Coupled Dynamic Systems and Le Chatelier's Principle in Noise Control

by
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K. J. Becker

[Work supported by ONR and In-House Fundings.]
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Investigation of coupling an adjunct dynamic system to an externally driven dynamic system - - a master dynamic system - - reveals that the response of the adjunct dynamic system affects the pre-coupled response of the master dynamic system. The responses, in the two dynamic systems when coupled, are estimated by the stored energies and , respectively. Since the adjunct dynamic system, prior to coupling was with zero stored energy, , the pre-coupled stored energy in the master dynamic system is expected to be reduced to when coupling is instituted; i.e., one expects . In this case a beneficial noise control of the master dynamic system would result from the coupling. It is argued that the change in the disposition of the stored energies as just described may not be the only change. The coupling may influence the external input power into the master dynamic system which may interfere with the expected noise control. Indeed, the coupling may influence the external input power such that the expected beneficial noise control may not materialize. An example of this kind of noise control reversal is cited.
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Abstract

Investigation of coupling an adjunct dynamic system to an externally driven dynamic system - a master dynamic system - reveals that the response of the adjunct dynamic system affects the pre-coupled response of the master dynamic system. The responses, in the two dynamic systems when coupled, are estimated by the stored energies \( (E_s) \) and \( (E_o) \), respectively. Since the adjunct dynamic system, prior to coupling was with zero \( (0) \) stored energy, \( E_s^o = 0 \), the pre-coupled stored energy \( (E_o^o) \) in the master dynamic system is expected to be reduced to \( (E_o) \) when coupling is instituted; i.e., one expects \( E_o < E_o^o \). In this case a beneficial noise control of the master dynamic system would result from the coupling. It is argued that the change in the disposition of the stored energies as just described may not be the only change. The coupling may influence the external input power into the master dynamic system which may interfere with the expected noise control. Indeed, the coupling may influence the external input power such that the expected beneficial noise control may not materialize. An example of this kind of noise control reversal is cited.
Preface

This report is based on an accepted oral paper that was to be given at the 75th Anniversary Celebration, 147th Meeting of the Acoustical Society of America which was held at the Sheraton New York Hotel and Towers 24-28 May 2004. It was to be given, but was not. This misgiving occurred due to time constraint, mostly contributed by mismanagement of computer projections (power point) of preceding papers that got out of hand. The paper 2aSA5 was scheduled for delivery from 10:00 to 10:15. There was to be a scheduled Break from 10:15 to 10:30. The preceding paper 2aSA4 was still going strong at 10:18. The presenting authors of paper 2aSA5 refused to present the paper out-of-slot in the middle of the Break period. There is no particular blame to be singled out. There is, however, a recognition that unless the problems associated with the new forms of presentations are nipped in the bud early enough, chaos, in the scheduling of papers at the ASA Meetings, will reign supreme.

In this report it is intended to present the narrative on the left side and the viewgraphs on the right side. This form will enable the reader to follow the intended material in paper 2aSA5 as if he had attended the presentation. We hope that this will disseminate satisfactorily the information contained in the paper as if it were given.
The statement of Le Chatelier’s Principle is rephrased for the noise control community on the first viewgraph, V1. Is the statement relevant to noise control engineering?

Note: Le Chatelier’s Principle is a familiar dogma in physical chemistry. The extension of this Principle into noise control is in its infancy. Nonetheless, there are many examples in noise control engineering in which the essence of this Principle is validated. The awareness of this validity may help avoid pronouncements that are optimistic beyond the realm of reality.
LE CHATELIER’S PRINCIPLE

IN NOISE CONTROL

Le Chatelier’s Principle rephrased:

When a dynamic system is changed in order to induce a beneficial noise control, the modified dynamic system will act to mitigate the expected benefit.
A statement of the example to be presented to illustrate Le Chatelier's Principle, is stated in the second viewgraph, V2.

Note: This example lies within the area now termed Structural Fuzzies, so named by the initiator of this subject matter, Christian Soize of ONERA, France.
An Example:

A master dynamic system is coupled to a reverberant adjunct dynamic system in order to achieve a beneficial noise control. The master dynamic system experiences noise control that is less beneficial than that induced by an adjunct dynamic system that is effectively a sink (i.e., a non-reverberant adjunct dynamic system).
The master dynamic system pre-coupling is depicted atop the third viewgraph, V3. The master dynamic system is defined by the modal density \( V_0(\omega) \equiv V_0 \), the mass \( M_0 \) and the loss factor \( \eta_0(\omega) \equiv \eta_0 \). The loss factor of \( \eta_0(\omega) \) relates, by definition, the power \( \Pi_0(\omega) \equiv \Pi_0 \) dissipated to the stored energy \( E_0^0(\omega) \equiv E_0^0 \) in the uncoupled master dynamic system. On the other hand, by the conservation of energy (power) the power \( \Pi_0(\omega) \) dissipated in this master dynamic system is equal to the external power input \( \Pi_0(\omega) \equiv \Pi_0 \). The relationship and the power equality may be expressed in the form

\[
\Pi_0(\omega) = \eta_0(\omega) [\omega E_0^0(\omega)] = \Pi_0(\omega)
\]

where \( (\omega) \) is the frequency. An adjunct dynamic system is coupled to the master dynamic system as shown. The adjunct dynamic system is defined by the modal density \( V_s(\omega) \equiv V_s \), the mass \( M_s \) and the loss factor \( \eta_s(\omega) \equiv \eta_s \). The loss factor \( \eta_s(\omega) \) relates, by definition, the power \( \Pi_s(\omega) \equiv \Pi_s \) dissipated to the stored energy \( E_s(\omega) \equiv E_s \) in the adjunct dynamic system. On the other hand, by the conservation of energy (power) the power \( \Pi_s(\omega) \) is equal to the net power \( \Pi_s(\omega) \equiv \Pi_s \) that is transferred to the adjunct dynamic system. The relationship and the power equality may be expressed in the form

\[
\Pi_s(\omega) = \eta_s(\omega) [\omega E_s(\omega)] = \Pi_s(\omega)
\]

The net power \( \Pi_s(\omega) \) may also be defined in the form

\[
\Pi_s(\omega) = \eta_s(\omega) [\omega E_o(\omega)]
\]
\[ \Pi_s = \eta_I(\omega E_o) = \eta_s(\omega E_s) \quad ; \quad Z^s_o = (E_s / E_o) \quad , (V3.1) \]

\( \eta_I \) induced loss factor of the master dynamic system; induced by the coupling

\( Z^s_o \) global coupling strength: \( (\eta_I / \eta_s) = Z^s_o \quad , (V3.2) \)
where the loss factor \( \eta_j(\omega) = \eta_j \) is designated induced. As indicated in this third viewgraph, V3, considerations are focused on a master dynamic system that is externally driven; the adjunct dynamic system is not externally driven. A global coupling strength is defined, then, as the ratio of the stored energy in the adjunct dynamic system to the stored energy in the coupled master dynamic system. From Equations (2) and (3) one finds that the global coupling strength \( \frac{E^s_s(\omega)}{E^s_o(\omega)} = \left[ \frac{\eta_j(\omega)}{\eta_s(\omega)} \right] \).

The ratio \( \frac{E^s_s(\omega)}{E^s_o(\omega)} = \frac{\eta_j(\omega)}{\eta_s(\omega)} \) is the higher the stronger is the coupling and the higher is the ratio of the modal densities \( \frac{v_s(\omega)}{v_o(\omega)} = \frac{v^s_s}{v^s_o} \).

Note: In the statistical energy analysis (SEA) format

\[ \frac{E_s}{E_o} = \frac{v_s}{v_o} \left( \frac{\epsilon_s}{\epsilon_o} \right) = \left( \frac{\eta_{os}}{\eta_s + \eta_{os}} \right) \]

where \( \eta_{os} \) is the coupling loss factor from the adjunct dynamic system to the master dynamic system and \( \epsilon_o \) and \( \epsilon_s \) are the stored modal energies in the master and in the adjunct dynamic systems, respectively. Clearly from Equation (5), if one wishes to render the global coupling strength high, the coupling loss factor \( \eta_{os} \) need to approach or exceed the loss factor \( \eta_s \). Then, the modal coupling strength \( \zeta^s_o = \left( \frac{\epsilon_s}{\epsilon_o} \right) \) approaches its maximum value of unity. In addition, to heighten the global coupling strength further, the ratio of the modal densities \( \frac{v_s}{v_o} \) needs be high. In that situation the adjunct dynamic system stores most of the stored energy in the combined dynamic system. If the loss factor \( \eta_s \) can be maintained simultaneously adequate to the task of gobbling the net power \( (\Pi_s) \), noise control benefit may be accrued. To examine the validity of this contention a more detailed analysis is required.
\[ \Pi^0_e = \Pi^0 = \eta_o(\omega E^0_o) \]

\[ E^0_o, \nu_o, M_o, \eta_o \]

\( \eta_o \) loss factor of the master dynamic system

\[ \Pi_s = \eta_I(\omega E_o) \]

\[ E_o, \nu_o, \{m_c, k_c, G\} \]

\[ M_o, \eta_o \]

\[ \Pi_s = \eta_s(\omega E_s) \]

\[ E_s, \nu_s \]

\[ M_s, \eta_s \]

\( \eta_s \) loss factor of the adjunct dynamic system

\[ \Pi_s = \eta_I(\omega E_o) = \eta_s(\omega E_s) \quad ; \quad Z^s_o = (E_s / E_o) \quad , (V3.1) \]

\( \eta_I \) induced loss factor of the master dynamic system; induced by the coupling

\[ Z^s_o \] global coupling strength: \( \left( \eta_I / \eta_s \right) = Z^s_o \quad . (V3.2) \]
For this purpose a noise control parameter $\xi(\omega) = \xi$ is defined in a way that a beneficial noise control is achieved if this parameter is small compared with unity; the smaller the parameter is, the more is the beneficial noise control. A noise control reversal occurs if the noise control parameter exceeds unity. Can a noise control reversal ever occur under the considerations here conducted?!

Note: In the statistical energy analysis (SEA) format

$$\xi_o = \eta_o (\eta_o + \eta_l)^{-1} ; \quad \eta_l = (v_s/v_o) \eta_s \eta_{os} (\eta_s + \eta_{os})^{-1}$$

In SEA a consistency condition relates $(\eta_{os})$ to $(\eta_{so})$ in the form

$$(\eta_{os} v_s) = (\eta_{so} v_o)$$

where $(\eta_{so})$ is the coupling loss factor from the master dynamic system to the adjunct dynamic system. It is observed that if the coupling is weak; $\eta_s \gg \eta_{os}$, then the induced loss factor $(\eta_l)$ is essentially independent of $(\eta_s)$. On the other hand, if the coupling is strong; $\eta_s \ll \eta_{os}$, then $(\eta_l)$ is essentially proportional to $(\eta_s)$. Either way, $(\eta_l)$ is proportional to the ratio of the modal densities $(v_s/v_o)$. The higher this ratio is, the higher is the induced loss factor $\eta_l$. Thus a fact, nonetheless, if $\eta_l \gg \eta_o$, the beneficial contribution of $\xi_o$ to the noise control parameter is significant.
Hi

\[ \Pi^o = \eta_o(\omega E^o) \]
\[ E^o, \nu_o \]
\[ M_o, \eta_o \]

\[ \Pi_s = \eta_s(\omega E_s) \]
\[ E_s, \nu_s \]
\[ \{m_c, k_c, G\} \]
\[ M_s, \eta_s \]

noise control parameter

\[ \xi = \left[ \frac{E_s}{E^o} \right] = \pi \xi_o \quad , (V4.1) \]

external input power ratio \( \pi = \left[ \frac{\Pi_s}{\Pi^o_s} \right] \quad , (V4.2) \)

loss factor ratio \( \xi_o = \eta_o \left[ \eta_o + \eta_I \right]^{-1} \quad , (V4.3) \)

\[ \eta_I = \eta_s Z^s_o \quad ; \quad \xi_o = \left[ 1 + (\eta_s / \eta_o) Z^s_o \right]^{-1} \quad . (V4.4) \]
In this fifth viewgraph, V5, an attempt is made to evaluate the ratio of the external power inputs $\pi(\omega)[=\pi]$. This ratio is of the external power input $\Pi_e(\omega)[=\Pi_e]$ into the coupled master dynamic system to the external power input $\Pi_e^o(\omega)[=\Pi_e^o]$ in the absence of the coupling. Under the assumption that the mass of the adjunct dynamic system is small compared with that of the master dynamic system

$$\pi = \frac{\Pi_e}{\Pi_e^o} = (1 + Z_o^s)$$

[cf. Equations (V5.1)-(V5.7)].

Note: Employing Equations (6), (8) and (V4.4) one derives

$$\pi = 1 + \left(\frac{v_s}{v_o}\right)\eta_{os}(\eta_s + \eta_{os})^{-1} > 1$$

indicating that $(\pi)$ exceeds unity, thereby, contributing contrarily to the factor $(\xi_o)$ in the noise control parameter $(\xi)$. In particular, for weak coupling; $\eta_{os} << \eta_s$, one derives

$$\pi = \pi_w \Rightarrow 1 + \left(\frac{v_s}{v_o}\right)\left(\frac{\eta_{os}}{\eta_s}\right) < 1 + \left(\frac{v_s}{v_o}\right)$$

whereas, for strong coupling; $\eta_{os} >> \eta_s$, one derives

$$\pi = \pi_s \Rightarrow 1 + \left(\frac{v_s}{v_o}\right) > \pi_w > 1$$

Thus, strong coupling enhances $(\pi)$ more than does weak coupling. Moreover, if $(\frac{v_s}{v_o})$ is high compared with unity, $(\pi_s)$ may be similarly high. Beware, were $(\pi)$ to be high, in excess of unity, it may negate effectively the beneficial noise control contributed by the factor $(\xi_o)$ in the noise control parameter $(\xi)$!
\[ \Pi_e = \Pi_o = \eta_o(\omega E_o) \]
\[ E_o, \nu_o \]
\[ M_o, \eta_o \]
\[ \Pi_e \approx S_f(\pi/2)[\nu_o / M_o] \quad (V5.1) \]

\[ \Pi_e \approx S_f(\pi/2)[\nu_s^s / M_o^s] \quad (V5.2) \]

\[ \Pi_s = \eta_I(\omega E_o) \]
\[ E_o, \nu_o \]
\[ \{m_c, k_c, G\} \]
\[ M_o, \eta_o \]
\[ \Pi_s = \eta_s(\omega E_s) \]
\[ E_s, \nu_s \]
\[ M_s, \eta_s \]

modified modal density

\[ \nu_s^s = [\nu_o + \zeta_o^s \nu_s] = \nu_o [1 + Z_o^s] \quad (V5.3) \]

\[ \zeta_o^s \text{ modal coupling strength: } Z_o^s = (\nu_s / \nu_o) \zeta_o^s \quad (V5.4) \]

assume: \( (M_o / M^s_o) \approx 1 \quad (V5.5) \)

\[ \Pi_e \approx S_f(\pi/2)[\nu_o / M_o][1 + Z_o^s] \quad (V5.6) \]

\[ \pi = [\Pi_e / \Pi_o] = [1 + Z_o^s] \quad (V5.7) \]
The noise control parameter ($\xi$) is evaluated from Equations (V4.3), (V4.4) and (V5.7) to be

$$\xi = \left(\frac{E_o}{E_o^o}\right) = \pi \xi_o = \left(1 + Z_o^s\right)\eta_o\left(\eta_o + \eta_I\right)^{-1} \quad , (12a)$$

or equivalently to be

$$\xi = \left(1 + Z_o^s\right)\left[1 + \left(\frac{\eta_o}{\eta_o}\right)Z_o^s\right]^{-1} \quad , (12b)$$

The first example is illustrated assuming that the adjunct dynamic system is merely a sink. A sink is characterized such that it is incapable of storing energy; for an adjunct dynamic system that is a sink, $(Z_o^s)$ is equal identically to zero; $Z_o^s = (E_s/E_o) = 0$. For a sink then, the ratio $(\pi)$, of the external power inputs, is equal to unity; $(\pi) = 1$, and, therefore, from Equation (12) one obtains

$$\xi \Rightarrow \xi_o = \eta_o\left(\eta_o + \eta_I\right)^{-1} \quad ; \quad \eta_I = \left(\eta_s Z_o^s\right) \quad . (13)$$

Note: From Equations (6) and (13) one finds that in (SEA) the induced loss factor $\eta_I$ may be equated in the form

$$\eta_I = \left(\nu_s/\nu_o\right)\eta_s\eta_o\left(\eta_s + \eta_o\right)^{-1} \quad . (14a)$$

It transpires that for a sink the loss factor $(\eta_s)$ is such that the coupling is always weak; namely $\eta_s \gg \eta_o$. Then Equation (14) assumes the form

$$\eta_I \Rightarrow \left(\nu_s/\nu_o\right)\eta_o = \eta_{so} \quad , (15)$$

where use is made of Equation (7). Thus, when the adjunct dynamic system is a sink, the loss factor of the coupled master dynamic system increases by the coupling loss factor $(\eta_{so})$. When coupling to the sink is implemented, the loss factor of the master dynamic system increases from that of $(\eta_o)$ to $(\eta_o + \eta_{so})$.  

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noise control parameter

$$\xi = \left[ E_o / E_o^o \right] = \pi \Rightarrow 1 + Z_s^o \right] \left[ 1 + (\eta_s / \eta_o) Z_s^o \right]^{-1}$$

Case 1. merely a sink: $\left[ E_s / E_o^o \right] = Z_s^o \Rightarrow 0$

$$\xi \Rightarrow \xi_o = [\eta_o] [\eta_o + \eta_I]^{-1} ; \quad \pi = 1$$

loss factor in master increases by the induced loss factor: $\eta_I = \eta_s Z_s^o$
In this seventh viewgraph, V7, two cases are investigated. In both these cases; case 2 and case 3, the global coupling strength is assumed to be in excess of unity; \( Z_o^s \gg 1 \). In case 2 the loss factor (\( \eta_s \)) in the adjunct dynamic system is assumed to exceed the loss factor (\( \eta_o \)) in the master dynamic system; \( \eta_s > \eta_o \). The noise control parameter is beneficial to the degree that (\( \eta_s \)) exceeds (\( \eta_o \)). This is a reasonable result. It is of interest to consider that were one to assume (falsely) that the ratio (\( \pi \)) of the external power inputs is unity, the noise control benefit would be greatly (and falsely) enhanced. In case 3 the loss factor (\( \eta_s \)) in the adjunct dynamic system is assumed to be less than the loss factor (\( \eta_o \)) in the master dynamic system; \( \eta_s < \eta_o \). The noise control parameter shows deficient result -- a noise control reversal! Again, it is of interest to consider that were one to assume (falsely) that the ratio (\( \pi \)) of the external power inputs is unity, the noise control benefit would be greatly (and falsely) recovered.

A corollary to cases 2 and 3 is afforded by equating (\( \eta_s \)) to (\( \eta_o \)). Under this condition the noise control parameter is neutral, registering a value of unity. Indeed, in this case the increase in the external power input to the coupled master dynamic system; i.e., (\( \Pi_e \)), as compared with that to the isolated master dynamic system; i.e., (\( \Pi_e^o \)), balances the increase in the loss factor of the coupled master dynamic system; i.e., (\( \eta_o + \eta_f \)), as compared with that of the isolated master dynamic system; (\( \eta_o \)). This balance renders \( \xi \Rightarrow 1 \). Once again, were one to assume (falsely) that the ratio (\( \pi \)) of the external power inputs is unity, the noise control benefit would be greatly (and falsely) enhanced.
noise control parameter

\[ \xi = \frac{E_s}{E_o} = \pi \quad \xi_o \Rightarrow [1 + Z_o^s] [1 + (\eta_s / \eta_o) Z_o^s]^{-1} \]

in subsequent cases: \[ \frac{E_s}{E_o} = Z_o^s \gg 1 \]

Case 2. \[ \frac{\eta_s}{\eta_o} > 1 \quad \xi \Rightarrow \frac{\eta_o}{\eta_s} < 1 \]

Were \( \pi \Rightarrow 1 \) then \( \xi \Rightarrow \eta_o [1 + (\eta_o / \eta_s) Z_o^s]^{-1} \ll 1 \)

Case 3. \[ \frac{\eta_s}{\eta_o} < 1 \quad \frac{(\eta_s / \eta_o) Z_o^s} > 1 \]

\[ \xi \Rightarrow \frac{\eta_o}{\eta_s} > 1 \quad \text{a noise control reversal.} \]

Were \( \pi \Rightarrow 1 \) then \( \xi \Rightarrow \eta_o [1 + (\eta_s / \eta_o) Z_o^s]^{-1} \ll 1 \)

A corollary to cases 2 and 3: \( \eta_o = \eta_s \)

\[ \xi \Rightarrow 1; \quad \pi \Rightarrow [1 + Z_o^s] > 1; \quad \xi_o \Rightarrow [1 + Z_o^s]^{-1} < 1 \]

Were \( \pi \Rightarrow 1 \) then \( \xi \Rightarrow [1 + Z_o^s]^{-1} \ll 1 \)
It is, then, suggested: Noise control engineers beware, Le Chatelier's Principle is there to undermine your predictions were you not careful to recognize that Principle's existence.

Note: If one couples an adjunct dynamic system on to a master in order to achieve noise control, one needs to ensure that most of the stored energy \( E \); \( E = (E_o + E_s) \), in the combined dynamic system (master + adjunct) resides in the adjunct dynamic system; i.e., \( (E_s/E_o) \gg 1 \). However, this condition is not enough, one must ensure also that the adjunct dynamic system has a loss factor \( \eta_s \) high enough to participate effectively in the dissipation of the external power input \( (\Pi_e) \). This external power input exceeds the external power input \( (\Pi_e^o) \) that is imparted to the master dynamic system in the absence of the coupling. The effective dissipation of \( (\Pi_e) \) is thus imperative. The eighth viewgraph, V8, is introduced, for the purpose of evaluating this statement.
noise control parameter

\[ \xi = \frac{E_o}{E'_o} = \pi \quad \xi_o \Rightarrow [1 + Z_s^o] [1 + (\eta_s / \eta_o) Z_s^o]^{-1} \]

in subsequent cases: \[ \frac{E_s}{E_o} = Z^s_o >> 1 \]

Case 2. \[ \frac{\eta_s}{\eta_o} > 1 \quad \xi \Rightarrow \frac{\eta_o}{\eta_s} < 1 \]

Were \( \pi \Rightarrow 1 \) then \( \xi \Rightarrow \eta_o [1 + (\eta_o / \eta_s) Z_s^o]^{-1} \ll 1 \)

Case 3. \[ \frac{\eta_s}{\eta_o} < 1 \quad \left[ (\eta_s / \eta_o) Z_s^o \right] \gg 1 \]

\[ \xi \Rightarrow \frac{\eta_o}{\eta_s} > 1 \quad \text{a noise control reversal.} \]

Were \( \pi \Rightarrow 1 \) then \( \xi \Rightarrow \eta_o [1 + (\eta_s / \eta_o) Z_s^o]^{-1} \ll 1 \)

A corollary to cases 2 and 3: \( \eta_o = \eta_s \)

\[ \xi \Rightarrow 1; \quad \pi \Rightarrow [1 + Z_s^o] > 1; \quad \xi_o \Rightarrow [1 + Z_s^o]^{-1} < 1 \]

Were \( \pi \Rightarrow 1 \) then \( \xi \Rightarrow [1 + Z_s^o]^{-1} \ll 1 \)
From Equations (V4.1) and (V4.3), (V5.7) and (V8.1) the ratio of the initial loss factor \( \eta_o \) to the effective loss factor \( \eta_e \) of the dynamic system (master + adjunct) may be established in the form

\[
\frac{\eta_o}{\eta_e} = \xi = \left[ 1 + \left( \frac{\eta_I}{\eta_o} \right) \right] \left[ 1 + \left( \frac{\eta_I}{\eta_s} \right) \right]^{-1}
\]

(16)

It follows that if \( \eta_o \) is less than \( \eta_s \) then

\[
\xi < 1 \quad ; \quad \eta_o < \eta_s
\]

(17a)

and if \( \eta_o \) exceeds \( \eta_s \) then

\[
\xi > 1 \quad ; \quad \eta_o > \eta_s
\]

(17b)

which reemphasize Cases 2 and 3, respectively, as presented in the preceding viewgraph, V7. Moreover, if \( \eta_o = \eta_s \) then

\[
\xi = 1 \quad ; \quad \eta_o = \eta_s
\]

(18)

The validity of Equations (17) and (18) are independent of the value of the induced loss factor \( \eta_I \). In (SEA) the induced loss factor may be cast in the form

\[
\eta_I = (\eta_{so})[\eta_s(\eta_{os} + \eta_s)]^{-1} \leq \eta_{so}
\]

(19)

The equality occurs only if the coupling is weak; i.e., only if \( \eta_{os} \ll \eta_s \). Weak coupling characterizes the case, for example, when the adjunct dynamic system is a sink. [cf. Equation (15) and the case; Case 1, depicted in the sixth viewgraph, V6.]
\[ \eta_v(\omega E_o) = \eta_e(\omega E) \quad ; \quad \eta_v = \eta_o + \eta_I \quad ; \quad \eta_v = \eta_e(1 + Z_s^o) \]

, (V8.1)

\( \eta_v \) the virtual loss factor of the coupled master
dynamic system

\( \eta_e \) the effective loss factor of the coupled dynamic
system (master + adjunct)

noise control parameter

\[ \xi = \frac{E_o}{E_o^o} = \pi \xi_o \]

\[ \Rightarrow \quad [1 + Z_s^o] \left[ \frac{\eta_o}{\eta_v} \right] \Rightarrow \left[ \frac{\eta_o}{\eta_e} \right] \]

, (V8.2)
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