Nonadiabatic effects in a self-consistent Hartree model for electrons under an ac electric field in multiple quantum wells

By deriving a dynamical differential equation for the electron distribution function in the presence of a nonadiabatic sequential-tunneling current under an ac electric field through a multiple-quantum-well system, the self-consistent Hartree model is generalized for the calculation of electronic states with the inclusion of nonadiabatic effects (dependence on the time derivative of the applied ac electric field) in quantum wells. The influence of different doping profiles, temperatures, and amplitudes of an applied ac electric field on the electron distribution function and sequential tunneling are studied. This work provides a fully quantum-mechanical explanation to the previously proposed current-surge model to a leading-order approximation.
Nonadiabatic effects in a self-consistent Hartree model for electrons under an ac electric field in multiple quantum wells

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I. INTRODUCTION

Recently, transient transport properties of semiconductor quantum wells,\textsuperscript{1-3} superlattices\textsuperscript{4,5} and quantum dots\textsuperscript{6} have been of great interest. Various nonlinear properties having their origin in the Coulomb interaction have attracted a great deal of attention.\textsuperscript{7-10} Self-sustained current oscillations and multistability have been predicted in tunneling currents through doped semiconductor superlattices and multiple quantum wells (MQW's). They are attributed to the dynamics of domain walls separating the electric-field domains. In addition, oscillations in the tunneling-current transient have been predicted in MQW's, even in the absence of electric-field domains due to nonadiabatic effects.\textsuperscript{10} The nonadiabatic effects discussed in this paper are associated with the fact that a transient conduction current depends not only on an electric field but also on its time derivative due to quantum-well capacitive coupling. The quantum-well capacitance is of the order of picofarads. However, the resistance of the MQW sample with a thick barrier between the wells used in this study is of the order of teraohms due to the extremely small sequential-tunneling current. As a result, the nonadiabatic effect occurs on a time scale of seconds, which makes electron tunneling depend on the time derivative of the applied electric field in addition to the field itself for low ac frequencies of the order of a few hertz.\textsuperscript{11,12}

In this paper, we consider the sequential-tunneling transport of electrons in an MQW system in the presence of an applied ac electric field. We assume that the lattice temperature is kept constant and the electrons are in thermal equilibrium with the lattice, so that the electron temperature is the same as that of the lattice. The sequential tunneling of electrons (of the order of nanoseconds) through a thick barrier between adjacent quantum wells in an MQW system is a very slow process compared to the coherent tunneling of electrons (of the order of sub-picoseconds) through a thin barrier in a superlattice system. However, electrons during the sequential-tunneling process still "see" an instantaneous electric field because of \(\tau_s \approx \frac{2 \pi}{\Omega} \) for low ac frequency \(\Omega\) (of the order of a few hertz) with \(\tau_s\) being the sequential-tunneling time (of the order of nanoseconds).

Adiabatic electrons in an MQW system with an applied ac electric field stay in the equilibrium states with a constant Fermi level, since \(\tau_s \ll \tau_\text{relax}\), with \(\tau_\text{relax}\) being the energy-relaxation time due to the very-long sequential-tunneling time within which an equilibrium state can be established by the much faster inelastic scattering of electrons inside the quantum well. However, the electron density can vary with time if the electrons in the quantum well stay in the nonadiabatic state.\textsuperscript{10,13,14} For the nonadiabatic state, the nonadiabatic effects cause the Fermi level in the "equilibrium" state to change with time under an ac electric field. As a result, a charge-density fluctuation in the quantum well will modify the Hartree potential in the surrounding barrier region, and thus greatly affect the sequential tunneling of electrons through the barrier. Simultaneously, the charge-density fluctuation also modifies the electronic states in the quantum well within the self-consistent Hartree model.\textsuperscript{15}

For the quantum-well sample considered in this study, the second-subband edge is 83.4 meV above the first-subband edge (see Table I), while the Fermi level is only 14.2 meV above the first-subband edge (see Table I). As a result, the second subband is completely unpopulated at temperatures below 40 K. Therefore, we have neglected the tunneling contribution from the unpopulated second subband. The well-known negative differential conductance (NDC) phenomenon can be seen if the second subband in the quantum well is brought into consideration for electron tunneling. For the multisubband case, the NDC occurs at a field strength where the first-subband edge in the preceding well is aligned with the second-subband edge in the next tilted well (there is a very narrow spectral density for both quantum wells below 40 K).

<table>
<thead>
<tr>
<th>(V_o) (meV)</th>
<th>(L_w) (Å)</th>
<th>(L_B) (Å)</th>
<th>(n_{2D}) (\times 10^{11}\text{cm}^{-2})</th>
<th>(e_w)</th>
<th>(m_w) ((m_e))</th>
<th>(m_B) ((m_e))</th>
</tr>
</thead>
<tbody>
<tr>
<td>331</td>
<td>80</td>
<td>300</td>
<td>4.0</td>
<td>12.0</td>
<td>11.2</td>
<td>0.067</td>
</tr>
</tbody>
</table>

TABLE I. Parameters of GaAs/Al\(_0.1\)Ga\(_{0.9}\)As MQW sample used for numerical calculations with well depth \(V_o\), well width \(L_w\), barrier thickness \(L_B\), electron density \(n_{2D}\), well (barrier) relative dielectric constant \(e_w\) (\(e_B\)), and well (barrier) electron effective mass \(m_w\) (\(m_B\)) with \(m_e\) being the free-electron mass.
cause of the very thick barrier between them). On the other hand, the NDC phenomenon also occurs in quantum wells with a single subband. This is due to a lesser overlap between the quantum-well quasiparticle spectral functions (whose width depends not only on the interwell coupling but also on the disorder self-energy) as the applied electric field increases when the Fermi energy is not too far from the top of the barrier. For the sample with barrier thickness \( L_B = 300 \, \text{Å} \) considered in this paper, the required field strength for the multisubband NDC phenomenon is 27.8 kV/cm. However, the maximum field strength employed in this study is only 1 kV/cm. Consequently, we have only included the first subband and neglected the NDC effect in our model where the Fermi energy is well below the top of the barrier. Moreover, the field-domain effect in an MQW system is expected to be very small under low electric fields for coherent-tunneling cases or below 40 K for sequential-tunneling cases and is neglected in this paper since it becomes significant only for a large-tunneling current. The thick-barrier-layer sample used in this study is to limit the dark sequential-tunneling cases or below 40 K for sequential-tunneling cases.

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\[
\frac{d}{dt} \delta p_k(t) = \frac{d}{dt} \left( \frac{e}{\mathcal{V}} \left[ f^{\mu_0}(k) - f_0^{\mu_0}(E_k) \right] \right) = -\frac{e}{\mathcal{V}} \left( \frac{d\Delta E_k}{dt} \right) - \frac{\partial f_0^{\mu_0}(E_k)}{\partial E_k}, \tag{4}
\]

where \( \mathcal{V} \) is the volume of the sample. The local charge-density fluctuation is a result of the change of the electron distribution in energy space with respect to the equilibrium state even when \( n_{2D} \) is a constant.

III. COHERENT- AND SEQUENTIAL-TUNNELING MODELS

For an MQW system with thick-barrier layers, the adiabatic sequential-tunneling current density flowing in the \( z \) direction (growth direction and perpendicular to the quantum-well layers) is found to be

\[
J^{\mu_0}(t) = \frac{2e}{\mathcal{V}} \sum_k v_k^z T[E_k, E_0] \left[ f^{\mu_0}(E_k) - f_0^{\mu_0}(E_k) + eE_0 L_B \right], \tag{5}
\]

where \( L_B \) is the thickness of the barrier between two adjacent quantum wells, \( v_k^z \) is the group velocity of quasibound- or continuum-state electrons in the \( z \) direction, and \( T[E_k, E_0] \) is the quantum-mechanical transmission of electrons through the biased barrier. If \( \Omega \tau_p \gg 1 \), with \( \tau_p \) being the electron sequential-tunneling time, \( T[E_k, E_0] \) has to be found by solving a time-dependent Schrödinger equation. Otherwise, \( T[E_k, E_0] \) can be calculated from a static Schrödinger equation at each time \( t \) if \( \Omega \tau_p \ll 1 \). We will be only interested in the latter case with \( \Omega \tau_p \ll 1 \) hereafter.

In the limit of small barrier thickness and weak field, i.e., \( eE_0 L_B \ll E_k \), Eq. (5) yields a coherent-tunneling current that takes the same form as that obtained using the regular Boltzmann's equation \((d\Delta E_k/dt=0)\) under the relaxation-time approximation:

\[
J^{\mu_0}(t) = \frac{2e^2}{\mathcal{V}} \sum_k (v_k^z)^2 \tau_p \left[ \frac{\partial f^{\mu_0}(E_k)}{\partial E_k} \right], \tag{6}
\]

where we have set \( T[E_k, E_0] = v_k^z \tau_p / L_B \) for scattering-limited miniband-state electrons at very low electric field (with mean free path \( v_k^z \tau_p \) smaller than \( L_B \)), and \( \tau_p \) is the momentum-relaxation time of electrons. Consequently, the conductance that is proportional to \( J^{\mu_0}(t)/E_0 \) becomes independent of \( E_0 \) in this situation.

If we replace \( f^{\mu_0}(E_k) \) to the leading-order approximation by the equilibrium value \( f_0^{\mu_0}(E_k) \) for faster electron energy relaxation processes due to inelastic scattering of electrons as compared to the electron sequential tunneling, and replace the electron group velocity \( v_k^z \) by a drift velocity \( v_d^z [E_0] \) (a statistically averaged group velocity) of electrons in a bulk material, Eq. (5) reduces to Levine's sequential-tunneling model\(^{20}\)

\[
J^{\mu_0}(t) = \frac{2e}{\mathcal{V}} v_d^z [E_0] \sum_k T[E_k, E_0] \left[ f^{\mu_0}(E_k) - f_0^{\mu_0}(E_k) + eE_0 L_B \right], \tag{7}
\]

where \( v_d^z [E_0] = (e\tau_p/m^*) E_0 \), the momentum-relaxation time \( \tau_p \) is given by

\[
\tau_p = \frac{m^* v_d}{e \sqrt{E_0^2 + E_0^0}} \tag{8}
\]

\(^{m^*} \) is the effective mass of electrons, \( \nu_d \) is the electron saturation velocity, and \( E_0 \) is the saturation electric field. In Eq. (7), \( J^{\mu_0}[E_0] \) can be equivalently viewed as a three-dimensional tunneling-electron density that depends on \( E_0 \), \( T \), and \( n_{2D} \). Obviously, the conductance that is proportional to \( J^{\mu_0}[E_0]/E_0 \) becomes dependent on \( E_0 \) in this situation.

IV. CURRENT-SURGE MODEL

From now on, we limit ourselves to an electrical-quantum limit where only the ground subband of the narrow quantum well is occupied by electrons at low temperatures and low electron densities. The electron kinetic energy of the ground subband (measured from the edge) is given by \( E_k = h^2 k^2 / 2m^* \). In the current-surge model, we assume that \( \Delta E_k \) is associated with the fluctuation of the chemical potential of electrons in the quantum well (independent of individual electron state), instead of the local fluctuation of electron kinetic energy for each electron state. By writing \( \Delta E_k = -\Delta \mu = \mu_0(n_{2D}, T) - \mu(t) \) for the global chemical-potential fluctuation, where \( \mu(t) \) and \( \mu_0(n_{2D}, T) \) are, respectively, the transient chemical potential for electron density \( n_e(t) \) and that for an equilibrium electron gas in quantum wells, we get

\[
\frac{d\Delta E_k}{dt} = \frac{\partial \Delta E_k}{\partial t} - \frac{\partial \mu}{\partial n_e} \frac{dn_e}{dt}. \tag{9}
\]

We further introduce a spatially averaged space-charge field \( \tilde{E}_m(t) \) which is defined by\(^{10,13,14}\)

\[
\tilde{E}_m(t) = e \tilde{E}_m(t) L_B, \tag{10}
\]

where \( \tilde{E}_m(t) \) measures the reduction of the electron chemical potential in quantum wells. If we use Levine's sequential-tunneling model in Eq. (7), we find the change in the current density due to the existence of this space-charge field \( \tilde{E}_m(t) \),

\[
\Delta J_m(t) = J^{\mu_0} - e \tilde{E}_m \left[ E_0 + E_0^0 \right] - J^{\mu_0}[E_0]. \tag{11}
\]

where \( J^{\mu_0}[E_0] \) has been given in Eq. (7). In Eq. (11), the first term can be viewed as an equivalent capture current flowing into the quantum well, while the second term can be regarded as a sequential-tunneling current flowing out of the quantum well.

For a quantum well, the electron density will be constant if the conduction currents flowing in and out of the well are equal. The variation of the charge density in the well is cre-
Eqs. (4), (9), (10), and (12), we finally arrive at the following

\[ \frac{d}{dt} \delta \rho(t) = \nabla \cdot \sum_k \delta n_k(t) = A \Delta J_{\text{res}}(t), \]  

where \( A \) is the cross-sectional area of the MQW sample. The left-hand side of Eq. (12) represents the charge increase inside the well, while the right-hand side of the equation stands for the net increase in charges due to a nonadiabatic change in the current flowing into the quantum well. Combining Eqs. (4), (9), (10), and (12), we finally arrive at the following equation derived previously as the current-surge model:

\[ L_B C_{\text{QW}} \frac{d}{dt} e \phi(t) = L_B C_{\text{QW}} \frac{d}{dt} \phi(t) - \Delta J_{\text{res}}(t), \]  

where the quantum-well capacitance per unit area is \( C_{\text{QW}} = (m^* e^2 / \pi \hbar^2) \phi_0^2(0) \). Here, we have employed in Eq. (13) the fact that \([\partial \mu / \partial n_d](\alpha n_d(t)) = e L_B [d \phi_0(t)/dt] \) for the capacitance coupling of the quantum well to an external ac electric field.\(^{10,13}\) The fast inelastic scattering in quantum wells ensures that electrons are in an "equilibrium" state. However, Eq. (13) causes a shaking Fermi level for the equilibrium state on a macroscopic time scale.

\section{V. NONADIABATIC EFFECTS IN A SELF-CONSISTENT HARTREE MODEL}

As mentioned in the Introduction, electrons in quantum wells see only the instantaneous ac electric field during their sequential-tunneling process if \( \Omega \tau \ll 1 \). In this case, the ground-state electron wave function \( \phi_1(z,t) \) inside the quantum well within the self-consistent Hartree model is determined by

\[ -\frac{\hbar^2}{2m^*(z)} \frac{d}{dz} \left( \frac{d}{dz} \phi_1(z,t) \right) - e E_b(z,t) + U_{\text{QW}}(z) + \nu_{\text{H}}(z,t) \phi_1(z,t) = E_1(t) \phi_1(z,t), \]  

where \( E_1(t) \) is the time-dependent ground-subband edge, the electron effective mass \( m^*(z) \) takes \( m_w \) in the well and \( m_B \) in the barrier, and \( U_{\text{QW}}(z) \) is zero inside the well but \( V_0 \) outside the well. For the adiabatic state, we have \( \delta \rho(t) = 0 \) for the cases with nonadiabatic effects. The Hartree potential \( \nu_{\text{H}}(z,t) \) in Eq. (14) can be found from the Poisson equation

\[ \frac{d}{dz} \left[ \varepsilon(z) \frac{d}{dz} \nu_{\text{H}}(z,t) \right] = \frac{e^2}{\varepsilon_0} \left[ N_{\text{D}}(z) - n_e(z,t) \right], \]  

where \( N_{\text{D}}(z) \) is the static profile of donor doping for the single quantum well, \( n_e(z,t) = \phi_1(z,t)^2 n_e(r) \) is the density function, and

\[ n_e(t) = n_{2D} + \frac{\delta \rho(t)}{e} L_W = n_{2D} + \rho_{2D} \int_0^\infty dE \delta f(E,t), \]

where \( \rho_{2D} = (m_w / \pi \hbar^2) \) is the density of states for two-dimensional electrons in the quantum well and \( \delta f(E,t) \) represents the local fluctuation of the electron distribution function in energy space. Here, the number of electrons in the quantum well is not a constant due to the nonadiabatic current flowing. Moreover, we find from Eqs. (4), (9), and (16) that

\[ L_B \frac{d}{dt} \delta \rho(t) = e L_B \int_0^\infty dE \frac{d}{dt} \delta f(E,t) + e^2 L_B^2 \rho_{2D} \frac{d}{dt} \phi_0(t) \]

Applying Eq. (12) and using Eqs. (11) and (17), we find the following integral equation for \( \delta f(E,t) \) by using Levine's model in Eq. (7)

\[ \int_0^\infty dE \left[ \frac{\partial \delta f_0^a(E)}{\partial E} - \left( \frac{e \rho_{2D}}{2 L_W^2} \right) \nu_{\text{d}} E_0 \right] \delta f_0^a(E) + \delta f(E,t) \]

In Eq. (18), \( \nu_{\text{H}}(z,t) \) and \( E_1(t) \) are written simply as \( \nu_{\text{H}} \) and \( E_1 \). The adiabatic quantities \( \nu_{\text{H}}^a(z,t) \) and \( E_1^a(t) \) can be obtained by simply setting \( \delta \rho(t) = 0 \) in Eq. (15) and \( \nu_{\text{H}}(z,t) = \nu_{\text{H}}^a(z,t) \) in Eq. (14). Moreover, the fluctuation of the drift velocity \( \delta v_{\text{d}} [\delta f] \) introduced in Eq. (18) is calculated to be

\[ \delta v_{\text{d}} [\delta f] = \left( \frac{\rho_{2D}}{n_{3D}} \right) \int_0^\infty dE \delta f(E,t) \]  

Finally, Eq. (18) leads us to the dynamical differential equation for \( \delta f(E,t) \),

\[ \int_{-\infty}^\infty \frac{d}{dt} \delta \rho(t) = \frac{\rho_{2D}}{n_{3D}} \int_0^\infty dE \delta f(E,t) \sqrt{\frac{2E}{m_w}}. \]
\[
\frac{\partial}{\partial t} \delta f(E,t) - eL_B \frac{\partial f^{\mu_0}(E)}{\partial t} - \frac{1}{L_w} \frac{\partial f^{\mu_0}(E)}{\partial E} = 0
\]

where the initial condition is chosen to be \( \delta f(E,t) = 0 \) at \( t = 0 \) if the ac electric field is applied to the system after \( t = 0 \). \( \delta f(E,t) \) has a lower bound that is set by the condition \( \delta f(E,t) + f^{\mu_0}(E) = 0 \).

For small \( \Delta \mu \), the first term in Eq. (20) can be approximated to the leading order by

\[
\frac{\partial}{\partial t} \delta f(E,t) \approx -\frac{\partial \Delta \mu}{\partial t} \left[ -\frac{\partial f^{\mu_0}(E)}{\partial E} \right].
\]

Similarly, a part of the third term in Eq. (20) can be approximated as

\[
T[E+E_1,\varepsilon_b;\varepsilon_H][f^{\mu_0}(E) + \delta f(E,t)] + \frac{1}{L_w} \left[ (\varepsilon[H]^{(0)} + 2\nu_{\parallel}[\varepsilon[H]^{(0)}]f^{\mu_0}(E)
\right.
\]

\[
- f^{\mu_0}(E + e\varepsilon_bL_B) = 0,
\]

where \( \Delta \mu = -e\varepsilon_b(t)L_B \). Eq. (20) results in the current-surge model in Eq. (13), where

\[
\phi_{N_B+1}(t) = \left[ \begin{array}{c} \phi_{N_B}(t) \\ -1-i\tilde{k} - (1/2E_d)[E+E_1(t) - U_{N_B+1}^{(0)} + e\varepsilon_b(t)N_B - \nu_{\parallel}[\varepsilon[H]^{(0)}]f^{\mu_0}(E)] \end{array} \right] \phi_{N_B+1}(t)
\]

where \( \tilde{k} = (\Delta/\hbar)\sqrt{2m_0(E+e\varepsilon_b(t)L_B)} \). From the solution of Eq. (24) we find the quantum-mechanical transmission of electrons from

\[
T[E+E_1,\varepsilon_b;\varepsilon_H] = \frac{1}{|S|^2} \sqrt{\frac{E+e\varepsilon_b(t)L_B}{E}},
\]

where \( |S|^2 = |a|^2 + |b|^2 + 2Re(ab^*)/4 \). The solution of Eq. (24) ensures that the transmission coefficient in Eq. (26) depends on the barrier thickness and height in an exponential way. Here, the two complex numbers \( a \) and \( b \) are defined by the starting boundary condition of Eq. (24).

VI. NUMERICAL RESULTS AND DISCUSSION

We choose a GaAs/Al\(_{x}\)Ga\(_{1-x}\)As MQW sample for numerical calculations. Some sample parameters can be found in Tables I and II. Other parameters include \( E_{ex} = 0.05 \) kV/cm, \( v_T = 2 \times 10^6 \) cm/sec, \( \varepsilon = 2 \) kV/cm, and \( T_p = 4 \) sec. Different doping profiles have been considered, in-
due to the large increase of electron density inside the quantum wells and the saturation of $\mathcal{E}(t)$ with respect to $\mathcal{E}_0(t)$, we find a small enhancement in maximum (i.e., $\mathcal{I}_s(t)$) for nonadiabatic [solid curve] and adiabatic [$\mathcal{I}_s(t)$=0, dashed curve], and an applied ac electric field $\mathcal{E}_{ac}(t)$ (right scale) for $T$=40 K and $\mathcal{E}_{ac} = 1$ kV/cm with uniform doping inside GaAs quantum wells. On the other hand, we find a large enhancement in $\mathcal{I}(t)$=0 and $\mathcal{E}_s(t)$=0 indicate electrons moving out of and into the quantum wells, respectively. Moreover, $\delta \mathcal{E}(E,t)$ will be in phase with $\mathcal{E}(E,t)$ since it is proportional to $-\delta f(E,t)$ that itself is proportional to $\mathcal{E}_{ac}(t)$.

Figure 2 displays the calculated adiabatic Hartree potential [in panel (a)] and the change of Hartree potential [in panel (b)] in the nonadiabatic state from the Poisson equation (15) as functions of position $z$ for different doping profiles at $t/T_p = 0.25$. Here, $T$=40 K, $\mathcal{E}_{ac} = 1$ kV/cm, and the quantum well sits in the range of 300 A≤$z$≤380 A. From Fig. 2(a) we find that the absolute value of the adiabatic Hartree potential becomes smallest for the uniform-doping case. The center $\delta$ doping in the quantum well causes the conduction band edge to bend down at the well center, while the edge $\delta$ doping makes the conduction band edge bend up there, as shown in Fig. 3(a). With the total potential seen by the electrons being the sum of the adiabatic Hartree potential $V_H(z,t)$ plus the change $\Delta V_H(z,t)$ plus the quantum-well potential $U_{QW}(z)$, the out-of-phase nature of Figs. 3(a) and 3(b) will result in the band bending seen in Fig. 3(a) being substantially suppressed by the nonadiabatic effects in Fig. 3(b). However, the nonadiabatic effects with edge $\delta$ doping produces two positive spikes [solid curve in Fig. 3(b)] at the

=3 sec). These features are a result of the induced space-charge field $\mathcal{E}_{ac}(t)$.

The reduction of electron population around $E$ = $\mu_d(n_{2D}, T)$ can be described by the space-charge field $\mathcal{E}_{ac}(t)$ defined in Eq. (23). We display $\mathcal{E}_{ac}(t)$ (solid curve, left scale) in Fig. 2(a), along with $\mathcal{E}_s(t)$ (dashed curve, right scale) as functions of $t$ for uniform doping. From the figure we see that $\mathcal{E}_{ac}(t)$ and $\mathcal{E}_s(t)$ are nearly in phase with each other, except for a slight phase shift. This is a direct result of oscillations in the change of the charge density $\delta p(t)$ in the quantum well, as shown in Fig. 2(b), where both $\delta p(t)$ (solid curve, left scale) and $\delta \mathcal{E}(E)$ (dashed curve, right scale) are plotted as functions of $t$. Since $\mathcal{E}_{ac}(t)$ describes the reduction of charge density in the quantum well, we expect $\delta p(t)$ to be nearly out of phase with $\mathcal{E}_{ac}(t)$ or $\mathcal{E}_s(t)$, as can be seen from Figs. 2(a) and 2(b). The situations with $\delta p(t)$<0 and $\delta p(t)$>0 indicate electrons moving out of and into the quantum well, respectively. Moreover, $\delta \mathcal{E}(E,t)$ will be in phase with $\mathcal{E}(E,t)$ since it is proportional to $-\delta f(E,t)$ itself is proportional to $\mathcal{E}_{ac}(t)$.
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Figure 3. Calculated position dependence of adiabatic Hartree potentials \( V_\text{H}^{(0)}(z,t) \) in panel (a) and change of nonadiabatic Hartree potentials \( \Delta V_\text{H}(z,t) \) in panel (b) at \( t/T_p = 0.25 \), with \( T=40 \text{ K} \) and \( \varepsilon_{ac} = 1 \text{ kV/cm} \) for edge \( \delta \)-doping (solid curves), center \( \delta \)-doping (dashed curves), and uniform doping (dotted curves) in the quantum well.

Figure 4 indicates the effects of an ac electric-field strength \( \varepsilon_{ac} \) [in panel (a)] and temperature \( T \) [in panel (b)] on the charge-density fluctuations \( \delta \rho(t) \) as a function of \( t \) in a uniformly doped quantum well. In Fig. 4(a) we find that fluctuations \( \delta \rho(t) \) increase with \( \varepsilon_{ac} \) at \( T=40 \text{ K} \), with the negative peak (electrons removed from the quantum well) being saturated at \( \varepsilon_{ac} = 5 \text{ kV/cm} \). In Fig. 4(b), as \( T \) increases \( \delta \rho(t) \) is enhanced when it is negative (electrons removed from the quantum well), but reduced when it is positive (electrons added to the well) at \( \varepsilon_{ac} = 1 \text{ kV/cm} \).

Figure 4 only shows us the global fluctuation of the charge density in the quantum well. In order to gain further insight into the local change in the electron distribution function, we display \( \delta \phi(E,t) \) in Fig. 5 at \( t/T_p = 0.25 \) with uniform doping for different values of \( \varepsilon_{ac} \) [in panel (a)] and \( T \).

From Fig. 5(b) it is clear that \( \delta \phi(E,t) \) always shows a negative minimum at \( \mu_{\text{eff}}(n_{2D},T) \), since it is proportional to \( \delta f^{\text{eq}}(E) \partial E \). Because the Fermi surface broadens with increasing \( T \), we find from Fig. 5(b) that the negative minimum is partially suppressed and broadened (solid curve) when \( T=40 \text{ K} \) as compared to that (dashed curve) at \( T=20 \text{ K} \). From Fig. 5(a) we find that the negative minimum is enhanced when \( \varepsilon_{ac} \) is increased. The cusp (dashed curve) in Fig. 5(a) is a result of zero occupation of electrons in a specific state with kinetic energy \( E \) in the ground subband.

Figures 6(a) and 6(b) present nonadiabatic effects on the Hartree potentials in the uniformly doped quantum well at \( T=40 \text{ K} \) and \( \varepsilon_{ac} = 1 \text{ kV/cm} \). From Fig. 6(a) we find that the positive peak in the adiabatic Hartree potential \( V_\text{H}^{(0)}(z,t) \) (dashed curve) at the center of the quantum well is greatly suppressed by the nonadiabatic effects (solid curve) at \( t/T_p = 0.25 \), leaving two positive spikes at the edges of the quantum well.

FIG. 3. Calculated position dependence of adiabatic Hartree potentials \( V_\text{H}^{(0)}(z,t) \) in panel (a) and change of nonadiabatic Hartree potentials \( \Delta V_\text{H}(z,t) \) in panel (b) at \( t/T_p = 0.25 \), with \( T=40 \text{ K} \) and \( \varepsilon_{ac} = 1 \text{ kV/cm} \) for edge \( \delta \)-doping (solid curves), center \( \delta \)-doping (dashed curves), and uniform doping (dotted curves) in the quantum well.

FIG. 4. Time dependence of calculated charge-density fluctuations \( \delta \rho(t) \) in the uniformly doped quantum well. In panel (a), we set \( T=40 \text{ K} \) with \( \varepsilon_{ac} = 1 \text{ kV/cm} \) (solid curve) and \( \varepsilon_{ac} = 5 \text{ kV/cm} \) (dashed curve). In panel (b), we set \( \varepsilon_{ac} = 1 \text{ kV/cm} \) with \( T=40 \text{ K} \) (solid curve) and \( T=20 \text{ K} \) (dashed curve).

FIG. 5. Time evolution of calculated charge-density fluctuations \( \delta \rho(t) \) for different values of \( \varepsilon_{ac} \) and \( T \) in the uniformly doped quantum well.

FIG. 6. Nonadiabatic effects on the Hartree potentials in the uniformly doped quantum well at \( T=40 \text{ K} \) and \( \varepsilon_{ac} = 1 \text{ kV/cm} \).
FIG. 5. Calculated change of nonadiabatic distribution functions $\delta f(E, t)$ at $t/T_p = 0.25$ for electrons in the uniformly doped quantum well. In panel (a), we set $T = 40$ K with $E_a = 1$ kV/cm (solid curve) and $E_a = 5$ kV/cm (dashed curve). In panel (b), we set $E_a = 1$ kV/cm with $T = 40$ K (solid curve) and $T = 20$ K (dashed curve).

In conclusion, we have derived a dynamical differential equation for the nonadiabatic electron distribution function with sequential-tunneling current flowing through an MQW system. Using this equation, we generalized the self-consistent Hartree model for the calculation of the electronic states with the inclusion of nonadiabatic effects in a quantum well. We have also studied the effects of different doping profiles, temperatures, and amplitudes of applied ac electric field on the nonadiabatic electron sequential tunneling. Finally, we display in Fig. 7 $f(E, t)$ at $t/T_p = 0.25$ (dotted curve) and 0.75 (dashed curve), as well as the equilibrium distribution $f_{eq}^{\rho 0}(E)$ (solid curve) in panel (a) and $\log_{10}|I_{ad}(t)|$ as a function of $E_k(t)$ in panel (b). From Fig. 7(a) we see $f(E, t)$ resembles the equilibrium distribution $f_{eq}^{\rho 0}(E)$ with a shaking Fermi level with time (shaking up at $t/T_p = 0.75$ and shaking down at $t/T_p = 0.25$). Compared with the adiabatic electron sequential-tunneling current [thin solid curve with $\Delta J_{ad}(t) = 0$ in Fig. 7(b)], the symmetry of $\log_{10}|I_{ad}(t)|$ with respect to the positive (electrons being removed) and negative (electrons being added) extreme values of $E_k(t)$ is broken in the case with nonadiabatic effects (thick solid curve). A small offset of $\log_{10}|I_{ad}(t)|$ with respect to $E_k(t) = 0$ can be seen by comparing thick and thin solid curves.

VII. CONCLUSIONS AND REMARKS

FIG. 6. Calculated position dependence of Hartree potentials for nonadiabatic (solid curve) and adiabatic (dashed curve) in panel (a) at $T = 40$ K and $E_a = 1$ kV/cm with uniform doping inside the quantum well, and nonadiabatic Hartree potentials $V_H(z, t)$ in panel (b) for $t/T_p = 0.25$ (solid curve) and $t/T_p = 0.75$ (dashed curve).
Finally, we have connected the present quantum-statistical theory to the previously proposed current-surge model with a leading-order approximation.

In this paper, only the self-consistent Hartree model is employed. The exchange interaction between electrons and the field-domain effect are expected to be very small at $T=40$ K and $\varepsilon_{ac}=1$ kV/cm and have been neglected.

The time scale for observing the nonadiabatic effects requires

$$\tau_{e} \ll \tau_{t} \ll t < R_{t} C_{QW} \Lambda, \quad 2\pi/\Omega,$$

where $R_{t} = (Z_{B}/A)[\partial J^{\mu}[\varepsilon_{b}]/\partial \varepsilon_{b}]^{-1}$ is the differential tunneling resistance, depending on $\varepsilon_{b}$ and $T$. Here, $\Omega \tau_{e} \ll 1$ excludes the energy-drift effect, leaving only the momentum-drift effect in the system. Further, $\Omega \tau_{e} \ll 1$ ensures that the electrons see only an instantaneous ac electric field during the sequential-tunneling process. Finally, $t < R_{t} C_{QW} \Lambda$ ensures the observation of the nonadiabatic effects inside the quantum well. Assuming $\tau_{e} = 1$ ps corresponds to a homogeneous energy-level broadening of 1 meV, leading to $\Omega < 10^{12}$ Hz from $\Omega \tau_{e} \ll 1$. Therefore, only a momentum drift exists for low ac frequency $\Omega \ll 1$ Hz. The tunneling time $\tau_{t}$ can be estimated from $\tau_{t} \sim e^{[J^{\mu}[\varepsilon_{b}]\Lambda]}$. For a superlattice, we take $J^{\mu}[\varepsilon_{b}]\Lambda = 1$ $\mu$A, leading to $\tau_{e} = 0.1$ ps and $\Omega \ll 10^{13}$ Hz from $\Omega \tau_{e} \ll 1$. For an MQW system, we take $J^{\mu}[\varepsilon_{b}]\Lambda = 10$ pA, leading to $\tau_{e} = 10$ ns and $\Omega \ll 10^{8}$ Hz.

This justifies the calculation of the quantum-mechanical transmission of electrons through a biased barrier using the time-independent Schrödinger equation in an MQW system with $\Omega \ll 1$ Hz. Difficulties in observing the nonadiabatic effects may come from the small quantum-well capacitance $C_{QW} \ll 10$ pF in the requirement $t < R_{t} C_{QW} \Lambda$. For a superlattice, we take $R_{t} = 10^{4}$ ohm, and then $t < 10^{-7}$ sec is required (impossible to observe with $\Omega \ll 1$ Hz). For an MQW system, on the other hand, we take $R_{t} = 10^{1}$ ohm, which implies $t < 1$ sec (very easy to observe with $\Omega \ll 1$ Hz).


