DISSIPATION OF WAVE ENERGY BY COHESIVE SEDIMENTS

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Wave energy dissipation by bottom muds is studied. A dissipation mechanism which contains explicit expressions of wavenumber modification due to a viscous bottom fluid is incorporated into a nonlinear wave shoaling model. This formulation allows a preliminary look at mud-induced energy decay without cumbersome root-finding numerics. Several cases of linear and nonlinear wave transformation over sloping and flat bottoms are run with the modified model. It is found that nonlinear energy exchange between spectral components, and a general trend toward spectral whiteness, is exacerbated in the presence of the mud layer. Strongest dissipation occurs only in the lower frequencies, contradicting field measurements of mud-induced decay which show dissipation over the entire frequency range. Thus, the need for a more comprehensive formulation of the interfacial dynamics between the water and the underlying mud layer is apparent.

1. Introduction

Most studies of bottom sediment-induced dissipation of waves have involved waves propagating over cohesionless (usually sandy) bottoms. These environments induce purely frictional dissipation, and thus tend to be relatively minor compared to the dissipation due to wave breaking, form drag over bottom ripples, and other dissipative mechanisms. Additionally, the effect of bottom friction is usually noticeable only over fair wide continental shelves, such as the eastern coast of the U.S.

However, cohesive sediments make up much of the sedimentary environments in many coastal oceans, particularly those influenced by the outflow from major rivers. In the Gulf of Mexico, the outflow of sediment from the Mississippi River influences many of the oceanographic processes present, including the exacerbation of energy dampening of surface waves by cohesive bottom sediment. This, along with the relatively small wind wave fetch and limited swell entry from the Atlantic Ocean, causes the Gulf of Mexico to be a generally low energy environment. Thus, the passage of high-energy events such as hurricanes impart a transient change on the system which are then quickly restored to the low-energy equilibrium. Sheremet and Stone\(^1\) found that both low-frequency waves and high-frequency waves are quickly dampened in muddy areas after generation by frontal passages. While the low frequency waves would be long enough to interact with, and consequently dampened by, the bottom muds, the high frequency wave damping must be correlated with strong suspension events, implying a coupled system of wave-bottom mud interaction.

Our interest is to develop a wave prediction model which accounts for the dampening of waves by muddy bottoms. Presently, wave models used in a predictive capability such as WAM\(^2\) and SWAN\(^3,4\) assume cohesionless sediments for bottom damping. However, these models are incapable of replicating the extreme dissipation imparted by muds\(^1\). Thus, existing models must be augmented by a mechanism which accounts for dissipation by cohesive sediments.

2. Wave Dissipation by Bottom Muds

While there has been substantial work on the formulation of wave dissipation mechanisms due to bottom muds, this phenomenon has generally been studied outside of a wave prediction context. Work on this subject has generally hovered around two assumptions concerning the nature of the bottom mud. The early work by Gade\(^5\) assumes that the bottom mud acts like a highly viscous fluid, with dissipation occurring due to work done on the lower fluid by the upper fluid. Dalrymple and Liu\(^6\) expanded the work of Gade\(^5\) to deep water, requiring use of numerical methods to solve the resulting eigenvalue problem for the (complex) wavenumber \(k\). The viscous fluid paradigm has been pursued by others, most recently Wen and Liu\(^7\) and Ng\(^8\).

The other representation of the mud layer assumes that it is a viscoelastic material with a constant shear modulus and a constant viscosity (a so-called Voigt body). The viscosity serves to dampen the wave while the shear modulus shortens it. Both Macpherson\(^9\) and Hsiao and Shemin\(^10\)
use this model of the mud layer. Macpherson\textsuperscript{9} showed that, for any given shear modulus, the maximum damping occurred when the dimensionless kinematic viscosity and the dimensionless shear modulus were of the same order of magnitude; furthermore, he also showed that the overall maximum damping occurred when the shear modulus was zero, or in the limit of a purely viscous fluid. Other models of the mud layer have included poroelastic models\textsuperscript{11} and Bingham plastic models\textsuperscript{12}. Foda\textsuperscript{13} considered the mud bed as a stratified elastic material coupled with nonlinear sideband oscillations of the free surface waves.

In most of these theories, the solution of a complex linear dispersion relation was required. Dalrymple et al.\textsuperscript{14} pointed out the various difficulties of numerical solution of these dissipative dispersion relation, noting in particular the inadequacy of the zero-dissipation root as an initial guess. Since our eventual goal is a robust model capable of propagating wave energy over muddy bottoms, it was decided to use the mechanism of Ng\textsuperscript{8} as a first attempt at simulating mud-induced energy dissipation. The model of Ng\textsuperscript{8} reduces that of Dalrymple and Liu\textsuperscript{6} to a boundary layer solution, allowing explicit modifications to the zero-dissipation root of the dispersion relation to be developed and obviating the need for numerical root-finding in the complex plane. (We note here the work of Mendez and Losada\textsuperscript{15}, who use a perturbative approach to determining the root of the dissipative dispersion relation for various examples of energy dissipation; their method holds promise for use with other dissipation mechanisms).

2.1. Dissipation Model

As mentioned above, Ng\textsuperscript{8} developed an asymptotic variant of the mechanism of Dalrymple and Liu\textsuperscript{6} in order to arrive at analytic expressions for the mud-induced dissipation. The mud layer was assumed to be as thick as the mud Stokes boundary layer thickness, which is also the same order as the surface wave amplitude (linear wave theory). With these assumptions, the ordering of the two-layer boundary value problem was made explicit, and a wavenumber perturbation expansion in orders of boundary layer thickness possible. The first order wavenumber $k_1$ is given by the linear dispersion relation:

$$\omega^2 = gk_1 \tanh k_1 h$$

(1)

The higher order wavenumber $k_2$ is:

$$k_2 = -\frac{Bk_1}{\sinh k_1 h \cosh k_1 h + k_1 h}$$

(2)

where $B$ is a complex coefficient:

$$B = f \left( d, \delta_m, \gamma = \frac{\rho w}{\rho_m}, \zeta = \sqrt{\frac{\nu_m}{\nu_w}} \right)$$

(3)

where $d$ is the mud layer depth, $\delta_m$ is the mud boundary layer thickness, $\rho$ is the density and $\nu$ is the kinematic viscosity. The subscripts $w$ and $m$ denote water and mud, respectively. The full expression for $B$ is complicated and can be found in the appendix in Ng\textsuperscript{8}. The mechanism has a maximum damping when the mud layer thickness is around 1.5 times the mud Stokes boundary layer thickness, in agreement with the observations of Gade\textsuperscript{6} and Dalrymple and Liu\textsuperscript{6}.

The complex wavenumber affects the wave energy through the assumed form of the free surface elevation:

$$\eta = A_n e^{i \int k_r x - \omega_n t} e^{-i \int k_i x} dx$$

(4)

where the subscripts $r$ and $i$ denote real and imaginary parts, respectively.

2.2. Wave Model

Initial implementation and testing of the dissipation mechanism required modification of an existing wave model. We used the one-dimensional model of Kaibhat and Kirby\textsuperscript{16} as a basis for the modification. The model is capable of simulating wave transformation, nonlinear interaction and breaking in the frequency domain:

$$A_{n,x} + \frac{C_{gn,x}}{2C_{gn}} A_n + \alpha_n A_n =$$

$$\frac{i}{8\omega_n C_{gn}} \left[ \sum_{i=1}^{n-1} RA_i A_{n-i} e^{i\theta_{n-i}} + 2 \sum_{i=1}^{N-n} SA_i A_{n+i} e^{i\theta_{n+i-1}} \right]$$

(5)

where $C_{gn,n}$ is the group velocity of the frequency component with index $n$, $\theta$ denotes the phase mismatches between the interacting triad of waves, and $R$ and $S$ denote complicated interaction coefficients. The coefficient $\alpha_n$ is a variant of the narrow-banded surfzone dissipation mechanism of Thornton and Guza\textsuperscript{17} with an adjustable parameter to weight the distribution of the dissipation over the frequency range. Mud-induced dissipation would affect the interaction coefficients, the phase mismatches, the kinematics and the frequency dispersion.
3. Results

In this section we describe some tests which investigate the model's dissipative behavior in the face of other transformational processes.

3.1. Linear and Nonlinear Shoaling

We used the model (5) to simulate wave shoaling over a sloping bottom. The domain is designed to be a proxy of the cross-shore profile at Duck, NC, location of the U.S. Army Corps of Engineers Field Research Facility. The offshore depth is 13 m, and the slope is 0.01. The incident wave spectrum has a significant wave height $H_s = 1m$, a peak period $T_p = 10s$ and is of the TMA form (Bouws et al.\textsuperscript{18}) with a peakedness value of $\gamma = 20$, redolent of narrow banded swell.

We ran three cases: no mud, $\gamma = 0.5$, and $\gamma = 0.9$. Based on the dissipation mechanism of $\text{Ng}^8$, we expect the latter case to provide the most damping. For all simulations $\zeta = 10$.

We first run the model without nonlinear interactions; this allows a view of the impact of mud dissipation on the general characteristics of the spectrum. Figure 1 shows the results for all three cases for four water depths. As suspected, the largest damping is seen with $\gamma = 0.9$, indicating that damping is enhanced if the mud is less inertial to the velocities imparted by the waves. Additionally, the damping appears to be highly dependent on frequency, with shorter waves undergoing less damping. This is sensible since these waves undergo less interaction with the bottom than the lower frequency waves.

Next we investigate the case with nonlinear interactions activated; this is shown in Figure 2. Because of the extreme damping in shallow water and the high degree of variability of the nonlinear terms at the high frequencies, the model suffers from insufficient resolution in the nearshore areas, precluding a stable result in water depths less than two meters.

The presence of mud has a clear effect on the spectral shape. The no-mud case evidences the expected amplification of the higher harmonics of the spectral peak, which is endemic of triad interactions. Interestingly, however, this amplification occurs much earlier in the transformation process for the cases with mud, with $\gamma = 0.9$ displaying the quickest amplification. In the shallower water depths, the spectra in the no-mud case still retains a distinct peak, while the wave spectra propagating over mud devolve toward whiteness rather quickly. Again the spectra for the case of $\gamma = 0.9$ shows the fastest trend toward whiteness.

Figure 1. Linear wave shoaling over a 1/100 slope; results for four water depths.

3.2. Nonlinear Transformation over a Flat Bottom

The Gulf coast of Louisiana is characterized by relatively short waves distributed over a broad range of frequencies, propagating over an extremely shallow slope. A relevant estimate of $\gamma$ for the area is $\gamma = 0.75$, over a mud layer approximately 20cm deep.

We characterize the incident wave spectrum as a TMA spectrum with
whitening of the spectrum relative to the case without mud. Here, the low frequencies are primarily affected, reflective of the $kh$-dependence of the dissipation mechanism. However, as stated previously, Sheremet and Stone\(^1\) witnessed high frequency damping in their measurements. This damping is not seen here, a possible indication that the dissipation mechanism of Ng\(^2\) may not be entirely adequate for simulating processes in the area.

Figure 2. Nonlinear wave shoaling over a 1/100 slope; results for four water depths.

significant waveheight $H_s = 0.3m$, peak period $T_p = 7s$ and peakedness parameter $\tilde{\gamma} = 3.3$. We use a constant depth of 5m and an overall domain length of 1.2km. As before, we run the model with and without mud dissipation.

The resulting spectra at four locations in the domain are shown in Figure 3. The effect of wave nonlinearity is evident in the increase of the low frequency variance, but the presence of mud dissipation results in a general

Figure 3. Random wave propagation over flat bottom - spectral output at four points in domain
4. Discussion

A mud-induced wave energy dissipation mechanism was incorporated into a nonlinear wave shoaling model and run for several analytical cases. The dissipation mechanism of $N_g$ was used, since the wavenumber modification due to the viscous mud layer was expressible in an explicit form, thereby eliminating the need for a complex root-finding algorithm embedded in the model.

The test cases generally involved wave propagation over a sloping or flat bottom. For the case of nonlinear wave shoaling, it was found that the dissipation due to mud caused the wave spectra to tend toward dissipative whiteness earlier in the transformation process than if no mud was present. This trend is most noticeable when the density of mud is close to the density of water; the mud’s inertial tendencies are reduced compared to heavier muds. For the case of nonlinear wave propagation over a flat bottom, strongest dissipation occurred in the lower frequencies, sensible since the interaction with the bottom is strongest in these frequencies.

While encouraging, it is not clear whether or not the dissipation mechanism of $N_g$ is sufficient for simulating energy decay in areas such as the Louisiana shelf, where Sheremet and Stone have found strong dissipative tendencies in both high and low frequencies. Further work will concentrate on interfacial dynamics between the water and mud layer, building on the work of Hill and Foda and Jamali et al.

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References

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