdots \leq \text{SNR}(t_{K}) \) (the variable \( n \) is omitted).

From (5) and (6), the resulting received symbols sequence become \( \begin{bmatrix} R(t_1), R(t_2), \ldots, R(t_{K/4}) \end{bmatrix} \). The sorted symbol \( R(t_i) \) are then put into a \( 4 \times (K/4) \) matrix as follows:
\[
\begin{bmatrix}
R(t_1) & R(t_2) & \cdots & R(t_{K/4}) \\
R(t_{K/2}) & R(t_{K/2+1}) & \cdots & R(t_{K/2+K/4}) \\
R(t_{K/4}) & R(t_{K/4+1}) & \cdots & R(t_{K/4+K/4}) \\
R(t_{K-1}) & R(t_{K-1+1}) & \cdots & R(t_{K-1+K/4})
\end{bmatrix}
\] (10)

The symbols are then taken out column by column to form a sequence
\[
\begin{bmatrix} R(d_1) & R(d_2) & \cdots & R(d_K) \end{bmatrix}
\] (11)

Hence, from [12] we derive that the de-interleaving matrix \( D \) and the interleaving matrix \( M \) are given by
\[
D = \begin{bmatrix} [1, d_1], [2, d_2], \ldots, [K, d_K] \end{bmatrix}
\] (12)
\[
M = \begin{bmatrix} [d_1, 1], [d_2, 2], \ldots, [d_K, K] \end{bmatrix}
\] (13)

IV. PAIRWISE ERROR PROBABILITY ANALYSIS

We investigate the upper bound of pairwise error probability (PEB) of the MIMO_OFDM systems in fig.1. The PEB \( P(C \to C') \) is defined as the probability that ML decoding of \( C \) erroneously decides \( C' \) in favor of the actually transmitted \( C \).

In [8], the PEB conditioned on \( H(k) \) is bounded by
\[
P(C \to C' \mid H(0), \ldots, H(K-1)) \leq \exp \left( -\frac{E_c d^2(\mathbf{C}, \mathbf{C'})}{4N_0} \right)
\] (14)

where
\[
d^2(\mathbf{C}, \mathbf{C'}) = \sum_{k=0}^{K-1} \| H(k)[C(k) - C'(k)] \|^2
\] (15)

Define
\[
W(k) = \begin{bmatrix} W_0^k, \ldots, W_{K-1}^k \end{bmatrix}^T
\]
\[
h(n) = [h_0[n,0], \ldots, h_{K}[n, L-1], h_{K}[n,0], \ldots, h_{K}[n, L-1]]^T
\] (16)

From [8]
\[
d^2(\mathbf{C}, \mathbf{C'}) = h^T(n) \Lambda \Lambda^H h(n)
\] (17)

Where
\[
\Lambda = \sum_{k=0}^{K-1} \Omega(k)[C(k) - C'(k)]\mathbf{H}(k)^H \mathbf{H}(k)
\]
\[
\Omega(k) = I + \mathcal{W}(k)
\] (18)

where \( \mathcal{W} \) denoting the Kronecker product, \( I \) stand for the \( N_t \times N_t \) identity matrix.

Let us suppose that \( \Lambda \) has rank \( r \) and denote its nonzero eigenvalues as \( \lambda_i, i = 1, \ldots, r \). We consider the expected PEP averaged over all channel realizations. The expected PEP is given by
\[
\mathbb{E}[P(C \to C')] \leq \left( \prod_{i=1}^{r} \lambda_i \right)^{-N_t} \left( \frac{E_c}{4N_0} \right)^{-N_t}
\] (19)

From (19), the first term \( \prod_{i=1}^{r} \lambda_i \) represents the coding gain achieved by the space-time code, and the second term \( \left( \frac{E_c}{4N_0} \right)^{-N_t} \) represents a diversity gain of the system.

It is clear that SNR, channel order and the number of receive antenna are the major factor affecting performance of the system proposed in this paper.

V. CHANNEL ESTIMATION AND PATTERN SYNCHRONIZATION

A. CSI Estimation

To detect the transmitted signal and perform adaptive interleaving, CSI is needed. The CSI could be estimated at the receiver with the aid of pilot symbols. A channel estimator by exploiting the time-and-frequency-domain correlations of the channel parameters is developed in [3],[5-6],[14-15]. In this paper, we use a method of simplified channel estimation proposed in [5] for CSI. The method exploit the optimum training sequences for channel estimation in OFDM with multiple transmit antennas. The optimal training sequences not only simplify the initial channel estimation, but also gain the best performance. The simplified channel estimation significantly reduces the complexity of the channel estimation at the expense of a negligible performance degradation. For simplicity, we assume that the modulation results in a constant-modulus signal[5].

B. Pattern Synchronization

To ensure that both the transmitter and the receiver are operating at the permutation, pattern synchronization is a challenging problem behind the adaptive interleaving systems. [11] proposed a quasi-closed-loop method to resolve the problem of pattern synchronization. Both the transmitter and the receiver use an identical predictive filter. Provided that both the receiver and the transmitter have the same copy of the CSI sequence, an identical de-interleaving pattern can be obtained at both sides.

VI. SIMULATIONS

A. Simulation Parameters

The entire OFDM channel bandwidth is divided into 32 sub-channels. We use RS(63, 47) code to simulate a 32 bit transmission OFDM systems. In our simulation, a 16 bit codeword is first generated at the transmitter. Then 31
redundant zero bits are appended to the codeword. The codeword length is hence increased to 47. The 47 bit codeword is then encoded using RS(63, 47) codes, giving an encoded codeword of length 63 bits, after which the 31 zero bits previously added are removed, giving a codeword of length 32. The 32 bit codeword is then transmitted. Similarly, RS(15,11) can be used to simulate 8 bit transmission OFDM systems.

One packet consisting of 8 OFDM blocks were transmitted for all simulations, and the first OFDM block transmitted is used for synchronization and channel estimation. We assume that the channel length $L$ of frequency selective fading channel is three. Therefore, the cyclic prefix for all cases are taken as 4. In the simulation, for each given SNR a total of 500 trials are taken and BPSK modulation is used.

B. Simulation results

Fig.2 show the performance of interleaving with different diversity gain. The MIMO-OFDM case has a performance improvement over the single antenna case. As can be seen from the results, for 2Tx-2Rx case at a BER of $10^{-2}$, adaptive interleaving offers a 5dB improvement compared with block interleaving.

A comparison is made between RS(63,47) coding and that of RS(15,11) coding. The results for 2Tx2Rx OFDM systems with adaptive interleaving is shown in Fig.3. From Fig.3, the RS(63,47) coding gave better BER response than RS(15,11) coding. This due to the fact that RS(63,47) coding can correct up to 8 bits of error while that of the RS(15,11) can only correct up to 2 bits of error.

VII. CONCLUSION

Canceling burst channel error is an important task in MIMO-OFDM systems. In this paper, we utilize the proposed adaptive interleaving scheme for MIMO-OFDM systems. The scheme could better exploit the instantaneous channel variation by rearranging the transmitted symbols according to the quasi-instantaneous channel SNR of the OFDM frame. The purpose is to break up a long burst channel error so that overall bit error probability is reduced. The upper bound of PEB of the MIMO_OFDM systems is investigated. The code gain and diversity gain of the proposed scheme are given. The result shows that the major factor affecting performance of the proposed scheme is SNR, channel order and the number of receive antenna. Extensive simulations show that significant SNR can be obtained with the proposed scheme.

REFERENCES